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1. (i)

$$\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^4+25}\right) \tag{1}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^4}}\right) \tag{2}$$

$$\Leftrightarrow \frac{3}{7} \tag{3}$$

(ii)

$$\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^5+25}\right) \tag{4}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{1}{n^5}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^5}}\right) \tag{5}$$

$$\Leftrightarrow 0 \tag{6}$$

(iii)

$$\lim_{n\to\infty}\left(\frac{-3n^5+2n^2+n+1}{-7n^4+25}\right) \tag{7}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n}{1}\cdot\frac{-3+\frac{2}{n^3}+\frac{1}{n^4}+\frac{1}{n^5}}{-7+\frac{25}{n^4}}\right) \tag{8}$$

$$\Leftrightarrow \infty \tag{9}$$

(iv)

$$\lim_{n\to\infty}\left(\frac{6n^3+2n-3}{9n^2+2}-\frac{2n^3+5n^2+7}{3n^2+3}\right) \tag{10}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{-18n^5-45n^4-63n^2-4n^3-10n^2-14+18n^5+6n^3-9n^2+18n^3+6n-}{(9n^2+2)\cdot(3n^2+3)}\right. \tag{11}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{-45n^4+20n^3-82n^2+6n-23}{27n^4+33n^2+6}\right) \tag{12}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-45+\frac{20}{n}-\frac{82}{n^2}+\frac{6}{n^3}-\frac{23}{n^4}}{27+\frac{33}{n^2}+\frac{6}{n^4}}\right) \tag{13}$$

$$\Leftrightarrow -\frac{45}{27} \Leftrightarrow -\frac{5}{3} \tag{14}$$

$$\tag{15}$$

(v)

$$\lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{2n^2+1}\cdot\sqrt{2n^2+n+1}}\right) \tag{16}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{4n^4+2n^3+4n^2++n+1}}\right) \tag{17}$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{\sqrt{9+\frac{1}{n^2}+\frac{1}{n^4}}-\frac{2}{n^2}+\frac{3}{n^4}}{\sqrt{4+\frac{2}{n}+\frac{4}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}}\right) \tag{18}$$

$$\Leftrightarrow \frac{\sqrt{9}}{\sqrt{4}} \Leftrightarrow \frac{3}{2} \tag{19}$$

2. a) (i)

$$a_0 = 1 \qquad s_0 = 1 \tag{20}$$

$$a_1 = \frac{2}{5} \qquad s_1 = \frac{7}{5} \tag{21}$$

$$a_2 = \frac{4}{25} \qquad s_2 = \frac{39}{25} \tag{22}$$

$$a_3 = \frac{8}{125} \qquad s_3 = \frac{203}{125} \tag{23}$$

$$a_4 = \frac{16}{625} \qquad s_4 = \frac{1031}{625} \tag{24}$$

$$\tag{25}$$

$$\text{Diese geometrische Reihe konvergiert, da q} = \frac{2}{5} \Rightarrow |q| < 1$$

$$\text{Sie konvergiert gegen } \sum_{i=0}^{\infty} \left(\frac{2}{5}\right) = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$$

(ii)

$$a_0 = 1 \qquad s_0 = 1 \tag{26}$$
$$a_1 = \frac{5}{2} \qquad s_1 = \frac{7}{2} \tag{27}$$
$$a_2 = \frac{25}{4} \qquad s_2 = \frac{39}{4} \tag{28}$$
$$a_3 = \frac{125}{8} \qquad s_3 = \frac{203}{8} \tag{29}$$
$$a_4 = \frac{625}{16} \qquad s_4 = \frac{1031}{16} \tag{30}$$
$$\tag{31}$$

$$\text{Diese geometrische Reihe divergiert, da q} = \frac{5}{2} \Rightarrow |q| > 1$$

(iii)

$$a_0 = 1 \qquad s_0 = 1 \tag{32}$$
$$a_1 = -\frac{2}{5} \qquad s_1 = \frac{3}{5} \tag{33}$$
$$a_2 = \frac{4}{25} \qquad s_2 = \frac{19}{25} \tag{34}$$
$$a_3 = -\frac{8}{125} \qquad s_3 = \frac{87}{125} \tag{35}$$
$$a_4 = \frac{16}{625} \qquad s_4 = \frac{451}{625} \tag{36}$$
$$\tag{37}$$

$$\text{Diese geometrische Reihe konvergiert, da q} = -\frac{2}{5} \Rightarrow |q| < 1$$

$$\text{Sie konvergiert gegen } \sum_{i=0}^{\infty} \left(-\frac{2}{5}\right) = \frac{1}{1-(-\frac{2}{5})} = \frac{5}{7}$$

b) (i)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-(-\frac{3}{10})} = \frac{10}{13}$$

(ii) $x = -\frac{3}{5}$

3. TODO

4. TODO