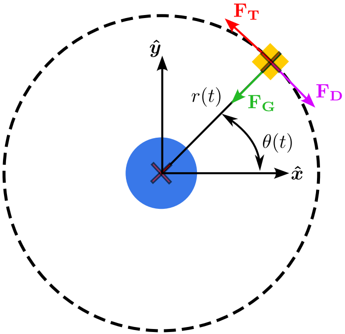
Baptiste Trotabas, Ph.D.

January 23th 2023

Determination of the satellite lifetime for a given orbit

$r(t)$: distance between the center of earth and satellite
$\theta(t)$: angle between the vector $\mathbf{r}$ and $\hat{\boldsymbol{x}}$
$\mathbf{F_G}$: gravity force
$\mathbf{F_D}$: drag force
$\mathbf{F_T}$: thrust force



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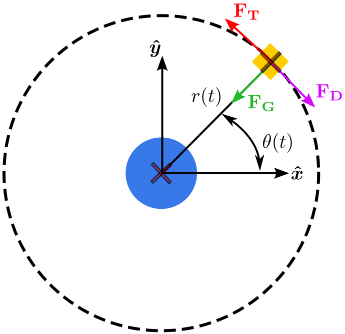

Assumption

The orbit stay in the plan defined by $(\hat{\boldsymbol{r}}, \hat{\boldsymbol{\theta}})$

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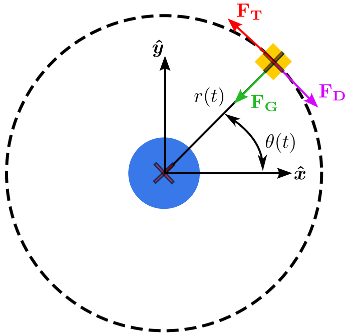
Determine $r(t)$ and $\theta(t)$ bo solving the following equations.



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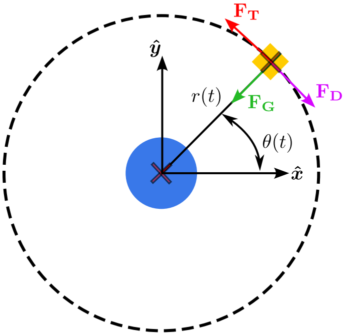

Assumption

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Determine $r(t)$ and $\theta(t)$ bo solving the following equations.

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 - \frac{\mu}{r^2} + \frac{\mathrm{d}r}{\mathrm{d}t}\frac{F_{T0}}{m\sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}} - \frac{\mathrm{d}r}{\mathrm{d}t}\frac{\rho(r)}{2m}C_d(r)S\sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}$$

$$\frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} = -\frac{2}{r}\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{F_{T0}}{m\sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}} - \frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\rho(r)}{2m}C_d(r)S\sqrt{\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2}$$

Workflow

Input parameters

$r_{earth} = 6371$ km
$\mu = 3.9\ 10^{14}\ \mathrm{m^3 s^{-2}}$

Physical constants

$m = 50$ kg
$H = 500$ km

Satellite and orbit parameters

$$\theta(0) = \frac{\pi}{2}$$
$$r(0) = r_{earth} + H$$
$$v_0 = \sqrt{\frac{\mu}{r(0)}}$$

Initialisation of variables

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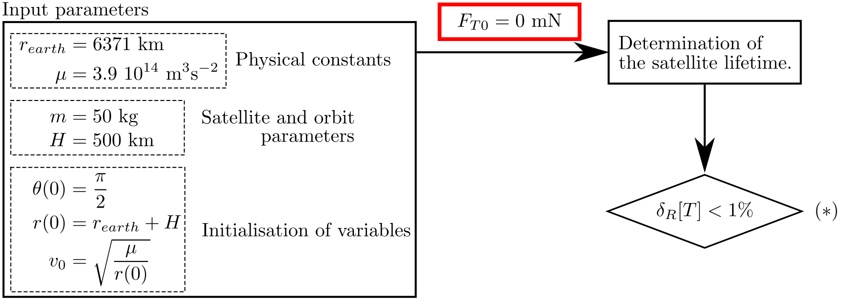
Initialisation of variables

