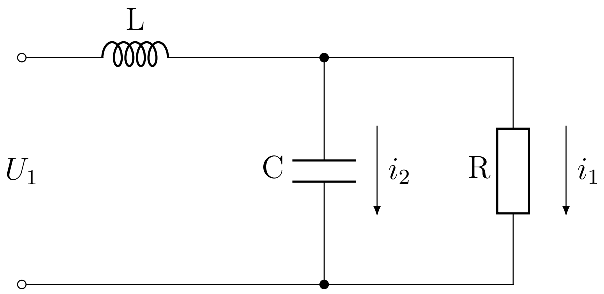


At t=0 we have $i_1(0) = i_{10}$ and $i_2(0) = i_{20}$.

$$U = const$$

Lets get graphs for $i_1(t), i_2(t)$ from analitic solution of differencial equations for the circuit.

$$\begin{cases} U_1 = L\frac{\partial i}{\partial t} + Ri_1 & \Rightarrow \quad 0 = \frac{\partial^2 i}{\partial t^2} + R\frac{\partial i_1}{\partial t} \\ U_1 = L\frac{\partial i}{\partial t} + \frac{1}{C}\int\limits_0^t i_2(\tau)d\tau & \Rightarrow \quad 0 = \frac{\partial^2 i}{\partial t^2} + \frac{1}{C}i_2 \end{cases} \tag{1}$$

Из первого и второго уравнения системы (1) получаем:

$$R\frac{\partial i_1}{\partial t} = \frac{1}{C}i_2 \Rightarrow \frac{\partial i_1}{\partial t} = \frac{1}{RC}i_2 \tag{2}$$

из первого уравнения системы (1):

$$\frac{\partial i_1}{\partial t} + \frac{\partial i_2}{\partial t} = -\frac{R}{L}i_1 + \frac{U_1}{L}$$

В этом уравнении заменим $\frac{\partial i_1}{\partial t}$ из (2):

$$\frac{\partial i_2}{\partial t} = -\frac{R}{L}i_1 - \frac{1}{RC}i_2 + \frac{U_1}{L}$$

$$\frac{\partial}{\partial t}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{RC} \\ -\frac{R}{L} & -\frac{1}{RC} \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{U_1}{L} \end{pmatrix} \tag{3}$$

Предполагая решение в виде $i(t) = e^{\lambda t}$ получаем характеристическое уравнение.

$$\begin{pmatrix} -\lambda & \frac{1}{RC} \\ -\frac{R}{L} & -\lambda - \frac{1}{RC} \end{pmatrix} = 0 \Rightarrow \lambda^2 + \lambda\frac{1}{RC} + \frac{1}{LC} = 0$$

$$D = b^2 - 4ac = \frac{1}{(RC)^2} - \frac{4}{LC}$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = -\frac{1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}} = -\alpha \pm j\omega$$

$$i_1(t) = A \cdot e^{\lambda_1 t} + B \cdot e^{\lambda_2 t} + \frac{U_1}{R}$$

$$i_2(t) = \left(A \cdot \lambda_1 e^{\lambda_1 t} + B \cdot \lambda_2 e^{\lambda_2 t}\right) \frac{1}{2\alpha}$$

При $t = 0$ получаем:

$$A + B + \frac{U_1}{R} = i_{10}$$

$$\lambda_1 \cdot A + \lambda_2 \cdot B = i_{20} \cdot 2\alpha$$

Для определения коэффициентов получаем матрицу

$$\begin{pmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} i_1(0) - \frac{U_1}{R} \\ i_2(0) \cdot 2\alpha \end{pmatrix}$$

