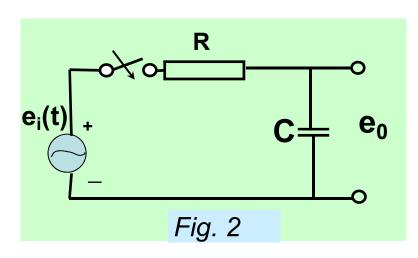


例: 电阻电容串联电路

图2中, R, C为已知常数, $e_i(t)$ 是输入; $e_o(t)$ 是输出,请列写关于电路输出 $e_o(t)$ 和输入 $e_i(t)$ 的方程



$$RCDe_0 + e_0 = e_i$$

$$T\frac{\mathrm{d}e_0}{\mathrm{d}t} + e_0 = e_i$$

解: 动态阻抗(复阻抗)法

电容
$$u_C = \frac{1}{CD}i_C$$

电阻
$$u_R = R i_R$$

$$e_0 = \frac{\frac{1}{CD}}{R + \frac{1}{CD}}e_i = \frac{1}{RCD + 1}e_i$$

$$RC \frac{\mathrm{d}e_0}{\mathrm{d}t} + e_0 = e_i$$

其中, T=RC称为电路的时间常数

由一阶微分方程描述的系统称为一阶系统





单边拉普拉斯变换简介

 \rightarrow 一个连续时间信号x(t)的单边拉普拉斯变换X(s)定义为

$$X(s) = \mathcal{L}[x(t)] = \int_0^\infty x(t)e^{-st} dt$$

$$x(t):[0,+\infty)\to \mathbb{R}$$
 (时域)

$$X(s):\mathbb{C}\to\overline{\mathbb{C}}$$
 (复频域)

 \rightarrow 单位脉冲信号 $\delta(t)$

$$\mathcal{L}[\delta(t)] = 1$$

➤ 单位阶跃信号1(t)

$$1(t) = \begin{cases} 1, & t \ge 0 \\ 0 & t < 0 \end{cases}$$
$$\mathcal{L}[1(t)] = \frac{1}{s}$$





单边拉普拉斯变换简介

▶ 单位斜坡信号t·1(t)

$$\mathcal{L}[t\cdot 1(t)] = \frac{1}{s^2}$$

▶ 单位抛物线信号0.5t²·1(t)

$$\mathcal{L}[0.5t^2 \cdot 1(t)] = \frac{1}{s^3}$$

 \rightarrow 单位正弦信号 $\sin\omega t \cdot 1(t)$

$$\mathcal{L}[\sin\omega t \cdot 1(t)] = \frac{\omega}{s^2 + \omega^2}$$

▶ 指数信号e^{-at}·1(t)

$$\mathcal{L}[e^{-at}\cdot 1(t)] = \frac{1}{s+a}$$





单边拉普拉斯变换简介

$$X(s) = \mathcal{L}[x(t)]$$

$$X_1(s) = \mathcal{L}[x_1(t)]$$

$$X_2(s) = \mathcal{L}[x_2(t)]$$

> 线性

$$aX_1(s)+bX_2(s) = \mathcal{L}[ax_1(t)+bx_2(t)]$$

> 时域平移

$$e^{-\tau s}X(s) = \mathcal{L}[x(t-\tau)\cdot 1(t-\tau)]$$

> 时域卷积

$$X_1(s)X_2(s) = \mathcal{L}\left[\int_0^t x_1(t-\tau)x_2(\tau)d\tau\right]$$

$$= \mathcal{L}\left[\int_0^t x_2(t-\tau)x_1(\tau)d\tau\right]$$

$$= \mathcal{L}[x_1(t)*x_2(t)] = \mathcal{L}[x_2(t)*x_1(t)]$$

▶ 时域微分

$$s^{n}X(s)-s^{n-1}x(0_{-})-s^{n-2}x^{(1)}(0_{-})-\cdots-x^{(n-1)}(0_{-})=\mathcal{L}[x^{(n)}(t)]$$

> 时域积分

$$\frac{X(s)}{s} = \mathcal{L}\left[\int_{0_{-}}^{t} x(\tau) d\tau\right]$$

> 初值定理,终值定理





> 零初始条件:

系统
$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \dots + a_0 y(t) = b_m x^{(m)}(t) + \dots + b_0 x(t)$$

x(t)为输入, y(t)为输出, a_n 不等于零, b_m 不等于零

其零初始条件为

$$y(0_{-}) = y^{(1)}(0_{-}) = \dots = y^{(n-1)}(0_{-}) = 0$$

$$x(0_{-}) = x^{(1)}(0_{-}) = \dots = x^{(m-1)}(0_{-}) = 0$$

- 传递函数的定义:传递函数是在零初始条件下,系统输出单边拉普拉斯变换除以输入单边拉普拉斯变换的商
- 因果系统:指当且仅当输入信号激励系统时,才会出现输出 (响应)的系统。因果系统的(响应)不会出现在输入信号 激励系统的以前时刻;也就是说系统的输出仅与当前与过去 的输入有关,而与将来的输入无关的系统



◆ 考虑如下由时域微分方程描述的 n 阶系统

$$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \dots + a_0 y(t) = b_m x^{(m)}(t) + \dots + b_0 x(t)$$

通常有 n≥m (因果系统)

在零初始条件下作拉普拉斯变换

$$a_n s^n Y(s) + a_{n-1} s^{n-1} Y(s) + \dots + a_1 s Y(s) + a_0 Y(s)$$

= $b_m s^m X(s) + b_{m-1} s^{m-1} X(s) + \dots + b_1 s X(s) + b_0 X(s)$

◆ 其传递函数是

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$Y(s) = G(s)X(s)$$

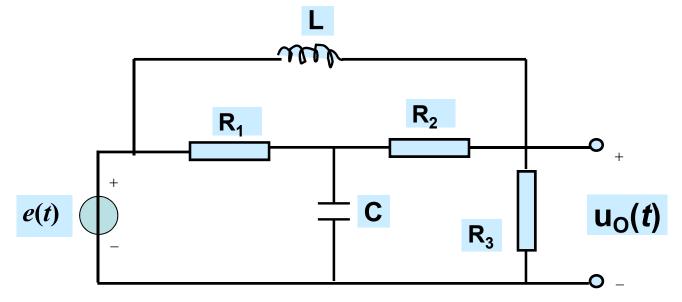
$$\longrightarrow \qquad \qquad X(s)$$

$$\longrightarrow \qquad \qquad Y(s)$$





◆ 例:如图电路中,输入e(t),输出 $u_{O}(t)$,求传递函数



$$R_{1}(R_{2} + R_{3})LC\ddot{u}_{o} + (R_{1}R_{2}R_{3}C + (R_{1} + R_{2} + R_{3})L)\dot{u}_{o} + (R_{1} + R_{2})R_{3}u_{o}$$

$$= (R_{1}R_{2}R_{3}C + R_{3}L)\dot{e} + R_{3}(R_{1} + R_{2})e$$

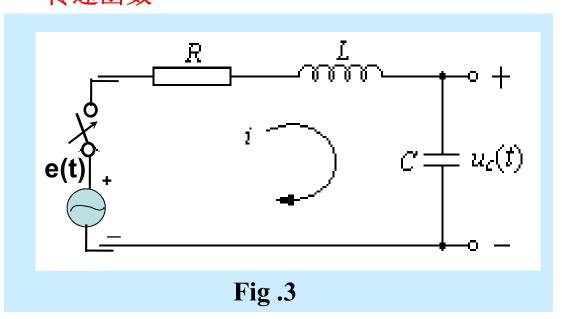
$$G(s) = \frac{U_{o}(s)}{E(s)} = \frac{\left(R_{1}R_{2}R_{3}C + R_{3}L\right)s + R_{3}\left(R_{1} + R_{2}\right)}{R_{1}\left(R_{2} + R_{3}\right)LCs^{2} + \left(R_{1}R_{2}R_{3}C + \left(R_{1} + R_{2} + R_{3}\right)L\right)s + \left(R_{1} + R_{2}\right)R_{3}}$$





