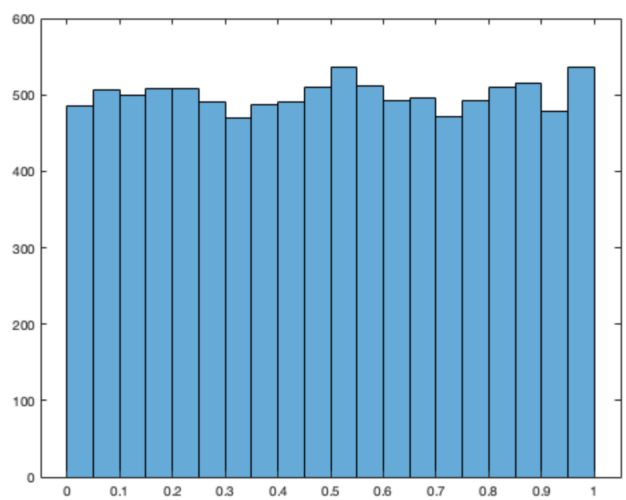
Contents


```
% Math 512 Proj 2
clear all
close all
```

1a

```
a = 7^5;
m = 2^31-1;

U_1a = zeros(10000,1);
U_1a(1) = mod(a,m);

for i = 1:length(U_1a)-1
    U_1a(i+1) = mod(a*U_1a(i),m) ;
end
U_1a = U_1a/m;
figure(1)
histogram(U_1a);
```



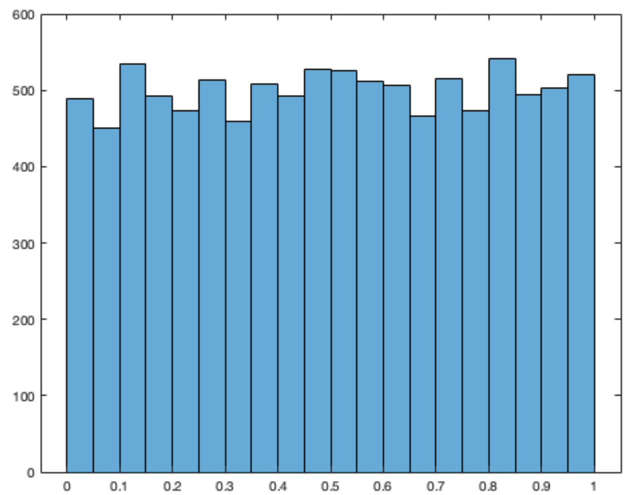
1b

```
a = 6;
m = 11;

x = zeros(1,22);
x(1) = 3;

for i = 1:length(x)-1
    x(i+1) = mod(a*x(i),m);
end

m = 10;
y = zeros(1,22);
y(1) = 3;
for i = 1:length(y)-1
```

```
    y(i+1) = mod(a*y(i),m);
end
```

1c

```
figure(2)
U_c = rand(10000,1);
histogram(U_c)
```

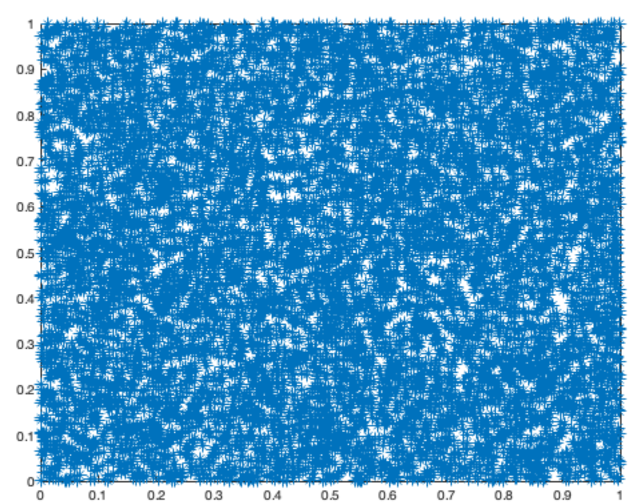


1d

the histogram from 1a presents a fixed distribution but the 1c distribution changes.

