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1. (i)

$$\lim_{n \rightarrow \infty} \left(\frac{-3n^4 + 2n^2 + n + 1}{-7n^4 + 25} \right) \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{n^4}{n^4} \cdot \frac{-3 + \frac{2}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}{-7 + \frac{25}{n^4}} \right) \Leftrightarrow \frac{3}{7}$$

(ii)

$$\lim_{n \rightarrow \infty} \left(\frac{-3n^4 + 2n^2 + n + 1}{-7n^5 + 25} \right) \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{1}{n^5} \cdot \frac{-3 + \frac{2}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}{-7 + \frac{25}{n^5}} \right) \Leftrightarrow 0$$

(iii)

$$\lim_{n \rightarrow \infty} \left(\frac{-3n^5 + 2n^2 + n + 1}{-7n^4 + 25} \right) \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{n}{1} \cdot \frac{-3 + \frac{2}{n^3} + \frac{1}{n^4} + \frac{1}{n^5}}{-7 + \frac{25}{n^4}} \right) \Leftrightarrow \infty$$

(iv)

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left(\frac{6n^3 + 2n - 3}{9n^2 + 2} - \frac{2n^3 + 5n^2 + 7}{3n^2 + 3} \right) \\ & \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{-45n^4 + 20n^3 - 82n^2 + 6n - 23}{27n^4 + 33n^2 + 6} \right) \\ & \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{n^4}{n^4} \cdot \frac{-45 + \frac{20}{n} - \frac{82}{n^2} + \frac{6}{n^3} - \frac{23}{n^4}}{27 + \frac{33}{n^2} + \frac{6}{n^4}} \right) \Leftrightarrow -\frac{45}{27} \Leftrightarrow -\frac{5}{3} \end{aligned}$$

(v)

$$\begin{aligned}
& \lim_{n \rightarrow \infty} \left(\frac{\sqrt{9n^4 + n^2 + 1} - 2n^2 + 3}{\sqrt{2n^2 + 1} \cdot \sqrt{2n^2 + n + 1}} \right) \\
& \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{\sqrt{9n^4 + n^2 + 1} - 2n^2 + 3}{\sqrt{4n^4 + 2n^3 + 4n^2 + n + 1}} \right) \\
& \Leftrightarrow \lim_{n \rightarrow \infty} \left(\frac{n^4}{n^4} \cdot \frac{\sqrt{9 + \frac{1}{n^2} + \frac{1}{n^4}} - \frac{2}{n^2} + \frac{3}{n^4}}{\sqrt{4 + \frac{2}{n} + \frac{4}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}} \right) \Leftrightarrow \frac{\sqrt{9}}{\sqrt{4}} \Leftrightarrow \frac{3}{2}
\end{aligned}$$

2. a) (i)

$$\begin{array}{ll}
a_0 = 1 & s_0 = 1 \\
a_1 = \frac{2}{5} & s_1 = \frac{7}{5} \\
a_2 = \frac{4}{25} & s_2 = \frac{39}{25} \\
a_3 = \frac{8}{125} & s_3 = \frac{203}{125} \\
a_4 = \frac{16}{625} & s_4 = \frac{1031}{625}
\end{array}$$

Diese geometrische Reihe konvergiert, da $q = \frac{2}{5} \Rightarrow |q| < 1$

Sie konvergiert gegen $\sum_{i=0}^{\infty} \left(\frac{2}{5}\right)^i = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$

(ii)

$$\begin{array}{ll}
a_0 = 1 & s_0 = 1 \\
a_1 = \frac{5}{2} & s_1 = \frac{7}{2} \\
a_2 = \frac{25}{4} & s_2 = \frac{39}{4} \\
a_3 = \frac{125}{8} & s_3 = \frac{203}{8} \\
a_4 = \frac{625}{16} & s_4 = \frac{1031}{16}
\end{array}$$

Diese geometrische Reihe divergiert, da $q = \frac{5}{2} \Rightarrow |q| > 1$

(iii)

$$\begin{array}{ll} a_0 = 1 & s_0 = 1 \\ a_1 = -\frac{2}{5} & s_1 = \frac{3}{5} \\ a_2 = \frac{4}{25} & s_2 = \frac{19}{25} \\ a_3 = -\frac{8}{125} & s_3 = \frac{87}{125} \\ a_4 = \frac{16}{625} & s_4 = \frac{451}{625} \end{array}$$

Diese geometrische Reihe konvergiert, da $q = -\frac{2}{5} \Rightarrow |q| < 1$

Sie konvergiert gegen $\sum_{i=0}^{\infty} \left(-\frac{2}{5}\right)^i = \frac{1}{1 - (-\frac{2}{5})} = \frac{5}{7}$

b) (i)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1 - (-\frac{3}{10})} = \frac{10}{13}$$

(ii)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} = \frac{5}{8} \Leftrightarrow 1 = (1-x) \cdot \frac{5}{8}$$

$$\Leftrightarrow 1 = \frac{5}{8} - \frac{5}{8}x \Leftrightarrow \frac{3}{8} = -\frac{5}{8}x \Leftrightarrow x = -\frac{3}{5}$$

3.

(i)

$\sum_{i=0}^{\infty} \left(\frac{5}{8}\right)^i$ konvergiert, da $q = \frac{5}{8} \Rightarrow |q| < 1$

$$\sum_{i=0}^{\infty} \left(\frac{5}{8}\right)^i = \frac{1}{1 - \frac{5}{8}} = \frac{8}{3}$$

(ii)

$$\sum_{i=2}^{\infty} \left(\frac{5}{8}\right)^i \text{ TODO}$$

(iii)

$$\sum_{i=1}^{\infty} \left(-\frac{5}{8}\right)^i \text{ TODO}$$

(iv)

$$\sum_{i=1}^{\infty} (-1)^i \cdot \left(\frac{5}{8}\right)^{i+2} \text{ TODO}$$

4.

(i)

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^5 \Rightarrow (1+0)^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

(ii)

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n+5} \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \cdot \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^5 \Rightarrow e \cdot 1^5 = e$$

(iii)

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{2n+3} \Rightarrow \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n\right)^2 \cdot \left(1 + \frac{1}{n}\right)^3 = e^2 \cdot 1^3 = e^2$$

(iv)

$$\sum_{i=3}^{\infty} \frac{1}{i(i+1)} \Leftrightarrow \sum_{i=3}^{\infty} \frac{1}{i} \cdot \frac{1}{i+1} \Leftrightarrow \sum_{i=3}^{\infty} \frac{1}{i} \cdot \sum_{i=4}^{\infty} \frac{1}{i} = \infty \cdot \infty = \infty$$