решая в Reduce-algebra

```
m:=mat((1,1),(lambda_1,lambda_2));
mf:=mat((i_10-u/R),(i_20));
coef:= 1/m*mf;
```

$$\begin{aligned} A &= -\tfrac{i_1(0)\lambda_2}{\lambda_1-\lambda_2} + \tfrac{i_2(0)\cdot 2\alpha}{\lambda_1-\lambda_2} + \tfrac{\lambda_2}{\lambda_1-\lambda_2}\tfrac{U}{R} \\ B &= \tfrac{i_1(0)\lambda_1}{\lambda_1-\lambda_2} - \tfrac{i_2(0)\cdot 2\alpha}{\lambda_1-\lambda_2} - \tfrac{\lambda_1}{\lambda_1-\lambda_2}\tfrac{U}{R} \end{aligned}$$

упростим

$$\lambda_1 - \lambda_2 = \sqrt{\frac{1}{(RC)^2} - \frac{4}{LC}} = j\sqrt{\frac{4}{LC} - \frac{1}{(RC)^2}} = 2j \cdot \mathcal{I}m = 2j\omega$$

еще три упрощения:

$$\frac{\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}}{\lambda_1 - \lambda_2} = -e^{-\alpha t}\left(\cos(\omega t) + \frac{\alpha}{\omega}\sin(\omega t)\right)$$

$$\frac{e^{\lambda_1 t} - e^{\lambda_2 t}}{\lambda_1 - \lambda_2} = e^{-\alpha t}\frac{\sin \omega t}{\omega}$$

$$\frac{\lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t}}{\lambda_1 - \lambda_2} = -e^{-\alpha t}\left(\cos(\omega t) - \frac{\alpha}{\omega}\right)$$

$$i_1(t) = \frac{i_1(0) - \frac{U}{R}}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{\lambda_1 \cdot t} + \lambda_1 e^{\lambda_2 \cdot t}\right) + \frac{i_2(0)2\alpha}{\lambda_1 - \lambda_2}\left(e^{\lambda_1 \cdot t} - e^{\lambda_2 \cdot t}\right) + \frac{U}{R} =$$

$$= e^{-\alpha t}\left[\left(i_1(0) - \frac{U}{R}\right)\left(\cos(\omega t) + \frac{\alpha}{\omega}\sin(\omega t)\right) + i_2(0)\frac{\sin(\omega t)}{\omega}\right] + \frac{U}{R} =$$

$$= \frac{e^{-\alpha t}}{\omega}\left[\left(i_1(0) - \frac{U}{R}\right)(\omega\cos(\omega t) + \alpha\sin(\omega t)) + i_2(0)\sin(\omega t)\right] + \frac{U}{R}$$

$$i_2(t) = RC\left\{-\alpha e^{-\alpha t}\left[\left(i_1(0) - \frac{U}{R}\right)\left(\cos(\omega t) + \frac{\alpha}{\omega}\sin(\omega t)\right) + i_2(0)\frac{2\alpha \cdot \sin(\omega t)}{\omega}\right] + \right.$$

$$\left. + e^{-\alpha t}\left[\left(i_1(0) - \frac{U}{R}\right)(-\omega\sin(\omega t) + \alpha\cos(\omega t)) + i_2(0)2\alpha\cos(\omega t)\right]\right\} =$$

$$= e^{-\alpha t}\frac{1}{2\alpha}\left[\left(i_1(0) - \frac{U}{R}\right)\left(-\omega - \frac{\alpha^2}{\omega}\right)\sin(\omega t) + i_2(0) \cdot 2\alpha\left(\cos(\omega t) - \frac{\alpha}{\omega}\sin(\omega t)\right)\right]$$

здесь заметим, что

$$\alpha^2 + \omega^2 = \frac{1}{(2RC)^2} + \frac{1}{LC} - \frac{1}{(2RC)^2} = \frac{1}{LC}; \quad \frac{1}{2\alpha} = RC; \quad \frac{\alpha^2 + \omega^2}{2a} = \frac{R}{L}$$

$$i_2(t) = \frac{e^{-\alpha t}}{\omega}\left[-\left(i_1(0) - \frac{U}{R}\right)\frac{R}{L}\sin(\omega t) + i_2(0)\frac{1}{2\alpha}(\omega\cos(\omega t) - \alpha\sin(\omega t))\right] =$$

$$\frac{e^{-\alpha t}}{\omega}\left[-\left(i_1(0) - \frac{U}{R}\right)\frac{R}{L}\sin(\omega t) + i_2(0)RC(\omega\cos(\omega t) - \alpha\sin(\omega t))\right]$$

