MMS

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1. Für Aufgabe 1 haben wir die folgenden Formeln aus der Vorlesung verwendet.

$$\int_{-\pi}^{\pi} (\sin(k\omega_0 x)\cos(m\omega_0 x))dx = 0, \text{ für } k,m \in \mathbb{Z} \tag{1}$$

$$\int_{-\pi}^{\pi} (\sin(mx)\sin(nx))dx = \delta_{m,n}\pi \tag{2}$$

$$\int_{-\pi}^{\pi} (\cos(mx)\cos(nx))dx = \delta_{m,n}\pi \tag{3}$$

Des weiteren noch:

$$\int \cos(kt)dt = \frac{\sin(kt)}{k} \tag{4}$$

$$\cos(t+\pi) = -\cos(t) \tag{5}$$

$$\sin(-t) = -\sin(t) \tag{6}$$

$$\cos(-t) = \cos(t) \tag{7}$$

a) $f(t) = \sin(t)$

$$\alpha_k = \frac{1}{\pi}\int_{-\pi}^{\pi} (\sin(t)\cos(kt))dt \underbrace{=}_{(1)} 0$$

$$\beta_k = \frac{1}{\pi}\int_{-\pi}^{\pi} (\sin(t)\sin(kt))dt$$

Hier gilt wegen (2):

$$\beta_1 = \frac{1}{\pi}\cdot\pi = 1$$

$$\beta_k = 0, \text{ für } k > 1$$

b) $f(t) = \cos(2t)$

$$\alpha_k = \frac{1}{\pi}\int_{-\pi}^{\pi} (\cos(2t)\cos(kt))dt$$

Hier gilt wegen (3):

$$\alpha_2 = \frac{1}{\pi} \cdot \pi = 1$$
$$\alpha_k = 0, \text{ für } k \neq 2$$

$$\beta_k = \frac{1}{\pi} \int_{-\pi}^{\pi} (\cos(2t)\sin(kt))dt \underbrace{=}_{(1)} 0$$

c) $f(t) = 1$

$$\begin{aligned}
\alpha_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} (\cos(kt))dt \\
&= \frac{1}{\pi} \left[\frac{\sin(kt)}{k} \right]_{-\pi}^{\pi} \\
&= \frac{1}{\pi} \left(\frac{\sin(k\pi)}{k} - \frac{\sin(-k\pi)}{k} \right) \\
&\underbrace{=}_{(6)} \frac{1}{\pi} \left(\frac{\sin(k\pi)}{k} + \frac{\sin(k\pi)}{k} \right) \\
&= \frac{1}{\pi} \left(2\frac{\sin(k\pi)}{k} \right) \\
&= \frac{2\sin(k\pi)}{k\pi}
\end{aligned}$$

$$\begin{aligned}
\beta_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} (\sin(kt))dt \\
&= \frac{1}{\pi} \left[\frac{\cos(kt)}{k} \right]_{-\pi}^{\pi} \\
&= \frac{1}{\pi} \left(\frac{\cos(k\pi)}{k} - \frac{\cos(-k\pi)}{k} \right) \\
&\underbrace{=}_{(7)} \frac{1}{\pi} \left(\frac{\cos(k\pi)}{k} - \frac{\cos(k\pi)}{k} \right) \\
&= 0
\end{aligned}$$

d) $f(t) = \cos(5t + \pi) + 3\sin(9t)$

$$\begin{aligned}
\alpha_k &= \frac{1}{\pi}\int_{-\pi}^{\pi}((\cos(5t+\pi) + 3\sin(9t))\cos(kt))dt \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)\cos(kt) + 3\sin(9t)\cos(kt))dt \\
&= \frac{1}{\pi}\left(\int_{-\pi}^{\pi}(\cos(5t+\pi)\cos(kt))dt\right. \\
&\qquad\qquad\left. + \int_{-\pi}^{\pi}(3\sin(9t)\cos(kt))dt\right) \\
&\underbrace{=}_{(1)} \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)\cos(kt))dt \\
&\underbrace{=}_{(5)} -\frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t)\cos(kt))dt
\end{aligned}$$

Und dann gilt aufgrund von (3):

$$\alpha_5 = -\frac{1}{\pi}\cdot\pi = -1$$

$$\alpha_k = 0, \text{ für } k \neq 5$$

$$\begin{aligned}
\beta_k &= \frac{1}{\pi}\int_{-\pi}^{\pi}((\cos(5t+\pi) + 3\sin(9t))\sin(kt))dt \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)\sin(kt) + 3\sin(9t)\sin(kt))dt \\
&= \frac{1}{\pi}\left(\int_{-\pi}^{\pi}(\cos(5t+\pi)\sin(kt))dt\right. \\
&\qquad\qquad\left. + \int_{-\pi}^{\pi}(3\sin(9t)\sin(kt))dt\right) \\
&\underbrace{=}_{(5)} \frac{1}{\pi}\left(-\int_{-\pi}^{\pi}(\cos(5t)\sin(kt))dt\right. \\
&\qquad\qquad\left. + \int_{-\pi}^{\pi}(3\sin(9t)\sin(kt))dt\right) \\
&\underbrace{=}_{(1)} \frac{1}{\pi}\int_{-\pi}^{\pi}(3\sin(9t)\sin(kt))dt \\
&= \frac{3}{\pi}\int_{-\pi}^{\pi}(\sin(9t)\sin(kt))dt
\end{aligned}$$

Und dann gilt aufgrund von (2):

$$\beta_9 = \frac{3}{\pi}\cdot\pi = 3$$

$$\beta_k = 0, \text{ für } k \neq 9$$