$$\boxed{F_{T0} = 0 \text{ mN}}$$

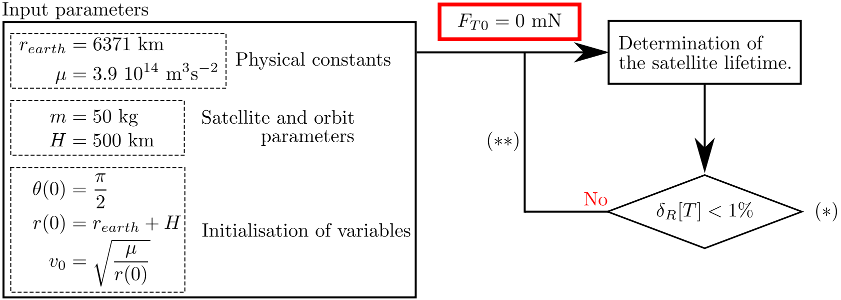
Determination of the satellite lifetime.

Workflow



$(*)$ Comparison of the lifetime obtained with STELA with that previously determined.

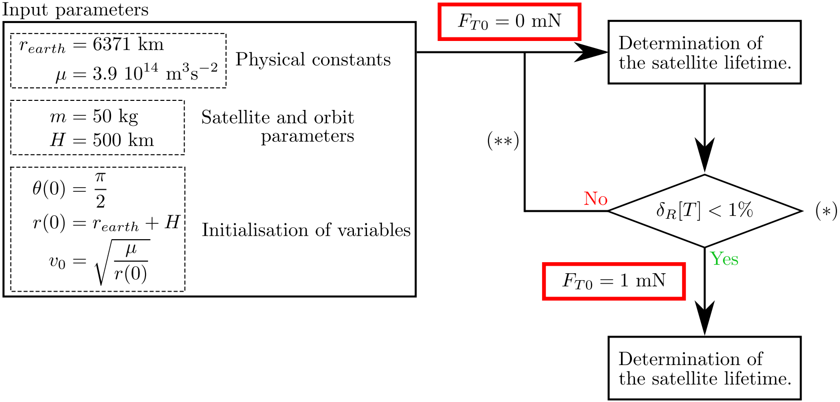
Workflow



(*) Comparison of the lifetime obtained with STELA with that previously determined.

(**) Verify the numerical methods and coefficients used

Workflow



(∗) Comparison of the lifetime obtained with STELA with that previously determined.

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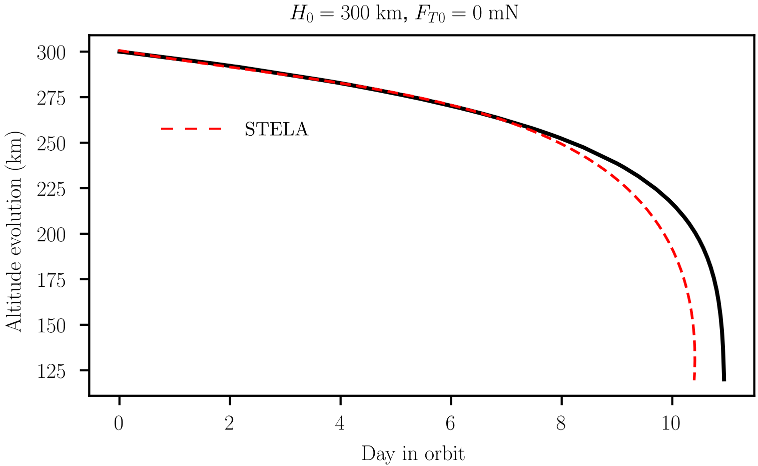
Comparison with STELA (1/2)



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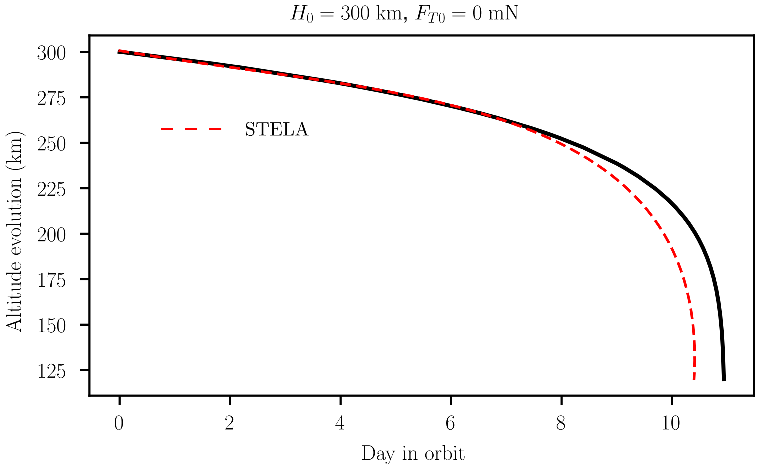

