Parallel Algorithms and Programming

Parallel Algorithms in Distributed Memory Systems (Part 2)

Thomas Ropars

Email: thomas.ropars@univ-grenoble-alpes.fr

Website: tropars.github.io

In this lecture

- Performance of parallel distributed algorithms
- Stencil algorithms
- Some major concepts
    - Greedy algorithm
    - Bulk communication
    - block-cyclic allocation

Introduction

About the performance of distributed parallel algorithm

Assumptions

- A distributed memory system
    - Processes communicate by sending and receiving messages
- A **virtual ring** topology

Main costs to consider

- The cost of running computation
    - floating point operations in our context
- The cost of moving data
    - from the memory to the processors
    - between nodes (over the interconnection network)

Cost of a distributed parallel algorithm

- A computational term:

$$\text{nb of flops} \times \text{time per flop}$$

- A bandwidth term:

$$\text{amount of data moved} \times \frac{1}{bandwidth}$$

- A latency term:

$$\text{nb of messages} \times latency$$

Cost of a distributed parallel algorithm

We should remember that (in general):

$$\text{time per flop} \ll \frac{1}{bandwidth} \ll latency$$

Consequence

- Minimizing communication is important for performance

- It is also important for energy efficiency
  - Moving data from/to DRAM or over the interconnection network are the most energy consuming operations

About memory accesses

Cost of moving data between the memory and the processor

- Can often be ignored
    - Cost mostly hidden by hardware prefetchers

Prefetchers

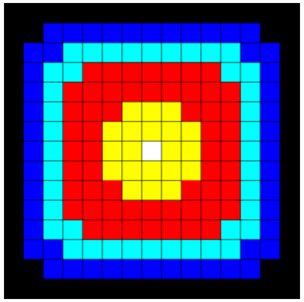
- Hardware mechanisms to load data in the cache before it is needed
- Based on the memory access pattern of the program
- Works well for regular access patterns
    - It is the case for the programs studied in this lecture

Stencil algorithms

About stencil algorithms

A common class of parallel applications

- Operate on cells that divide a discrete domain



Main properties of the cells

- Cells can hold one or multiple values
- Cells are in general organized in a N-dimensional grid (often 2D or 3D)
    - They are non-overlapping
    - They are of equal size (load balancing)
- Cells can be distributed over multiple nodes to execute a program in parallel
    - Deal with large problems

About stencil algorithms

Definition of a stencil

- Iterative algorithm
- Applies a pre-defined function to update the value of the cells based on the value of the neighboring cells

- Specifying a stencil boils down to defining:
    - The **location of a cell's neighbors**
    - The **function used to update cell values**

- The combination of both forms a *stencil*
    - Applied to all cells in the domain

2-dimensional domains

In the following, we consider stencils applying to 2-dimensional domains

- In this case, a cell can have at most 8 neighbors
  - Cardinal coordinates: N, NE, E, SE, S, SW, W, NW.

NW	N	NE
W	Cell	E
SW	S	SE

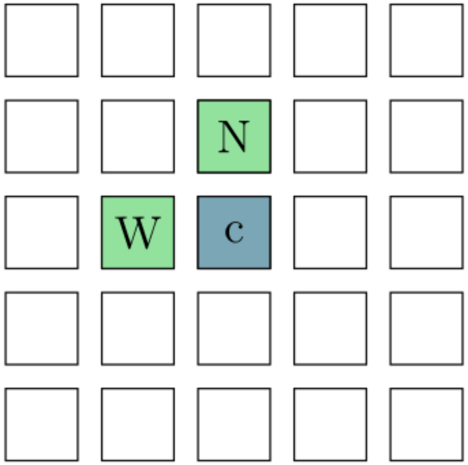
- 9-point stencil: The stencil function uses as input the value of the cell and of the 8 neighbors

A case study

A stencil algorithm

Definition of the stencil

$$c_{new} = Update(c_{old}, W_{new}, N_{new})$$



- The new value of a cell (c_{new}) depends on the previous value of the cell (c_{old}) and the already updated value of the `West` and `North` neighbors.

