MMS

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1. 1) a) $z_1 = 1 + j\sqrt{3}$ $z_2 = 1 - j$

$$\begin{aligned} z_1 + z_2 &= 1 + 1 + j\sqrt{3} - j \\ &= 2 + j(\sqrt{3} - 1) \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= 1 - 1 + j\sqrt{3} + j \\ &= j(\sqrt{3} + 1) \end{aligned}$$

$$\begin{aligned} z_1 \cdot z_2 &= (1 + j\sqrt{3})(1 - j) \\ &= 1 - j + j\sqrt{3} + \sqrt{3} \\ &= (1 + \sqrt{3}) + j(\sqrt{3} - 1) \end{aligned}$$

$$\begin{aligned} z_1 / z_2 &= \frac{(1 + j\sqrt{3})}{(1 - j)} \\ &= \frac{1 + \sqrt{3}}{1 + 1} + j\frac{\sqrt{3} - 1}{1 + 1} \\ &= \frac{1 + \sqrt{3}}{2} + j\frac{\sqrt{3} - 1}{2} \end{aligned}$$

$$\begin{aligned} z_1^* \cdot z_2 &= (1 - j\sqrt{3})(1 - j) \\ &= 1 - j - j\sqrt{3} - \sqrt{3} \\ &= (1 - \sqrt{3}) - j(\sqrt{3} + 1) \end{aligned}$$

$$\begin{aligned} z_1 / z_2^* &= \frac{(1 + j\sqrt{3})}{(1 + j)} \\ &= \frac{1 + \sqrt{3}}{1 + 1} + j\frac{\sqrt{3} - 1}{1 + 1} \\ &= \frac{1 + \sqrt{3}}{2} + j\frac{\sqrt{3} - 1}{2} \end{aligned}$$

b) $z_1 = 2 + 3j \; z_2 = 3 - 5j$

$$\begin{aligned} z_1 + z_2 &= 2 + 3 + 3j - 5j \\ &= 5 - 2j \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= 2 - 3 + 3j + 5j \\ &= -1 + 8j \end{aligned}$$

$$\begin{aligned} z_1 \cdot z_2 &= (2 + 3j)(3 - 5j) \\ &= 6 - 10j + 9j + 15 \\ &= 21 - j \end{aligned}$$

$$\begin{aligned} z_1 / z_2 &= \frac{(2 + 3j)}{(3 - 5j)} \\ &= \frac{6 - 15}{9 + 25} + j\frac{9 + 10}{9 + 25} \\ &= \frac{-9}{34} + j\frac{19}{34} \end{aligned}$$

$$\begin{aligned} z_1^* \cdot z_2 &= (2 - 3j)(3 - 5j) \\ &= 6 - 10j - 9j - 15 \\ &= -9 - 19j \end{aligned}$$

$$\begin{aligned} z_1 / z_2^* &= \frac{2 + 3j}{3 + 5j} \\ &= \frac{6 + 15}{9 + 25} + j\frac{9 - 10}{9 + 25} \\ &= \frac{21}{34} - k\frac{1}{34} \end{aligned}$$

c) $z_1 = 4 - 5j \; z_2 = 4 + 5j$

$$\begin{aligned} z_1 + z_2 &= 4 + 4 + 5j - 5j \\ &= 8 \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= 4 - 4 - 5j - 55j \\ &= -10j \end{aligned}$$

$$\begin{aligned} z_1 \cdot z_2 &= (4 - 5j)(4 + 5j) \\ &= 16 + 20j - 20j + 25 \end{aligned}$$

$$= 41$$

$$\begin{aligned} z_1/z_2 &= \frac{(4-5j)}{(4+5j)} \\ &= \frac{16-25}{16+25} + j\frac{-20-20}{16+25} \\ &= \frac{-9}{41} - j\frac{40}{41} \end{aligned}$$

$$\begin{aligned} z_1^* \cdot z_2 &= (4+5j)(4+5j) \\ &= 16 + 20j + 20j - 25 \\ &= -9 + 40j \end{aligned}$$

$$\begin{aligned} z_1/z_2^* &= \frac{4-5j}{4-5j} \\ &= \frac{16+25}{16+25} + j\frac{-20+20}{16+25} \\ &= 1 \end{aligned}$$

