Color

Wojciech Matusik MIT EECS

Many slides courtesy of Victor Ostromoukhov, Leonard McMillan, Bill Freeman, Fredo Durand

6.837 Final Exam

- **Thursday, December 20, 1:30 to 4:30 PM** in **Walker Gym**

Assignment 1

- Due Wednesday
- Deadline is absolute

Does color puzzle you?

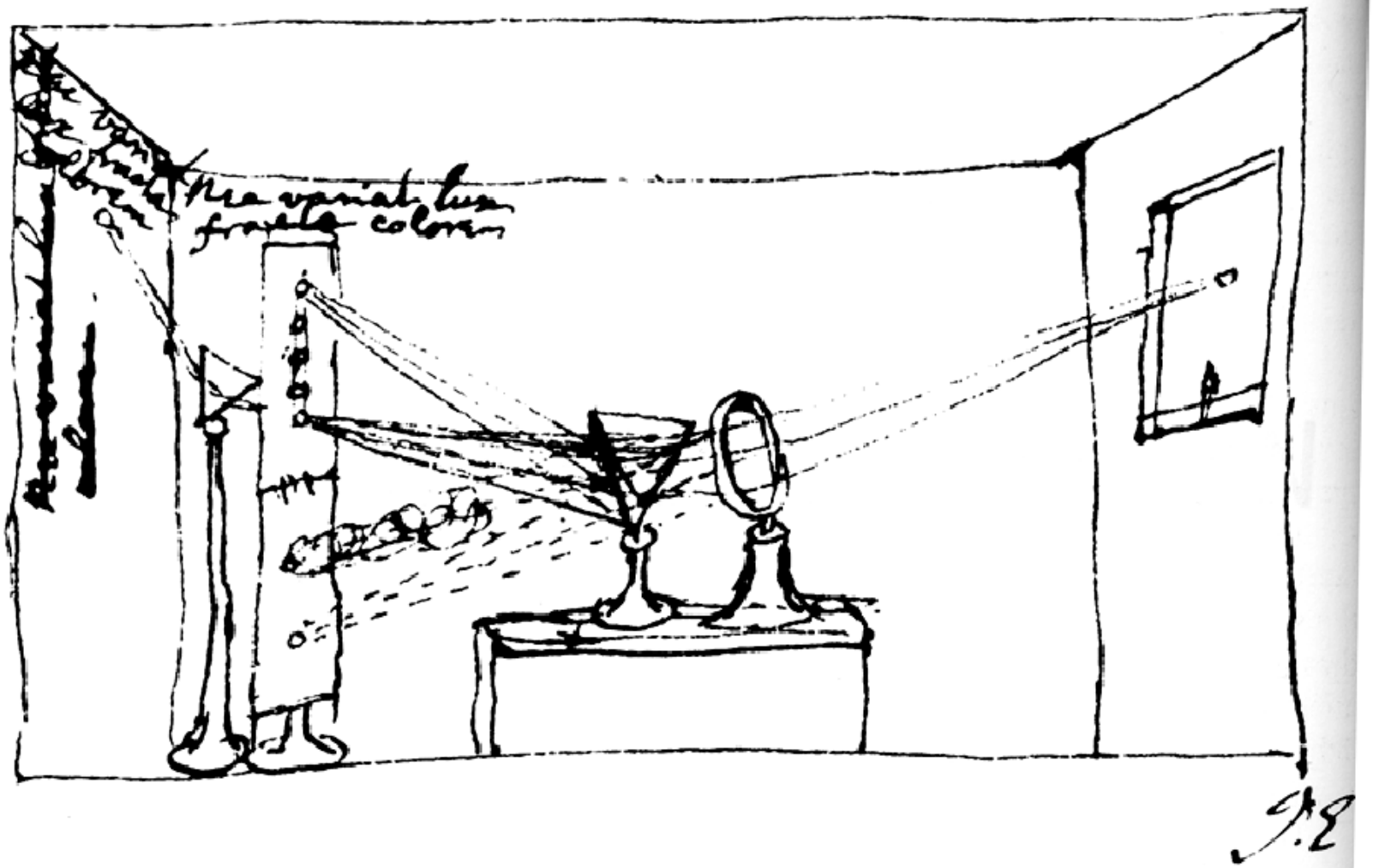
Answer

- It’s all linear algebra

Plan

- Spectra
- Cones and spectral response
- Color blindness and metamers
- Color matching
- Color spaces

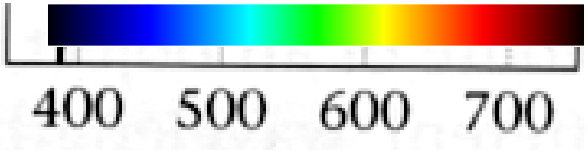
Color



4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

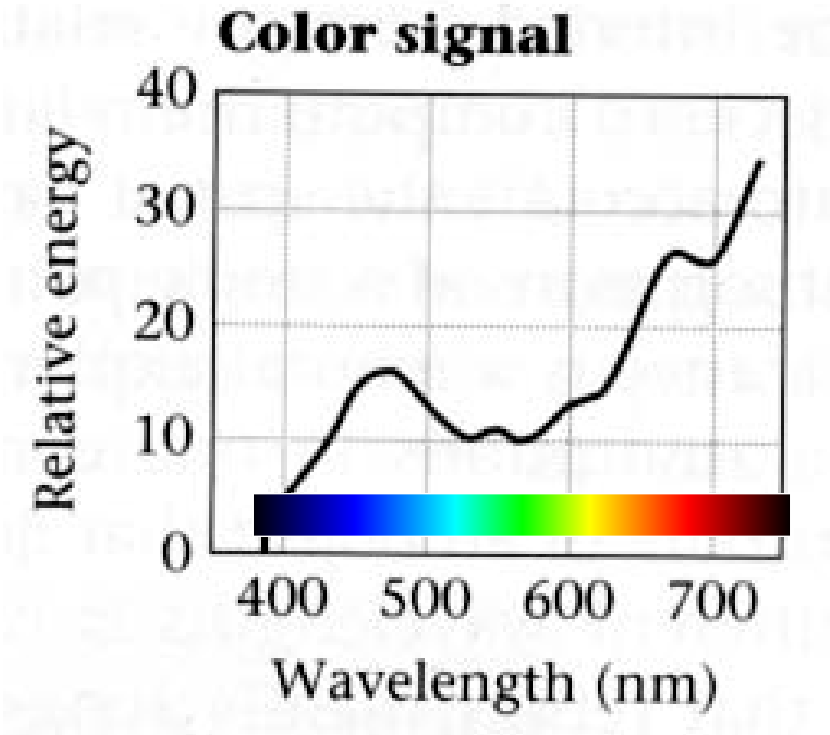
From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Spectrum



Light is a wave
Visible: between 450 and 700nm

Spectrum



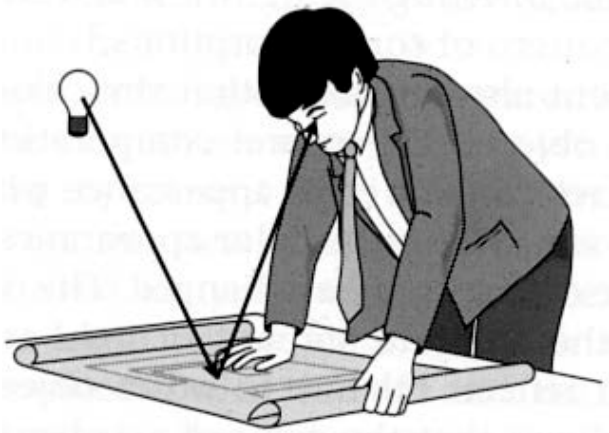
Light is characterized by its spectrum:

amount of energy at each wavelength

This is a full distribution:

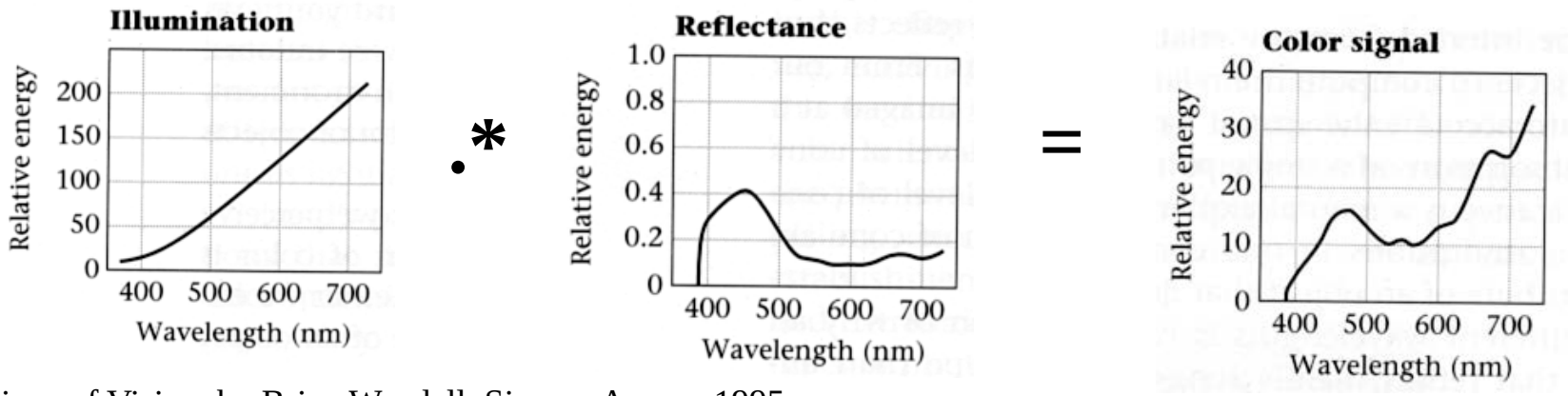
one value per wavelength (infinite number of values)

Light-matter interaction



Where spectra come from:
- light source spectrum
- object reflectance (aka spectral albedo)
get multiplied wavelength by wavelength

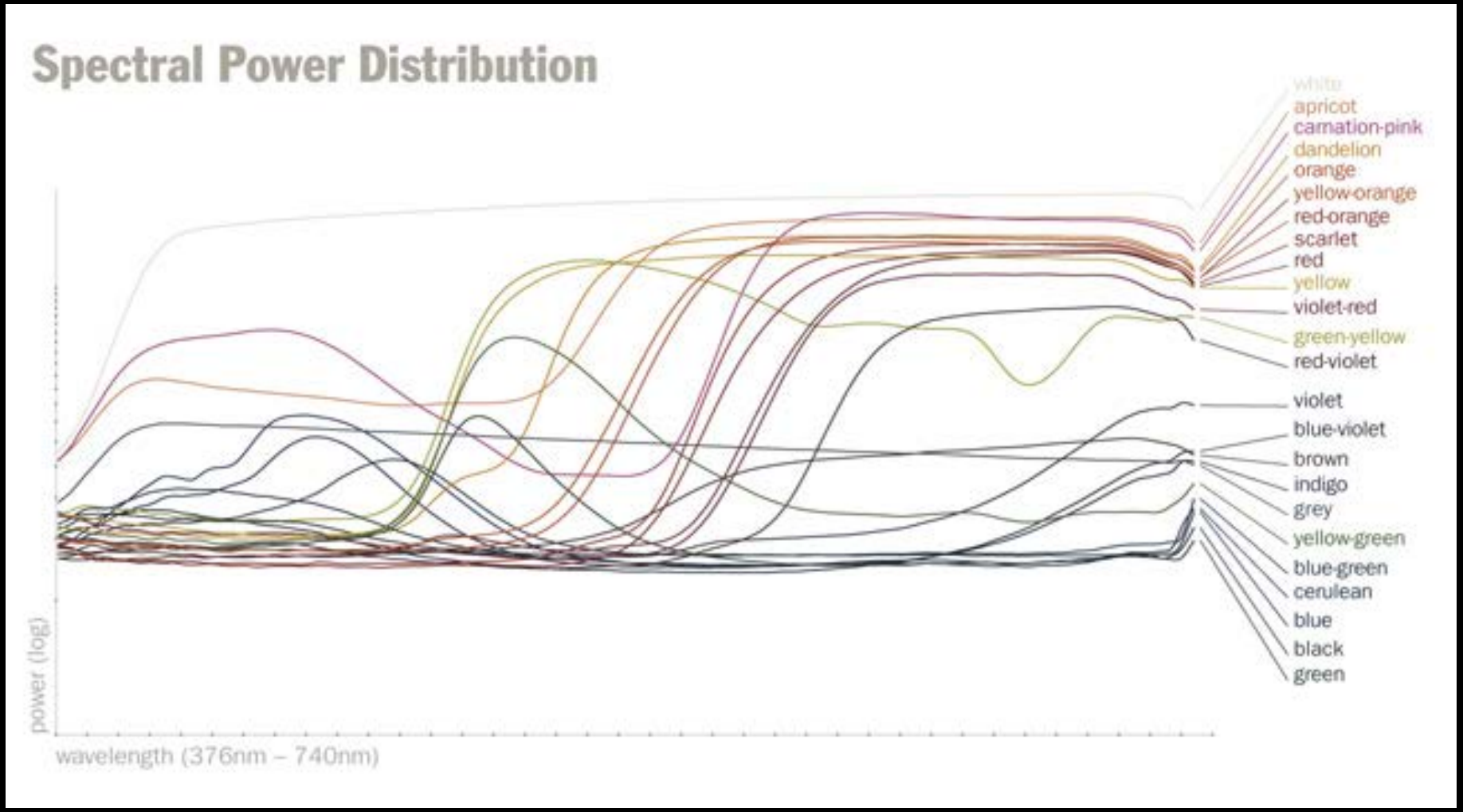
There are different physical processes that explain this multiplication
e.g. absorption, interferences



Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

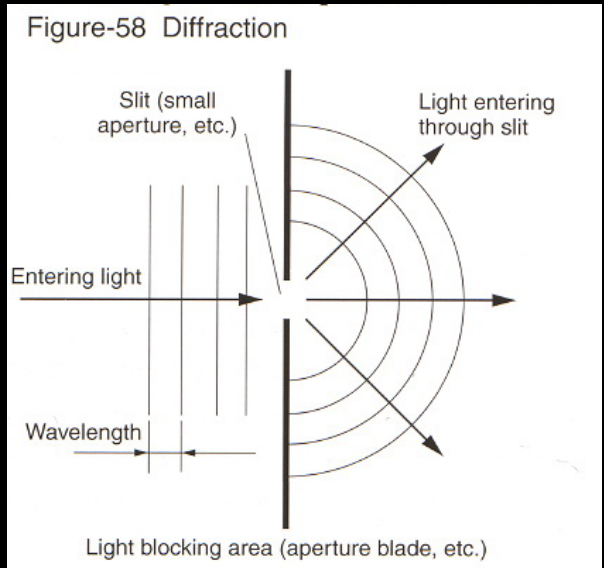
Crayons

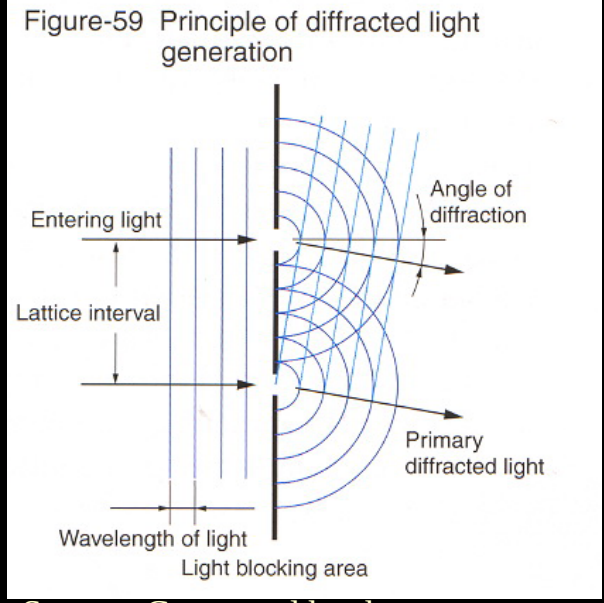
- http://www.photo-mark.com/notes/2011/sep/20/crayon-colors/



Spectrum demo

- Diffraction grating:
  - shifts light as a function of wavelength
  - Allows you to see spectra
  - In particular, using a slit light source, we get a nice band showing the spectrum
- See the effect of filters
- See different light source spectra

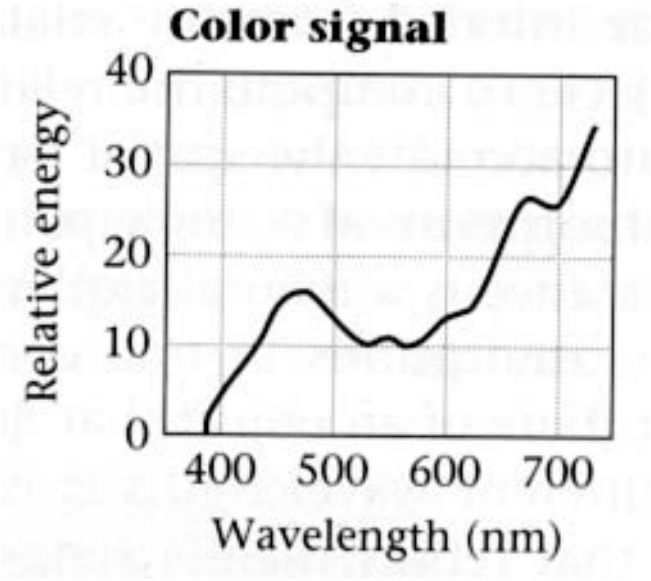




Source: Canon red book

Questions?

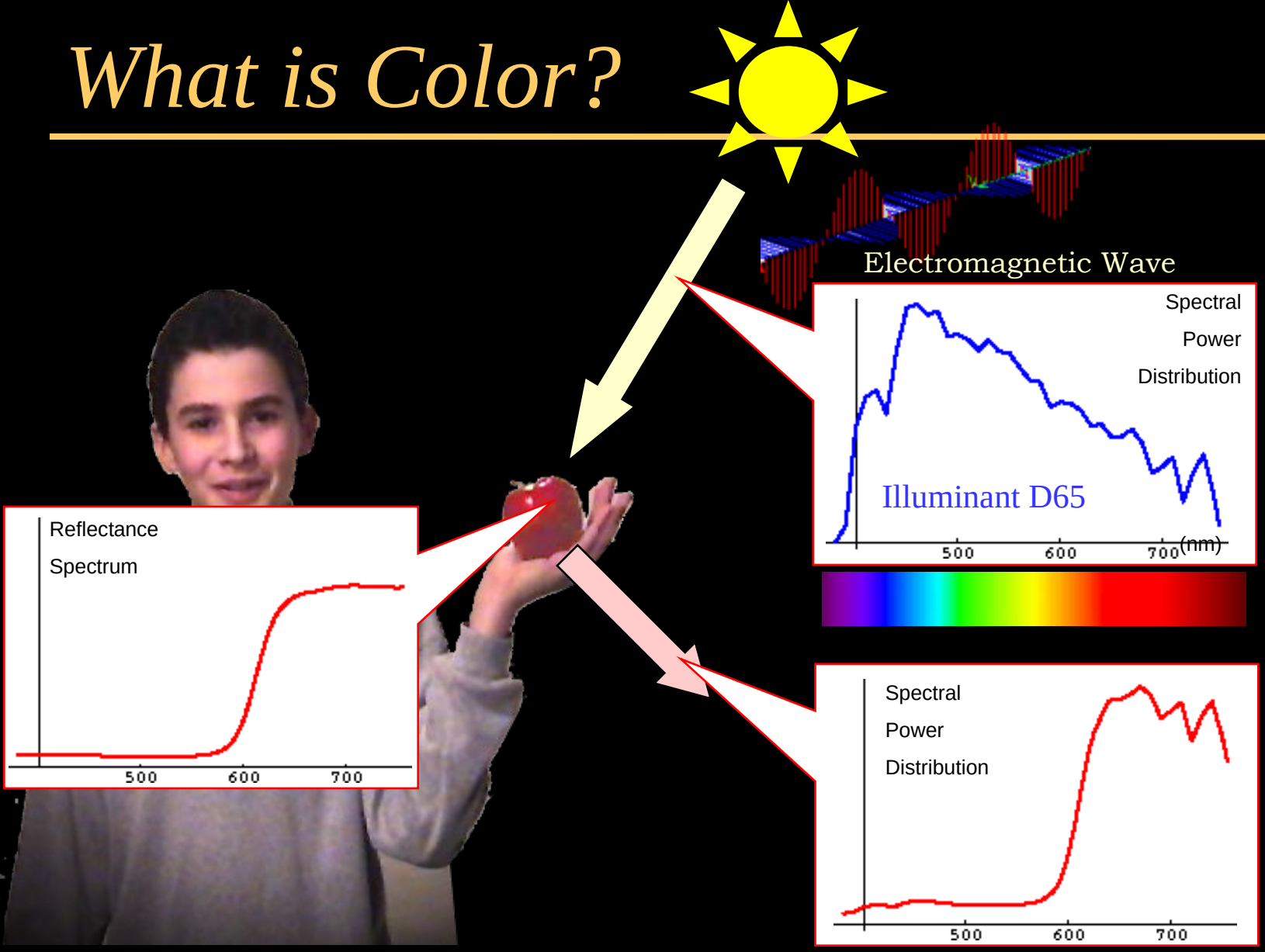
- So far, physical side of colors: **spectra**
  - an infinite number of values (one per wavelength)



Plan

- Spectra
- **Cones and spectral response**
- Color blindness and metamers
- Color matching
- Color spaces

What is Color?



Slide from Victor Ostromoukhov

What is Color?

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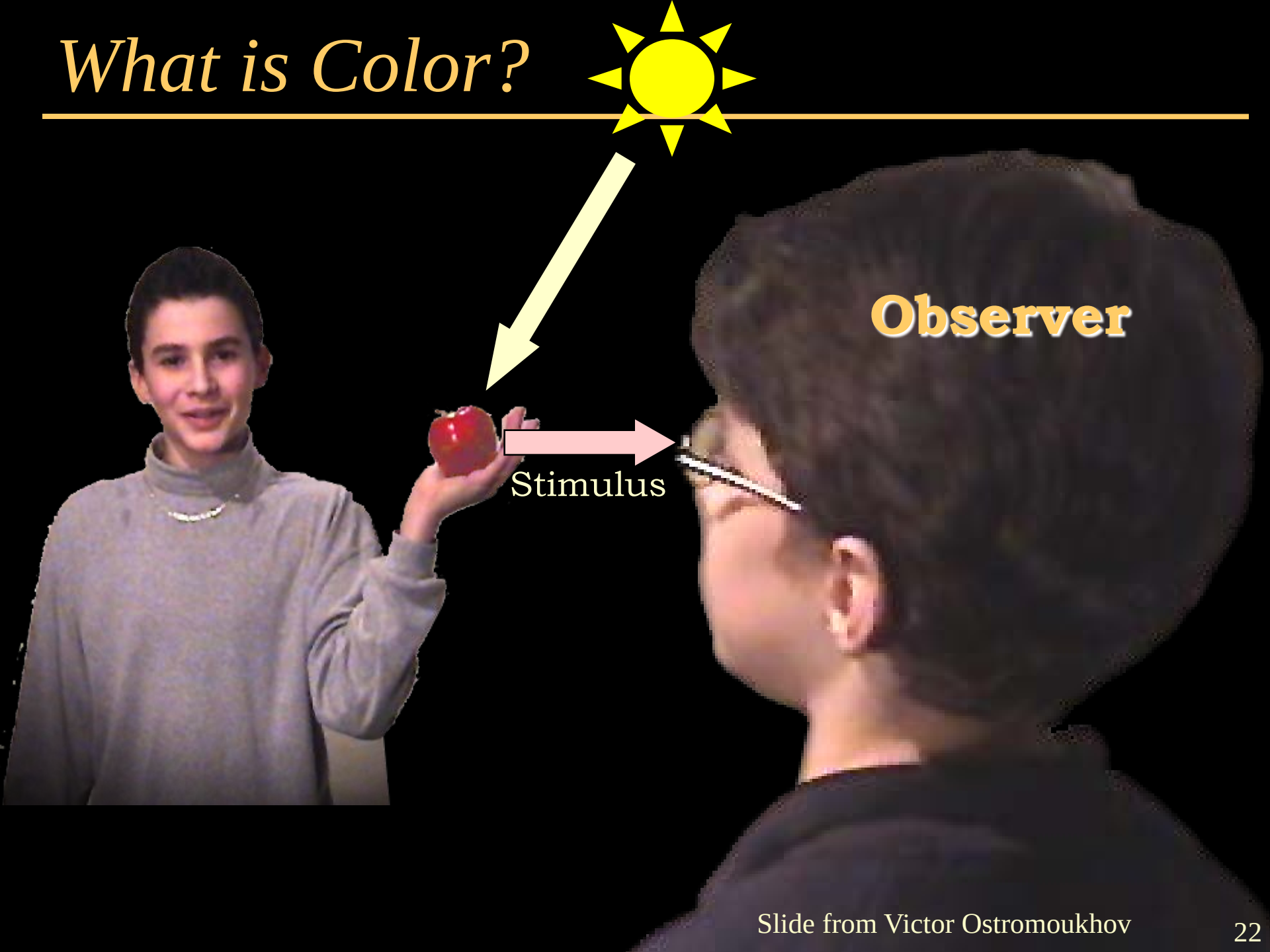
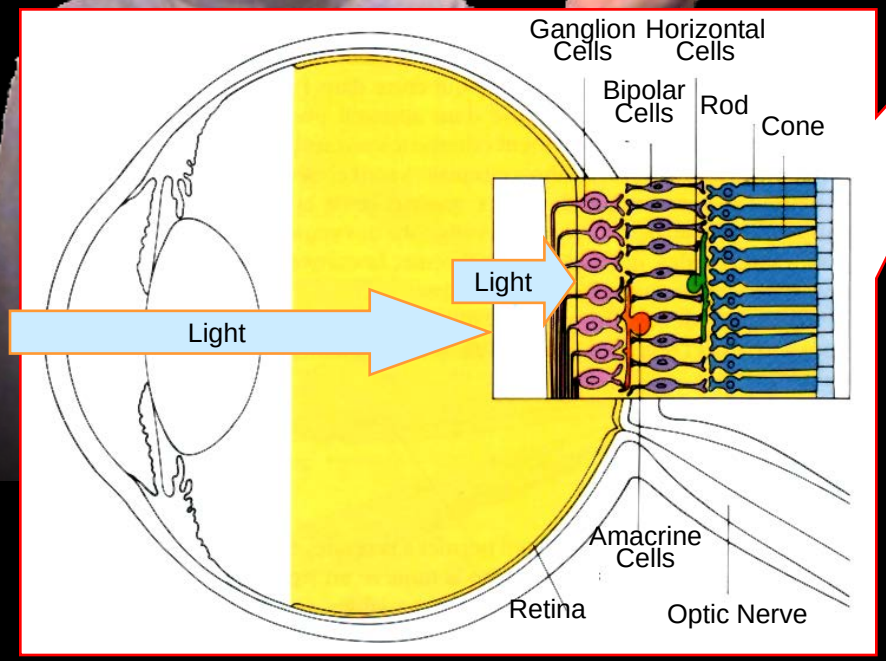
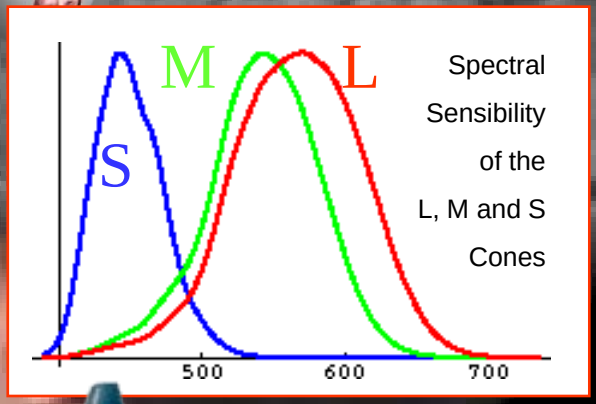
What is Color?