Time in orbit : 10 days 22 h 28 min 44.596 s

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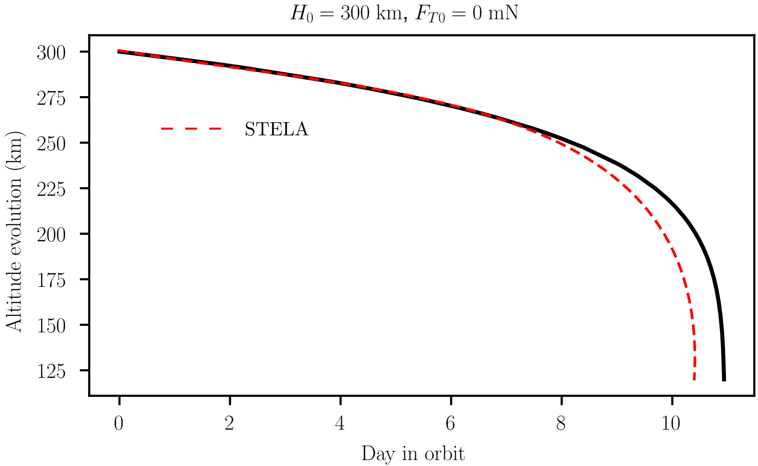
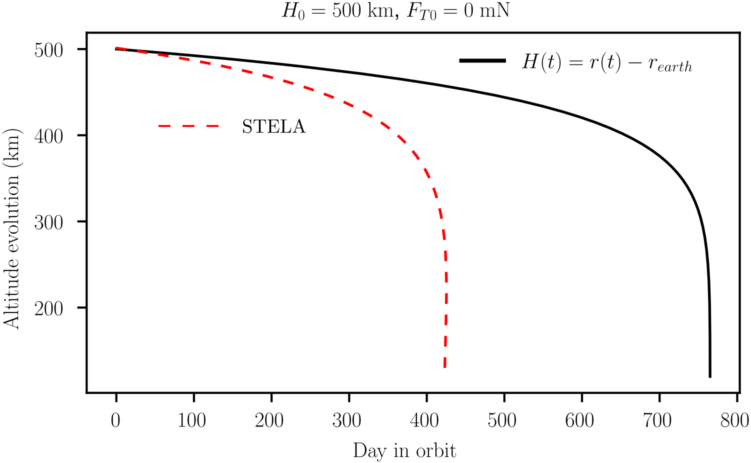
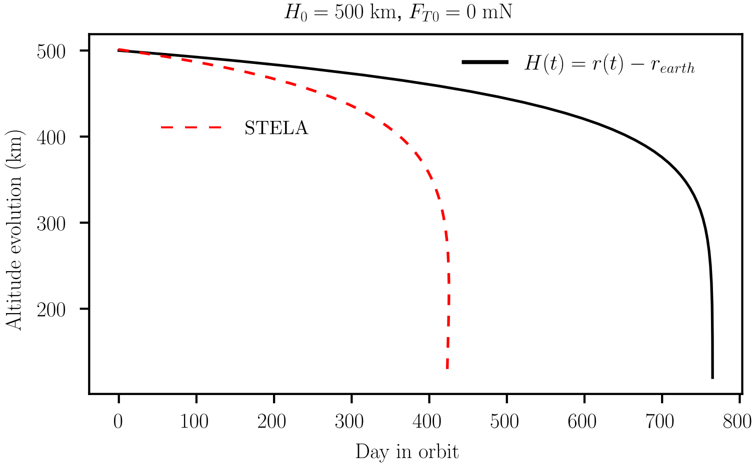


Time in orbit : 10 days 22 h 28 min 44.596 s
Time in orbit : 10 days 5 h 45 min 5.847 s (STELA)

Comparison with STELA (2/2)