d) $z_1 = j$ $z_2 = -2 - 4j$

$$\begin{aligned} z_1 + z_2 &= -2 + j - 4j \\ &= -2 - 3j \end{aligned}$$

$$\begin{aligned} z_1 - z_2 &= j + 2 + 4j \\ &= 2 + 5j \end{aligned}$$

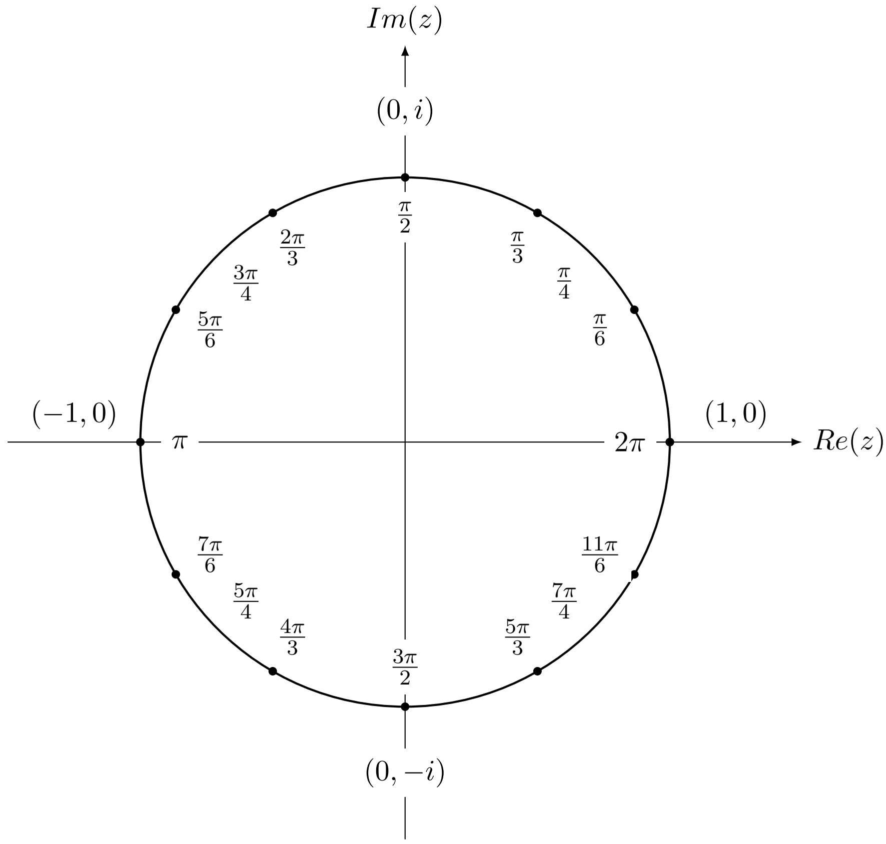
$$\begin{aligned} z_1 \cdot z_2 &= j(-2-4j) \\ &= 4 - 2j \end{aligned}$$

$$\begin{aligned} z_1/z_2 &= \frac{j}{(-2-4j)} \\ &= \frac{-4}{4+16} + j\frac{-2}{4+16} \\ &= -\frac{-1}{5} - j\frac{1}{10} \end{aligned}$$

$$\begin{aligned} z_1^* \cdot z_2 &= -j(-2-4j) \\ &= -4 + 2 \end{aligned}$$

$$\begin{aligned} z_1/z_2^* &= \frac{j}{-2+4j} \\ &= \frac{4}{4+16} + j\frac{-2}{4+16} \\ &= \frac{1}{5} - j\frac{1}{10} \end{aligned}$$

2) $e^{i\theta}$ stellt einen Vektor in der komplexen Zahlenebene da, der um einen Winkel θ um den Einheitskreis rotiert worden ist. Mit der eulerschen Formel $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ bekommt man die kartesischen Koordinaten des rotierten Vektors. Da π genau eine halbe Rotation um den Einheitskreis ist bekommt man $e^{i\theta} = -1$. Dies kann man dann noch umformen um Eulers Identität $e^{i\theta} + 1 = 0$ zu erhalten.



2.