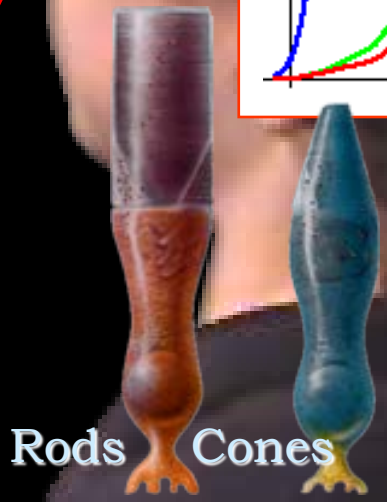


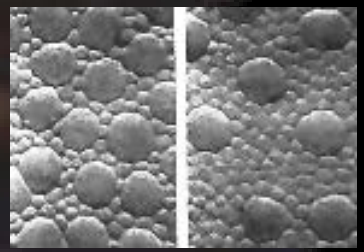
Slide from Victor Ostromoukhov

What is Color?







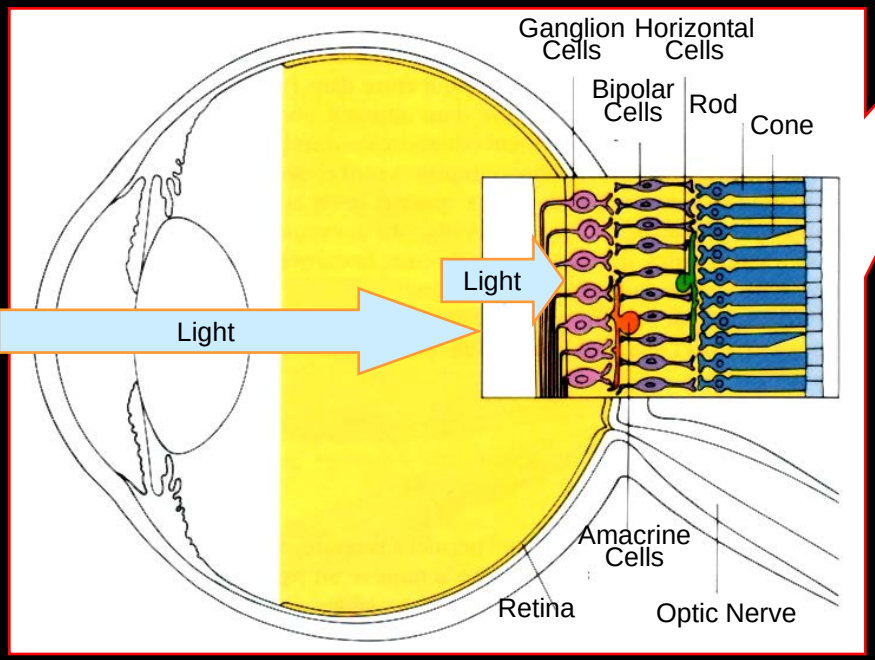


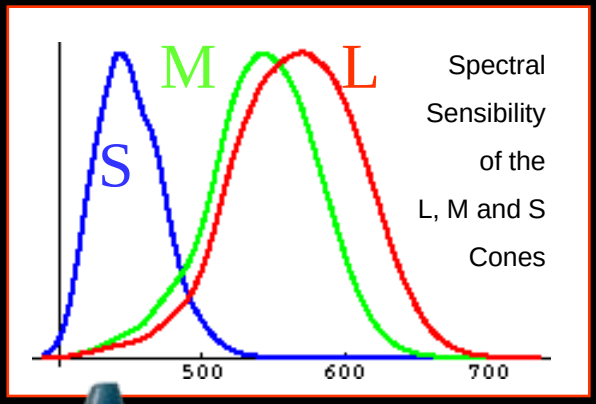
Distribution of Cones and Rods

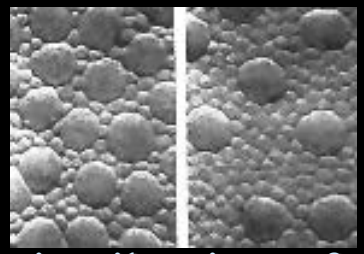
Slide from Victor Ostromoukhov

Cones

- We focus on low-level aspects of color
  - Cones and early processing in the retina
- We won't talk about rods (night vision)







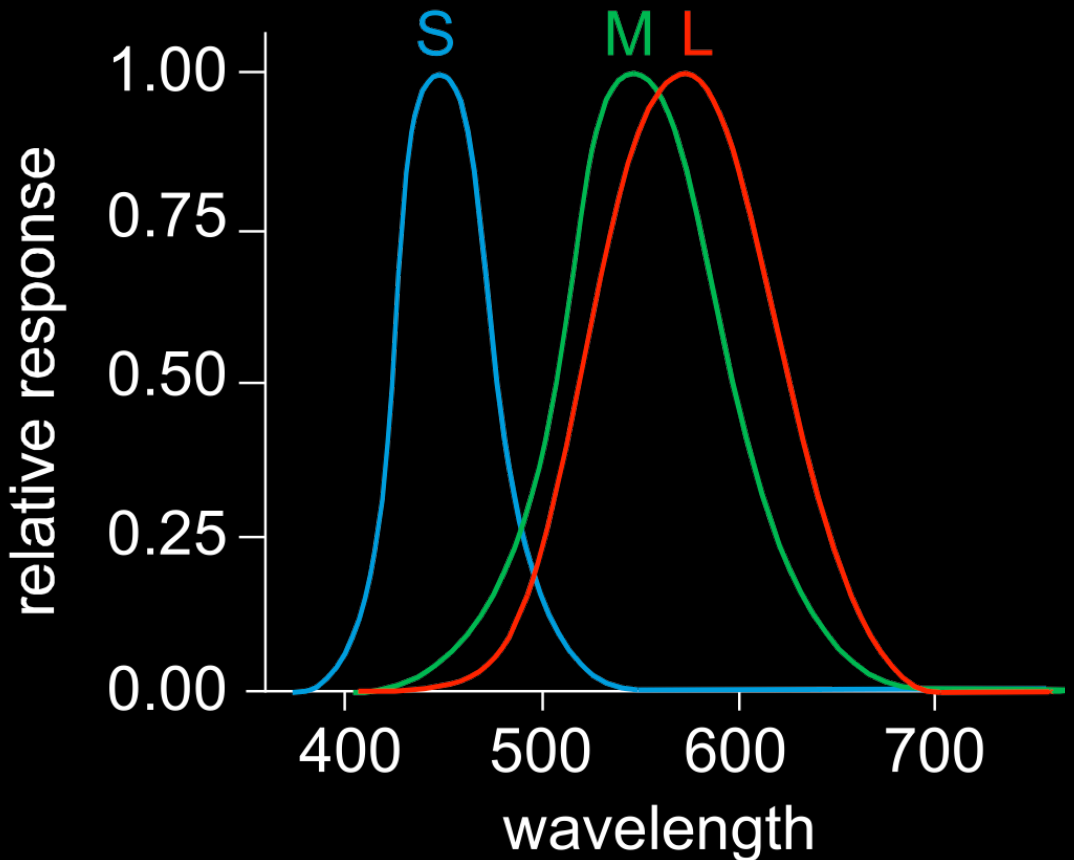
Distribution of Cones and Rods

Summary (and time for questions)

- Spectrum: infinite number of values
  - can be multiplied
  - can be added
- Light spectrum multiplied by reflectance spectrum
  - spectrum depends on illuminant
- Human visual system is complicated

Cone spectral sensitivity

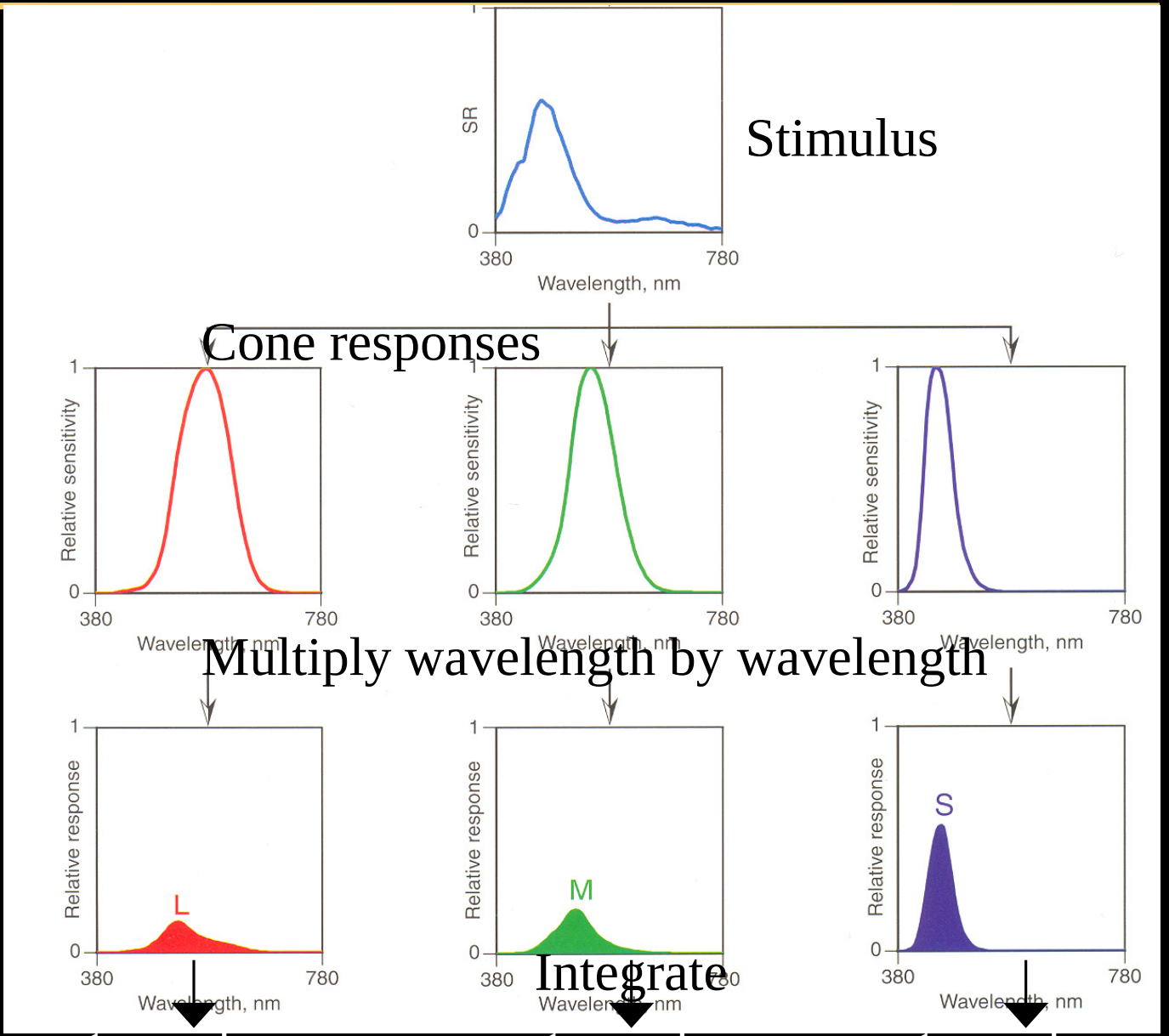
- Short, Medium and Long wavelength
- Response for a cone
  $= \int_{\lambda} \text{stimulus}(\lambda) * \text{response}(\lambda)\, d\lambda$