1e

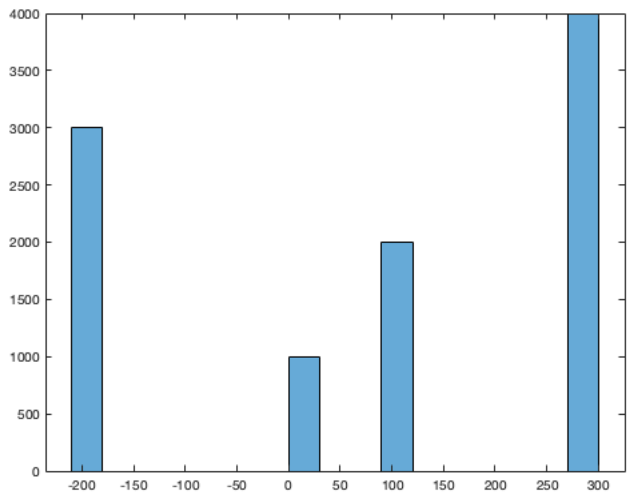
```
u = U_1a(1:9999);
v = U_1a(2:10000);
figure(3)
plot(u,v,LineStyle='none',Marker='*')


% note that since u_{n+1} = a*u_n mod m, we have that a*u_n - u_{n+1} = k*m
% where k is a non-negative integer. Therefore, u_{n+1} = a*u_n - k*m.
```



2

```
X_2 = 300+zeros(10000,1);

X_2(U_1a <= 0.6) = X_2(U_1a <= 0.6) - 500;
X_2(U_1a <= 0.3) = X_2(U_1a <= 0.3) + 300;
X_2(U_1a <= 0.1) = X_2(U_1a <= 0.1) - 85; % this method would have roundoff error when there are more possible values and smaller probabilities
```

```
figure(4)
histogram(X_2)
```

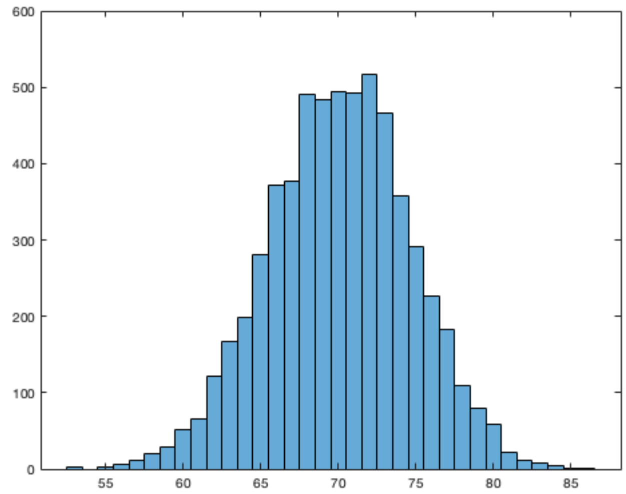


3a

generate uniform random variables

```
a = 7^5;
m = 2^31-1;
% parameters for binomial distribution
n = 100;
p = 0.7;

tic
U_3a = zeros(6000*n,1);
U_3a(1) = U_1a(end)*m; % to make it different from the first set of uniform random variables

for i = 1:length(U_3a)-1
    U_3a(i+1) = mod(U_3a(i)*a,m);
end

U_3a = U_3a/m;

% generate Binomial using Bernoulli and uniform
Bin = zeros(6000,1);

for i = 1: length(Bin)
    tempU = U_3a(1+(i-1)*100:100*i); % take a segment of U
    Ber = tempU <= p; % Bernoulli Random Variable
    Bin(i) = sum(Ber);
end
toc
figure(5)
histogram(Bin)

F_bin_70_data = sum(Bin<=70)/6000;

% compute the theoretical probability Bin <= 70
s = 0;
for i = 0:70
    s = s + factorial(n)/(factorial(i)*factorial(n-i))*p^i*(1-p)^(n-i);
end

F_bin_70_thry = s;

% the empirical probability is pretty close to the theoretical probability
```

```
Elapsed time is 0.055957 seconds.
```

