Learning Hawkes Processes from a Handful of Events

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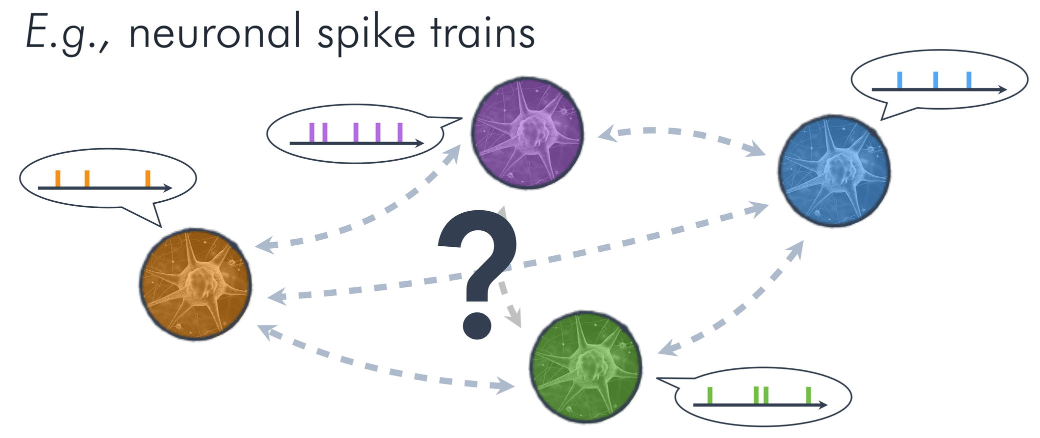
Our code is available on Github!

github.com/trouleau/var-hawkes

Motivation

- We want to learn the *causal relationships* in a network of discrete events (time series).
  E.g., neuronal spike trains



- *Multivariate Hawkes Processes* (MHPs) are widely used to model mutually exciting patterns in discrete events.
- But experimental *data might be scarce*…

Question: How to identify the causal structure of the network when only **small data** is available?

Model

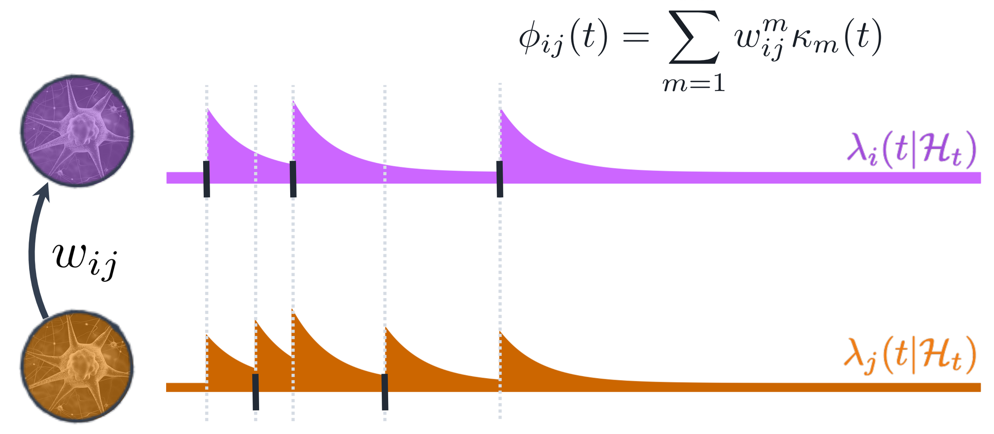
- A Multivariate Hawkes Process is a point process with intensity

$$\lambda_i(t) = \mu_i + \sum_{j=1}^{D} \int_0^t \phi_{ij}(t-\tau)dN_j(\tau).$$

<u>Exogenous intensity:</u> constant, independent of the past

<u>Endogenous intensity:</u> due to excitation from past events, with excitation kernel

$$\phi_{ij}(t) = \sum_{m=1}^{M} w_{ij}^m \kappa_m(t)$$



Contribution

1 We model the parameters of an MHP as latent variables, and we develop a probabilistic interpretation of existing maximum likelihood methods.

