ALA 02 17.04.2014

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17. April 2014

1. (i)

$$\begin{aligned}
&\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^4+25}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^4}}\right)\\
\Leftrightarrow\ &\frac{3}{7}
\end{aligned}$$

(ii)

$$\begin{aligned}
&\lim_{n\to\infty}\left(\frac{-3n^4+2n^2+n+1}{-7n^5+25}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{1}{n^5}\cdot\frac{-3+\frac{2}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}{-7+\frac{25}{n^5}}\right)\\
\Leftrightarrow\ &0
\end{aligned}$$

(iii)

$$\begin{aligned}
&\lim_{n\to\infty}\left(\frac{-3n^5+2n^2+n+1}{-7n^4+25}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{n}{1}\cdot\frac{-3+\frac{2}{n^3}+\frac{1}{n^4}+\frac{1}{n^5}}{-7+\frac{25}{n^4}}\right)\\
\Leftrightarrow\ &\infty
\end{aligned}$$

(iv)

$$\begin{aligned}
&\lim_{n\to\infty}\left(\frac{6n^3+2n-3}{9n^2+2}-\frac{2n^3+5n^2+7}{3n^2+3}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{-45n^4+20n^3-82n^2+6n-23}{27n^4+33n^2+6}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{-45+\frac{20}{n}-\frac{82}{n^2}+\frac{6}{n^3}-\frac{23}{n^4}}{27+\frac{33}{n^2}+\frac{6}{n^4}}\right)\\
\Leftrightarrow\ &-\frac{45}{27}\Leftrightarrow\ -\frac{5}{3}
\end{aligned}$$

(v)

$$\begin{aligned}
&\lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{2n^2+1}\cdot\sqrt{2n^2+n+1}}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{\sqrt{9n^4+n^2+1}-2n^2+3}{\sqrt{4n^4+2n^3+4n^2++n+1}}\right)\\
\Leftrightarrow\ &\lim_{n\to\infty}\left(\frac{n^4}{n^4}\cdot\frac{\sqrt{9+\frac{1}{n^2}+\frac{1}{n^4}}-\frac{2}{n^2}+\frac{3}{n^4}}{\sqrt{4+\frac{2}{n}+\frac{4}{n^2}+\frac{1}{n^3}+\frac{1}{n^4}}}\right)\\
\Leftrightarrow\ &\frac{\sqrt{9}}{\sqrt{4}}\Leftrightarrow\ \frac{3}{2}
\end{aligned}$$

2. a) (i)

$$\begin{aligned}
a_0 &= 1 & s_0 &= 1\\
a_1 &= \frac{2}{5} & s_1 &= \frac{7}{5}\\
a_2 &= \frac{4}{25} & s_2 &= \frac{39}{25}\\
a_3 &= \frac{8}{125} & s_3 &= \frac{203}{125}\\
a_4 &= \frac{16}{625} & s_4 &= \frac{1031}{625}
\end{aligned}$$

Diese geometrische Reihe konvergiert, da q $= \frac{2}{5} \Rightarrow |q| < 1$

$$\text{Sie konvergiert gegen } \sum_{i=0}^{\infty} \left(\frac{2}{5}\right) = \frac{1}{1-\frac{2}{5}} = \frac{5}{3}$$

(ii)

$$\begin{aligned} a_0 &= 1 & s_0 &= 1 \\ a_1 &= \frac{5}{2} & s_1 &= \frac{7}{2} \\ a_2 &= \frac{25}{4} & s_2 &= \frac{39}{4} \\ a_3 &= \frac{125}{8} & s_3 &= \frac{203}{8} \\ a_4 &= \frac{625}{16} & s_4 &= \frac{1031}{16} \end{aligned}$$

$$\text{Diese geometrische Reihe divergiert, da q} = \frac{5}{2} \Rightarrow |q| > 1$$

(iii)

$$\begin{aligned} a_0 &= 1 & s_0 &= 1 \\ a_1 &= -\frac{2}{5} & s_1 &= \frac{3}{5} \\ a_2 &= \frac{4}{25} & s_2 &= \frac{19}{25} \\ a_3 &= -\frac{8}{125} & s_3 &= \frac{87}{125} \\ a_4 &= \frac{16}{625} & s_4 &= \frac{451}{625} \end{aligned}$$

$$\text{Diese geometrische Reihe konvergiert, da q} = -\frac{2}{5} \Rightarrow |q| < 1$$

$$\text{Sie konvergiert gegen } \sum_{i=0}^{\infty} \left(-\frac{2}{5}\right) = \frac{1}{1-(-\frac{2}{5})} = \frac{5}{7}$$

b) (i)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-(-\frac{3}{10})} = \frac{10}{13}$$

(ii)

$$\frac{1}{1-x} = \frac{5}{8} \quad \Leftrightarrow \quad 1 = (1-x) \cdot \frac{5}{8}$$

$$\Leftrightarrow \; 1 = \frac{5}{8} - \frac{5}{8}x \quad \Leftrightarrow \quad \frac{3}{8} = -\frac{5}{8}x \quad \Leftrightarrow \quad x = -\frac{3}{5}.$$

3.

4.