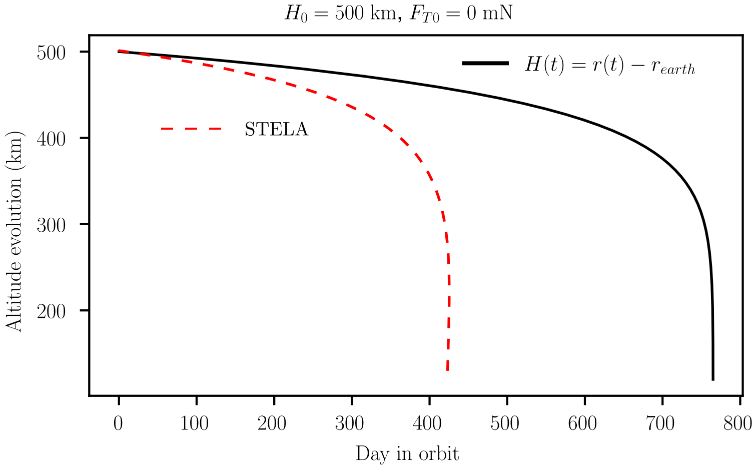


Comparison with STELA (2/2)



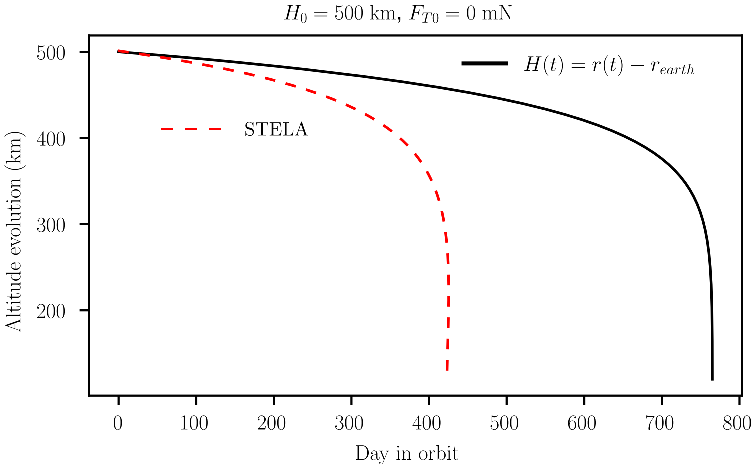
Time in orbit : 2 years 35 days 0 h 22 min 2.851 s

Comparison with STELA (2/2)



Time in orbit : 2 years 35 days 0 h 22 min 2.851 s
Time in orbit : 1 years 49 days 17 h 11 min 11.216 s (STELA)

Comparison with STELA (2/2)



Time in orbit : 2 years 35 days 0 h 22 min 2.851 s
Time in orbit : 1 years 49 days 17 h 11 min 11.216 s (STELA)
Why such a difference ?

Altitude where thrust compensates for drag

According to the theory (and because the orbit is circular), we can determine the altitude $H^\diamond$ such as

$$F_{T0} - \frac{\rho(H^\diamond)}{2} C_d S \frac{\mu}{r_{earth} + H^\diamond} = 0$$

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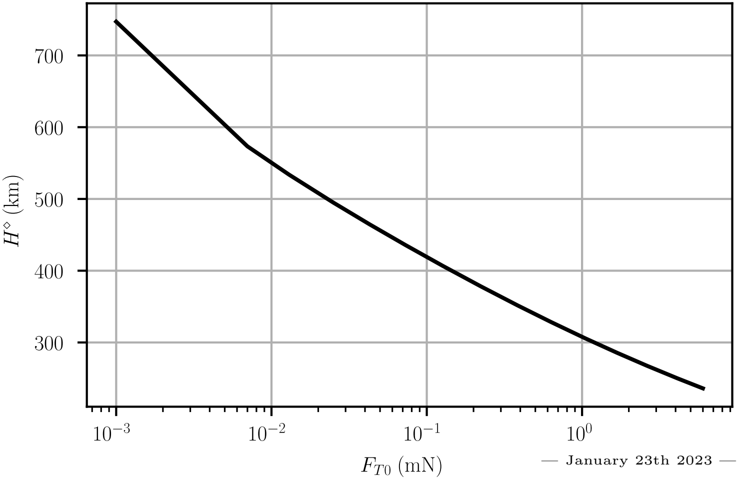

Thank you for your attention