Some comments

Stencil of practical importance

- At the root of some numerical algorithms

Case of the border cells

- Might not have `West` or `North` neighbors
- A modified `Update` function is applied to these cells
    - For instance, using a constant value for the non-existent cells.

Main assumptions that we make

- $n \times n$ cells
- p processors
    - Unidirectional ring

Greedy algorithm

```
*Greedy* = the algorithm tries to put all processors to work as
early as possible
```

First version (assuming $n = p$)

- Each processor stores 1 row
  - Row i is stored on processor P_i
- As soon as a processor P_i has computed a value, it sends it to processor P_{i+1}.

Notation used in the following

- A is the domain on which the stencil is applied
- $A_{x,y}$ (or $A[x][y]$) is the y-th element on row x

Parallel execution of the greedy algorithm

1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9

- The number associated with each cell is the step in which the cell is updated
  - In step k, the k-th anti-diagonal can be computed.

Description of the algorithm

Some special cases (in terms of communication)

- Processor P_0 does not receive `North` values from another processor
- Processor P_{n-1} does not need to send its updated values

General algorithm

- At iteration $i + j$, processor P_i performs the following operations:
    1. Receives $A_{i-1,j}$ from P_{i-1};
    2. Computes $A_{i,j}$;
    3. Sends $A_{i,j}$ to P_{i+1}.

Detailed description of the algorithm

```
double A[n]; /* one row of A, assumed to be already initialized*/
double north = 0; /* to recv North values */

int rank = my_rank();
int P = num_procs();

if (rank == 0){
  A[0] = Update(A[0], NULL, NULL);
  Send(A[0], rank+1);
}
else{
  Recv(north, rank-1);
  A[0] = Update(A[0], NULL, north);
}

for(j=1; j<n; j++){
  if (rank == 0){
    A[j] = Update(A[j], A[j-1], NULL);
    Send(A[j], rank+1);
  }
  else if(rank == P-1){
    Recv(north, rank-1);
    A[j] = Update(A[j], A[j-1], north);
  }
  else{
    Send(A[j-1], rank+1);
    ||
    Recv(north, rank-1);
    A[j] = Update(A[j], A[j-1], north);
  }
}
```

Generalizing the algorithm

What if $n > p$?

- How to assign rows to processors ?

First solution

- Assigning a set of consecutive rows to the same processors

Generalizing the algorithm

What if $n > p$?

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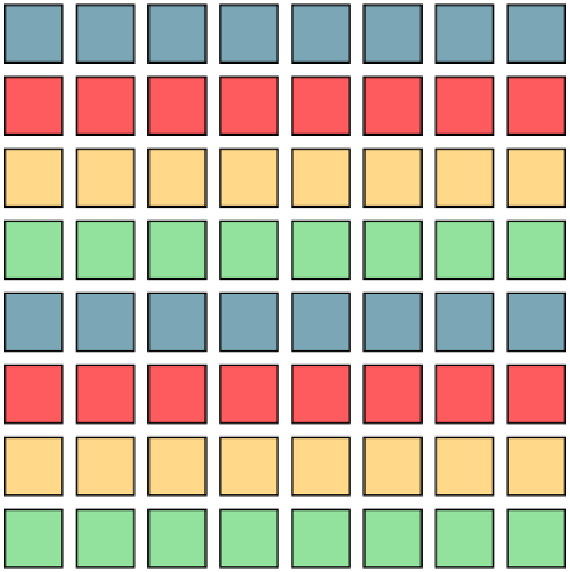
First solution

- Assigning a set of consecutive rows to the same processors
  - Problem: The algorithm is not greedy anymore
    - P_1 can start working when P_0 has computed the first value on each row assigned to it
    - P_1 waits at least $\frac{n}{p}$ steps
    - P_2 waits at least $2 \times \frac{n}{p}$ steps
    - **Processors do not start computing as early as possible**

Cyclic row assignment

Solution to ensure that each processor starts as early as possible



- Row j of the domain is assigned to processor P_k with $k = j \mod p$

Performance of the algorithm

Extra notation

- b -- time to transfer one cell: $b = \frac{sizeof(cell)}{B}$
- w -- cost of executing the stencil `Update()` function for one cell

Time to run one step of the algorithm

- 3 operations need to be run by one process
    1. Receiving `north` value for step k
    2. Computing one cell
    3. Sending value for step $k+1$
- Comment: The send operation (at iteration n) can occur in parallel with the reception (for iteration $n+1$)

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$$T_{step}(p, n) = w + L + b$$

Performance of the algorithm

Total number of steps

When does the last processor computes its last cell?