Cone response

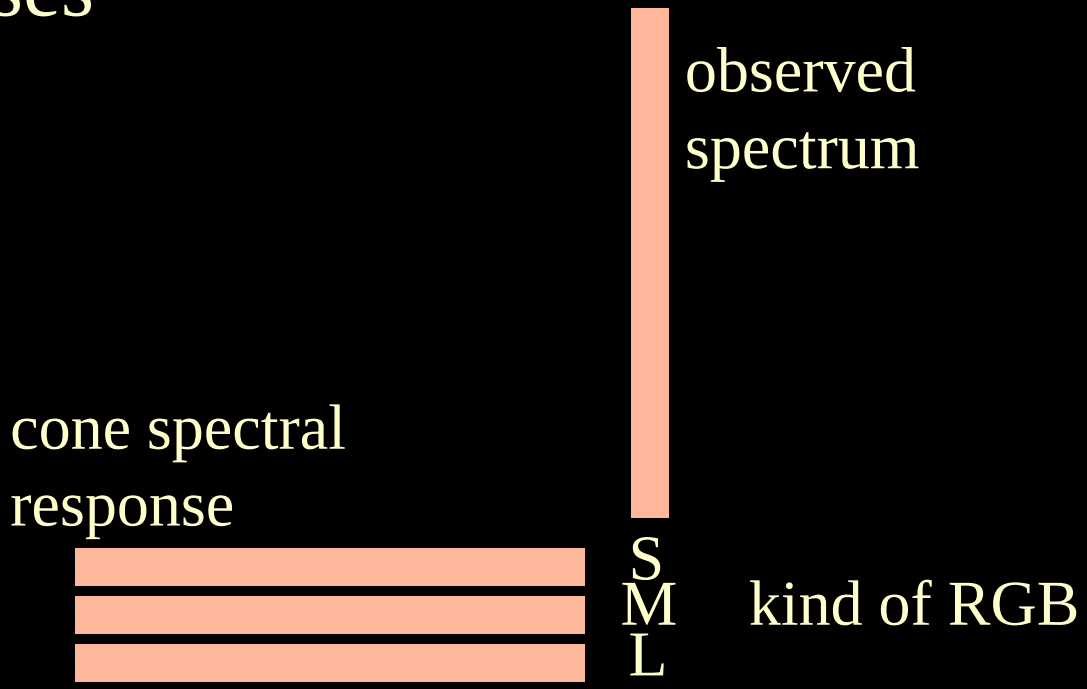
Start from infinite number of values (one per wavelength)

End up with 3 values (one per cone type)



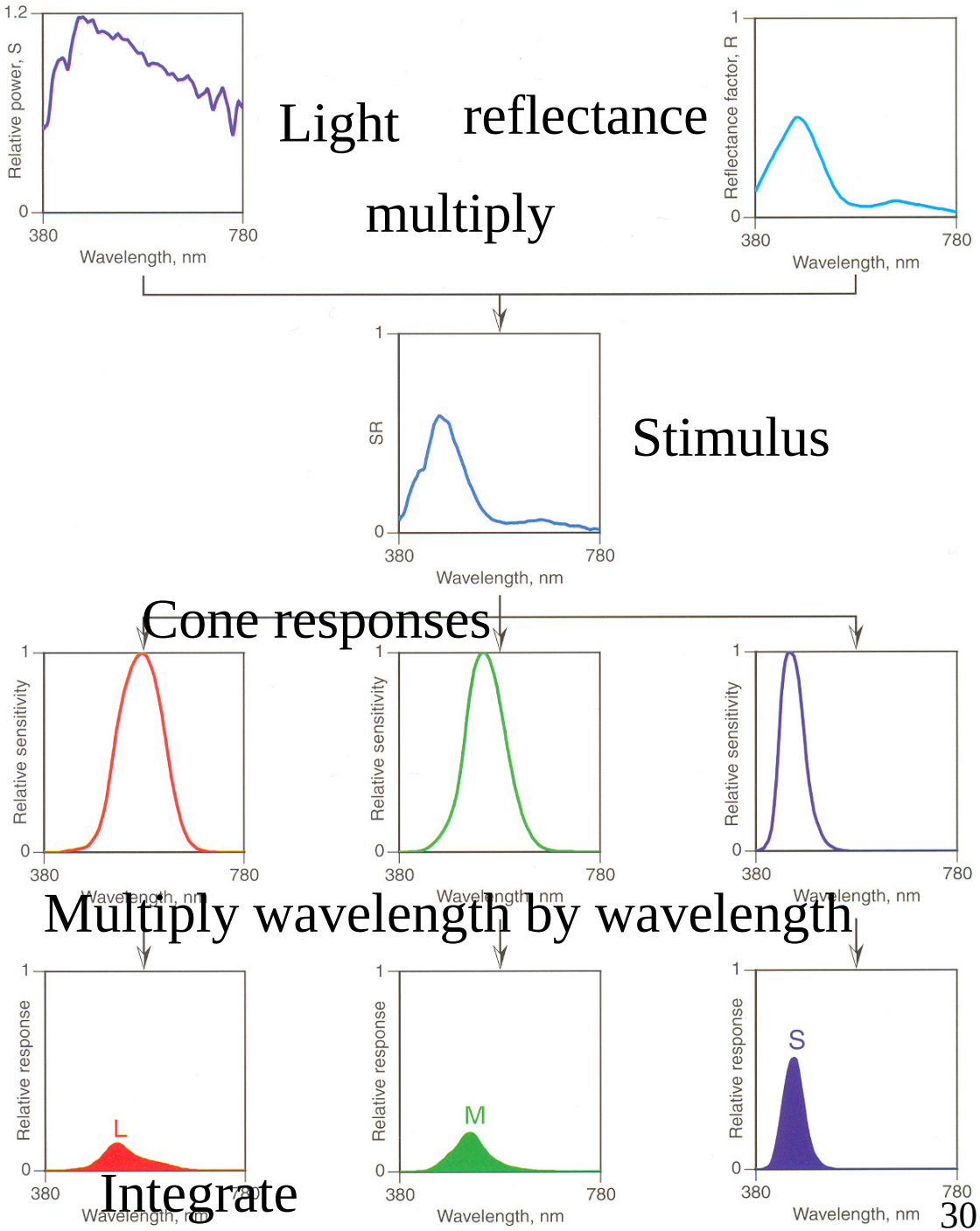
For matrix lovers

- Spectrum: big long vector size N where N=∞
- Cone response: 3xN matrix of individual responses



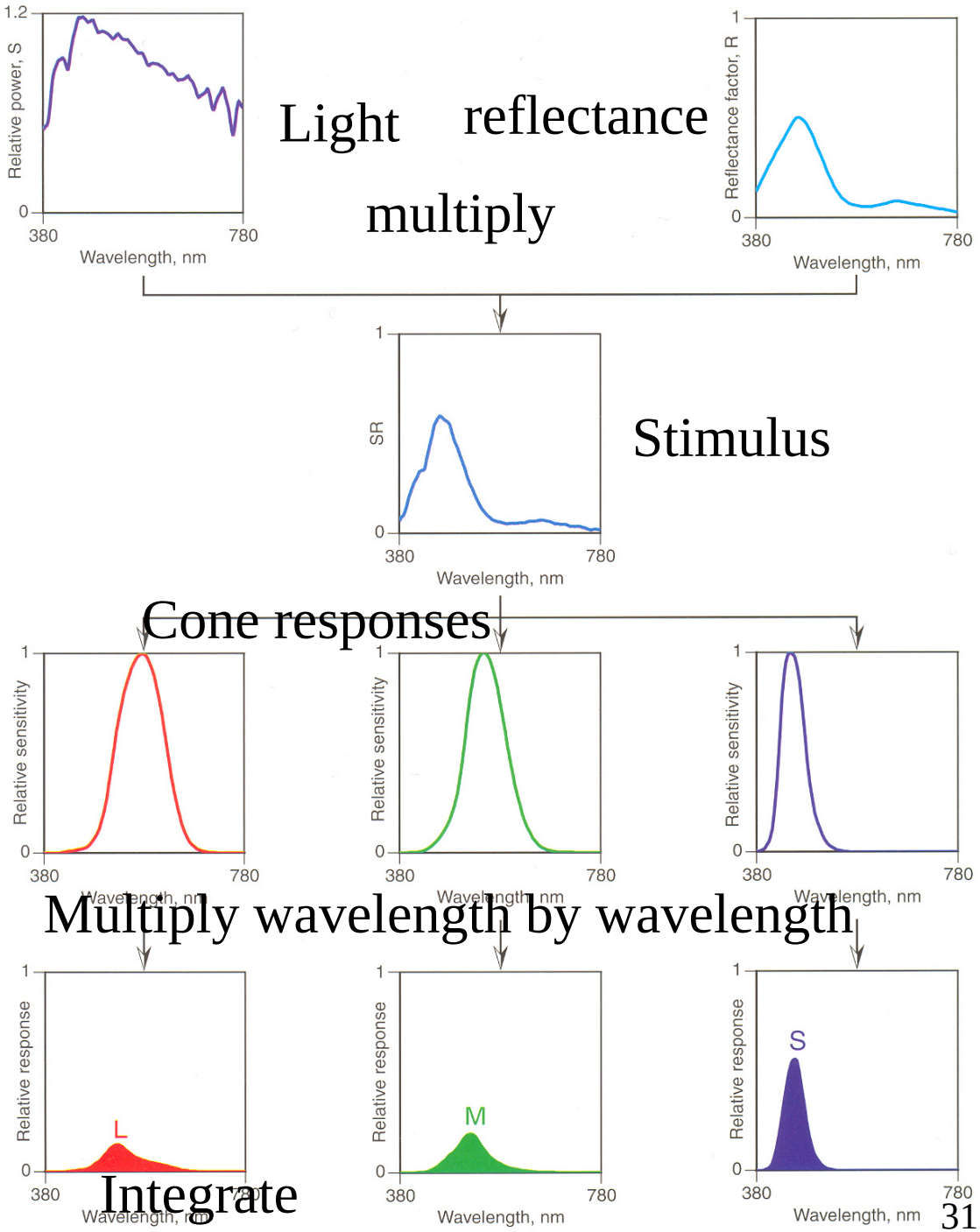
Big picture

- It’s all linear!

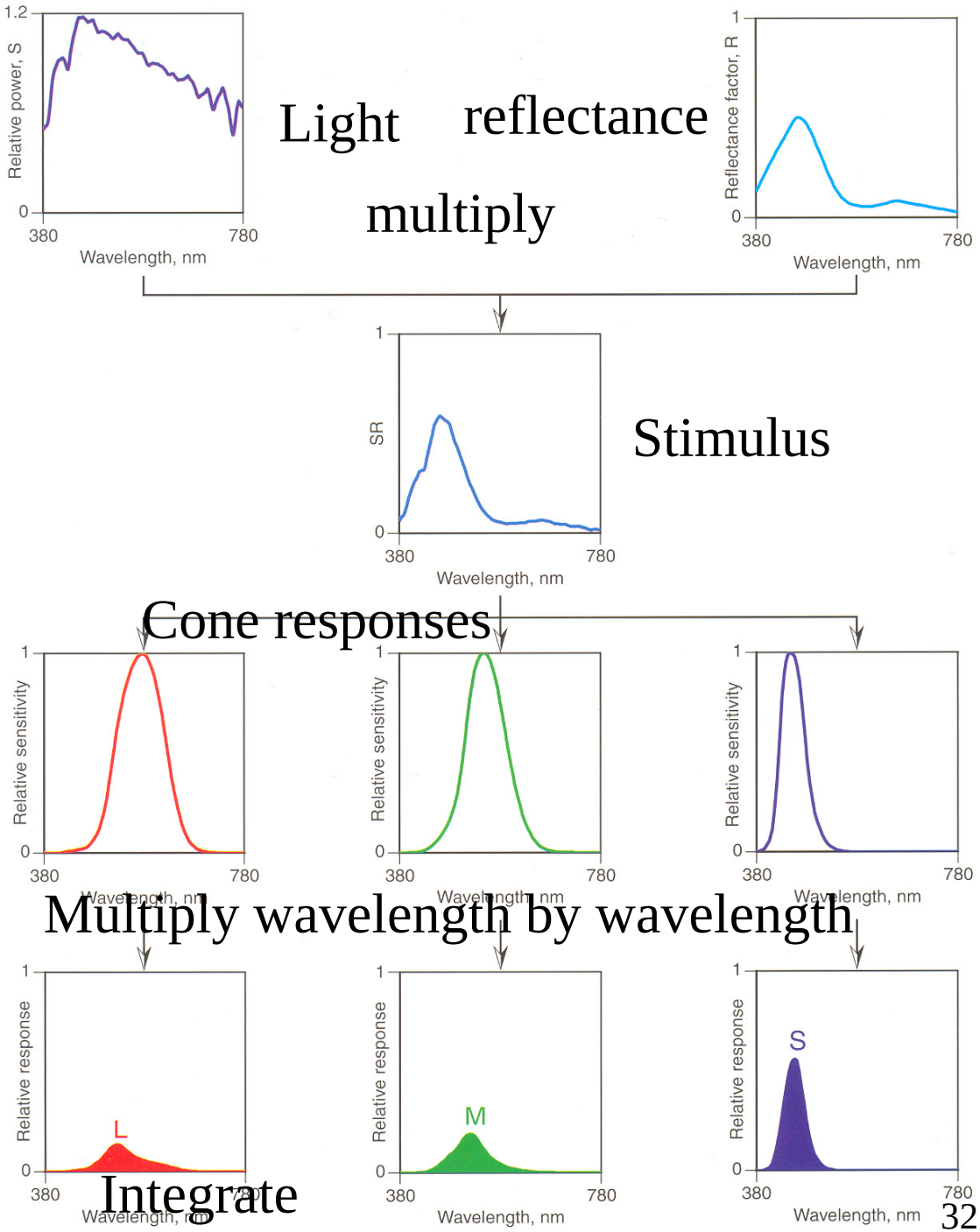


Big picture

- It's all linear!
  - multiply
  - add
- But
  - non-orthogonal basis
  - infinite dimension
  - light must be positive
- Depends on light source