例:电阻电感电容(RLC)串联电路

在图3中, R, L, C 为已知常数, e(t) 是输入; $u_c(t)$ 是输出。请列写电路的传递函数



解: 动态阻抗(复阻抗)法

电容
$$U_C(s) = \frac{1}{Cs}I_C(s)$$

电感
$$U_L(s) = LsI_L(s)$$

$$U_{C}(s) = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}}E(s)$$

$$\frac{U_{C}(s)}{E(s)} = \frac{1}{LCs^{2} + RCs + 1}$$

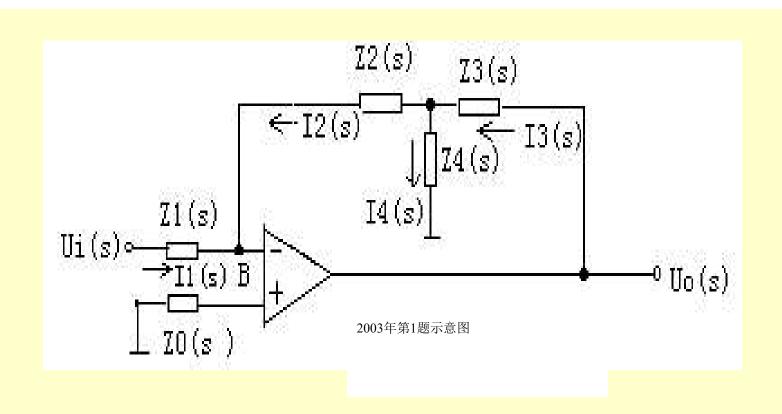
该系统为二阶系统





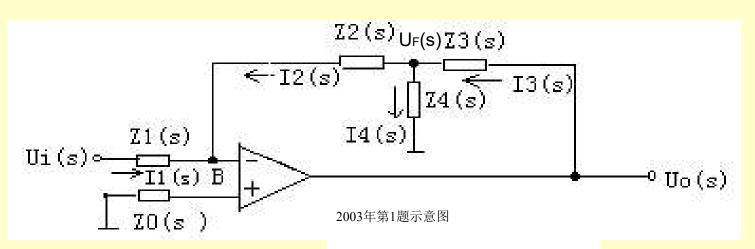
电路系统的机理建模(I/O微分方程模型)

一. 1. 求理想运算放大器的传递函数,结构图如下: (2003年)









解: 结点法

$$i_1 + i_2 = 0$$
 (虚断)

$$i_3 = i_2 + i_4$$

$$\begin{cases} \frac{U_{i}(s) - U_{B}(s)}{Z_{1}(s)} + \frac{U_{F}(s) - U_{B}(s)}{Z_{2}(s)} = 0\\ \frac{U_{O}(s) - U_{F}(s)}{Z_{3}(s)} = \frac{U_{F}(s) - U_{B}(s)}{Z_{2}(s)} + \frac{U_{F}(s)}{Z_{4}(s)} \end{cases}$$

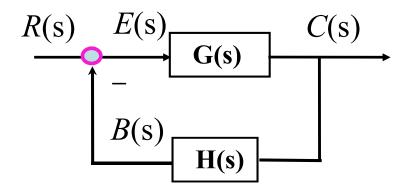
$$U_B(s) = 0$$
 (虚短)

消去 $U_F(s)$

得:
$$\frac{U_{o}(s)}{U_{i}(s)} = -\frac{Z_{2}(s)Z_{3}(s) + Z_{3}(s)Z_{4}(s) + Z_{2}(s)Z_{4}(s)}{Z_{1}(s)Z_{4}(s)}$$







•开环系统的传递函数(简称开环传函)

$$G_{o}(s) = \frac{B(s)}{E(s)} = G(s)H(s)$$

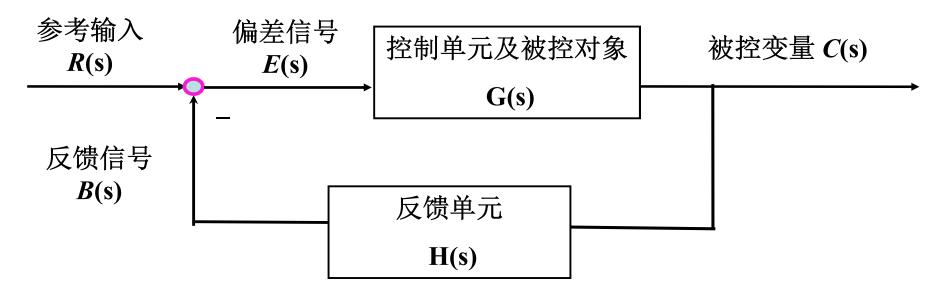
•闭环系统的传递函数(简称闭环传函)

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{G(s)}{1 + G_o(s)}$$





• 对于典型的闭环控制系统

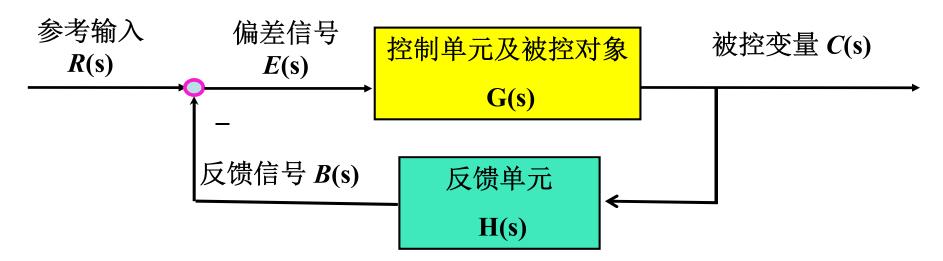


• 整个系统的传递函数 – 被控变量 C(s) 与参考输入 R(s) 的比值。

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$







• 开环传递函数 – 对于任意给定的反馈环,反馈通路输出变量 B(s) 与误差信号 E(s) 的比值(注意:系统仍然是闭环控制系统)。

$$G(s) = \frac{B(s)}{E(s)} = G(s)H(s)$$

• 前向通路传递函数 - 被控变量 C(s) 与误差信 E(s) 的比值

$$G_f(s) = \frac{C(s)}{E(s)} = G(s)$$

$$G_b(s) = \frac{B(s)}{C(s)} = H(s)$$





方块图的基本元素

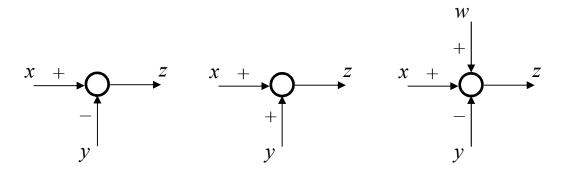


-1

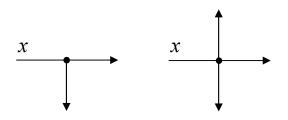
方框(环节)表示子系统或对信号的变换

 \xrightarrow{x} $\xrightarrow{x(t)}$ $\xrightarrow{X(s)}$

带单向箭头的线段(信号线)表示信号及其流向



比较点(综合点)表示对两个 或以上信号的加减运算

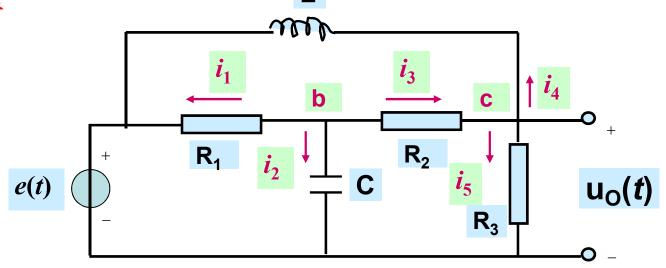


引出点表示同一个信号被引至多个不同的位置使用





◆ 例:如图电路中,输入e(t),输出 $u_O(t)$,要求用方块图表示电路模型



结点方法:

对于节点b
$$i_1 + i_2 + i_3 = 0$$

对于电容C
$$i_1 = C \frac{\mathrm{d}u_b}{\mathrm{d}t}$$

$$\begin{array}{c|c} I_2(s) \hline 1 \\ \hline Cs \end{array} \begin{array}{c} U_b(s) \\ \hline \end{array}$$

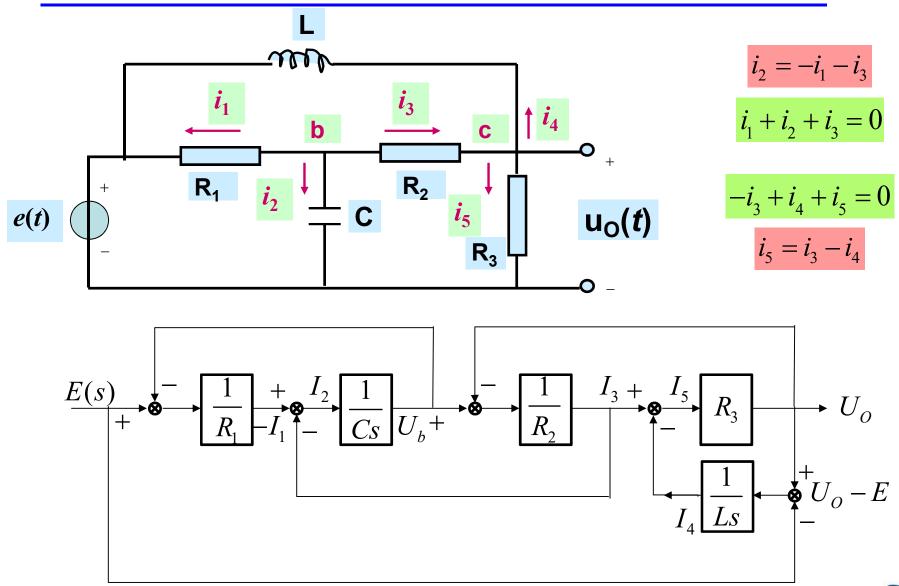
对于节点c
$$-i_3 + i_4 + i_5 = 0$$

对于电感C
$$u_O - e = L \frac{di_4}{dt}$$

$$U_O(s) - E(s)$$
 1 $I_4(s)$









串联 (cascade)

$$U(s) = U_1(s)$$

$$H_1(s)$$

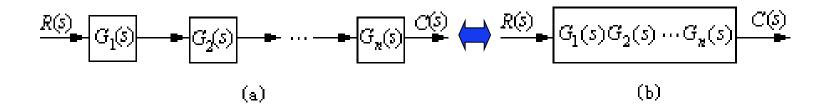
$$Y_1(s) = U_2(s)$$

$$H_2(s)$$

$$Y(s) = Y_2(s)$$

$$Y(s) = Y_2(s) = H_2(s)U_2(s) = H_2(s)Y_1(s) = H_2(s)H_1(s)U(s)$$

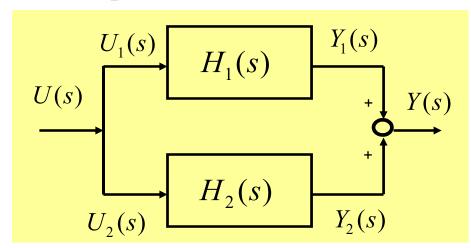
$$G(s) = \frac{Y(s)}{U(s)} = H_2(s)H_1(s)$$





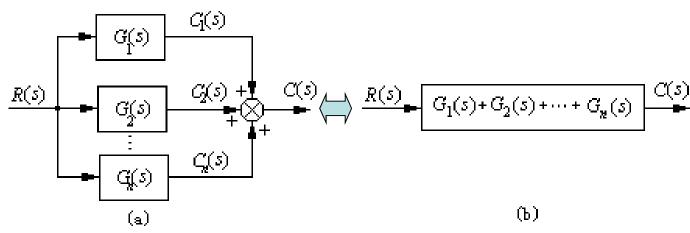


并联(parallel)



$$G(s) = \frac{Y(s)}{U(s)} = H_1(s) + H_2(s)$$

$$Y(s) = Y_1(s) + Y_2(s) = H_1(s)U_1(s) + H_2(s)U_2(s) = (H_1(s) + H_2(s))U(s)$$



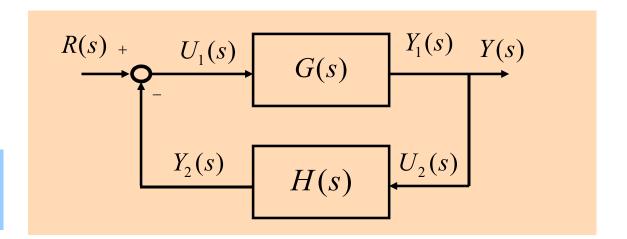




负反馈

(negative feedback)

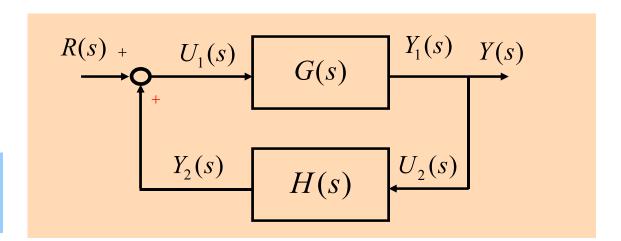
$$\Phi(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



正反馈

(positive feedback)

$$\Phi(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)}$$





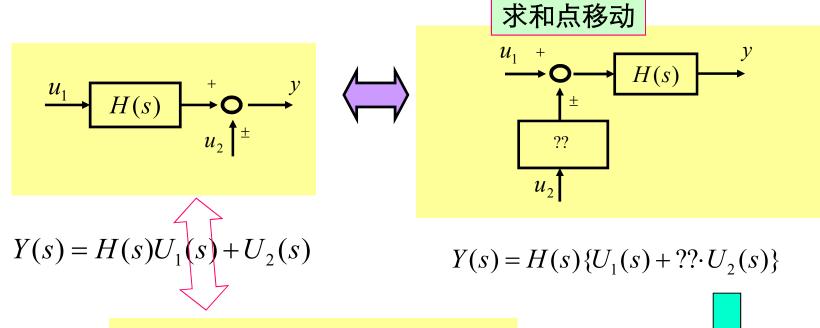


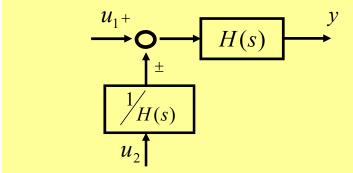
求和点移动 H(s)H(s) $Y(s) = H(s)(U_1(s) \pm U_2(s))$ $Y(s) = H(s)U_1(s) \pm ?? \cdot U_2(s)$ $=H(s)U_1(s)\pm H(s)U_2(s)$?? = H(s) $Y(s) = H(s)U_1(s) \pm H(s)U_2(s)$ H(s)