Введем новые переменные $\tilde{i}_1(t) = i_1(t) - \dfrac{U}{R}$, $\tilde{i}(t) = i(t) - \dfrac{U}{R}$
тогда:

$$\tilde{i}_1(t) = \frac{e^{-\alpha t}}{\omega}\left[\tilde{i}_1(0)(\omega\cos(\omega t) + \alpha\sin(\omega t)) + i_2(0) \cdot 2\alpha\sin(\omega t)\right]$$

$$i_2(t) = \frac{e^{-\alpha t}}{\omega}\left[-\tilde{i}_1(0)\frac{\alpha^2 + \omega^2}{2\alpha}\sin(\omega t) + i_2(0)(\omega\cos(\omega t) - \alpha\sin(\omega t))\right]$$

$$\tilde{i}(t) = \frac{e^{-\alpha t}}{\omega}\left[\omega\tilde{i}_1(0)\left(\omega\cos(\omega t) + \frac{\alpha^2-\omega^2}{2\alpha}\sin(\omega t)\right) + i_2(0)\left(\omega\cos(\omega t) + \alpha\sin(\omega t)\right)\right]$$

Построить графики аналитического решения для токов $i_1(t)$, $i_2(t)$ и $i(t)$ с помощью программы **gnuplot**. Пример построения в программе **gnuplot** представлен ниже:

```
u=24.
r=1200.
c=277.777/1000000000.
l=0.04
w= sqrt(1./l/c - 1./4./r/r/c/c)
a=1./2./r/c
i10=0.
i20=0.

set xrange [0:0.004]
plot exp(-a*x)*((i10-u/r)*(a/w*sin(w*x)+cos(w*x))+i20*2*a/w*sin(w*x))+u/r, \
exp(-a*x)/w*((i10-u/r)*(w*cos(w*x) + (a*a-w*w)/2/a*sin(w*x)) + \
i20*(w*cos(w*x) + a*sin(w*x)))
```

Получить численное решение дифференциальных уравнений с помощью программы **octave**. Пример численного решения в программе **octave** представлен ниже:

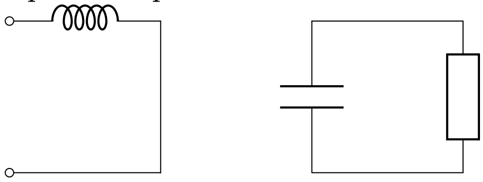
```
function xdot = f (x,t)
l=0.04;
c=0.0000002777;
r=1200;
u=24;
xdot(1) = 1/(r*c)*x(2);
xdot(2) = -r/l*x(1) - 1/(r*c)*x(2) + u/l;
endfunction
x = lsode("f", [0.0,0.0], (t = linspace(0,0.004,200000)'));
set term dumb;
plot(t,x);
```

Смоделировать решение дифференциальных уравнений с помощью программы **ngspice**. Пример моделирования решения с помощью **ngspice** представлен ниже:

```
check
*
V1 1 0 24
L1 1 2 0.04 ic=0
C1 2 0 0.2777u ic=0
R1 2 0 1200
.control
```

```
tran 0.000000001 0.004 uic
.endc
.end
```

Сравнить решения.