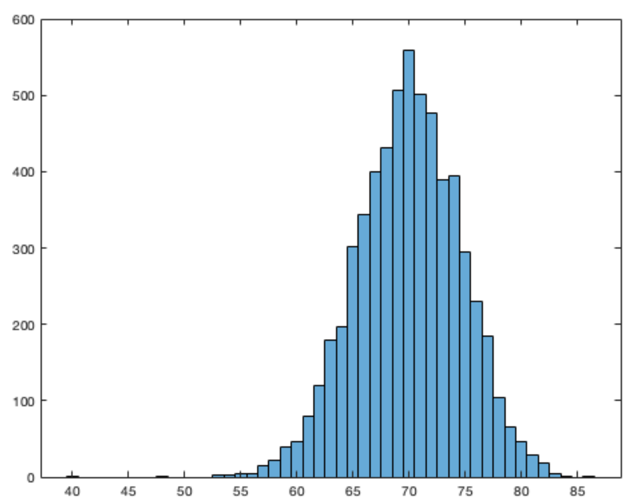


3b

Using inverse transform to generate binomial random variable

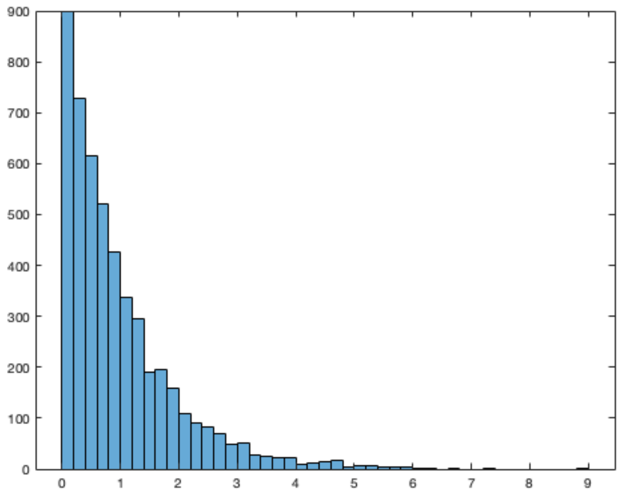
```
% generate uniform random variables
tic
U_3b = zeros(6000,1);
U_3b(1) = U_3a(end);

for i = 1:length(U_3b)-1
    U_3b(i+1) = mod(a*U_3b(i),m);
end
U_3b = U_3b/m;
% we will do the inverse transformation by computing the probability of
% binomial less than or equal to x with x = 100, 99, 98,...,0
Bin_inv = zeros(6000,1);
cdf_Bin = zeros(n+1,1);

cdf_Bin(1) = (1-p)^n;
Bin_inv(U_3b<=cdf_Bin(1)) = 0; % this is an unnecessary step for the purpose of readability
for i = 1:length(cdf_Bin)-1
    cdf_Bin(i+1) = cdf_Bin(i) + factorial(n)/(factorial(n-i)*factorial(i))*p^i*(1-p)^(n-i); % roundoff error will be a factor at the tail portion of tl
    Bin_inv(U_3b>cdf_Bin(i) & U_3b<= cdf_Bin(i+1)) = i;
end
toc

figure(6)
histogram(Bin_inv)
```

```
Elapsed time is 0.005621 seconds.
```