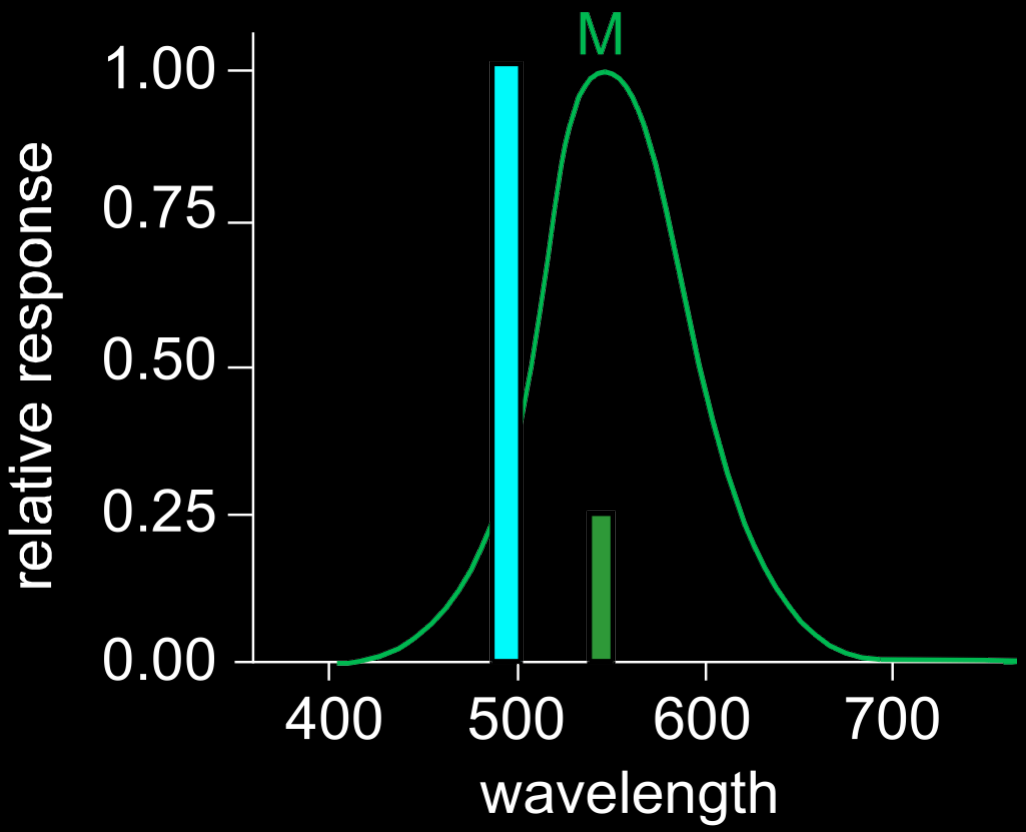


Questions?


Relative power, S
1.2
0
380
780
Wavelength, nm
Light
reflectance
multiply
Reflectance factor, R
1
0
380
780
Wavelength, nm
SR
1
0
380
780
Wavelength, nm
Stimulus
Cone responses
Relative sensitivity
1
0
380
780
Wavelength, nm
Multiply wavelength by wavelength
Relative response
L
M
S
Integrate

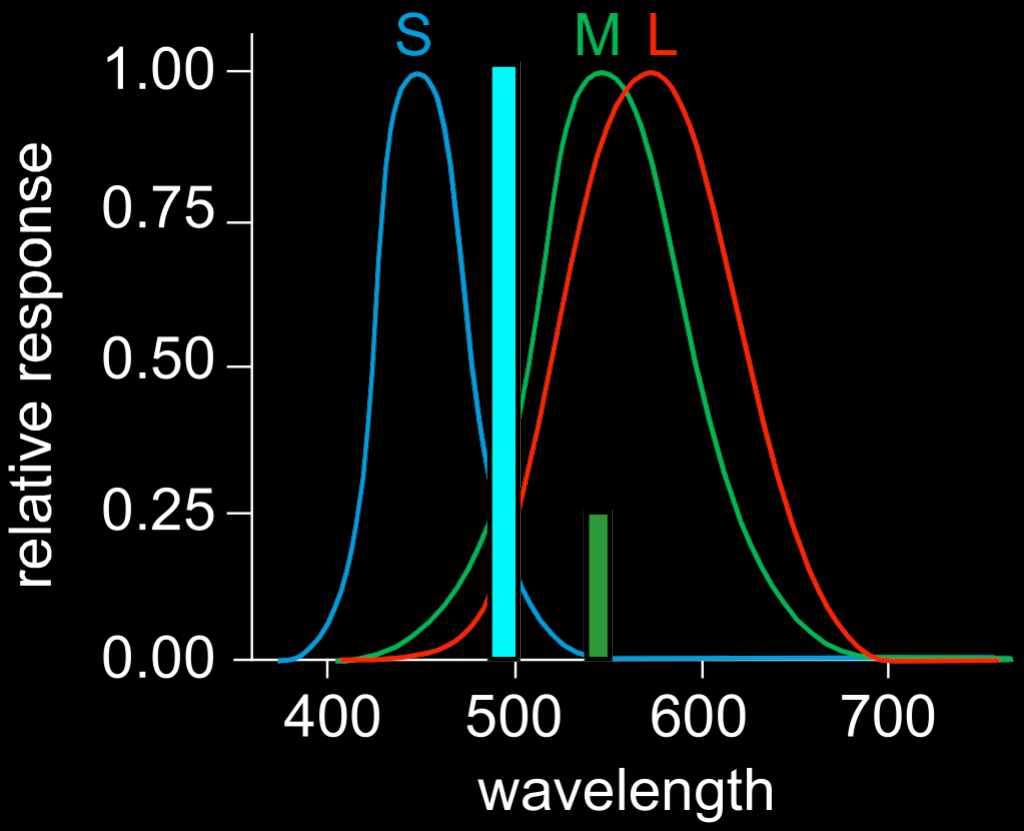
A cone does not "see" colors

- Different wavelength, different intensity
- Same response



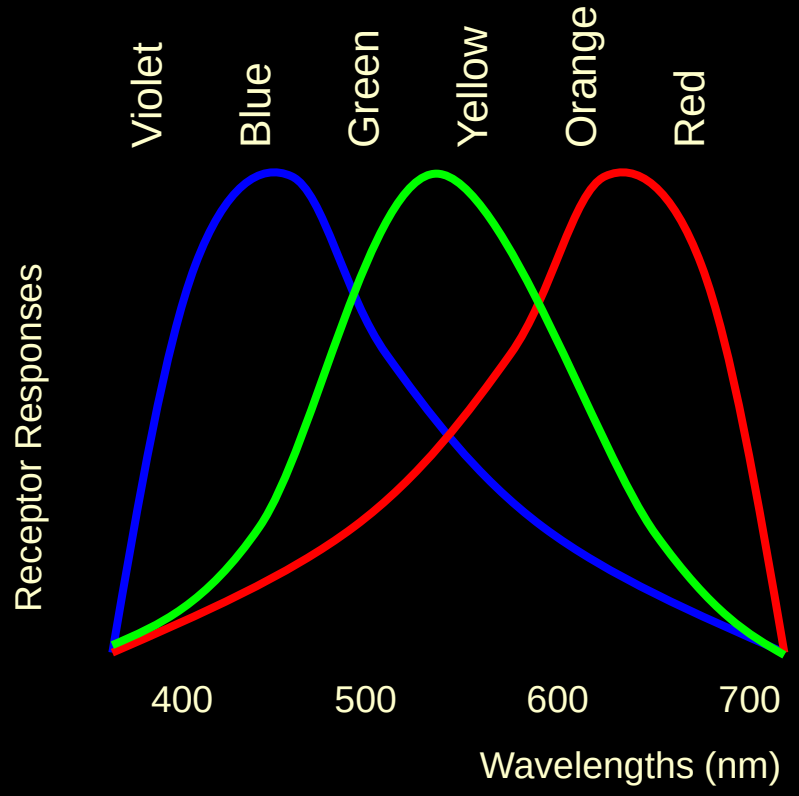
Response comparison

- Different wavelength, different intensity
- But different response for different cones

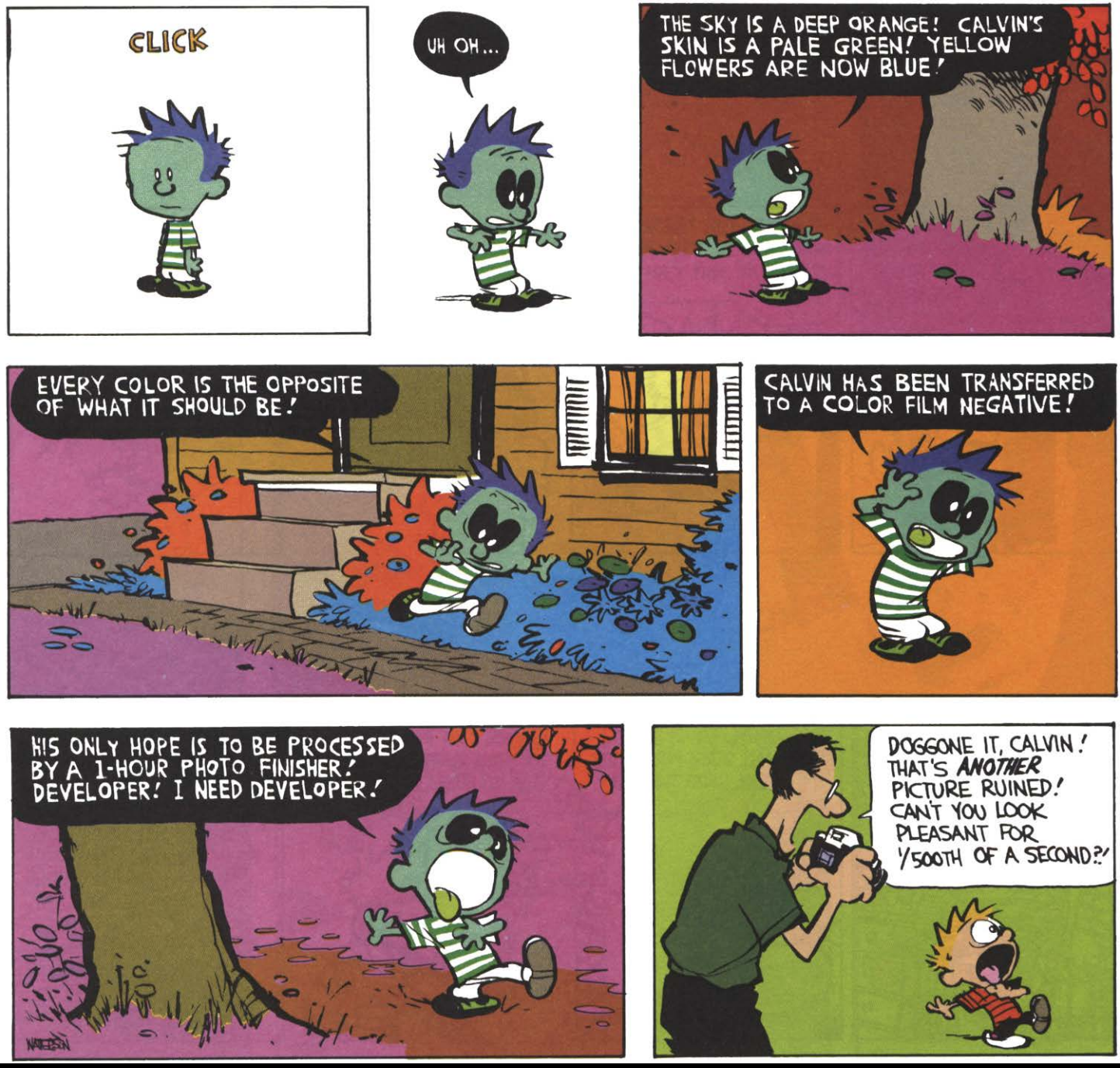


von Helmholtz 1859: Trichromatic theory

- Colors as relative responses (ratios)



Questions?

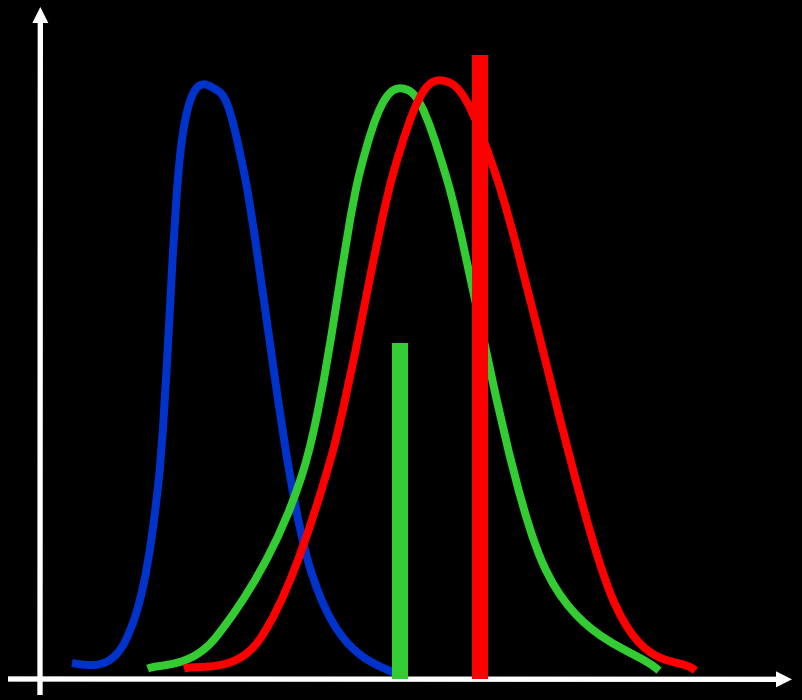

CLICK
UH OH...
THE SKY IS A DEEP ORANGE! CALVIN'S SKIN IS A PALE GREEN! YELLOW FLOWERS ARE NOW BLUE!
EVERY COLOR IS THE OPPOSITE OF WHAT IT SHOULD BE!
CALVIN HAS BEEN TRANSFERRED TO A COLOR FILM NEGATIVE!
HIS ONLY HOPE IS TO BE PROCESSED BY A 1-HOUR PHOTO FINISHER! DEVELOPER! I NEED DEVELOPER!
DOGGONE IT, CALVIN! THAT'S ANOTHER PICTURE RUINED! CAN'T YOU LOOK PLEASANT FOR 1/500TH OF A SECOND?!