2 We assume that each latent variable is sampled from its own prior with some unknown hyper-parameter.

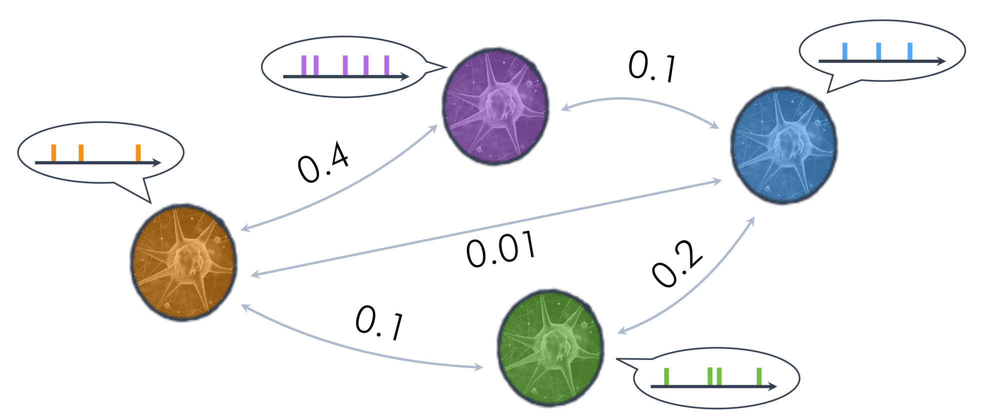
3 We introduce, VI-MHP, a new Bayesian approach for learning the causal structure of an MHP in the absence of large volume of data. VI-MHP can optimize over thousands of hyper-parameters efficiently.

Classic Maximum Likelihood Estimation Framework

- *Maximum Likelihood Estimation (MLE)*:

MLE estimates the parameters of the model by maximizing the log-likelihood of a sequence $\mathcal{S}$ w.r.t. the parameters with a regularization term $\mathcal{R}$

$$\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{W}} = \underset{\boldsymbol{\mu}\geq 0, \boldsymbol{W}\geq 0}{\operatorname{argmin}} -\log p(\mathcal{S}|\boldsymbol{\mu}, \boldsymbol{W}) + \frac{1}{\alpha}\mathcal{R}(\boldsymbol{\mu}, \boldsymbol{W}).$$



Challenges

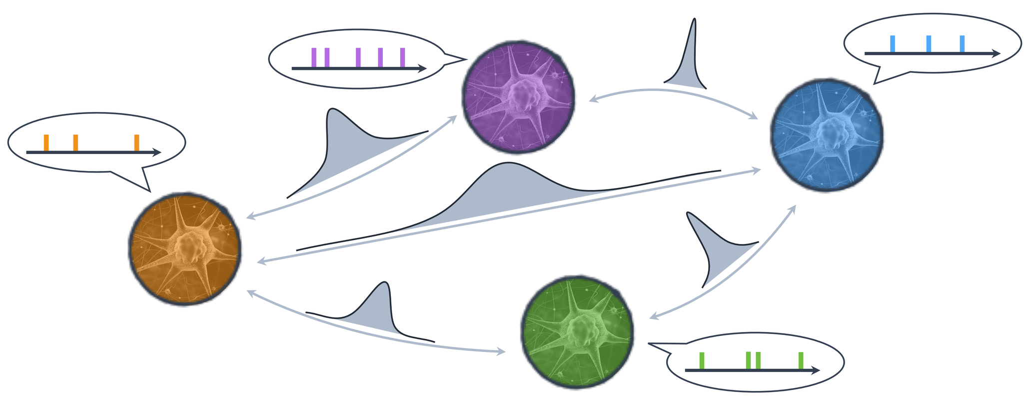
- *Small data* amplifies the *risk of overfitting*.
- It is desirable to control the effect of the penalty with *an independent hyper-parameter* for each of the $MD^2 + D$ *parameters*.

But finding all of them with grid-search requires exponentially many grid points… ***Impractical!***

Variational EM for Multivariate Hawkes Processes

- *Bayesian framework*:

Parameters are assumed to be random variables drawn from a prior $p_{\boldsymbol{\alpha}}(\boldsymbol{\mu}, \boldsymbol{W})$. The goal is to find the posterior $p_{\boldsymbol{\alpha}}(\boldsymbol{\mu}, \boldsymbol{W}|\mathcal{S})$ over the parameters.



Challenges

- *Computing the posterior* is computationally intractable.
- Finding priors, i.e., *tuning hyper-parameters* $\boldsymbol{\alpha}$ with grid search is impossible for ~1000 hyper-parameters.

Solution

- Use *variational inference* to *estimate the posterior* and to find a lower bound on the marginal likelihood.
- Find the $MD^2 + D$ hyper-parameters by *maximizing the variational lower bound* on the marginal likelihood w.r.t. the hyper-parameters.

- *Variational Inference (VI)* approximates the posterior by a variational distribution $q_{\boldsymbol{\gamma}}(\boldsymbol{\mu}, \boldsymbol{W})$ parameterized by the variational parameters $\boldsymbol{\gamma}$.
- VI finds the best $\boldsymbol{\gamma}$ by minimizing the KL-divergence between the variational distribution $q_{\boldsymbol{\gamma}}(\boldsymbol{\mu}, \boldsymbol{W})$ and the posterior $p_{\boldsymbol{\alpha}}(\boldsymbol{\mu}, \boldsymbol{W}|\mathcal{S})$, which is equivalent to maximizing the ELBO

$$\text{ELBO}(q_{\boldsymbol{\gamma}}, \boldsymbol{\alpha}) = \mathbb{E}_{q_{\boldsymbol{\gamma}}}[\log p_{\boldsymbol{\alpha}}(\boldsymbol{\mu}, \boldsymbol{W}, \mathcal{S})] - \mathbb{E}_{q_{\boldsymbol{\gamma}}}[\log q_{\boldsymbol{\gamma}}(\boldsymbol{\mu}, \boldsymbol{W})].$$

The ELBO is a lower bound on the marginal likelihood $\text{ELBO}(q_{\boldsymbol{\gamma}}, \boldsymbol{\alpha}) \leq \log p_{\boldsymbol{\alpha}}(\mathcal{S})$.