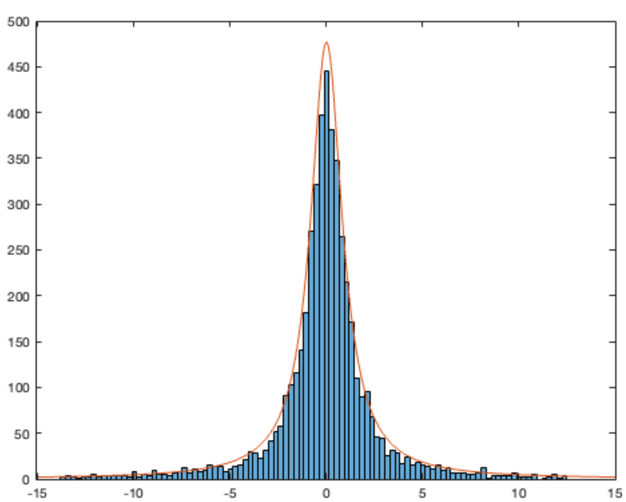
4

```
%generate uniform random variables
U_4 = zeros(5000,1);
U_4(1) = U_3b(end);

for i = 1:length(U_4)-1
    U_4(i+1) = mod(a*U_4(i),m);
end
U_4 = U_4/m;

EXP = -log(1-U_4);
figure(7)
histogram(EXP)
```



5

```
U_5 = zeros(5000,1);
U_5(1) = U_4(end);

for i = 1:length(U_5)-1
    U_5(i+1) = mod(a*U_5(i),m);
end
U_5 = U_5/m;

Cauchy = rmoutliers(tan(pi*(U_5-0.5)),'percentile',[2.5 97.5]);% we remove outliers so that the histogram can be properly displayed

figure(8)
histogram(Cauchy,100)
hold on

x = -15:0.1:15;
y = 1500./(pi*(1+x.^2)); % we scale the density for better
plot(x,y)
```

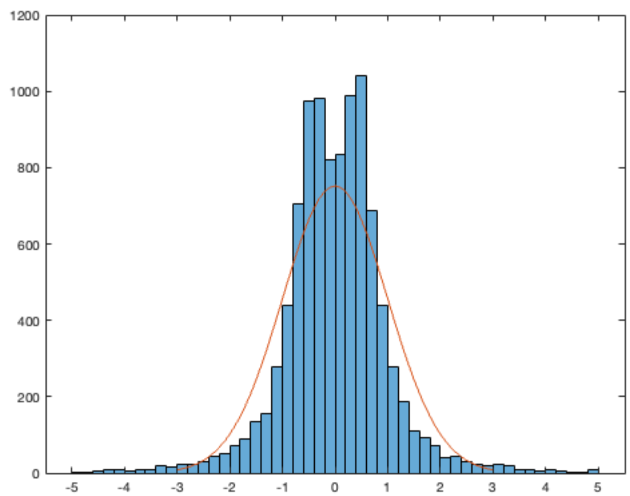


6a

generate uniform random variables

```
U_6 = rand(100000,1);
% we are not using the modulo-generated psuedorandom number because of the
% linear correlation between consecutive uniform random variables

% Gaussian density
G_x = -3:0.1:3;
G_y = 300*(2*pi)^0.5*exp(-0.5*G_x.^2);

% Box-Muller Method
tic
Z_BM = zeros(10000,1);
for i = 1:length(Z_BM)/2
    Z_BM(2*i-1) = (-2*log(U_6(2*i-1)))^(-0.5)*cos(U_6(2*i)*2*pi);
    Z_BM(2*i) = (-2*log(U_6(2*i-1)))^(-0.5)*sin(U_6(2*i)*2*pi);
end
toc
figure(9)
histogram(Z_BM,[-5:0.2:5])
hold on
plot(G_x,G_y)
```

```
Elapsed time is 0.003288 seconds.
```