уравнение цепи на интервале $(n+\gamma)T \le t \le (n+1)T$

$$RC\frac{dU_C}{dt} + U_C = 0$$

для уравнения получаем решение

$$U_C(t) = B_c e^{-\frac{1}{RC}t}$$

аналогично для тока через резистор

$$i_1(t) = i_1(n+\gamma)e^{-\frac{1}{RC}t}$$

$$i_1(n+1) = i_1(n+\gamma)e^{-\frac{1}{RC}(1-\gamma)\cdot T}$$

для цепи с индуктивностью

$$U = L\frac{di_L}{dt}$$

решение уравнения:

$$i(t) = i(n+\gamma) + \frac{U}{L}\cdot t$$

0.1 Laplace way

$$\begin{cases} Li_1' + Li_2' + Rx & = & U \\ Li_1' + Li_2' + \frac{1}{C}\int i_2 d\tau & = & U \end{cases}$$

$$\begin{cases} i_1' & = & \frac{1}{RC}i_2 \\ i_2' & = & \frac{U}{L} - \frac{R}{L}i_1 - \frac{1}{RC}i_2 \end{cases}$$

Делаем замену

$$\frac{U}{L} \doteqdot \frac{U}{L}\cdot\frac{1}{p}; \quad i_1 \doteqdot \tilde{I}_1; \quad i_2 \doteqdot \tilde{I}_2; \quad i_1' = \tilde{I}_1 \cdot p - i_1(0); \quad i_2' = p\tilde{I}_2 - i_2(0)$$

$$\begin{cases} p\cdot\tilde{I}_1 - i_1(0) &= \frac{1}{RC}\cdot\tilde{I}_2 \\ p\cdot\tilde{I}_2 - i_2(0) &= \frac{U}{L}\cdot\frac{1}{p} - \frac{R}{L}\cdot\tilde{I}_1 - \frac{1}{RC}\cdot\tilde{I}_2 \end{cases}$$

$$\begin{cases} p\cdot I_1 - \frac{1}{RC}\cdot I_2 &= i_1(0) \\ \frac{R}{L}\cdot I_1 + \left(p+\frac{1}{RC}\right)I_2 &= i_2(0) + \frac{U}{L}\cdot\frac{1}{p} \end{cases}$$

$$\Delta = \begin{vmatrix} p & -\frac{1}{RC} \\ \frac{R}{L} & p+\frac{1}{RC} \end{vmatrix} = p\left(p+\frac{1}{RC}\right) = p^2 + \frac{1}{RC}\cdot p + \frac{1}{LC} = p^2 + \frac{1}{RC}\cdot p + \frac{1}{LC} =$$

$$= p^2 + 2\cdot\frac{1}{2RC}p + \frac{1}{4R^2C^2} - \frac{1}{4R^2C^2} + \frac{1}{LC} = \left(p+\frac{1}{2RC}\right)^2 - \frac{1}{4R^2C^2}\left(1-\frac{4R^2C}{L}\right) =$$

$$= \left(p+\frac{1}{2RC}\right)^2 + \frac{1}{4R^2C^2}\left(\frac{4R^2C}{L}-1\right)$$

$$\Delta_1 = \begin{vmatrix} i_1(0) & -\frac{1}{RC} \\ i_2(0)+\frac{U}{L}\cdot\frac{1}{p} & \left(p+\frac{1}{RC}\right) \end{vmatrix} = i_1(0)\left(p+\frac{1}{RC}\right) + \frac{1}{RC}\left(i_2(0)+\frac{U}{L}\cdot 1p\right) =$$

$$= \frac{1}{RC}(i_1(0)+i_2(0)) + i_1(0)\cdot p + \frac{U}{RLC}\cdot 1p$$

$$\Delta_2 = \begin{vmatrix} p & i_1(0) \\ \frac{R}{L} & i_2(0)+\frac{U}{L}\cdot\frac{1}{p} \end{vmatrix} = p\left(i_2(0)+\frac{U}{L}\cdot 1p\right) - i_1(0)\frac{R}{L} = i_2(0)p + \frac{U}{L} - \frac{R}{L}i_1(0)$$

$$I_1(p) = \frac{\frac{1}{RC}(i_1(0)+i_2(0)) + i_1(0)\cdot p + \frac{U}{RLC}\cdot\frac{1}{p}}{\left(p+\frac{1}{2RC}\right)^2 + \frac{1}{4R^2C^2}\left(\frac{4R^2C}{L}-1\right)}$$

$$I_2(p) = \frac{\frac{U}{L} - \frac{R}{L}i_1(0) + i_2(0)\cdot p}{\left(p+\frac{1}{2RC}\right)^2 + \frac{1}{4R^2C^2}\left(\frac{4R^2C}{L}-1\right)}$$