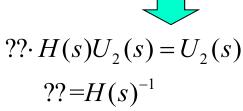


 u_2







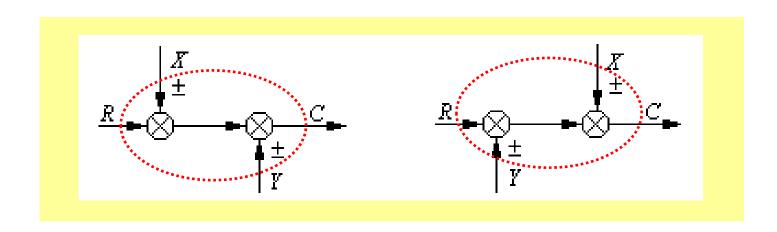


原则:在变换前后的方块图中,同名"变量对"之间的传递函数不变





> 相邻两个求和点前后移动的等效变换



$C = R \pm X \pm Y$

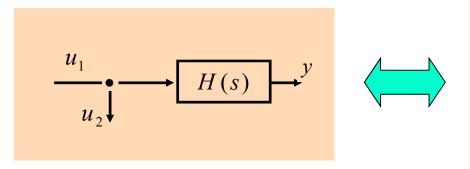
相邻多个求和点可以任意换位

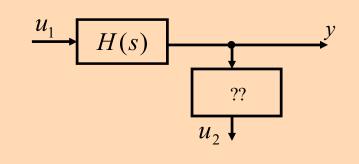
原则:在变换前后的方块图中,同名"变量对"之间的传递函数不变

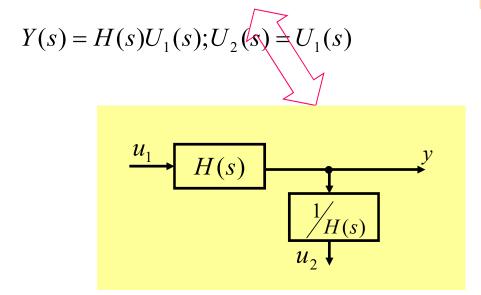


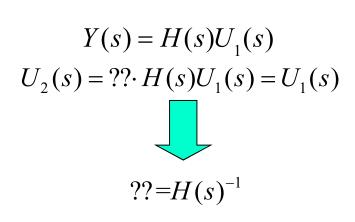


引出点移动



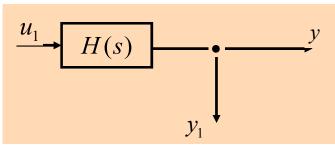


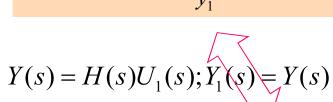


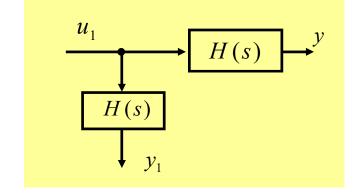




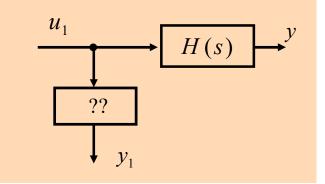








引出点移动



$$Y(s) = H(s)U_1(s)$$
$$Y_1(s) = U_1(s) \cdot ?? = Y(s)$$

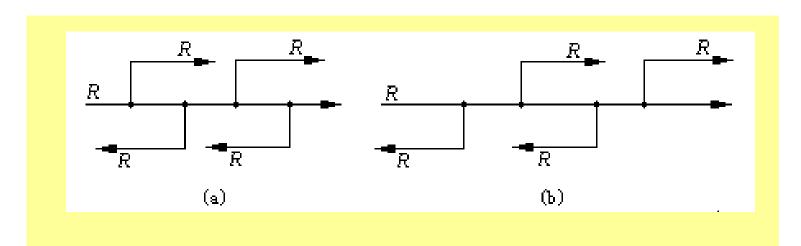


$$??=H(s)$$





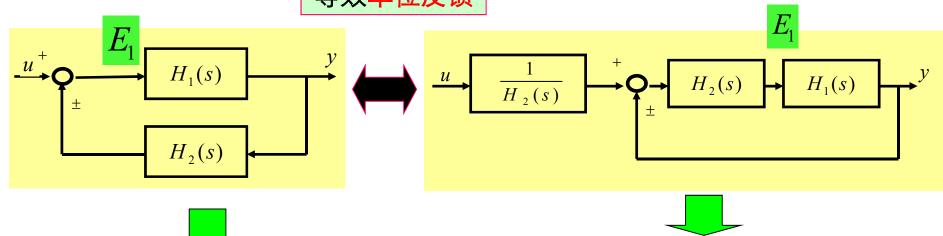
▶相邻多个引出点可以任意换位







等效单位反馈



$$G(s) = \frac{Y(s)}{U(s)} = \frac{H_1(s)}{1 \mp H_1(s)H_2(s)}$$



$$G(s) = \frac{Y(s)}{U(s)} = \frac{H_1(s)}{1 \mp H_1(s)H_2(s)}$$

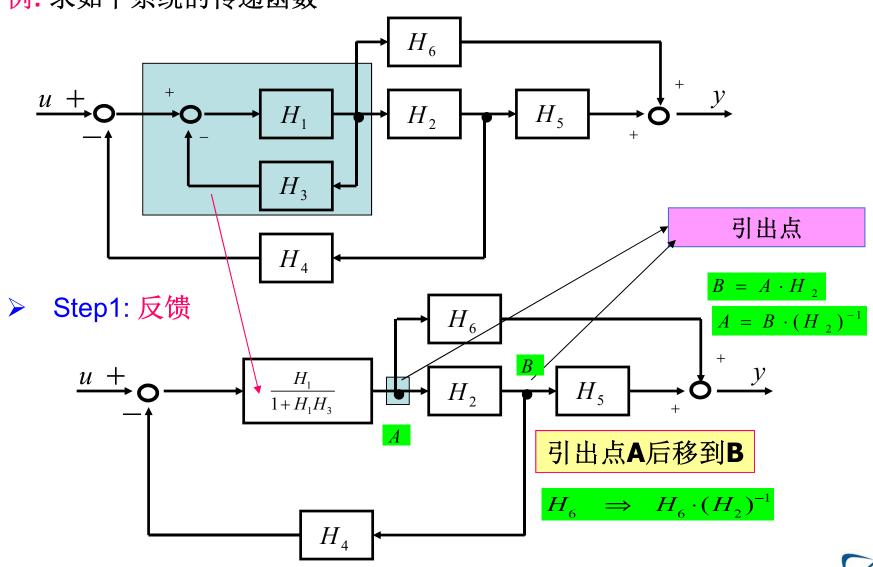
$$G(s) = \frac{1}{H_2(s)} \cdot \frac{H_1(s)H_2(s)}{1 \mp H_1(s)H_2(s)}$$

$$E_1(s) = U(s) \pm H_2(s)Y(s)$$

$$E_1(s) = \left(U(s) \frac{1}{H_2(s)} \pm Y(s)\right) H_2(s)$$
$$= U(s) \pm H_2(s) Y(s)$$

