MMS

Vanessa Closius, Jonas Tietz, Tronje Krabbe

1. November 2018

1. Für Aufgabe 1 haben wir die folgenden Formeln aus der Vorlesung verwendet.

$$\int_{\frac{T}{2}}^{\frac{T}{2}} (\sin(k\omega_0 x)\cos(m\omega_0 x))dx = 0, f\ddot{u}rk, m \in \mathbb{Z} \tag{1}$$

$$\int_{-\pi}^{\pi} (\sin(mx)\sin(nx))dx = \delta_{m,n}\pi \tag{2}$$

$$\int_{-\pi}^{\pi} (\cos(mx)\cos(nx))dx = \delta_{m,n}\pi \tag{3}$$

Des weiteren noch:

$$\int \cos(kt)dt = \frac{\sin(kt)}{k} \tag{4}$$

$$\cos(t+\pi) = -\cos(t) \tag{5}$$

a) $f(t) = \sin(t)$

$$\alpha_k = \frac{1}{\pi}\int_{-\pi}^{\pi} (\sin(t)\cos(kt))dt$$

Für $k \neq 1$ gilt $\alpha = 0$, da sie orthogonal zu einander sind. Für $k = 1$ gilt auch $\alpha = 0$

$$\beta_k = \frac{1}{\pi}\int_{-\pi}^{\pi} (\sin(t)\sin(kt))dt$$

$$= \frac{1}{\pi}\left(\int_{-\pi}^{\pi} (\cos(t)\cos(kt))dt - \int_{-\pi}^{\pi} (\cos(t+kt))dt\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2}\int_{-\pi}^{\pi} (\cos(t-kt))dt + \frac{1}{2}\int_{-\pi}^{\pi} (\cos(t+kt))dt - \int_{-\pi}^{\pi} (\cos(t+kt))dt\right)$$

$$= \frac{1}{\pi}\left(\frac{1}{2}\int_{-\pi}^{\pi} (\cos(t-kt))dt - \frac{1}{2}\int_{-\pi}^{\pi} (\cos(t+kt))dt\right)$$

$$\int_{-\pi}^{\pi} (\cos(t-kt))dt = \begin{cases} 0 & k \neq 1 \\ 2\pi & k = 1 \end{cases}$$

$$\int_{-\pi}^{\pi} (\cos(t - kt))dt = 0$$

$$\beta_1 = \frac{1}{\pi} \cdot \frac{1}{2} \cdot 2\pi = 1$$

$$\beta_k = 0, für k > 1$$

b) $f(t) = \cos(2t)$

$$\alpha_k = \frac{1}{\pi} \int_{-\pi}^{\pi} (\cos(2t)\cos(kt))dt$$

$$\alpha_2 = \frac{1}{\pi}\pi = 1$$

$$\alpha_k = 0, für k \neq 2$$

$$\beta_k = \frac{1}{\pi} \int_{-\pi}^{\pi} (\cos(2t)\sin(kt))dt = 0$$

c) $f(t) = 1$

$$\begin{aligned}
\alpha_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} (\cos(kt))dt \\
&= \frac{1}{\pi} \left[\frac{\sin(kt)}{k} \right]_{-\pi}^{\pi} \\
&= \frac{1}{\pi} \left(\frac{\sin(-k\pi)}{k} - \frac{\sin(k\pi)}{k} \right) \\
&= \frac{1}{\pi} \left(-\frac{\sin(k\pi)}{k} - \frac{\sin(k\pi)}{k} \right) \\
&= \frac{1}{\pi} \left(-2\frac{\sin(k\pi)}{k} \right) \\
&= -\frac{2\sin(k\pi)}{k\pi}
\end{aligned}$$

$$\beta_k = \frac{1}{\pi} \int_{-\pi}^{\pi} (\sin(kt))dt = 0$$

d) $f(t) = \cos(5t + \pi) + 3\sin(9t)$

$$\begin{aligned}
\alpha_k &= \frac{1}{\pi}\int_{-\pi}^{\pi}((\cos(5t+\pi) + 3\sin(9t))cos(kt))dt \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)cos(kt) + 3\sin(9t)cos(kt))dt \\
&= \frac{1}{\pi}\left(\int_{-\pi}^{\pi}(\cos(5t+\pi)cos(kt))dt\right. \\
&\qquad\qquad\qquad\qquad \left. + \int_{-\pi}^{\pi} 3\sin(9t)cos(kt))dt\right) \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)cos(kt))dt \\
&= \frac{-1}{\pi}\int_{-\pi}^{\pi}(\cos(5t)cos(kt))dt
\end{aligned}$$

$$\alpha_5 = -\frac{1}{pi}pi = -1$$

$$\alpha_k = 0, f\ddot{u}rk \neq 5$$

$$\begin{aligned}
\beta_k &= \frac{1}{\pi}\int_{-\pi}^{\pi}((\cos(5t+\pi) + 3\sin(9t))sin(kt))dt \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi}(\cos(5t+\pi)sin(kt) + 3\sin(9t)sin(kt))dt \\
&= \frac{1}{\pi}\left(\int_{-\pi}^{\pi}(\cos(5t+\pi)sin(kt))dt\right. \\
&\qquad\qquad\qquad\qquad \left. + \int_{-\pi}^{\pi} 3\sin(9t)sin(kt))dt\right) \\
&= \frac{1}{\pi}\left(-\int_{-\pi}^{\pi}(\cos(5t)sin(kt))dt\right. \\
&\quad \left. + \int_{-\pi}^{\pi} 3\sin(9t)sin(kt))dt\right) \\
&= \frac{1}{\pi}\int_{-\pi}^{\pi} 3\sin(9t)sin(kt))dt \\
&= \frac{3}{\pi}\int_{-\pi}^{\pi} \sin(9t)sin(kt))dt
\end{aligned}$$

$$\beta_9 = \frac{3}{\pi}\pi = 3$$

$$\beta_k = 0, f\ddot{u}rk \neq 9$$