Plan

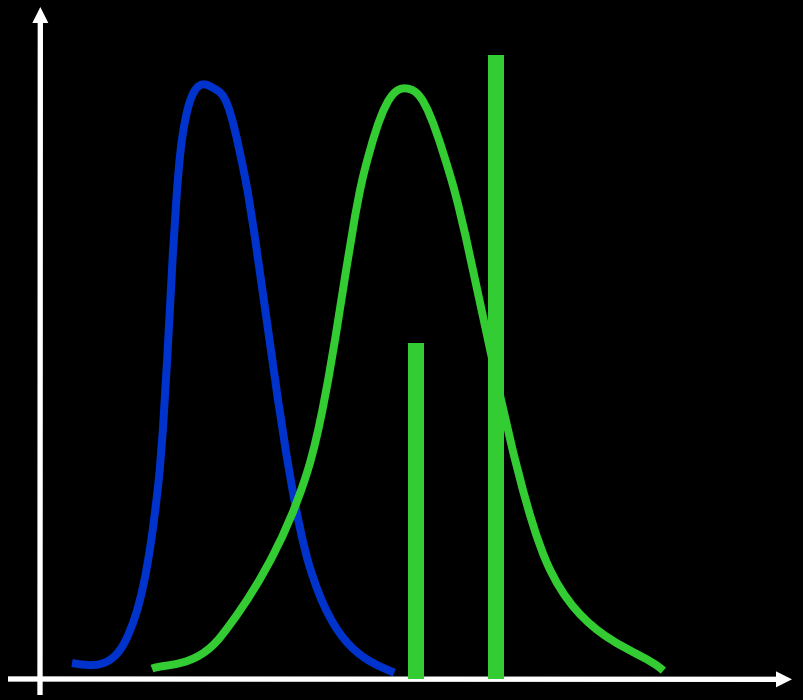
- Spectra
- Cones and spectral response
- **Color blindness and metamers**
- Color matching
- Color spaces

Color blindness

- Classical case: 1 type of cone is missing (e.g. red)
- Makes it impossible to distinguish some spectra



differentiated

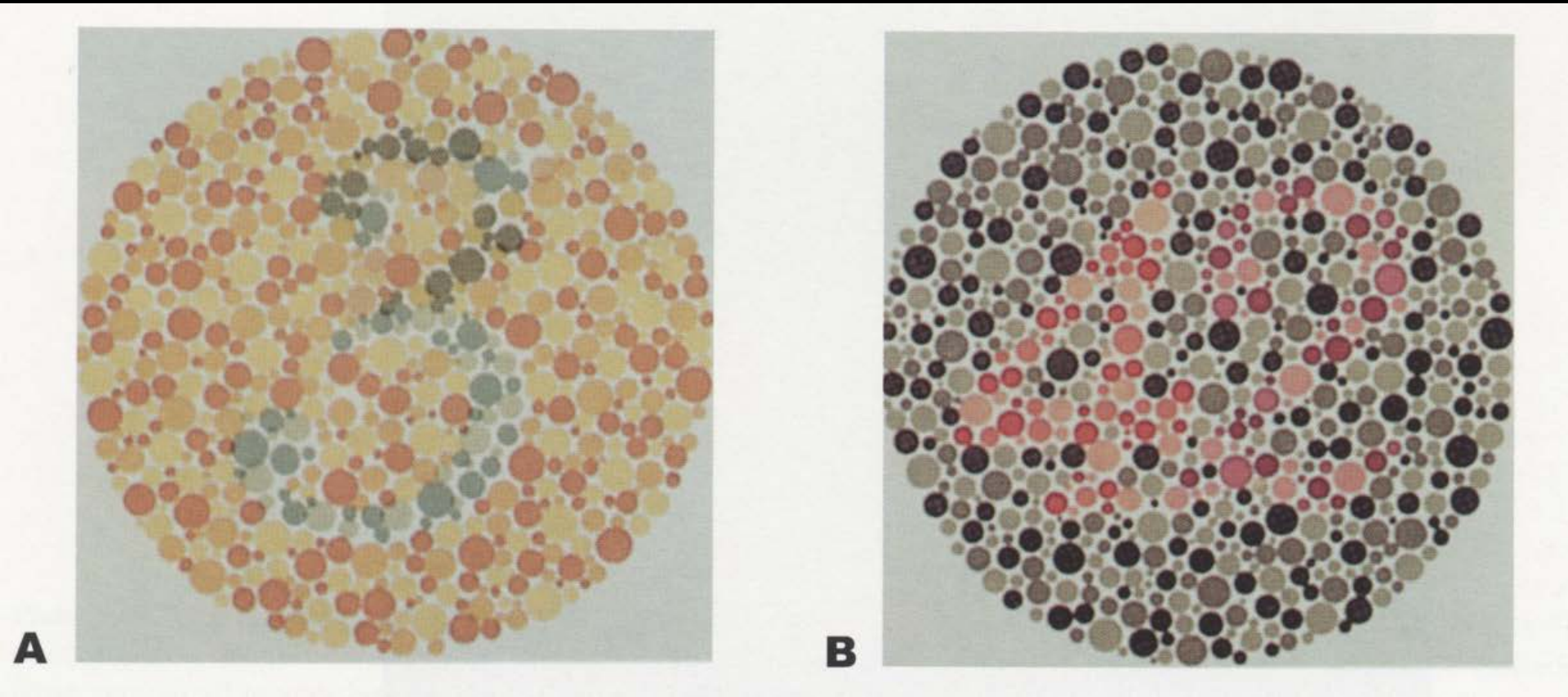


Same responses

Color blindness – more general

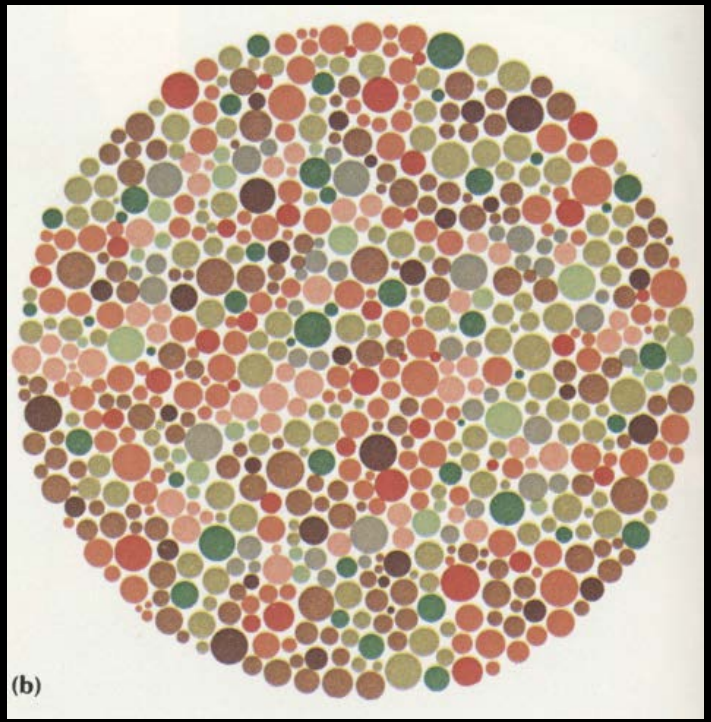
- Dalton
- 8% male, 0.6% female
- Genetic
- Dichromate (2% male)
  - One type of cone missing
  - L (protanope), M (deuteranope), S (tritanope)
- Anomalous trichromat
  - Shifted sensitivity

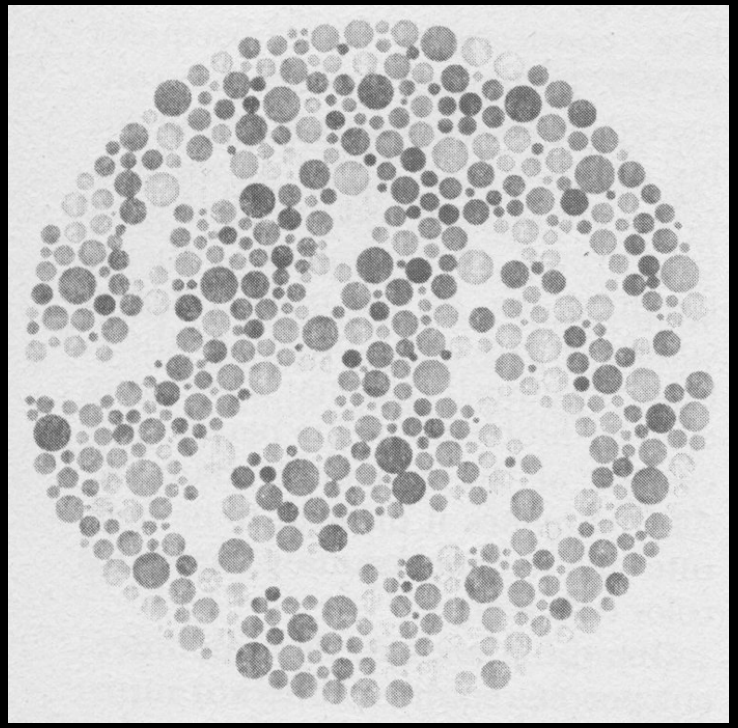
Color blindness test



Color blindness test

- Maze in subtle intensity contrast
- Visible only to color blinds
- Color contrast overrides intensity otherwise



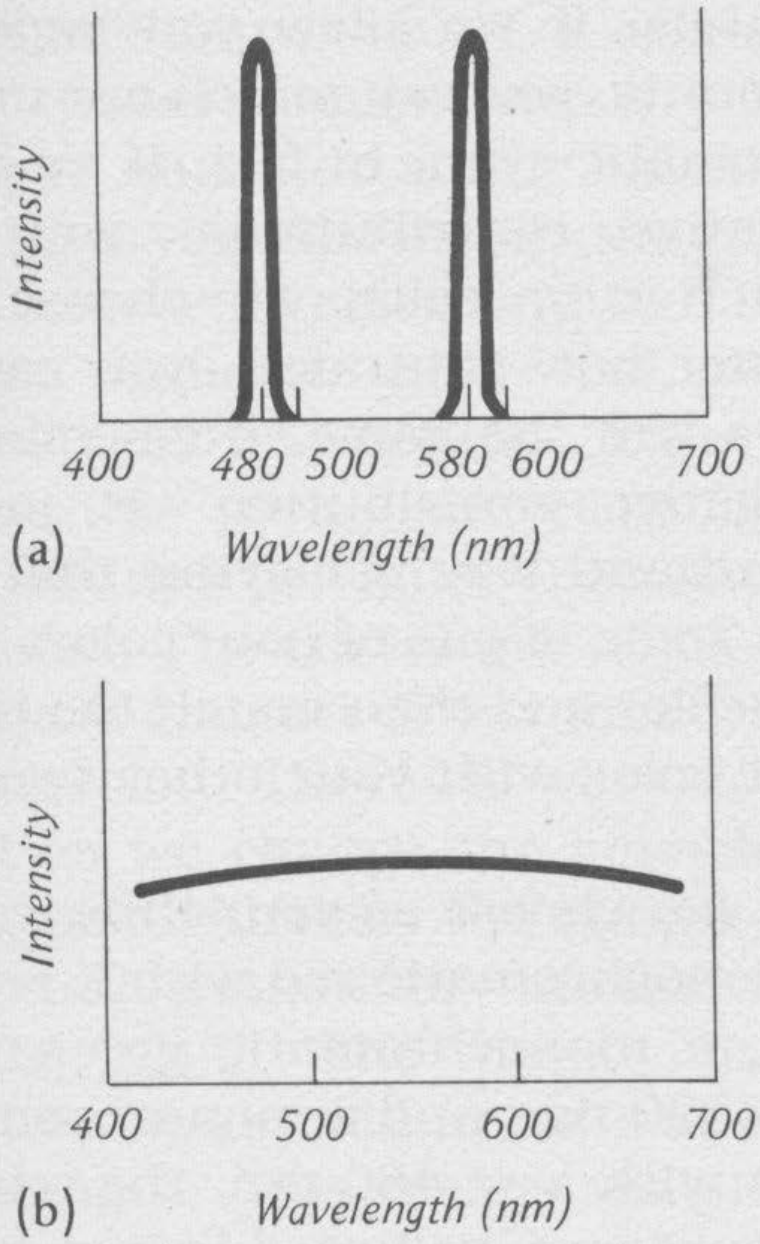


Questions?