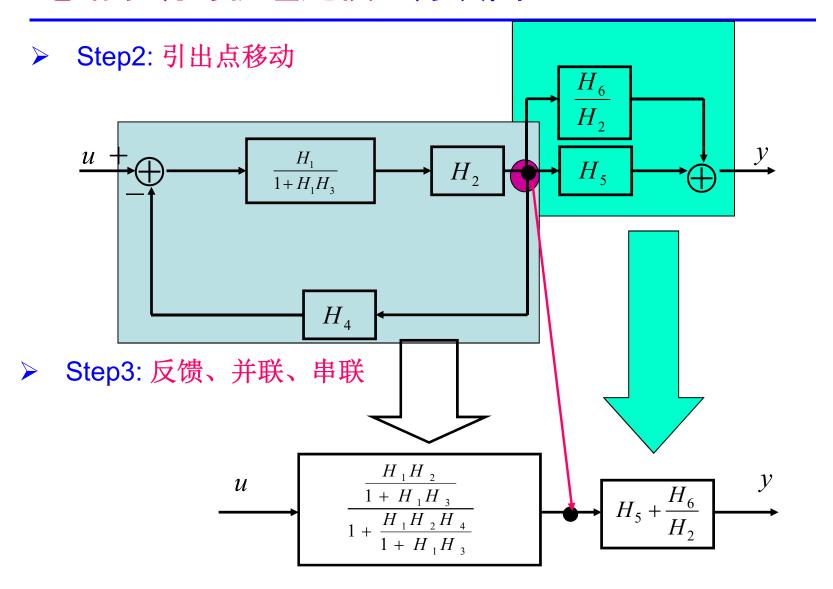


例: 求如下系统的传递函数





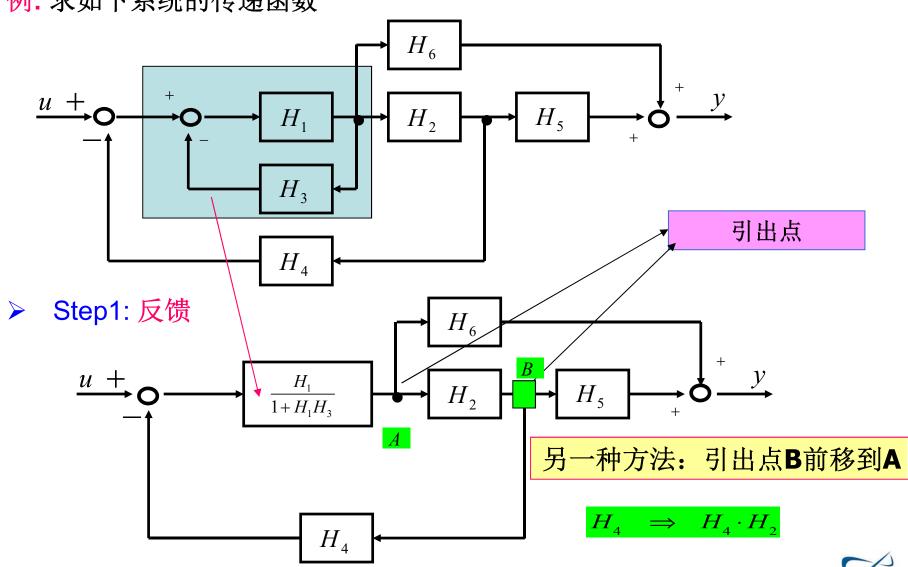






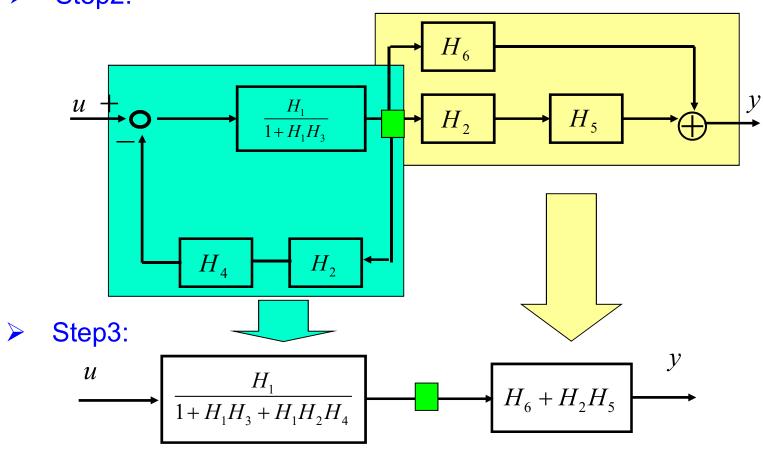


例: 求如下系统的传递函数





> Step2:

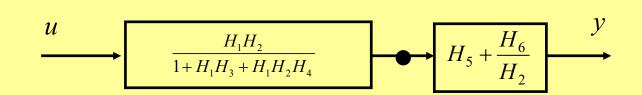




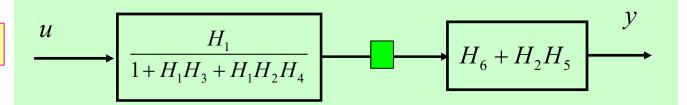


◆ 获得传递函数





引出点B前移



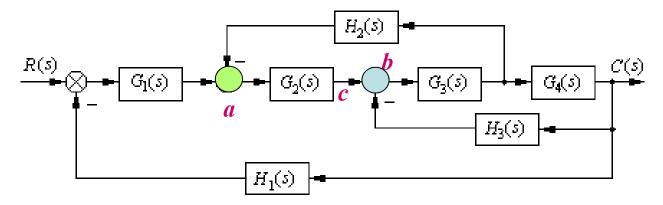


$$\frac{Y(s)}{U(s)} = \frac{\frac{H_1 H_2}{1 + H_1 H_3}}{1 + \frac{H_1 H_2 H_4}{1 + H_1 H_3}} \left(H_5 + \frac{H_6}{H_2}\right) = \frac{H_1 H_2 H_5 + H_1 H_6}{1 + H_1 H_3 + H_1 H_2 H_4}$$



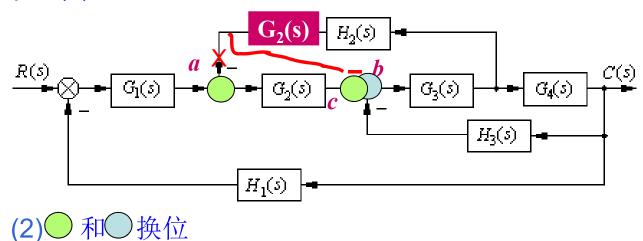


例: 求如下系统的传递函数



2个求和点: *a* , *b*

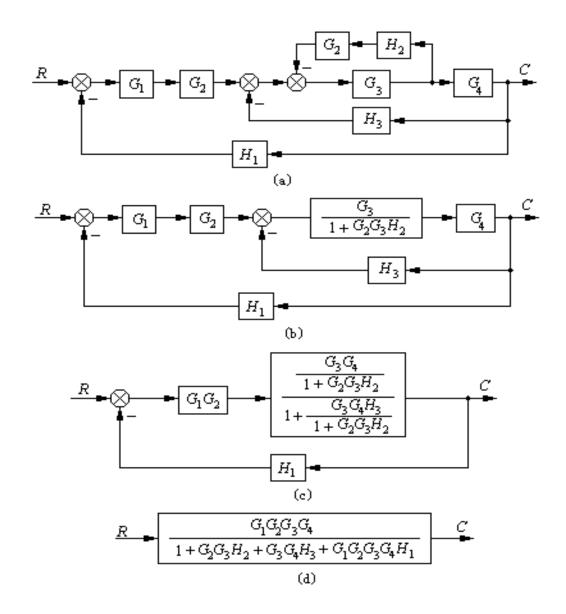
➤ Step1: (1) 将 ○ 从a移动到c







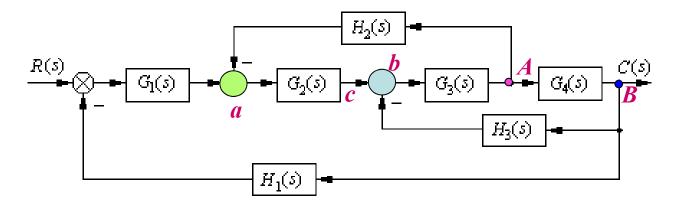
Step2: 串级控制







例: 求如下系统的传递函数



其它3种解法:

