Summary of current Beaver population dynamics models

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The model

We modeled the log-densities of beavers ($X = \ln(D)$) using a Gompertz model. The standard interpretation of the Gompertz model is that there is an siz-eindependent component of mortality as well as a component of mortality that increases exponentially with population size. For population i in year j with a vector of covariates ($\mathbf{Z}$) measured in year j we modeled

$$\begin{aligned} X_{i,j} &= a + u_i + (1-b)X_{i,j} + \boldsymbol{\beta}\mathbf{Z}_j + \varepsilon_{i,j} \\ u_i &\sim \text{Norm}\left(0,\, \sigma_a\right) \\ \varepsilon_{i,j} &\sim \text{Norm}\left(0,\, \sigma_i\right). \end{aligned}$$

The intrinsic growth rate is given by a and the random effect u_i allows this term to vary from population to population. The strength of density dependence is given by b, when $b = 0$ the model is density independent. The effects of extrinsic covariates on population growth are determined by the vector $\boldsymbol{\beta}$ and $\varepsilon_{i,j}$ is the extrinsic variability that is not otherwise accounted for in the model.

The priors we used were (need to double check these)

$$\begin{aligned} a &\sim \text{Norm}(0,\, 10) \\ \beta &\sim \text{Norm}(0, 10) \\ b &\sim \text{Unif}(-4,\, 2) \\ \sigma_a &\sim \text{Unif}(0,\, 20) \\ \sigma_i &\sim \text{Unif}(0,\, 20) \end{aligned}$$

Model fits

We fit our models in JAGS with a burnin of 10^4 draws followed by 10^6 draws from the posterior. We then thinned these draws by 100 for a total of 10^4 posterior samples. Some aspects of model fits are described below:

Below are the posterior estimates of environmental covariates.

The estimate of the average log-intrinsic growth rate across populations, a, (reported as mean (standard deviation)) is -0.44 (0.09) and the strength of density dependence, b is 0.62 (0.07). There was a great deal of variation in a from population to population, below we plot these estimates.

There is quite a bit of variability here that we are unable to explain, we should think this over. We fit a model with covariates to and one without covariates. The difference in the DIC values between the models is 12.9 suggesting that including covariates leads to a more parsimonious description of the data.

Model.name	DIC
Gompertz model	5.37554043293947
Gompertz model + PDSI	-7.54848477269156
Gompertz model + PPT	8.16107652245672

Now we check the prediction error by holding out the last 30% of the observerations for each population, refitting the models, then try to predict those missing observations. We calculated the average prediction error for each population as Error $= 1/n \sum_t^n \left| D_t - \hat{D}_t \right|$, where $\hat{D}_t$ is the estimate (mean population density)

