P'

S → aS / bA.

A → ~~ε~~B / d

~~B → CB / cC~~

~~C → εε~~

Elimination of null productions :- → modify only production.

A → AB / a.

B → ∧.

→ Find set of nullable variables.

$W_i(A) = \{ A \to \wedge \in P \}$ → include all non-terminal that generate ∧.

$W_{i+1}(A) = W_i(A) \cup \{ A / A \to B \text{ and } B \in W_i \}$.

$P' = \{$ if $B \to A_1 A_0 - - A_n \in P$ and $A_i \in W_i \}$

→ then write P including & excluding A.

eg: B → AC
A ∈ W_i } ⇒ B → Ac / c.

eg: S → aS / AB.
A → ∧ / bB.
B → ∧ / cAc.
C → cC

$W_1 = \{A, B\}$.
$W_2 = \{A, B\} \cup \{S\}$.
$= \{ABS\}$.

→ if S → aS / ABC then S is not nullable was all the non-terminal included should be null.

Teacher's Signature

eg. S → aS/a.
S → AB/A/B
A → bB/b
B → cAc/cC
C → cC

} once including once excluding

Elimination of Unit Production

A → B
B → C
C → e

} A → e
B → e
C → e.

Find var derived by another var.
→ for each non-terminal.

$W_i(A) = \{A\}$.
$W_{i+1}(A) = W_i(A) \cup \{B \mid A \to B \text{ and } B \in W_i(A)\}$

eg. S → AB/a.
A → b/B.
B → cD/c.
C → eD/d.
D → g.

$W_i(S) = S$. → any other non-terminal in single unit (→)
$W_i(A) = \{A\} \cup \{B\} \cup \{c\} = \{A, B, c\}$.
$W_i(B) = \{B, c\}$.
$W_i(c) = \{c\}$.
$W_i(D) = \{D\}$.

Production P'

if $B \leftarrow W_i(A)$

then if $B \to \alpha \in P$

add $A \to \alpha$ in P'

eg: $A \to B$, $B \to \alpha$. } $A \to \alpha$, $B \to \alpha$.

∴ in previous production

Production: P'

$S \to AB / a$.

$A \to b / cD / eD / d$.

$B \to cD / eD / d$.

$C \to eD / d$.

$D \to g$.

~~to~~ start ~~from~~ the smallest set.

Copy productions of C in B and then B in A (C included).

~~Normal forms.~~

Remove null

↓

Remove unit.

↓

Useless (NG).

↓

Useless (NR).

~~Normal forms.~~

Teacher's Signature

Normal Forms.

Chomsky Normal Form (CNF)

→ A grammar G is in CNF if all the produc
are in following form

$A \to BC$ (max 2 non-terminal)

or $A \to a$. (a single terminal).

① Simplify grammar.

② Remove intermediate terminal.
for each terminal a, introduce a new NT X
and a production $X \to a$. Replace a by X.

eg:- $S \to AB/a$.
$A \to b/cD/eD/d$.
$B \to cD/eD/dc$.
$C \to eDdB/a$.
$D \to g/c/b$.

$S \to AB$.
$S \to a$.
$A \to b$.
$A \to cD$.
$\to XD$. $X \to c$.
$A \to eD \to YD$ $Y \to e$.
$A \to d$.
$B \to XD/YD/ZC$ $Z \to d$.
$C \to YDZB$
$C \to a$
$D \to g/c/b$

③ Restricting number of NT

$A \to A_1 A_2 \dots A_n$

rewrite it as

$A \to A_1 Z_1$

$Z_1 \to A_2 Z_2$

$Z_2 \to A_3 Z_3$

$\vdots$

$Z_{n-2} = A_{n-1} A_n$

Also, for $C \to YDZB$

$C \to YC_1$

$C_1 \to DC_2$

$C_2 \to ZB$

eg:- $A \to aAcB / cDe$

$B \to \wedge / aA$

$D \to B / aDA / b$

$A \to aAcB$

$A \to XAYB$

$A \to YDZ$

$X \to a$

$Y \to c$

$Z \to e$

$W_1 = \{B\}$.

$W_2 = \{B\} \cup \{D\} = \{B, D\}$.

$A \to aAcB / aAc / cDe / ce$

$B \to aA$

$D \to aDA / aA / b / \cancel{B}$ ⟶ $\because B \to aA$ which is already included.

$X \to a \quad Y \to c \quad Z \to e$

$A \to XAYB$

↳ $A_1 \to xA_2 \quad A_2 \to AA_3 \quad A_3 \to YB$

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~~⟹~~ $A \to XAY$.

↳ $A_1 \to XA_2$ $\quad A_2 \to AY$.

~~⟹~~ $A \to YZ$.

$A \to YDZ$.

↳ $A \to YA_1$, $\quad A_1 \to DZ$.

$B \to xA$.

$D \to xD_1$, $\quad D_1 \to DA$.

$D \to xA$.

$D \to b$.

Greibeach Normal Form. (GNF)

A grammar G is in GNF if all the production are of form

$A \to a\alpha_1, \alpha_2 -- \alpha_n / a$.

$a \in \Sigma$

and

$\alpha_1, \alpha_2 -- \alpha_n \in NT$.

Procedure

1. Simplify grammar
2. Remove intermediate terminal (like CNF Rule-2)
3. if $A \to B\alpha \in P$ then $B \to \beta \in P$ then $A \to \beta\alpha \in P'$
4. if $A \to A\alpha / \beta$.

then

$A \to \beta A' / \beta$

$A' \to \alpha A' / \alpha$.

$A \to A\alpha$.

$\to A\alpha\alpha$.

$\to A\alpha\alpha\alpha$.