求和点b移动到求和点a附近

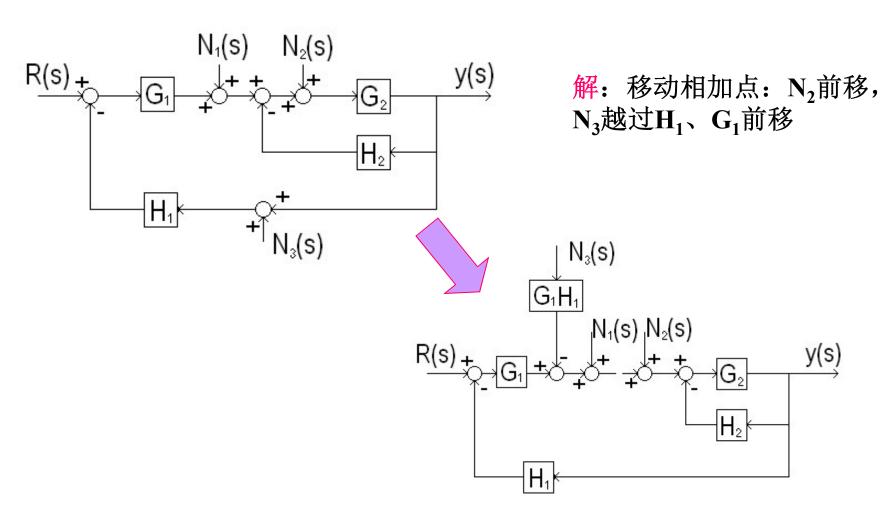
引出点A移动到引出点B附近

引出点B移动到引出点A附近





◆例: 求如图所示系统输出的表达式

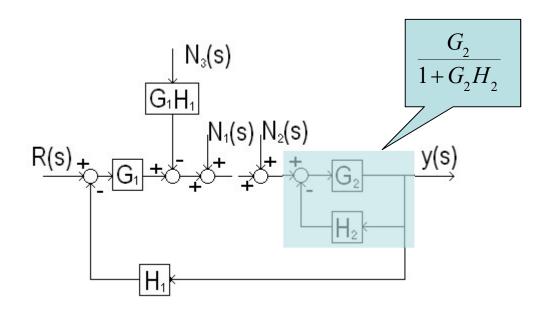






$$Y(s) = \frac{G_1 \frac{G_2}{1 + G_2 H_2}}{1 + G_1 H_1 \frac{G_2}{1 + G_2 H_2}} R(s) + \frac{\frac{G_2}{1 + G_2 H_2}}{1 + G_1 H_1 \frac{G_2}{1 + G_2 H_2}} (N_1(s) + N_2(s) - G_1 H_1 N_3(s))$$

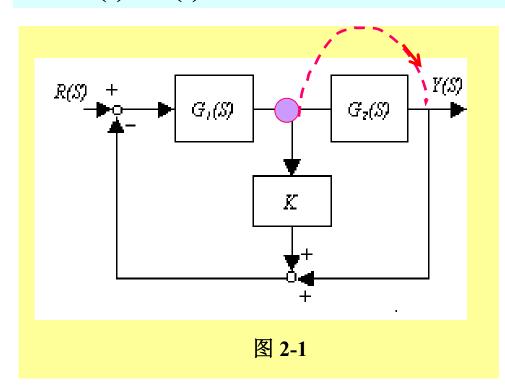
$$= \frac{G_2 N_1(s) + G_2 N_2(s) - G_1 H_1 G_2 N_3(s) + G_1 G_2 R(s)}{1 + G_2 H_2 + G_1 H_1 G_2}$$