- Links:
  - Vischeck shows you what an image looks like to someone who is colorblind.
  - http://www.vischeck.com/vischeck/
  - Daltonize, changes the red/green variation to brightness and blue/yellow variations.
  - http://www.vischeck.com/daltonize/
  - http://www.vischeck.com/daltonize/runDaltonize.php

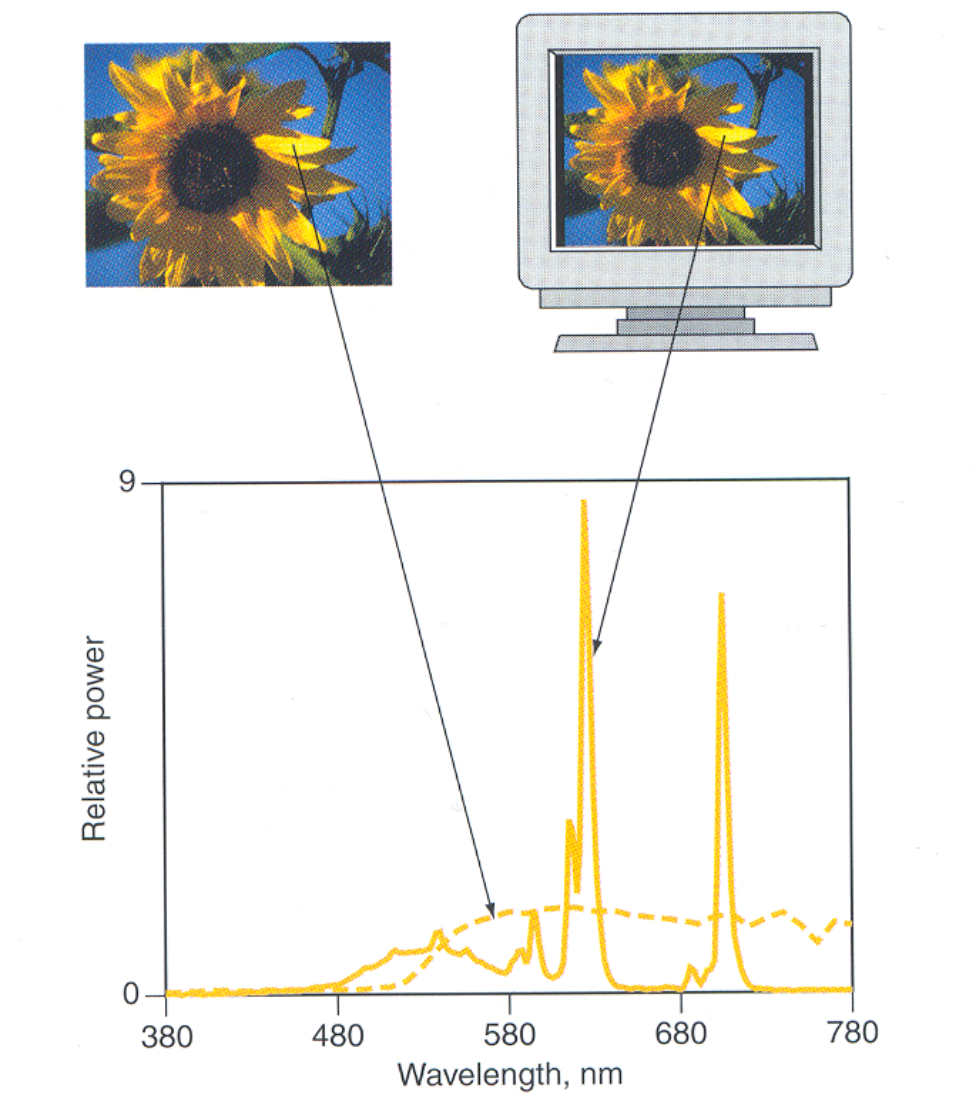
Metamers

- We are all color blind!
- These two different spectra elicit the same cone responses
- Called metamers



Good news: color reproduction

- 3 primaries are (to a first order) enough to reproduce all colors



The dashed line represents daylight reflecting from sunflower petals, while the solid line represents the light emitted by a color CRT display adjusted to match the color of the sunflower.

Recap

- Spectrum: infinite number of values
- projected according to cone spectral response => 3 values
- metamers: spectra that induce the same response (physically different but look the same)

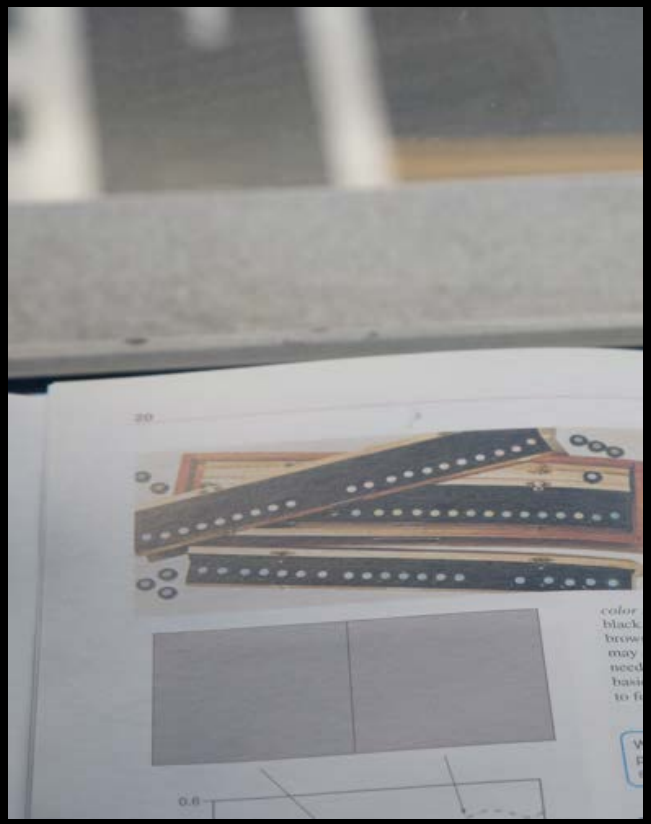
- Questions?

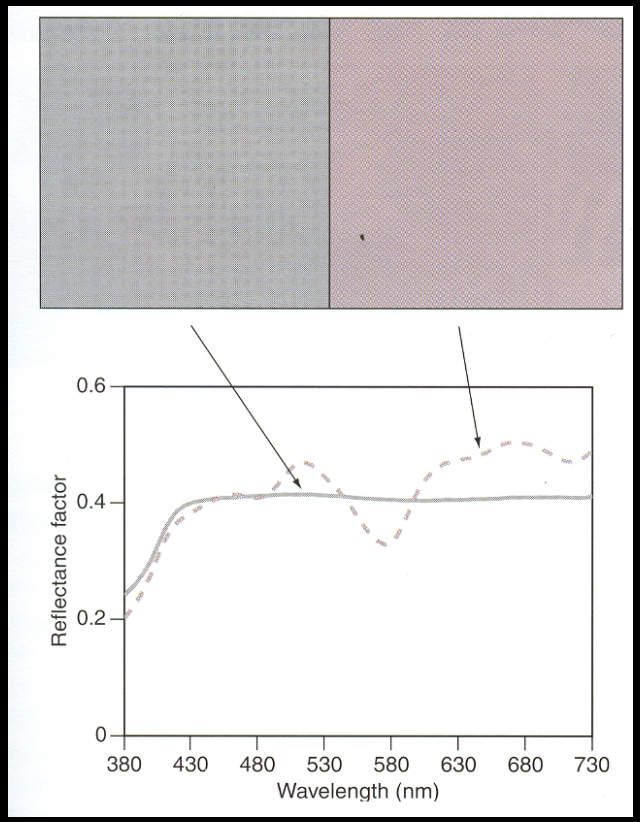
Metamerism & light source

- Metamers under a given light source
- May not be metamers under a different lamp

Illuminant metamerism example

- Two grey patches in Billmeyer & Saltzman's book
  look the same under daylight
  but different under neon or halogen
  (& my camera agrees ;-)





Daylight

Scan (neon)

Hallogen

Bad consequence: cloth matching

- Clothes appear to match in store (e.g. under neon)
- Don’t match outdoor

Recap

- Spectrum is an infinity of numbers
- Projected to 3D cone-response space
  - for each cone, multiply per wavelength and integrate
  - a.k.a. dot product
- Metamerism: infinite-D points projected to the same 3D point
  (different spectrum, same perceived color)
  - affected by illuminant
  - enables color reproduction with only 3 primaries

Questions?

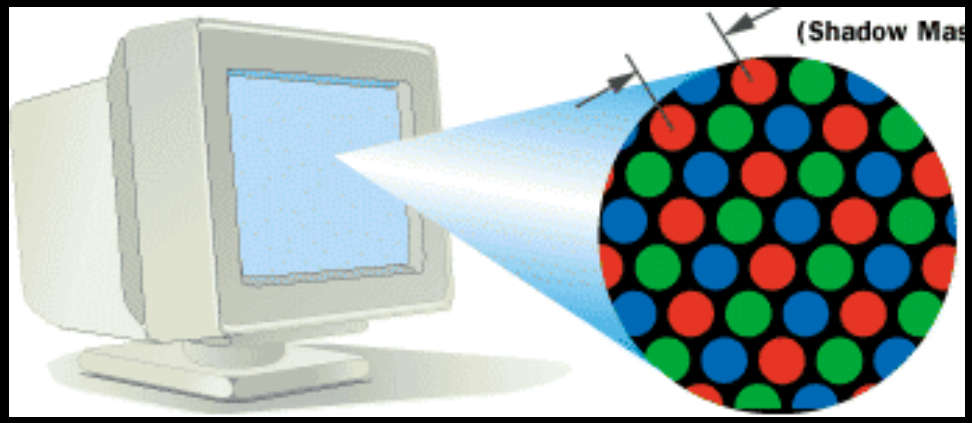
Meryon (a colorblind painter), *Le Vaisseau Fantôme*

Analysis & Synthesis

- Now let’s switch to technology
- We want to measure & reproduce color as seen by humans
- No need for full spectrum
- Only need to match up to metamerism

Analysis & Synthesis

- Focus on additive color synthesis
- We'll use 3 primaries (e.g. red green and blue) to match all colors



http://www.iriscam.info/PIXELS.html

- What should those primaries be?
- How do we tell the amount of each primary needed to reproduce a given target color?

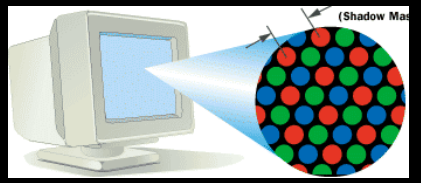
Warning

Tricky thing with spectra & color:

- Spectrum for the stimulus / synthesis
  - Light, monitor, reflectance
- Response curve for receptor /analysis
  - Cones, camera, scanner

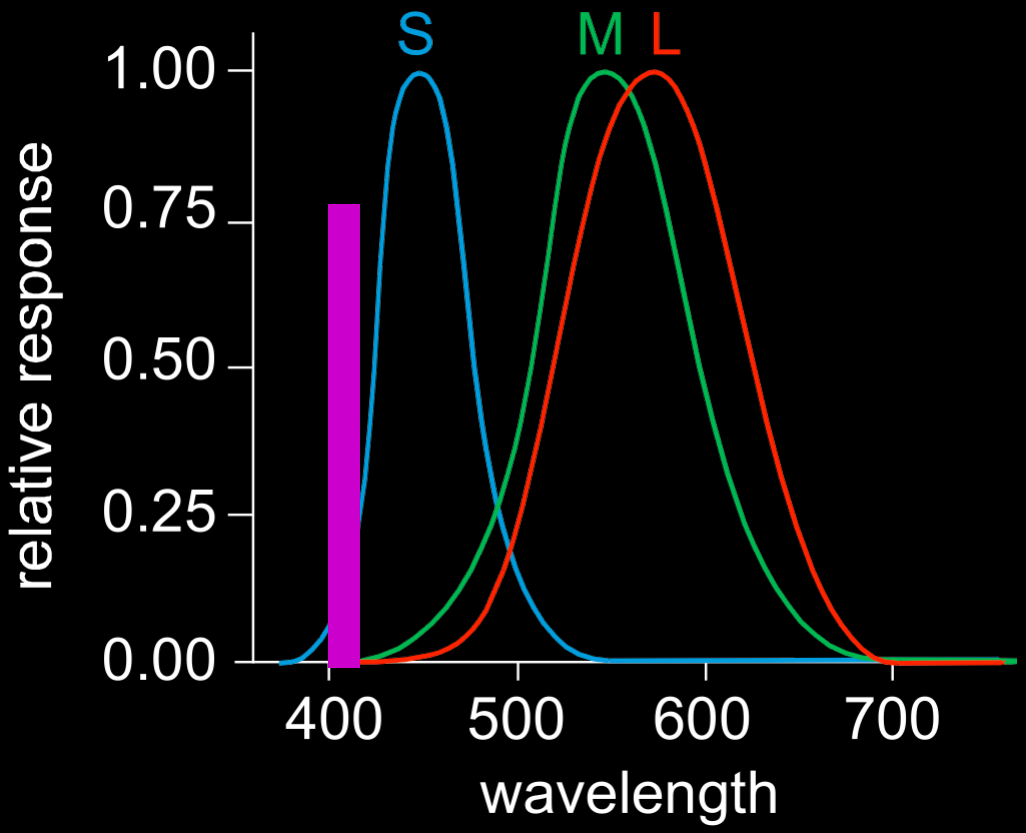
They are usually not the same

There are good reasons for this



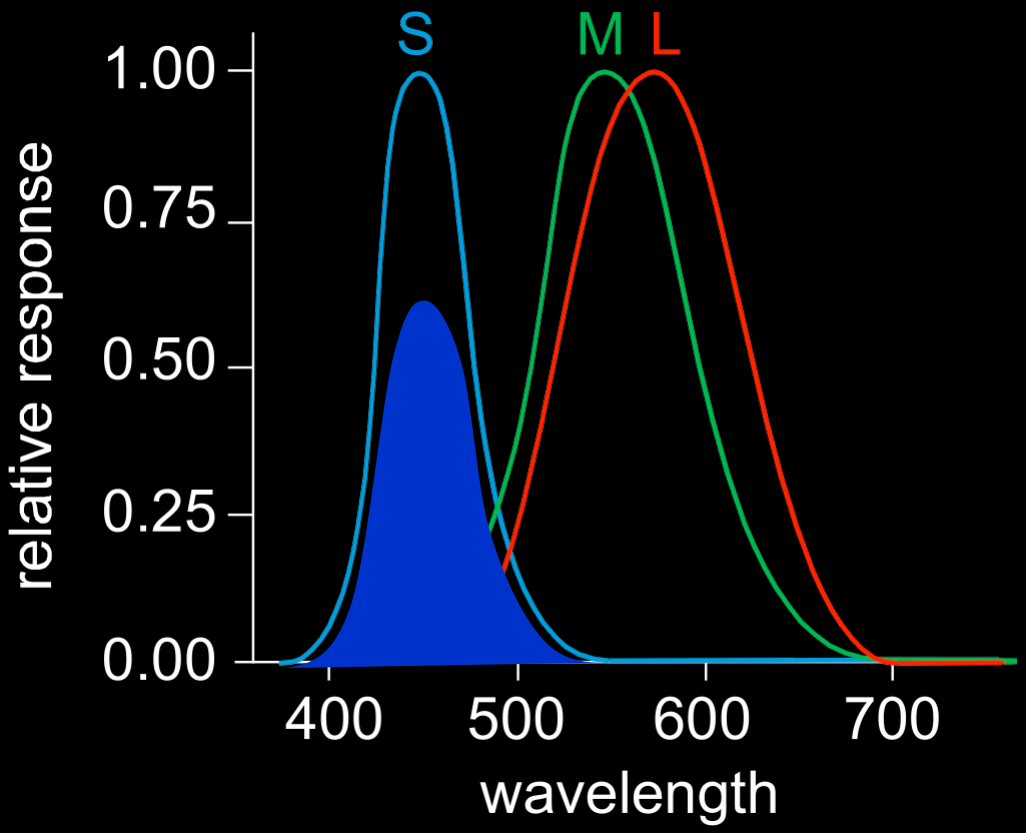
Additive Synthesis - wrong way

- Take a given stimulus and the corresponding responses s, m, l (here 0.5, 0, 0)



