MMS

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1. **1)** **a)** $z_1 = 1 + j\sqrt{3}$ $z_2 = 1 - j$

$$\begin{aligned} z_1 + z_2 &= 1 + 1 + j\sqrt{3} - j \\ &= 2 + j(\sqrt{3} - 1) \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= 1 - 1 + j\sqrt{3} + j \\ &= j(\sqrt{3} + 1) \end{aligned}$$

$$\begin{aligned} z_1 \cdot z_2 &= (1 + j\sqrt{3})(1 - j) \\ &= 1 - j + j\sqrt{3} + \sqrt{3} \\ &= (1 + \sqrt{3}) + j(\sqrt{3} - 1) \end{aligned}$$

$$z_1/z_2 = \frac{(1 + j\sqrt{3})}{(1 - j)} = \frac{1 + \sqrt{3}}{1 + 1}$$
$$+ j\frac{\sqrt{3} - 1}{1 + 1} = \frac{1 + \sqrt{3}}{2} + j\frac{\sqrt{3} - 1}{2}$$

$$\begin{aligned} z_1^* \cdot z_2 &= (1 - j\sqrt{3})(1 - j) \\ &= 1 - j - j\sqrt{3} - sqrt3 \\ &= (1 - \sqrt{3}) - j(\sqrt{3} + 1) \end{aligned}$$

$$z_1/z_2^* = \frac{(1 + j\sqrt{3})}{(1 + j)} = \frac{1 + \sqrt{3}}{1 + 1}$$
$$+ j\frac{\sqrt{3} - 1}{1 + 1} = \frac{1 + \sqrt{3}}{2} + j\frac{\sqrt{3} - 1}{2}$$

b) $z_1 = 2 + 3j$ $z_2 = 3 - 5j$

$$\begin{aligned} z_1 + z_2 &= 2 + 3 + 3j - 5j \\ &= 5 - 2j \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= 2 - 3 + 3j + 5j \\ &= -1 + 8j \end{aligned}$$

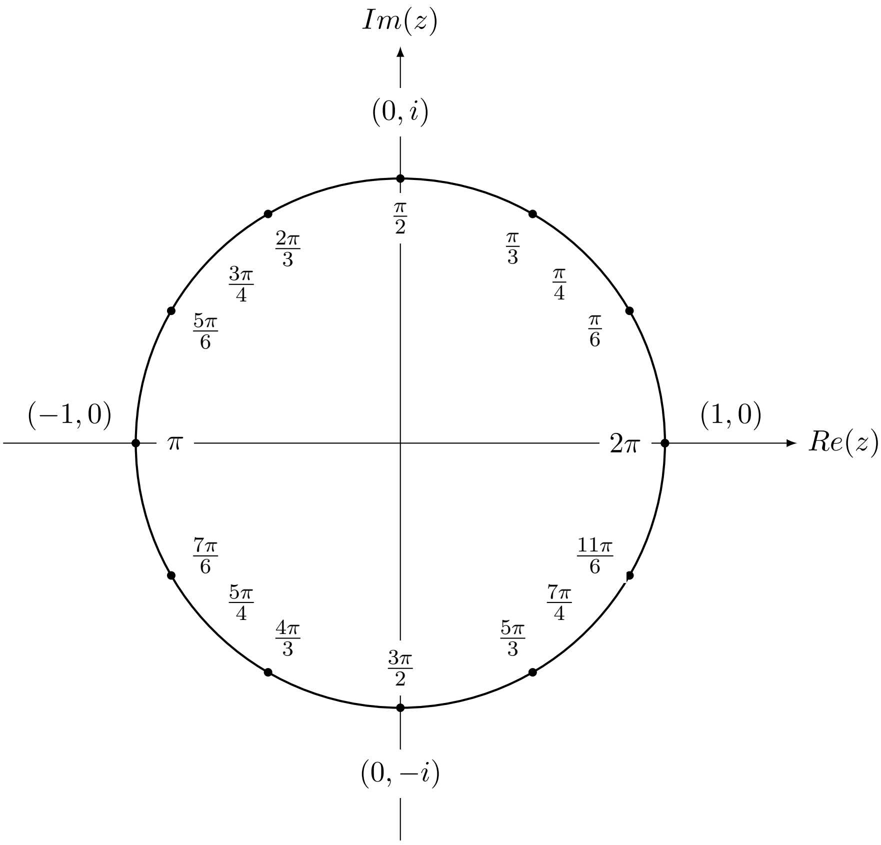
$$\begin{aligned} z_1 \cdot z_2 &= (2 + 3j)(3 - 5j) \\ &= 6 - 10j + 9j + 15 \\ &= 21 - j \end{aligned}$$

$$\begin{aligned} z_1 / z_2 &= \frac{(2 + 3j)}{(3 - 5j)} \\ &= \frac{6 - 15}{9 + 25} + j\frac{9 + 10}{9 + 25} \\ &= \frac{-9}{34} + j\frac{19}{34} \end{aligned}$$

$$\begin{aligned} z_1^* \cdot z_2 &= (2 - 3j)(3 - 5j) \\ &= 6 - 10j - 9j - 15 \\ &= -9 - 19j \end{aligned}$$

$$\begin{aligned} z_1 / z_2^* &= \frac{2 + 3j}{3 + 5j} \\ &= \frac{6 + 15}{9 + 25} + j\frac{9 - 10}{9 + 25} \\ &= \frac{21}{34} - k\frac{1}{34} \end{aligned}$$

2) $e^{i\theta}$ stellt einen Vektor in der komplexen Zahlenebene da, der um einen Winkel θ um den Einheitskreis rotiert worden ist. Mit der eulerschen Formel $e^{i\theta} = \cos(\theta) + i\sin(theta)$ bekommt man die kartesischen Koordinaten des rotierten Vektors. Da π genau eine halbe Rotation um den Einheitskreis ist bekommt man $e^{i\theta} = -1$. Dies kann man dann noch umformen um Eulers Identität $e^{i\theta} + 1 = 0$ zu erhalten.


$Im(z)$
$(0, i)$
$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
$(-1, 0)$
π
2π
$(1, 0)$
$Re(z)$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\frac{3\pi}{2}$
$(0, -i)$

1.