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When does the last processor computes its last cell?

- It takes $p-1$ steps before processor P_{p-1} starts computing its first cell
- From this point, processor P_{p-1} updates one cell per step
- Processor P_{p-1} has $\frac{n}{p} \times n$ cells to compute in total.

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$$T(n,p) = (p - 1 + \frac{n^2}{p}) \times (w + L + b)$$

Speedup achieved by the algorithm

Some comments:

- When n becomes large, the execution time is equal to:

$$T(n,p) = \frac{n^2}{p} \times (w + L + b)$$

- The sequential execution time is:

$$n^2 \times w$$

Speedup on p processors

$$Speedup = \frac{T_{seq}}{T_{par}} = \frac{n^2 \times w}{\frac{n^2}{p} \times (w + L + b)} = p \times \frac{w}{w + L + b} < p$$

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Can we do better?

- The network latency L can have a high impact on the performance of parallel algorithms
  - Let's try to reduce the latency term!

Bulk communication

- To reduce the impact of latency, the idea is to send less messages
    -

Bulk communication

- To reduce the impact of latency, the idea is to send less messages
    - We need to send larger messages
    - **A processor computes k values on one row before sending updates to the next processors**

New performance

- Execution time of one step:

$$T_{step} = k \times (w + b) + L$$

- Number of steps:
  - It takes $p - 1$ steps until processor P_{p-1} starts working
  - Processor P_{p-1} should run $\frac{n^2}{p \times k}$ such steps

Execution time:

$$T_{bulk}(p, n, k) = (p - 1 + \frac{n^2}{p \times k}) \times (k \times (w + b) + L)$$

- When n becomes large: $T_{bulk}(p, n, k) = \frac{n^2}{p} \times (w + b + \frac{L}{k})$

Discussion on performance

Some comments

- We obtain the expected result: the latency term is divided by k
- This solution does not perform that well when n is not large enough
  - In this case, the startup time cannot be ignored

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What values of k ensure that a processor is never idle ones it has started computing ?

- Case of processor P_0
  - P_0 has to process $\frac{n}{k}$ chunks before starting computing its second allocated row
  - It takes p steps until P_0 receives a first update from P_{p-1}

- Condition to be met:

$$p \leq \frac{n}{k} \Rightarrow k \leq \frac{n}{p}$$

Reducing the amount of communication

To further improve the performance we need to reduce the amount of data sent over the network.

-

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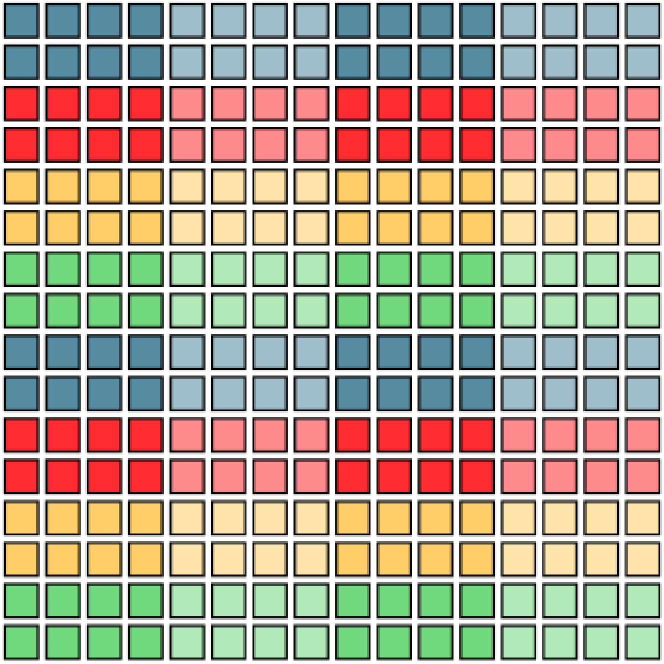
Solution

- Allocate blocks of r consecutive rows to the same processor
- **Concept of block-cyclic allocation**
  - Blocks of size $r \times k$
  - Total amount of communication is divided by r
    - Only the last row of each block is sent

Block-cyclic allocation



- Bloc-cyclic allocation with $p = 4$, $n = 16$, $k = 4$, and $r = 2$.
    - Each processor is associated with one color.
    - Light and dark colors are used to illustrate blocks.