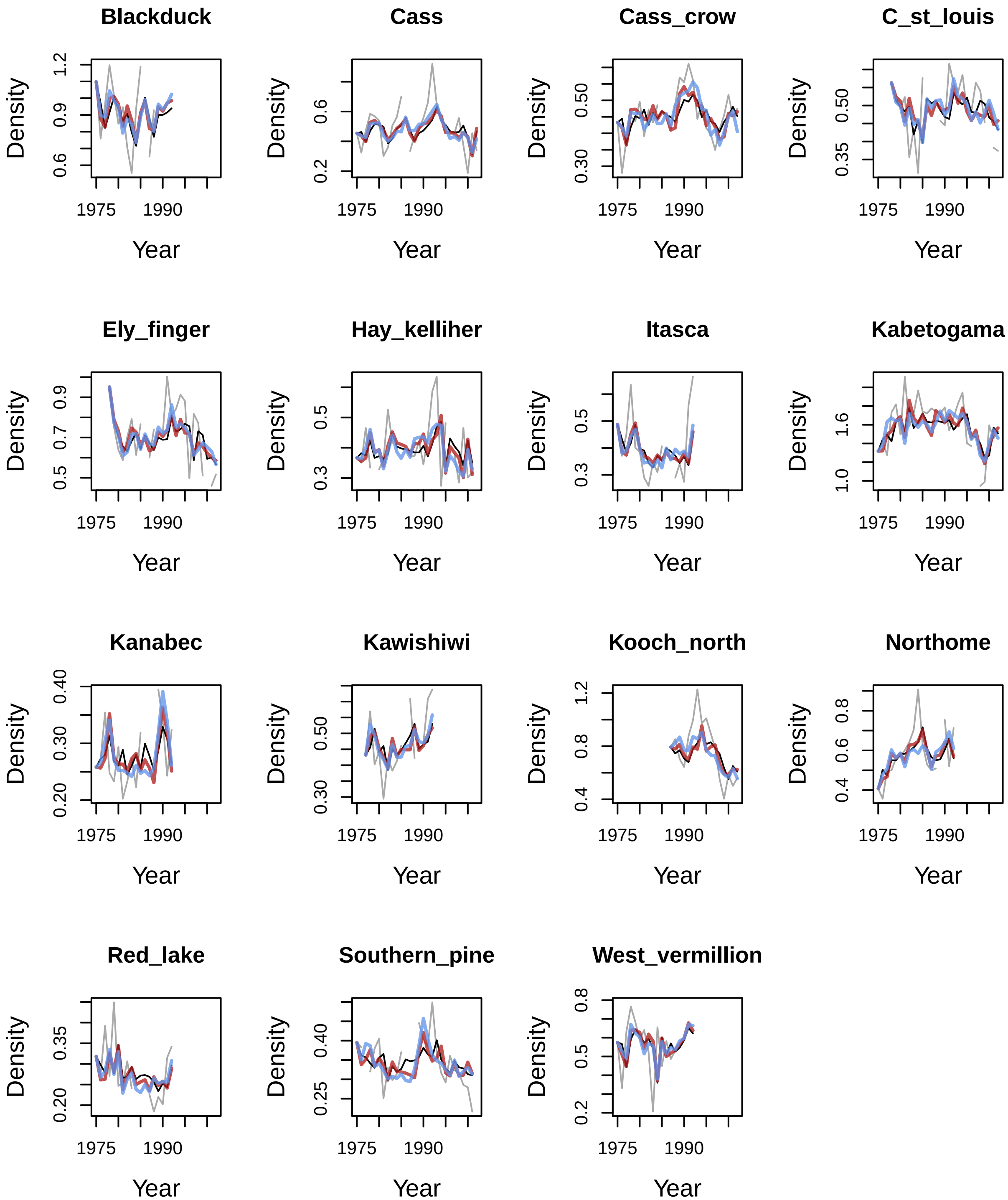

Figure 1: Gray lines are data, black lines are Gompertz model predictions, red lines are predictions of Gompertz with PDSI covariates, blue lines are predictions of Gompertz with PPT covariates.

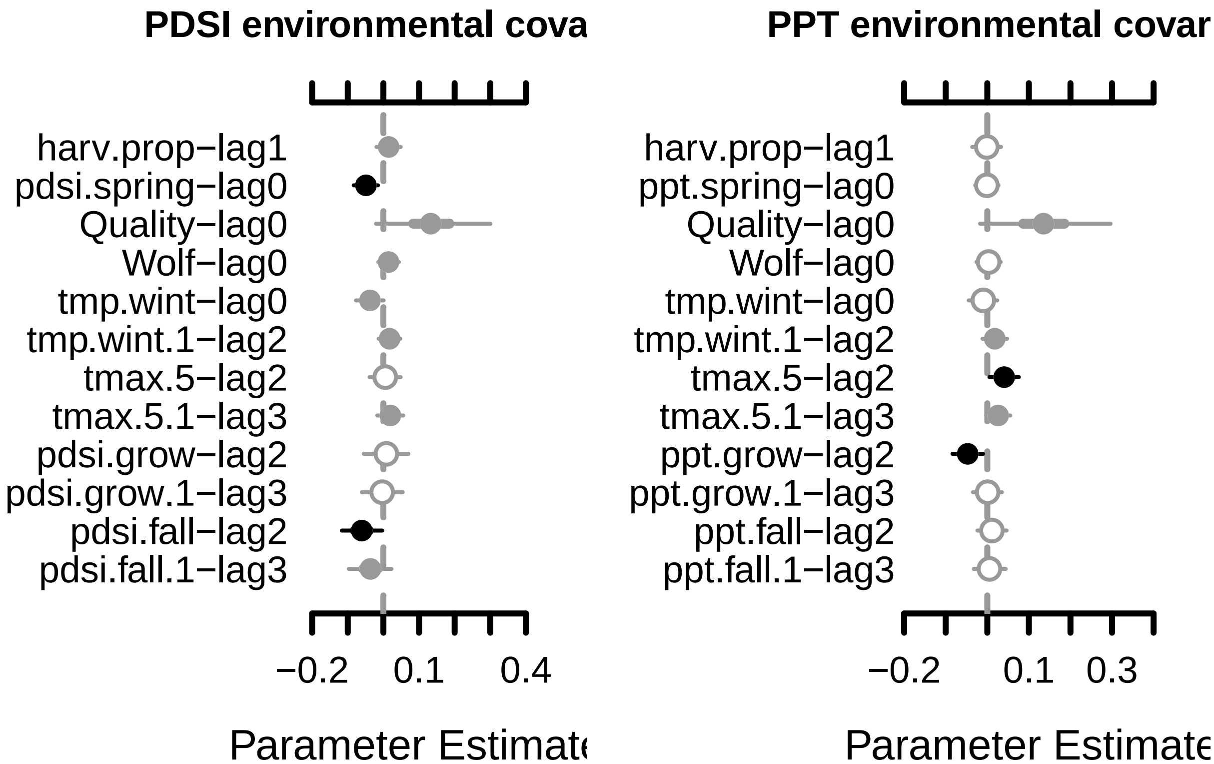

Figure 2: Posterior distributions of the environmental covariates.

