MMS

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1. a) $f(t) = \sin(t)$

$$\alpha_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(\sin(t)\cos(kt))dt$$

Für $k \neq 1$ gilt $\alpha = 0$, da sie orthogonal zu einander sind. Für $k = 1$ gilt auch $\alpha = 0$

$$\beta_k = \frac{1}{\pi}\int_{-\pi}^{\pi}(\sin(t)\sin(kt))dt$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{\pi}(\cos(t)\cos(kt))dt - \int_{-\pi}^{\pi}(\cos(t+kt))dt\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2}\int_{-\pi}^{\pi}(\cos(t-kt))dt + \frac{1}{2}\int_{-\pi}^{\pi}(\cos(t+kt))dt - \int_{-\pi}^{\pi}(\cos(t+kt))dt\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2}\int_{-\pi}^{\pi}(\cos(t-kt))dt - \frac{1}{2}\int_{-\pi}^{\pi}(\cos(t+kt))dt\right)$$

$$\int_{-\pi}^{\pi}(\cos(t-kt))dt = \begin{cases} 0 & k \neq 1 \\ 2\pi & k = 1 \end{cases}$$

$$\int_{-\pi}^{\pi}(\cos(t-kt))dt = 0$$

$$\beta_1 = \frac{1}{\pi}\cdot\frac{1}{2}\cdot 2\pi = 1$$

$$\beta_k = 0 f\ddot{u}r k > 1$$