- ***We maximize the ELBO over both*** $\boldsymbol{\gamma}$ ***and*** ***to*** $\boldsymbol{\alpha}$ ***increase the marginal likelihood of the data.***
- Since the parameters are non-negative, we use a *log-normal distribution* as the variational distribution to approximate the posterior.
- By interpreting regularization terms as unnormalized priors, we can choose priors that retain the desired properties of common penalties.

Variational EM Algorithm

until convergence **do**

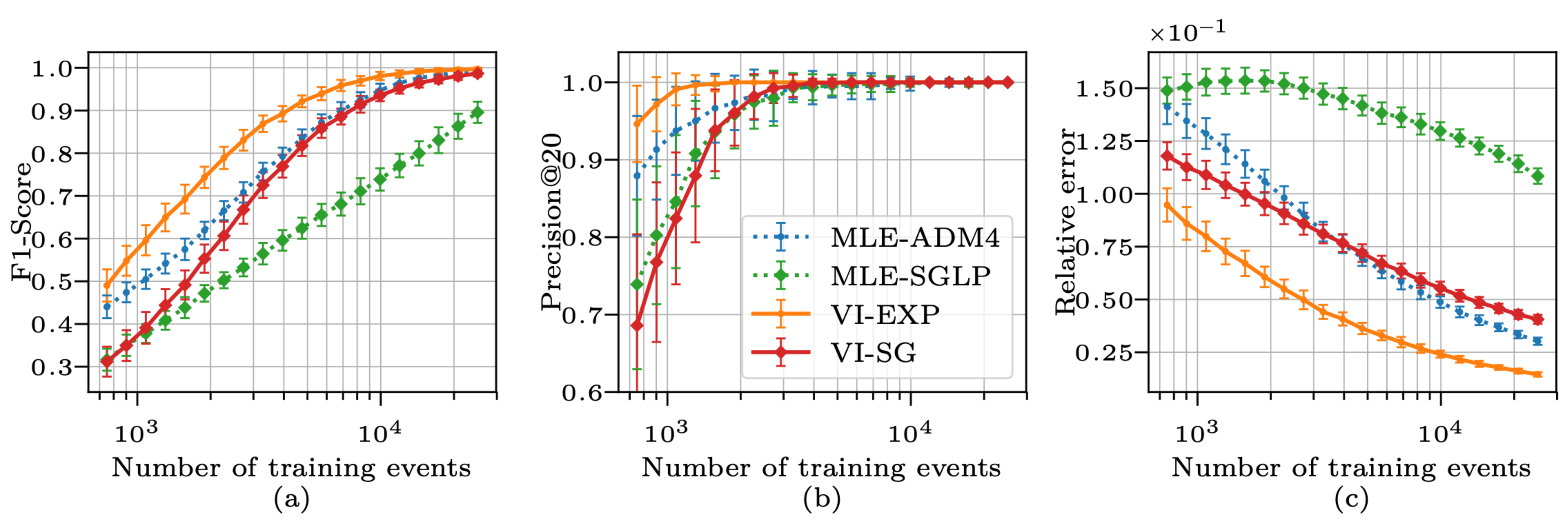
- *E-step:* Maximize the ELBO w.r.t. $\boldsymbol{\gamma}$
- *M-step:* Maximize the ELBO w.r.t. $\boldsymbol{\alpha}$ with a closed-form solution (which depends on the choice of prior).

done

Output: The variational parameters $\boldsymbol{\gamma}$.

Experimental Results

- Experiments on Synthetic data



Performance measured with respect to the number of training samples (averaged over 10 simulated observations over 30 random graphs, with D=50 fixed).

- Experiments on real dataset

Predictive log-likelihood for the models learned on several datasets with different number of events and dimentions.

Dataset	Statistics		Averaged predictive log-likelihood			
	#dim (D)	#events (N)	**VI-SG**	MLE-SGLP	**VI-EXP**	MLE-ADM4
Epidemics	54	5 349	**−2,06**	−3,03	−4,31	−4,61
Stock market	12	7 089	**−1,00**	−2,45	−2,82	−2,81
Enron email	143	74 294	−0,42	−1,01	**−0,23**	−0,40

For more information, check out our paper:
F. Salehi, W. Trouleau, M. Grossglauser, P. Thiran. **Learning Hawkes Processes from a Handful of Events.**
Proceedings of the 33rd Conference on Neural Information Processing Systems (NeurIPS 2019).