разлагаем на множители знаменатель, и заменяем переменные

$$\alpha = \frac{1}{2RC}; \omega = \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}-1}$$

$$\Delta = \left(p+\frac{1}{2RC}\right)^2 + \frac{1}{4R^2C^2}\left(\frac{4R^2C}{L}-1\right) = (p+\alpha)^2 - (-1)\cdot\omega^2 =$$

$$= (p+\alpha)^2 - (j\cdot\omega)^2 = (p+\alpha-j\cdot\omega)(p+\alpha+j\cdot\omega)$$

Разлагаем на множители по правилу:

$$\xi p + \zeta = \frac{A}{p+a} + \frac{B}{p+b}$$

если $p=-a$, то $-\xi a+\zeta = A(b-a)$
если $p=-b$, то $-\xi b+\zeta = B(b-a)$
В нашем случае:
$b=\alpha+i\omega; a=\alpha-i\omega; \xi=i_2(0); \zeta=\frac{U}{L}-\frac{R}{L}i_1(0) \Rightarrow$

$$\zeta-\xi a = \frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)(\alpha-j\omega)$$
$$\zeta-\xi b = \frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)(\alpha+j\omega)$$
$$b-a=2j\omega$$

$$I_2(p) = \frac{\frac{U}{L}-\frac{R}{L}i_1(0)+i_2(0)\cdot p}{\left(p+\frac{1}{2RC}\right)^2+\frac{1}{4R^2C^2}\left(\frac{4R^2C}{L}-1\right)} = \frac{\frac{U}{L}-\frac{R}{L}i_1(0)+i_2(0)\cdot p}{(p+\alpha-j\cdot\omega)(p+\alpha+j\cdot\omega)} =$$

$$= \frac{\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)(\alpha-j\omega)}{2j\omega}\frac{1}{(p+\alpha-j\omega)} - \frac{\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)(\alpha+j\omega)}{2j\omega}\frac{1}{(p+\alpha+j\omega)} =$$

$$= \frac{\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)\alpha}{2j\omega}\left[\frac{1}{p+\alpha-j\omega}-\frac{1}{p+\alpha+j\omega}\right] + \frac{i_2(0)}{2}\left[\frac{1}{p+\alpha-j\omega}+\frac{1}{p+\alpha+j\omega}\right] \risingdotseq$$

После обратного преобразования Лапласа

$$\risingdotseq \frac{\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)\alpha}{2j\omega}\left[e^{-\alpha t}\cdot e^{j\omega t}-e^{-\alpha t}\cdot e^{-j\omega t}\right] + \frac{i_2(0)}{2}\left[e^{-\alpha t}\cdot e^{j\omega t}+e^{-\alpha t}\cdot e^{-j\omega t}\right] =$$

$$= \frac{\left(\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)\alpha\right)e^{-\alpha t}}{\omega}\left[\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right] + i_2(0)e^{-\alpha t}\left[\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right]$$

$$i_2(t) = \left[\left(\frac{U}{L}-\frac{R}{L}i_1(0)-i_2(0)\alpha\right)\frac{\sin(\omega t)}{\omega}+i_2(0)\cos(\omega t)\right]e^{-\alpha t}$$

