

MMS

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1. a)

$$\begin{aligned}E\{aX + bY + c\} &= \iint (ax + by + c)p(x, y)dx dy \\&= \iint ax \cdot p(x, y) + by \cdot p(x, y) + c \cdot p(x, y)dx dy \\&= \iint ax \cdot p(x, y)dx dy + \iint by \cdot p(x, y)dx dy + \iint c \cdot p(x, y)dx dy \\&= \int ax \cdot p(x)dx + \int by \cdot p(y)dy + c \\&= a \int x \cdot p(x)dx + b \int y \cdot p(y)dy + c \\&= aE\{X\} + bE\{Y\} + c\end{aligned}$$

b)

$$\begin{aligned}E\{(X - \mu_x)^2\} &= E\{X^2 - 2X\mu_x + \mu_x^2\} \\&= E\{X^2\} - E\{2X\mu_x\} + \mu_x^2 \\&= E\{X^2\} - 2\mu_x E\{X\} + \mu_x^2 \\&= E\{X^2\} - 2E\{X\}E\{X\} + E\{X\}^2 \\&= E\{X^2\} - 2E\{X\}^2 + E\{X\}^2 \\&= E\{X^2\} - E\{X\}^2 \\&= E\{X^2\} - \mu_x^2\end{aligned}$$

c)

$$\begin{aligned}V\{aX + b\} &= E\{(aX + b)^2\} - E^2\{(aX + b)\} \\&= E\{(a^2X^2 + 2abX + b^2)\} - E\{(aX + b)\} \cdot E\{(aX + b)\} \\&= a^2E\{X^2\} + 2abE\{X\} + b^2 - (aE\{X\} + b) \cdot (aE\{X\} + b) \\&= a^2E\{X^2\} + 2abE\{X\} + b^2 - (a^2E^2\{X\} + 2abE\{X\} + b^2) \\&= a^2E\{X^2\} - a^2E^2\{X\} \\&= a^2 \cdot (E\{X^2\} - E^2\{X\}) \\&= a^2 \cdot V\{X\}\end{aligned}$$

d)

$$\begin{aligned}
E\{XY\} &= \iint xy \cdot p(x, y) dx dy \\
&= \iint xy \cdot \underbrace{p(x) \cdot p(y)}_{\text{independence}} dx dy \\
&= \int x \cdot p(x) dx \cdot \int y \cdot p(y) dy \\
&= E\{X\} \cdot E\{Y\}
\end{aligned}$$

e)

$$\begin{aligned}
V\{aX + bY\} &= E\{(aX + bY)^2\} - E^2\{(aX + bY)\} \\
&= E\{(a^2X^2 + 2abXY + b^2Y^2)\} - E\{(aX + bY)\} \cdot E\{(aX + bY)\} \\
&= a^2E\{X^2\} + 2abE\{XY\} + b^2E\{Y^2\} - \\
&\quad (aE\{X\} + bE\{Y\}) \cdot (aE\{X\} + bE\{Y\}) \\
&= a^2E\{X^2\} + 2abE\{XY\} + b^2E\{Y^2\} - \\
&\quad (a^2E^2\{X\} + \underbrace{2abE\{XY\}}_{\text{independence}} + b^2E^2\{Y\}) \\
&= a^2E\{X^2\} + E\{Y^2\} - a^2E^2\{X\} - b^2E^2\{Y\} \\
&= a^2 \cdot (E\{X^2\} - 2E^2\{X\}) + b^2 \cdot (E\{Y^2\} - 2E^2\{Y\}) \\
&= a^2 \cdot V\{X\} + b^2 \cdot V\{Y\}
\end{aligned}$$