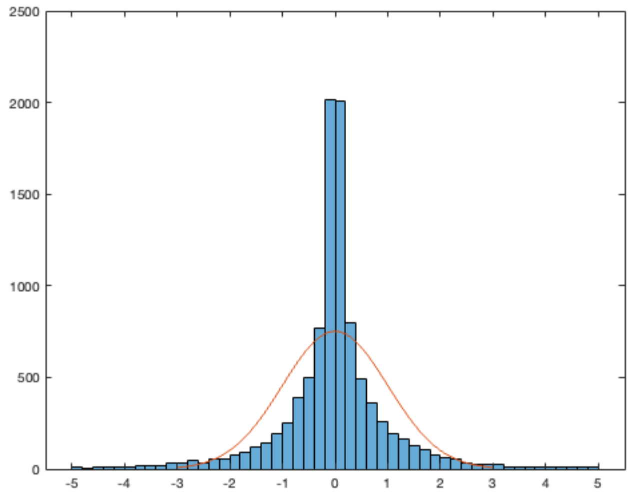


6b

Marsaglia-Bray Method

```
tic
Z_MB = zeros(10000,1);

i = 1;
ptr = 1; % pointer for uniform variables
while i <= length(Z_MB)
    x = U_6(ptr)*2-1;
    y = U_6(ptr+1)*2-1;
    s = x^2+y^2;
    if s<1
        Z_MB(i) = x*(-log(s)/s)^(-0.5);
        Z_MB(i+1) = y*(-log(s)/s)^(-0.5);
        i = i+2;
    end
    ptr = ptr+2;
end
toc
figure(10)
histogram(Z_MB,[-5:0.2:5])
hold on
plot(G_x,G_y)
```

```
Elapsed time is 0.003763 seconds.
```

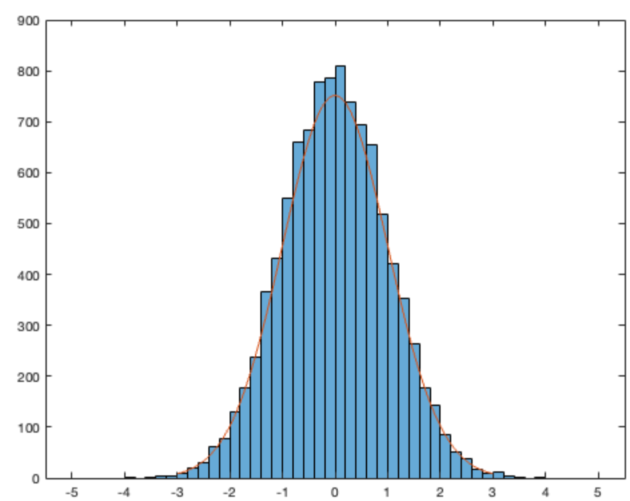


6c

acceptance-rejection method

```
tic
Z_ar = zeros(10000,1);

i = 1;
ptr = 1;
while i<= length(Z_ar)
    X1 = -log(U_6(ptr));
    X2 = -log(U_6(ptr+1));
    X3 = U_6(ptr+2);
    if X2 >= (X1-1)^2/2
        Z_ar(i) = abs(X1)*(-1)^(X3<=0.5);
        i = i+1;
    end
    ptr = ptr+3;
end
toc
figure(11)
histogram(Z_ar,[-5:0.2:5])
hold on
plot(G_x,G_y)
```

```
Elapsed time is 0.014233 seconds.
```

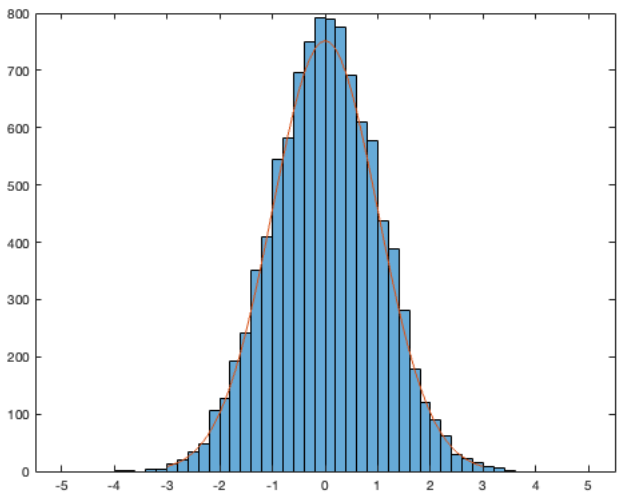


6d

```
tic
Z = randn(10000,1);
toc
figure(12)
```

```
histogram(Z,[-5:0.2:5])
hold on
plot(G_x,G_y)
```

```
Elapsed time is 0.000444 seconds.
```



Published with MATLAB® R2021b