заменяя координаты $\tilde{i}_1(t)=i_1(t)-\frac{U}{R}$, а также $\frac{R}{L}=\frac{\alpha^2+\omega^2}{2\alpha}$

$$i_2(t) = e^{-\alpha t}\left[\left(-\frac{\alpha^2+\omega^2}{2\alpha}\tilde{i}_1(0) - \alpha i_2(0)\right)\frac{\sin(\omega t)}{\omega} + i_2(0)\cos(\omega t)\right]$$

$$\frac{\partial}{\partial t}i_2(t) = -\alpha e^{-\alpha t}\left[\left(\frac{U}{L} - \frac{R}{L}i_1(0) - i_2(0)\alpha\right)\frac{\sin(\omega t)}{\omega} + i_2(0)\cos(\omega t)\right] +$$
$$+e^{-\alpha t}\left[\left(\frac{U}{L} - \frac{R}{L}i_1(0) - i_2(0)\alpha\right)\cos(\omega t) - i_2(0)\omega\sin(\omega t)\right] =$$
$$= e^{-\alpha t}\left[\left(-\left(\frac{U}{L} - \frac{R}{L}i_1(0) - \alpha i_2(0)\right)\frac{\alpha}{\omega} - i_2(0)\omega\right)\sin(\omega t) +\right.$$
$$\left. + \left(-\alpha i_2(0) + \frac{U}{L} - \frac{R}{L}i_1(0) - \alpha i_2(0)\right)\cos(\omega t)\right]$$

Из системы

$$\begin{cases} L(i_1' + i_2') + Ri_1 = U \\ i_1' = \frac{1}{RC}i_2 \end{cases} \Rightarrow i_1 = \frac{U}{R} - \frac{L}{R}\left(i_2' + \frac{1}{RC}i_2\right)$$

$$i_1(t) = \frac{U}{R} - \frac{L}{R}\left[\left(-\left(\frac{U}{L} - \frac{R}{L}i_1(0) - \alpha i_2(0)\right)\frac{\alpha}{\omega} - \omega i_2(0)\right)\sin(\omega t) +\right.$$
$$\left.+\left(\frac{U}{L} - \frac{R}{L}i_1(0) - 2\alpha i_2(0)\right)\cos(\omega t) + \frac{1}{RC}\left(\left(\frac{U}{L} - \frac{R}{L}i_1(0) - \alpha i_2(0)\right)\frac{\sin(\omega t)}{\omega} + i_2(0)\cos(\omega t)\right)\right] =$$

$$= \frac{U}{R} - \frac{L}{R}\left[\left(\left(-\frac{U - Ri_1(0)}{L} + \alpha i_2(0)\right)\frac{\alpha}{\omega} - \omega i_2(0) + 2\alpha\left(\frac{U - Ri_1(0)}{L} - \alpha i_2(0)\right)\frac{1}{\omega}\right)\sin(\omega t) +\right.$$
$$\left. + \left(\frac{U - Ri_1(0)}{L} - 2\alpha i_2(0) + 2\alpha i_2(0)\right)\cos(\omega t)\right]e^{-\alpha t} =$$
$$= \frac{U}{R} - e^{-\alpha t}\frac{L}{R}\left[\frac{U - Ri_1(0)}{L}\cos(\omega t) + \left(\frac{\alpha}{\omega}\left(\frac{2(U - Ri_1(0))}{L} - 2\alpha i_2(0) + \alpha i_2(0)\ -\right.\right.\right.$$
$$\left.\left. - \frac{U - Ri_1(0)}{L}\right) - \ \alpha i_2(0))\sin(\omega t)] =$$
$$= \frac{U}{R} - e^{-\alpha t}\frac{L}{R}\left[\frac{U - Ri_1(0)}{L}\cos(\omega t) + \left(\frac{\alpha}{\omega}\left(\frac{U - Ri_1(0)}{L} - i_2(0)\alpha\right) - i_2(0)\omega\right)\sin(\omega t)\right]$$

$$i_1(t) = \frac{U}{R} - e^{-\alpha t}\left[-\tilde{i}_1(0)\cos(\omega t) + \left(\frac{\alpha}{\omega}\left(-\tilde{i}_1(0)\right) - i_2(0)\frac{\alpha^2}{\omega}\frac{L}{R} - i_2(0)\omega\frac{L}{R}\right)\sin(\omega t)\right]$$

$$\tilde{i}_1(t) = e^{-\alpha t}\left[\tilde{i}_1(0)\cos(\omega t) + \frac{\alpha}{\omega}\tilde{i}_1(0) + \frac{2\alpha}{\omega}i_2(0))\sin(\omega t)\right]$$