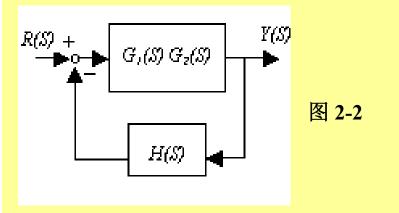


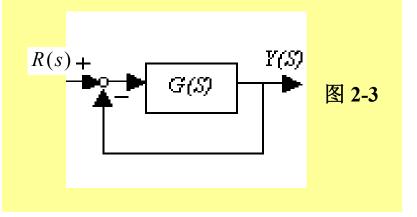




◆例: 系统框图见图2-1,要求将系统等效变换成图2-2、图2-3框图结构, 并求H(s),G(s)表达式

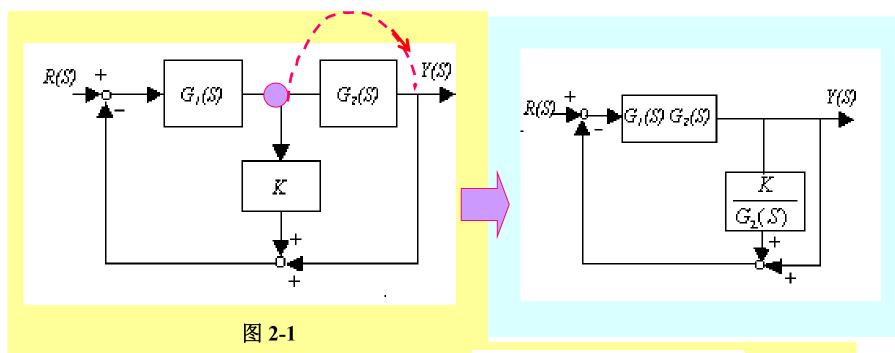




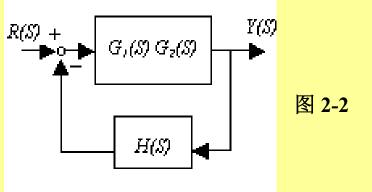






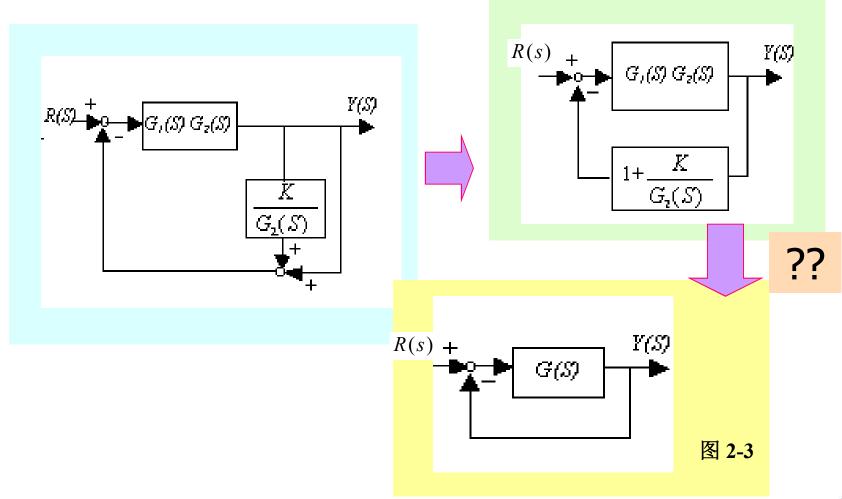


$$H(s) = 1 + \frac{K}{G_2(s)}$$



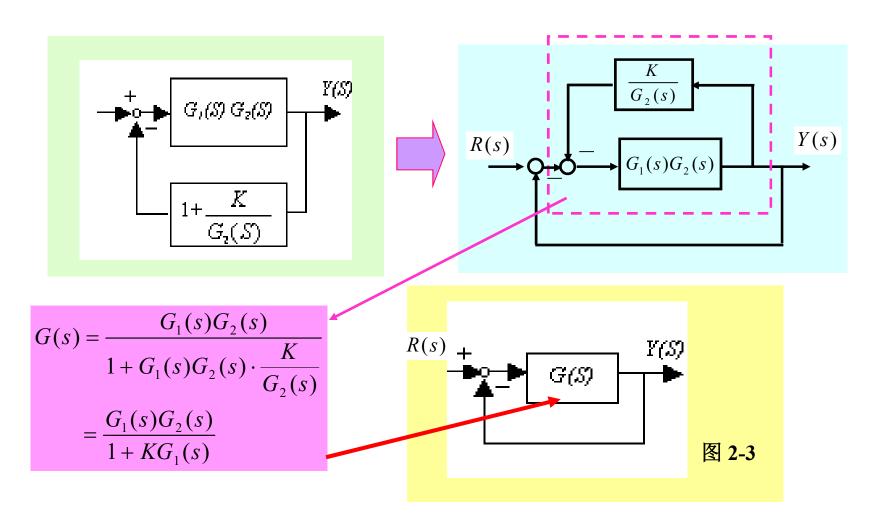














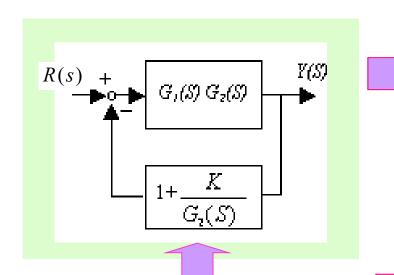


Y(S)

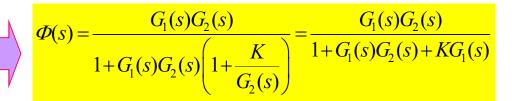
图 2-3

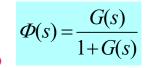
• 另一种方法求G(s)和H(s)

利用闭环传函相等(代数方法)



G(S)





$$\Phi(s) + \Phi(s)G(s) = G(s)$$

$$\Phi(s) = (1 - \Phi(s))G(s)$$

$$G(s) = \frac{\Phi(s)}{1 - \Phi(s)}$$

$$G(s) = \frac{\frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s) + KG_1(s)}}{1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s) + KG_1(s)}} = \frac{G_1(s)G_2(s)}{1 + KG_1(s)}$$



R(s) +



简化方块图求总传递函数的要点

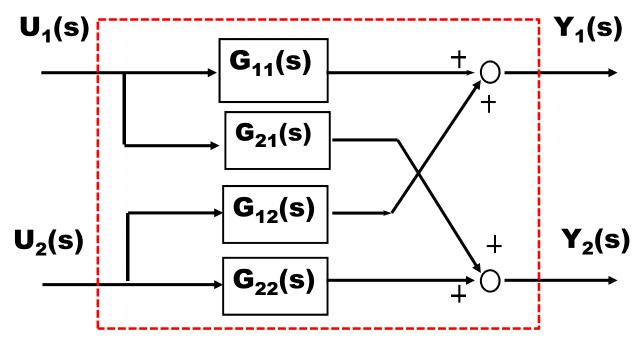
- 确定输入量与输出量。如果作用在系统上的输入量有多个(分别作用在系统的不同部位),则必须分别对每个输入量逐个进行结构变换,求得各自的传递函数;对于有多个输出量的情况,也应分别变换。
- 2. 若方块图中有交叉结构,按**同名变量对间传递函数不变 原则**,将交叉消除,化为无交叉的多回路
- 3. 对多回路结构,可由里向外进行变换(或按照要求进行方块图的简化),直至变换为一个等效的方框,即得到所求的传递函数。





多变量系统的传递函数矩阵表示

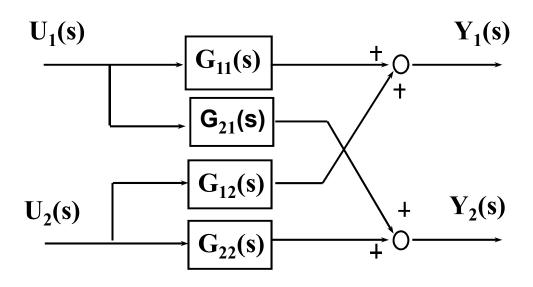
将描述单输入单输出(SISO)系统的传递函数推广到多输入多输出(MIMO)系统,就可用传递函数矩阵来描述多变量系统



如图所示两变量系统,当初始条件为零时, $Y_1=?Y_2=?$







写成矩阵形式:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

专递函数矩阵

如图所示两变量系统,当初始条件为零时,可以用拉氏变换式表示:

$$Y_{1}(s) = G_{11}(s)U_{1}(s) + G_{12}(s)U_{2}(s)$$

$$Y_{2}(s) = G_{21}(s)U_{1}(s) + G_{22}(s)U_{2}(s)$$

$$Y(s) = G(s)U(s)$$

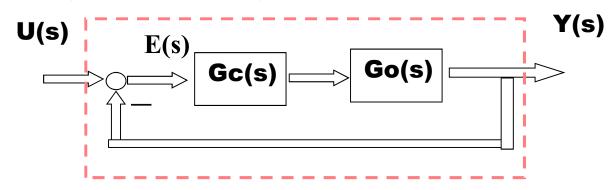
$$Y(s) = G(s)U(s)$$

传递函数矩阵G(s)拓宽了传递函数的概念,它适用于r个输入、m个输出的系统,这时的G(s)为mxr维矩阵,其元素G_{ii}(s)表示第j个输入对象i个输出的传递函数。





对于MIMO系统,在画方块图时,往往采用带箭头的双线表示信息流向



对于多变量系统的方块图运算,特别要注意乘法的前后次序不能颠倒

$$Y(s) = G(s)E(s) = G_0(s)G_c(s)E(s)$$

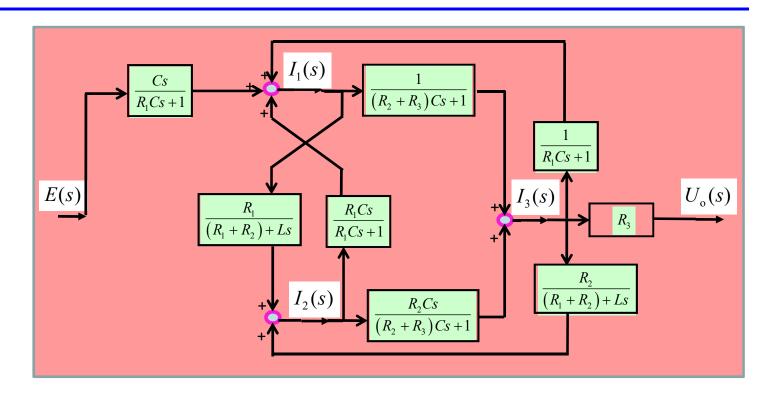
G(s)称为系统的开环 传递函数矩阵

$$Y(s) = \Phi(s)U(s) = [I + G(s)]^{-1} G(s)U(s)$$
$$= G(s)[I + G(s)]^{-1} U(s)$$

Φ(s)称为系统的 闭环传递函数矩阵







化简此方块图,几何方法非常繁 需要研究系统化的代数方法

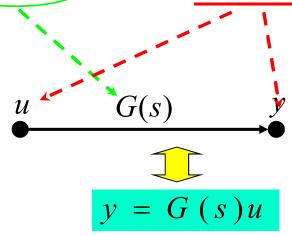
信号流图(SFG, Signal Flow Graph) 梅逊增益公式(简称梅逊公式)





信号流图(SFG)定义

- 信号流图是由节点和支路组成的信号传递网络
 - 节点表示系统中的变量(信号)
 - 系统元件的传递函数可以由连接两个节点的有向支路表示。



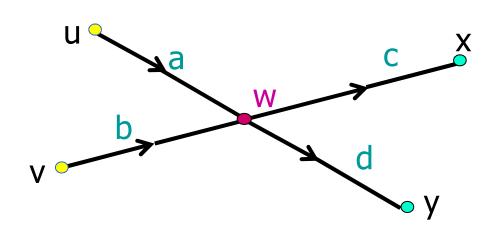
•连接两个节点的支路相当于单向乘法器:方向由箭头表示;乘法运算因子(传递函数或增益)置于相应的支路上。

