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1. (i)

$$\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^4+25}\right) \Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^4}}\right) \Leftrightarrow \frac{3}{7}$$

(ii)

$$\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^5+25}\right) \Leftrightarrow \lim_{n\to\infty}\left(\frac{1}{n^5}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^5}}\right) \Leftrightarrow 0$$

(iii)

$$\lim_{n\to\infty}\left(\frac{-3n^5+2n^2+n+1}{-7n^4+25}\right) \Leftrightarrow \lim_{n\to\infty}\left(\frac{n}{1}\cdot\frac{-3+\frac{2}{n^3}+\frac{1}{n^4}+\frac{1}{n^5}}{-7+\frac{25}{n^4}}\right) \Leftrightarrow \infty$$

(iv)

$$\lim_{n\to\infty}\left(\frac{6n^3+2n-3}{9n^2+2}-\frac{2n^3+5n^2+7}{3n^2+3}\right)$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{-45n^4+20n^3-82n^2+6n-23}{27n^4+33n^2+6}\right)$$

$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-45+\frac{20}{n}-\frac{82}{n^2}+\frac{6}{n^3}-\frac{23}{n^4}}{27+\frac{33}{n^2}+\frac{6}{n^4}}\right) \Leftrightarrow \quad -\frac{45}{27} \Leftrightarrow -\frac{5}{3}$$

(v)

$$\lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{2n^2+1}\cdot\sqrt{2n^2+n+1}}\right)$$
$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{4n^4+2n^3+4n^2++n+1}}\right)$$
$$\Leftrightarrow \lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{\sqrt{9+\frac{1}{n^2}+\frac{1}{n^4}}-\frac{2}{n^2}+\frac{3}{n^4}}{\sqrt{4+\frac{2}{n}+\frac{4}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}}\right)\Leftrightarrow \qquad \frac{\sqrt{9}}{\sqrt{4}}\Leftrightarrow\frac{3}{2}$$

2. a) (i)

$$\begin{aligned} a_0 &= 1 & s_0 &= 1 \\ a_1 &= \frac{2}{5} & s_1 &= \frac{7}{5} \\ a_2 &= \frac{4}{25} & s_2 &= \frac{39}{25} \\ a_3 &= \frac{8}{125} & s_3 &= \frac{203}{125} \\ a_4 &= \frac{16}{625} & s_4 &= \frac{1031}{625} \end{aligned}$$

Diese geometrische Reihe konvergiert, da q $= \frac{2}{5} \Rightarrow |q| < 1$

Sie konvergiert gegen $\sum_{i=0}^{\infty}\left(\frac{2}{5}\right) = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$

(ii)

$$\begin{aligned} a_0 &= 1 & s_0 &= 1 \\ a_1 &= \frac{5}{2} & s_1 &= \frac{7}{2} \\ a_2 &= \frac{25}{4} & s_2 &= \frac{39}{4} \\ a_3 &= \frac{125}{8} & s_3 &= \frac{203}{8} \\ a_4 &= \frac{625}{16} & s_4 &= \frac{1031}{16} \end{aligned}$$

Diese geometrische Reihe divergiert, da q $= \frac{5}{2} \Rightarrow |q| > 1$

(iii)

$$\begin{aligned} a_0 &= 1 & s_0 &= 1 \\ a_1 &= -\frac{2}{5} & s_1 &= \frac{3}{5} \\ a_2 &= \frac{4}{25} & s_2 &= \frac{19}{25} \\ a_3 &= -\frac{8}{125} & s_3 &= \frac{87}{125} \\ a_4 &= \frac{16}{625} & s_4 &= \frac{451}{625} \end{aligned}$$

Diese geometrische Reihe konvergiert, da q $= -\frac{2}{5} \Rightarrow |q| < 1$

Sie konvergiert gegen $\sum_{i=0}^{\infty} \left(-\frac{2}{5}\right) = \frac{1}{1-(-\frac{2}{5})} = \frac{5}{7}$

b) (i)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-(-\frac{3}{10})} = \frac{10}{13}$$

(ii)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} = \frac{5}{8} \quad \Leftrightarrow \quad 1 = (1-x) \cdot \frac{5}{8}$$

$$\Leftrightarrow \ 1 = \frac{5}{8} - \frac{5}{8}x \quad \Leftrightarrow \quad \frac{3}{8} = -\frac{5}{8}x \quad \Leftrightarrow \quad x = -\frac{3}{5}$$

3. (i)

$$\sum_{i=0}^{\infty} \left(\frac{5}{8}\right)^i \text{konvergiert, da q} = \frac{5}{8} \Rightarrow |q| < 1$$

$$\sum_{i=0}^{\infty} \left(\frac{5}{8}\right)^i = \frac{1}{1-\frac{5}{8}} = \frac{3}{8}$$

(ii)

$$\sum_{i=2}^{\infty} \left(\frac{5}{8}\right)^i = -\frac{89}{64} + \sum_{i=0}^{\infty} \left(\frac{5}{8}\right)^i = -\frac{89}{64} + \frac{1}{1-\frac{5}{8}} = \frac{245}{192}$$

(iii)

$$\sum_{i=1}^{\infty} \left(-\frac{5}{8}\right)^i = -1 + \sum_{i=0}^{\infty} \left(-\frac{5}{8}\right)^i = -1 + \frac{1}{1+\frac{5}{8}} = -\frac{5}{13}$$

(iv)

$$\begin{aligned} & \sum_{i=1}^{\infty} (-1)^i \cdot \left(\frac{5}{8}\right)^{i+2} \\ = & \left(\frac{5}{8}\right)^2 \cdot \sum_{i=1}^{\infty} \left(-\frac{5}{8}\right)^i \\ = & \frac{25}{64} \cdot \left(-1 + \sum_{i=0}^{\infty} \left(-\frac{5}{8}\right)^i\right) \\ = & \frac{25}{64} \cdot \left(-1 + \frac{1}{1+\frac{5}{8}}\right) \\ = & -\frac{125}{832} \end{aligned}$$

4.

(i)

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^5 \Rightarrow (1+0)^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \;=\; 1$$

(ii)

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n+5} \Rightarrow \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \cdot \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^5 \;\Rightarrow\; e \cdot 1^5 \;=\; e$$

(iii)

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{2n+3} \Rightarrow \left(\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n\right)^2 \cdot \left(1 + \frac{1}{n}\right)^3 \;=\; e^2 \cdot 1^3 \;=\; e^2$$

(iv)

$$\sum_{i=3}^{\infty} \frac{1}{i(i+1)} \;\Leftrightarrow\; \sum_{1=3}^{\infty} \frac{1}{i} \cdot \frac{1}{i+1} \Leftrightarrow \sum_{1=3}^{\infty} \frac{1}{i} \cdot \sum_{1=4}^{\infty} \frac{1}{i} \;=\; \infty \cdot \infty \;=\; \infty$$