0.2 Дискретное преобразование Лапласа

$$i_1(n+1) = U_c(n+1)/R$$
$$i_2(n+1) = i(n+1) - i_1(n+1)$$

$$U_c(n+1) = U_c(n+\gamma)e^{-\frac{(1-\gamma)T}{RC}}$$
$$i_1(n+1) = i_1(n+\gamma)e^{-\frac{(1-\gamma)T}{RC}}$$
$$i(n+1) = i(n+\gamma) + \frac{U}{L}\cdot(1-\gamma)T$$

$$U_c(n+\gamma) = R\cdot i_1(n+\gamma)$$

$$\tilde{i}(n+\gamma) = \frac{e^{-\alpha\gamma T}}{\omega}\left[\tilde{i}_1(n)\left(\omega\cos(\omega\gamma T) + \frac{\alpha^2-\omega^2}{2\alpha}\sin(\omega\gamma T)\right) + i_2(n)\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right)\right]$$

Выразим $n+1$ через n:

$$\tilde{i}[n+1] = \frac{U}{L}\cdot(1-\gamma)T +$$

$$+\frac{e^{-\alpha\gamma T}}{\omega}\left[\tilde{i}_1[n]\left(\omega\cos(\omega\gamma T) + \frac{\alpha^2-\omega^2}{2\alpha}\sin(\omega\gamma T)\right) + i_2[n]\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right)\right] =$$

$$= \frac{U}{L}\cdot(1-\gamma)T + \frac{e^{-\alpha\gamma T}}{\omega}\left[\tilde{i}_1[n]\left(\omega\cos(\omega\gamma T) + \frac{\alpha^2-\omega^2}{2\alpha}\sin(\omega\gamma T)\right) +\right.$$

$$\left. + \left(\tilde{i}[n] - \tilde{i}_1[n]\right)\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right)\right]$$

$$\tilde{i}_1[n+1] + \frac{U}{R} = \left(\frac{e^{-\alpha\gamma T}}{\omega}\left[\tilde{i}_1[n]\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right) + i_2[n]2\alpha\sin(\omega\gamma T)\right] + \frac{U}{R}\right)e^{-\frac{(1-\gamma)T}{RC}} =$$

$$= \frac{e^{-\alpha\gamma T - \frac{(1-\gamma)T}{RC}}}{\omega}\left(\tilde{i}_1[n]\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right) + \left(\tilde{i}[n] - \tilde{i}_1[n]\right)2\alpha\sin(\omega\gamma T)\right) + \frac{U}{R}e^{-\frac{(1-\gamma)T}{RC}}$$

На основании теоремы сдвига:

$$\left.\begin{array}{rcl} D\left\{i[n+1]\right\} & = & e^q I^*(q) - e^q i[0] \\ D\left\{i_1[n+1]\right\} & = & e^q I_1^*(q) - e^q i_1[0] \end{array}\right\}$$

$$e^q I^*(q) - e^q i[0] - \frac{U}{L}\cdot(1-\gamma)T\frac{e^q}{e^q-1} =$$

$$= \frac{e^{-\alpha\gamma T}}{\omega}\left[-\tilde{I}_1^*(q)\frac{\alpha^2+\omega^2}{2\alpha}sin(\omega\gamma T) + I^*(q)\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right)\right]$$

$$e^q\tilde{I}_1^*(q) - e^q i_1[0] + \frac{U}{R}\left(1 - e^{-\frac{(1-\gamma)T}{RC}}\right)\frac{e^q}{e^q-1} =$$

$$= \frac{e^{-\alpha\gamma T - \frac{(1-\gamma)T}{RC}}}{\omega}\left(\tilde{I}_1^*(q)\left(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)\right) + \left(I^*(q) - \tilde{I}_1^*(q)\right)\cdot 2\alpha\sin(\omega\gamma T)\right)$$