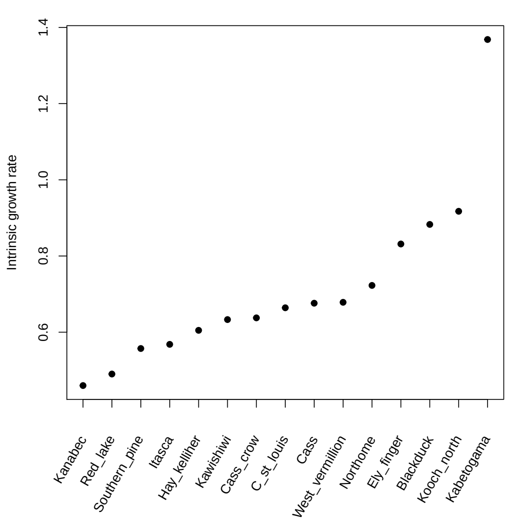

Figure 3: Add uncertainty in this figures. Intrinsic growth rate is exp(a). Estimates from model without any environmental covariates.

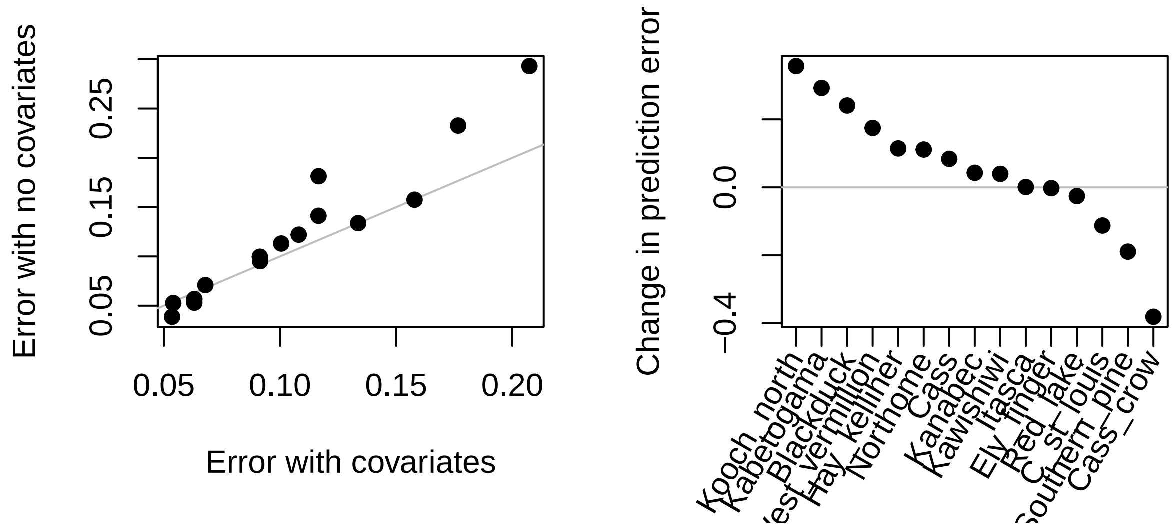

Figure 4: Left panel: Prediction error in models with and without covariates. Values that are lower with covariates fall above the gray one-to-one line. Right panel: Proportional changes in prediction error ((error with no covariates - error with covariates)/ error with no covariates) is positive if adding covariates reduces error. Values above the gray line have improved predictions when covariates are used.

predicted in year t. The sum in this equation is over all the datapoints held out of the model fitting, a number that varies from popualtion to population. We will compare the predictions made by a model without covariates to one with covariates. Below are the errors for each populuation.

We found that the average decrease in error when covariates were added to the model was 5%, this is not very large but it varied widely from an increase in error of 38% to a decrease in 36%. Cass Crow, Southern Pine, C St Louis, and Red Lake actually showed increases in prediction error when adding covariates. Kabetogama, Kooch North, and West Vermillion showed the greatest increase in predictabilty. Interestingly the Kabetogama and Kooch North populations had the highest intrinsic growth rates (a) as well, suggesting that these populations are the most reactive to environmental fluctuations.

Site-specific responses to environmental variables