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1. (i)

$$\lim_{n\to\infty} \left(\frac{-3n^4 + 2n^2 + n + 1}{-7n^4 + 25} \right) \tag{1}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{n^4}{n^4} \cdot \frac{-3 + \frac{2}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}{-7 + \frac{25}{n^4}} \right) \tag{2}$$

$$\Leftrightarrow \frac{3}{7} \tag{3}$$

(ii)

$$\lim_{n\to\infty} \left(\frac{-3n^4 + 2n^2 + n + 1}{-7n^5 + 25} \right) \tag{4}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{1}{n^5} \cdot \frac{-3 + \frac{2}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}{-7 + \frac{25}{n^5}} \right) \tag{5}$$

$$\Leftrightarrow 0 \tag{6}$$

(iii)

$$\lim_{n\to\infty} \left(\frac{-3n^5 + 2n^2 + n + 1}{-7n^4 + 25} \right) \tag{7}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{n}{1} \cdot \frac{-3 + \frac{2}{n^3} + \frac{1}{n^4} + \frac{1}{n^5}}{-7 + \frac{25}{n^4}} \right) \tag{8}$$

$$\Leftrightarrow \infty \tag{9}$$

(iv)

$$\lim_{n\to\infty} \left(\frac{6n^3 + 2n - 3}{9n^2 + 2} - \frac{2n^3 + 5n^2 + 7}{3n^2 + 3} \right) \tag{10}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{-45n^4 + 20n^3 - 82n^2 + 6n - 23}{27n^4 + 33n^2 + 6} \right) \tag{11}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{n^4}{n^4} \cdot \frac{-45 + \frac{20}{n} - \frac{82}{n^2} + \frac{6}{n^3} - \frac{23}{n^4}}{27 + \frac{33}{n^2} + \frac{6}{n^4}} \right) \tag{12}$$

$$\Leftrightarrow -\frac{45}{27} \ \Leftrightarrow \ -\frac{5}{3} \tag{13}$$

$$\tag{14}$$

(v)

$$\lim_{n\to\infty} \left(\frac{\sqrt{9n^4 + n^2 + 1} - 2n^2 + 3}{\sqrt{2n^2 + 1} \cdot \sqrt{2n^2 + n + 1}} \right) \tag{15}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{\sqrt{9n^4 + n^2 + 1} - 2n^2 + 3}{\sqrt{4n^4 + 2n^3 + 4n^2 + +n + 1}} \right) \tag{16}$$

$$\Leftrightarrow \lim_{n\to\infty} \left(\frac{n^4}{n^4} \cdot \frac{\sqrt{9 + \frac{1}{n^2} + \frac{1}{n^4}} - \frac{2}{n^2} + \frac{3}{n^4}}{\sqrt{4 + \frac{2}{n} + \frac{4}{n^2} + \frac{1}{n^3} + \frac{1}{n^4}}} \right) \tag{17}$$

$$\Leftrightarrow \frac{\sqrt{9}}{\sqrt{4}} \Leftrightarrow \frac{3}{2} \tag{18}$$

2. a) (i)

$$a_0 \ = \ 1 \qquad\qquad s_0 \ = \ 1 \tag{19}$$

$$a_1 \ = \ \frac{2}{5} \qquad\qquad s_1 = \frac{7}{5} \tag{20}$$

$$a_2 \ = \ \frac{4}{25} \qquad\qquad s_2 = \frac{39}{25} \tag{21}$$

$$a_3 \ = \ \frac{8}{125} \qquad\qquad s_3 = \frac{203}{125} \tag{22}$$

$$a_4 \ = \ \frac{16}{625} \qquad\qquad s_4 = \frac{1031}{625} \tag{23}$$

$$\tag{24}$$

Diese geometrische Reihe konvergiert, da q $= \dfrac{2}{5} \Rightarrow |q| < 1$

$$\text{Sie konvergiert gegen} \quad \sum_{i=0}^{\infty} \left(\frac{2}{5}\right) = \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$$

(ii)

$$
\begin{align}
a_0 &= 1 & s_0 &= 1 \tag{25} \\
a_1 &= \frac{5}{2} & s_1 &= \frac{7}{2} \tag{26} \\
a_2 &= \frac{25}{4} & s_2 &= \frac{39}{4} \tag{27} \\
a_3 &= \frac{125}{8} & s_3 &= \frac{203}{8} \tag{28} \\
a_4 &= \frac{625}{16} & s_4 &= \frac{1031}{16} \tag{29} \\
& & & \tag{30}
\end{align}
$$

$$\text{Diese geometrische Reihe divergiert, da q} = \frac{5}{2} \Rightarrow |q| > 1$$

(iii)

$$
\begin{align}
a_0 &= 1 & s_0 &= 1 \tag{31} \\
a_1 &= -\frac{2}{5} & s_1 &= \frac{3}{5} \tag{32} \\
a_2 &= \frac{4}{25} & s_2 &= \frac{19}{25} \tag{33} \\
a_3 &= -\frac{8}{125} & s_3 &= \frac{87}{125} \tag{34} \\
a_4 &= \frac{16}{625} & s_4 &= \frac{451}{625} \tag{35} \\
& & & \tag{36}
\end{align}
$$

$$\text{Diese geometrische Reihe konvergiert, da q} = -\frac{2}{5} \Rightarrow |q| < 1$$

$$\text{Sie konvergiert gegen} \quad \sum_{i=0}^{\infty} \left(-\frac{2}{5}\right) = \frac{1}{1 - \left(-\frac{2}{5}\right)} = \frac{5}{7}$$

b) (i)

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1 - \left(-\frac{3}{10}\right)} = \frac{10}{13}$$

(ii) $x = -\frac{3}{5}$

3. **TODO**

4. **TODO**