注意:增益可正可负





- > 节点还具有两种作用:
 - (1) 对所有来自于流入支路的信号作加法运算
 - (2) 将流入信号之和传输给所有的流出支路



$$w = au + bv$$

$$x = cw = c(au + bv)$$

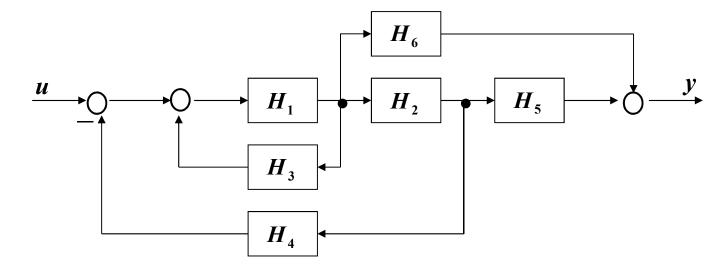
$$y = dw = d(au + bv)$$

▶ 因此,可以利用 SFG 表示输入输出关系

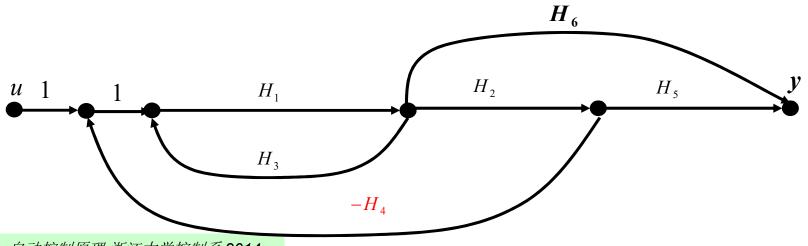




例: 试用信号流图表示如下系统



解:







$$x_{1} = u - H_{4}x_{4}$$

$$x_{2} = x_{1} + H_{3}x_{3}$$

$$x_{3} = H_{1}x_{2}$$

$$x_{4} = H_{2}x_{3}$$

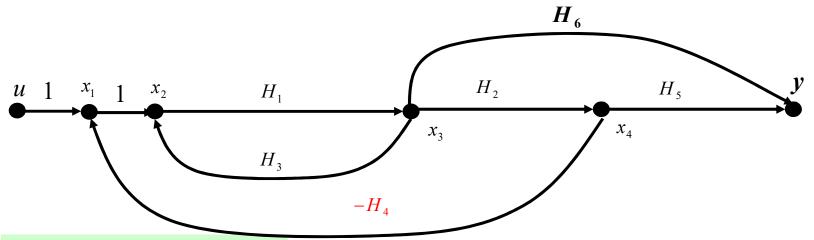
$$y = H_{6}x_{3} + H_{5}x_{4}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -H_4 \\ 1 & 0 & H_3 & 0 \\ 0 & H_1 & 0 & 0 \\ 0 & 0 & H_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & H_6 & H_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x = Ax + bu$$
$$y = Cx$$

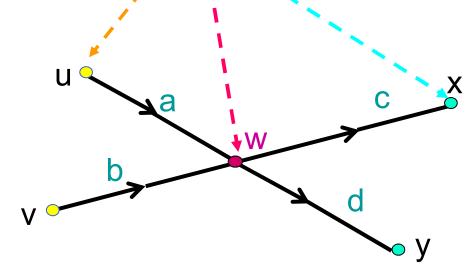
$$x = (I - A)^{-1}bu$$
$$y = C(I - A)^{-1}bu$$





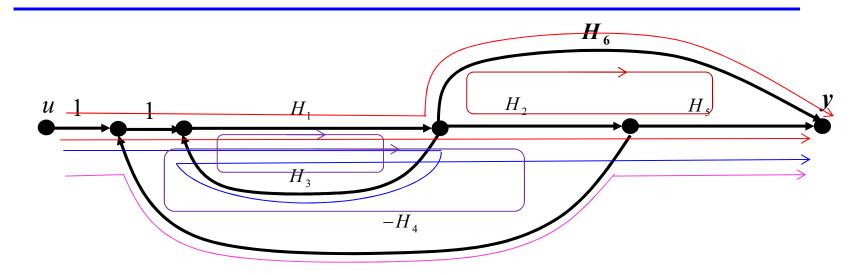


- ◆ 节点有三种类型
 - (1) 源节点 🤍 (独立节点、输入节点): 仅有流出支路
 - (2) 阱节点 ∕ (非独立节点、输出节点): 仅有流入支路
 - (3) 混合节点 (一般节点)









前向通路:从源节点到阱节点的一条可循箭头方向走通的有向路径,该路径与其上的节点相交不多于一次

前向通路增益: 前向通路中各支路增益的乘积

回路: 一条可循箭头方向走通的闭合有向路径,该路径与其上的节点相交不多于一次

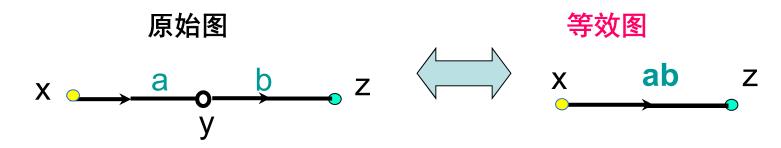
回路增益: 回路中各支路增益的乘积



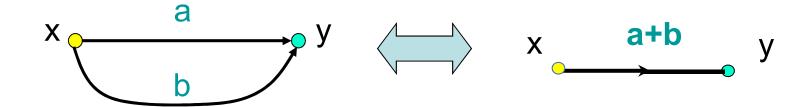


信号流图(SFG)的若干变换

◆ 串联通路

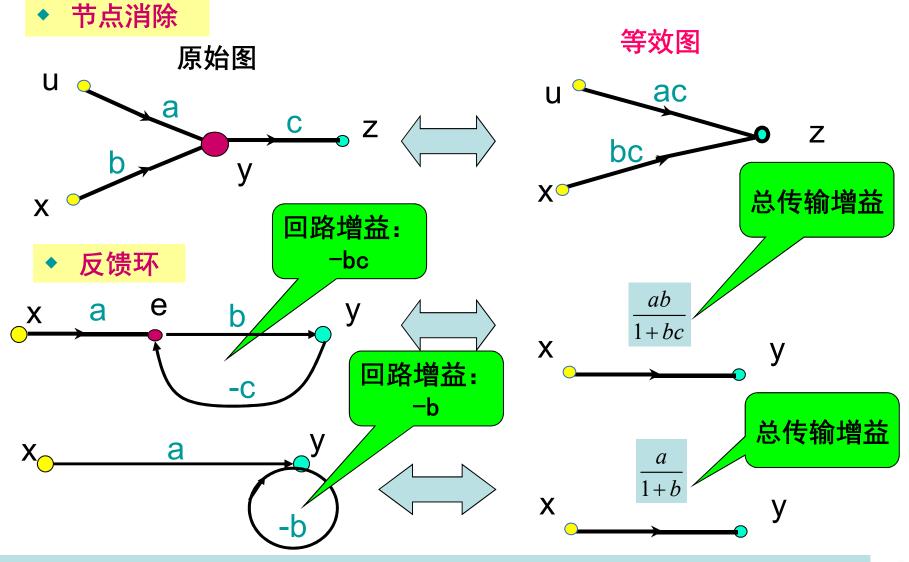


◆ 并联通路









回路增益相当于开环传函,总传输增益相当于从输入到输出的传递函数