$\to \beta\alpha\alpha\alpha$

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eg:- $S \to AB / b.$

$A \to cBd / gF$

$F \to a.$ ①

$B \to bB / e$

no $S \to AB / b$ ~~$X \to d$~~

$A \to eBX / gF \qquad X \to d.$

↳ don't remove c ∵ we need to replace it back

$F \to a.$

$B \to bB / c.$

$S \to cBXB / gFB / b.$

$A \to cBX / gF$

$F \to a.$

$B \to bB / c.$

eg:- $S \to Ab / c.$

$A \to Sa / d.$

$X \to a \qquad Y \to b.$

$S \to AY / c.$

$A \to SX / d.$

$S \to SXY / dY / c.$

$\overline{A} \quad \overline{A} \quad \overline{A} \quad \overline{\beta} \quad \overline{\beta}.$

$S \to dY / c.$

$S \to dYS' / cS'$

$S' \to XYS' / XY.$

$S' \to aYS' / aY.$

Rule 3 of GNF

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$A \to sx / d.$

$\Downarrow$

$A \to dyx / cx / dys'x / cs'x / d.$

Quiz:-

(1) $L = a^l b^n$ ~~$n \to \infty$~~ $|l - m| = 2.$

$l - m = 2 \qquad l = m + 2.$

or ~~$m \to \infty$~~ $\qquad m = l + 2.$

$a^m b^{m+2}$ or $a^{m+2} b^m.$

$a^m b^m (bb)$ or $(aa) a^m b^m$

↳ not regular.

→ pumping lemma.

(2) $L = a^{n+2} b^m c^{n+m}.$

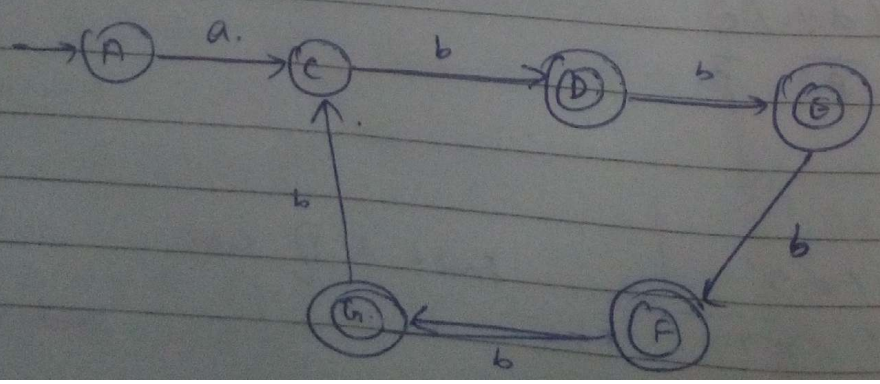
$= (aa) a^n b^m c^m c^n.$

$S \to aaA.$

$A \to aAc / B / \Lambda.$

$B \to bBc / \Lambda.$

(3) $L = a^m b^n \quad m > 0$ and n is not a multiple of 5



$A \to aA / aC$

$C \to bD / b.$

$D \to bE / b.$

$E \to bF / b.$

$F \to bG / b.$

$G \to bC.$

(4) $a^m b^n c^k$ where $|m-n| = k.$

$m - n = k \Rightarrow m = n + k$

OR $-m + n = k \Rightarrow n = k + m.$

$a^{n+k} b^n c^k \to a^k a^n b^n c^k.$

$a^m b^{k+m} c^k \to a^m b^m b^k c^k.$

] no regular grammar.

$S \to S_1 / S_2.$

$S_1 \to aS_1 c / A.$

$A \to aAb / bc.$

$S_2 \to CD.$

$C \to aCb / \wedge.$

$D \to bDc / \wedge$

(5) $S \to bSe / PQR.$

$P \to bPc / \wedge$

$Q \to cQd / \wedge.$

$R \to dRe / \wedge.$

$b^n b^k c^k c^m d^m d^p e^p e^m.$

$= b^{n+k} c^{k+m} d^{m+p} e^{p+m}$

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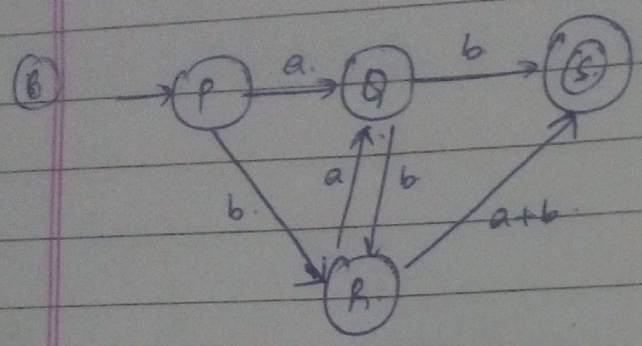
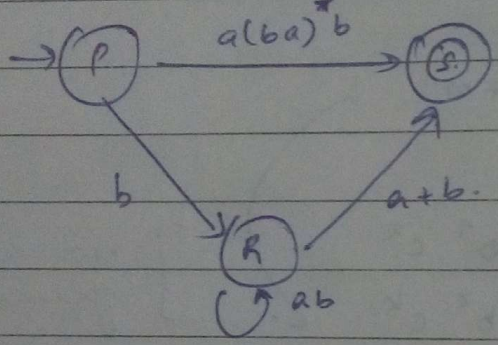
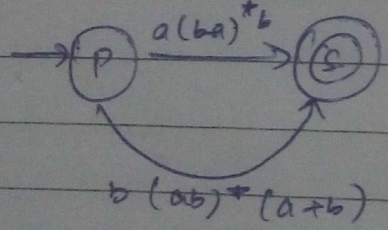
min string = bcde.





⇒



⇒ $a(ba)^*b + b(ab)^*(a+b)$