получаем систему уравнений относительно $I(q)$ и $\tilde{I}_1(q)$:

$$\begin{pmatrix} \frac{e^{-\alpha\gamma T}}{\omega}(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)) - e^q & -\frac{e^{-\alpha\gamma T}}{\omega}\frac{\alpha^2+\omega^2}{2\alpha}\sin(\omega\gamma T) \\ 2\alpha\frac{e^{-\alpha\gamma T - \frac{(1-\gamma)T}{RC}}}{\omega}\sin(\omega\gamma T) & \frac{e^{-\alpha\gamma T - \frac{(1-\gamma)T}{RC}}}{\omega}(\omega\cos(\omega\gamma T) - \alpha\sin(\omega\gamma T)) - e^q \end{pmatrix}\times$$

$$\times\begin{pmatrix} I^*(q) \\ \tilde{I}_1^*(q) \end{pmatrix} = \begin{pmatrix} -e^q i[0] - \frac{U}{L}\cdot(1-\gamma)T\frac{e^q}{e^q-1} \\ -e^q i_1[0] + \frac{U}{R}\left(1 - e^{-\frac{(1-\gamma)T}{RC}}\right)\frac{e^q}{e^q-1} \end{pmatrix}$$

$$-\alpha\gamma T - \frac{1-\gamma}{RC}T = -\alpha\gamma T - 2\alpha(1-\gamma)T = -\alpha(2-\gamma)T$$

$$\begin{pmatrix} \frac{e^{-\alpha\gamma T}}{\omega}(\omega\cos(\omega\gamma T) + \alpha\sin(\omega\gamma T)) - e^q & -\frac{e^{-\alpha\gamma T}}{\omega}\frac{\alpha^2+\omega^2}{2\alpha}\sin(\omega\gamma T) \\ 2\alpha\frac{e^{-\alpha(2-\gamma)T}}{\omega}\sin(\omega\gamma T) & \frac{e^{-\alpha(2-\gamma)T}}{\omega}(\omega\cos(\omega\gamma T) - \alpha\sin(\omega\gamma T)) - e^q \end{pmatrix}\times$$

$$\times\begin{pmatrix} I^*(q) \\ \tilde{I}_1^*(q) \end{pmatrix} = \begin{pmatrix} -e^q i[0] - \frac{U}{L}\cdot(1-\gamma)T\frac{e^q}{e^q-1} \\ -e^q i_1[0] + \frac{U}{R}\left(1 - e^{-\frac{(1-\gamma)T}{RC}}\right)\frac{e^q}{e^q-1} \end{pmatrix}$$

0.3 Предельное значение

Предельное значение $t \to \infty$ определяется по формуле:

$$\lim_{n\to\infty} f[n] = \lim_{q\to 0}(e^q - 1)F^*(q)$$

Предельное значение можно найти с помощью reduce-algebra

```
m:=mat((e^(-a*g*T)/w*(w*cos(w*g*T) + a*sin(w*g*T)) - e^q,
-e^(-a*g*T)/w*(a^2+w^2)/2/a*sin(w*g*T)),
(e^(-a*(2-g)*t)*2*a/w*sin(w*g*t),
e^(-a*(2-g)*t)/w*(w*cos(w*g*T) - a*sin(w*g*T))-e^q));

```

```
f:=mat((       -e^q*i0  -U/L*(1-g)*t*e^q/(e^q-1)      ),
    (-e^q*i20+U/R*(1-e^(-2*a*(1-g)*t))*e^q/(e^q-1)));

mm:=1/m*f;

limit(mm(1,1)*(e^q-1),q,0);
```