Performance of the block-cyclic-allocation version

- Execution time for one step
  - = time required to process one $r \times k$ block

$$T_{step} = r \times k \times w + k \times b + L$$

- Number of steps:
  - It takes $p - 1$ steps until processor P_{p-1} starts working.
  - Processor P_{p-1} should run $\frac{n^2}{p \times r \times k}$ such steps

Execution time

$$T_{block-cyclic}(n, p, r, k) = (p - 1 + \frac{n^2}{p \times r \times k}) \times (r \times k \times w + k \times b + L)$$

- When n becomes large: $T_{block-cyclic}(n, p, r, k) = \frac{n^2}{p} \times (w + \frac{b}{r} + \frac{L}{r \times k})$

Performance

Comments on this new version

- The block-cyclic allocation helps reducing both the bandwidth and the latency term
- However, if r is too large, the startup time is going to become too costly

Comments on the implementation

Concept of ghost cells

Problem

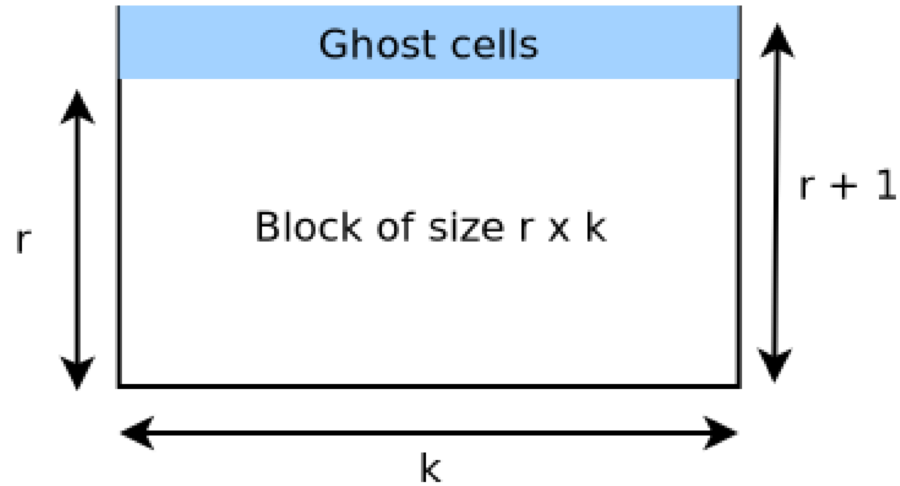
- We consider the case of a block allocation
  - A process computes over several consecutive rows
- Data are stored in different buffers (based on the figure presented in Slide 18)
  - The local domain: A (one allocation)
  - The values received from the `North` processor (*north vector*).

- **How to avoid having to describe a special case for the computation of the first row of each block?**
  - Special case: Read the north values from the *north vector* when applying `Update` to the first row of a block.

Concept of ghost cells

Solution: Allocating ghost cells

- Allocating a block of size $(r+1) \times k$ instead of $r \times k$
  - The extra row is used to receive data from north



Conclusion

- Study of stencil parallel algorithms in distributed environment
    - Impact of assignment of sub-domains to processes on performance

- Some general (and sometimes contradictory) principles to develop efficient algorithms in distribute shared memory:
    - **Sending data in bulk** to limit the impact of network latency
    - **Sending data early** to avoid having idle processors
    - **Assigning blocks of data to processors** to limit the amount of communication and have regular access patterns for the computation
    - **Applying cyclic data distribution** to increase load balancing between processors and reduce idle time.

References

- Section 4.1 and 4.3 of the book "Parallel Algorithms" (by Casanova, Robert, and Legrand).