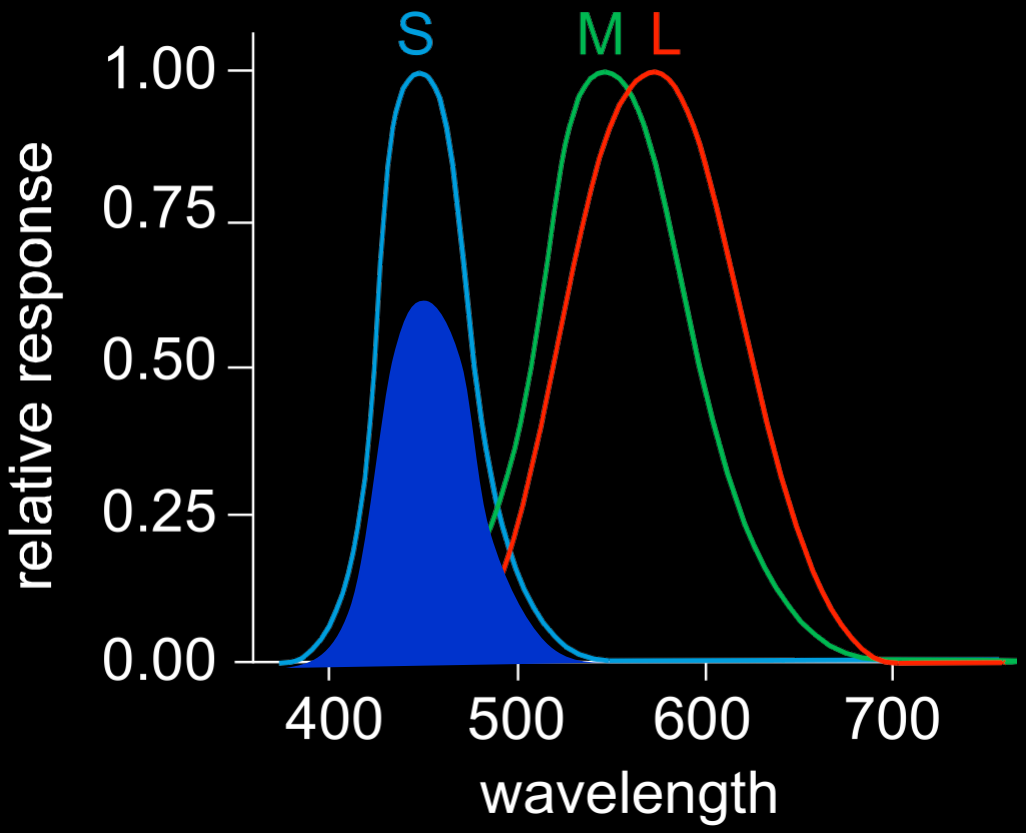
Additive Synthesis - wrong way

- Use it to scale the cone spectra (here 0.5 * S)
- You don’t get the same cone response!
  (here 0.5, 0.1, 0.1)



What's going on?

- The three cone responses are not orthogonal
- i.e. they overlap and "pollute" each other



Fundamental problems

- Spectra are infinite-dimensional
- Only positive values are allowed
- Cones are non-orthogonal/overlap

Summary

- Physical color
  - Spectrum
  - multiplication of light & reflectance spectrum
- Perceptual color
  - Cone spectral response: 3 numbers
  - Metamers: different spectrum, same responses
    - Color matching, enables color reproduction with 3 primaries
- Fundamental difficulty
  - Spectra are infinite-dimensional (full function)
  - Projected to only 3 types of cones
  - Cone responses overlap / they are non-orthogonal
    - Means different primaries for analysis and synthesis
  - Negative numbers are not physical

Questions?

Standard color spaces

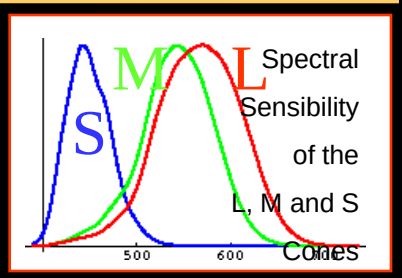
- We need a principled color space
- Many possible definition
  - Including cone response (LMS)
  - Unfortunately not really used,
    (because not known at the time)

- The good news is that color vision is linear and 3-dimensional, so any new color space based on color matching can be obtained using 3x3 matrix
  - But there are also non-linear color spaces
    (e.g. Hue Saturation Value, Lab)

Overview

- Most standard color space: CIE XYZ
- LMS and the various flavor of RGB are just linear transformations of the XYZ basis
  - 3x3 matrices

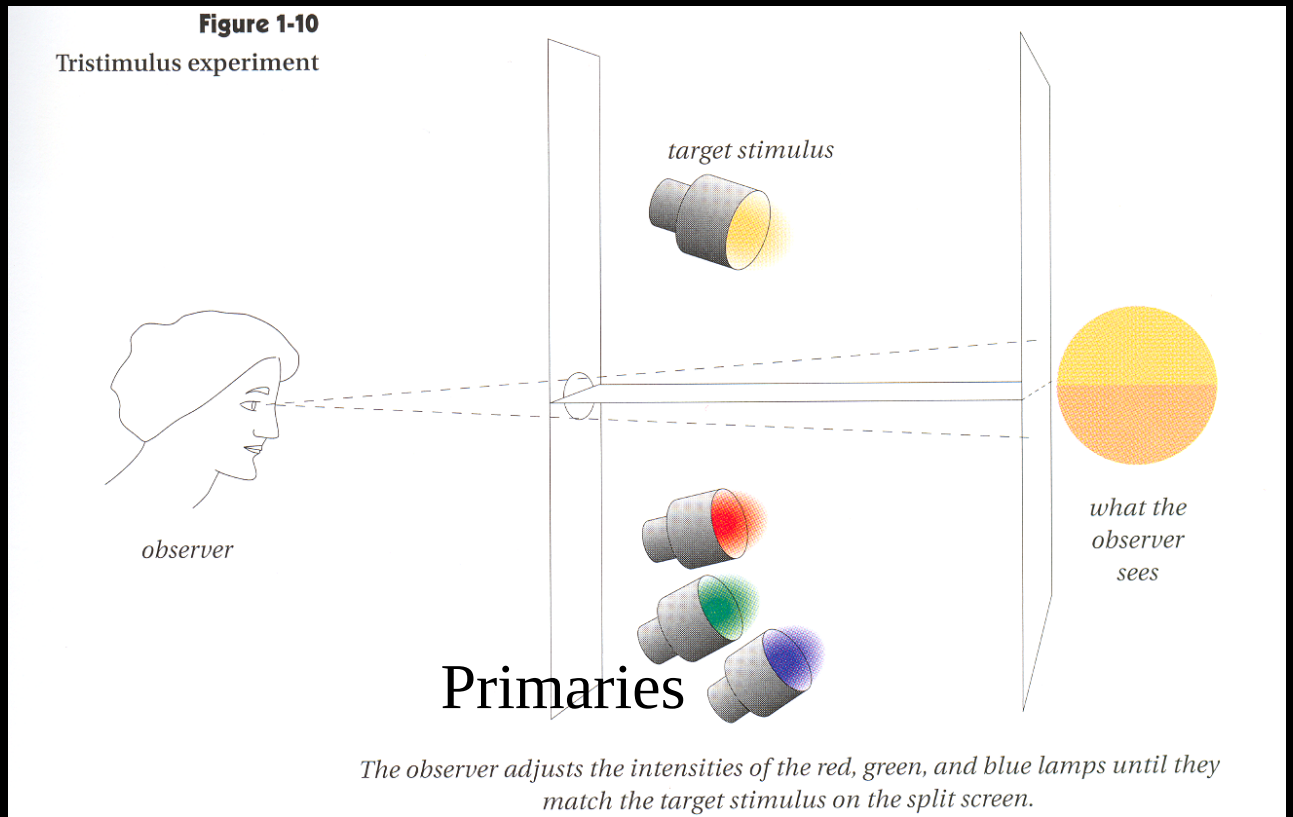
Why not measure cone sensitivity?



- Less directly measurable
  - electrode in photoreceptor?
  - not available when color spaces were defined
- Most directly available measurement:
  - notion of metamers & color matching
  - directly in terms of color reproduction:
    given an input color,
    how to reproduce it with 3 primary colors?
  - Commission Internationale de l'Eclairage (International Lighting Commission)
  - Circa 1920

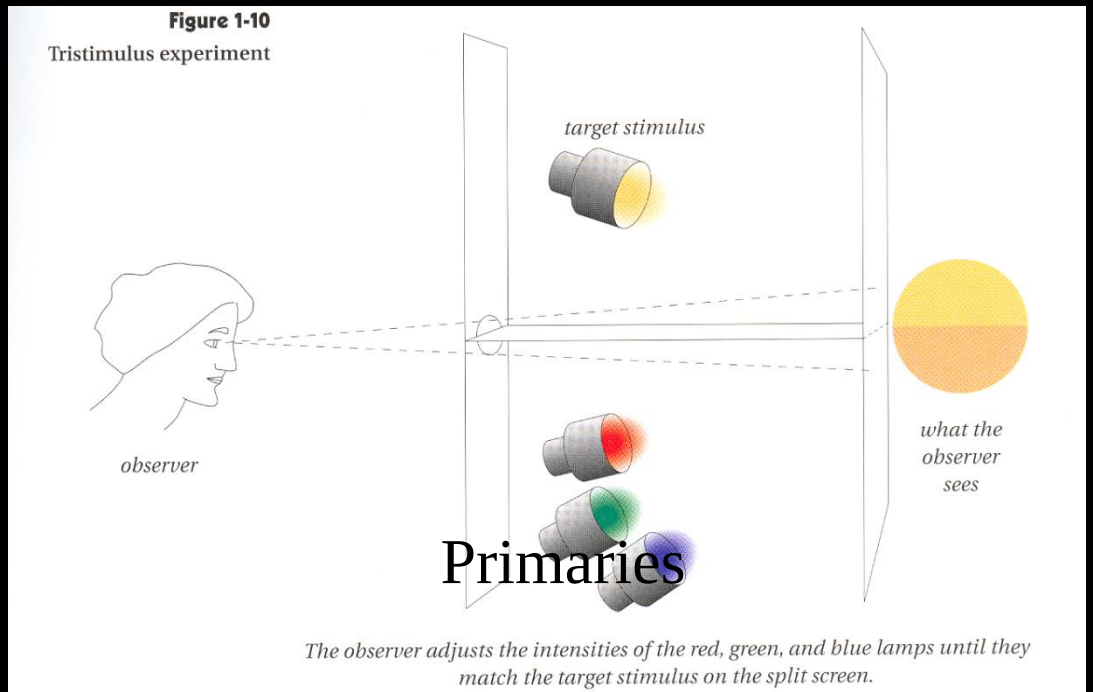
CIE color matching

- Choose 3 synthesis primaries
- Seek to match any monochromatic light (400 to 700nm)
  - Record the 3 values for each wavelength
- By linearity, this tells us how to match any light



CIE color matching

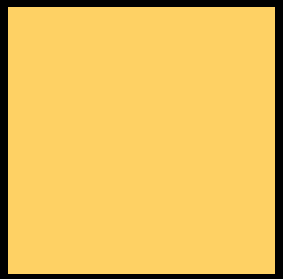
- Primaries (synthesis) at 435.8, 546.1 and 700nm
  - Chosen for robust reproduction, good separation in red-green
  - Don’t worry, we’ll be able to convert it to any other set of primaries (Linear algebra to the rescue!)
- Resulting 3 numbers for each input wavelength are called tristimulus values



Now, our interactive feature!

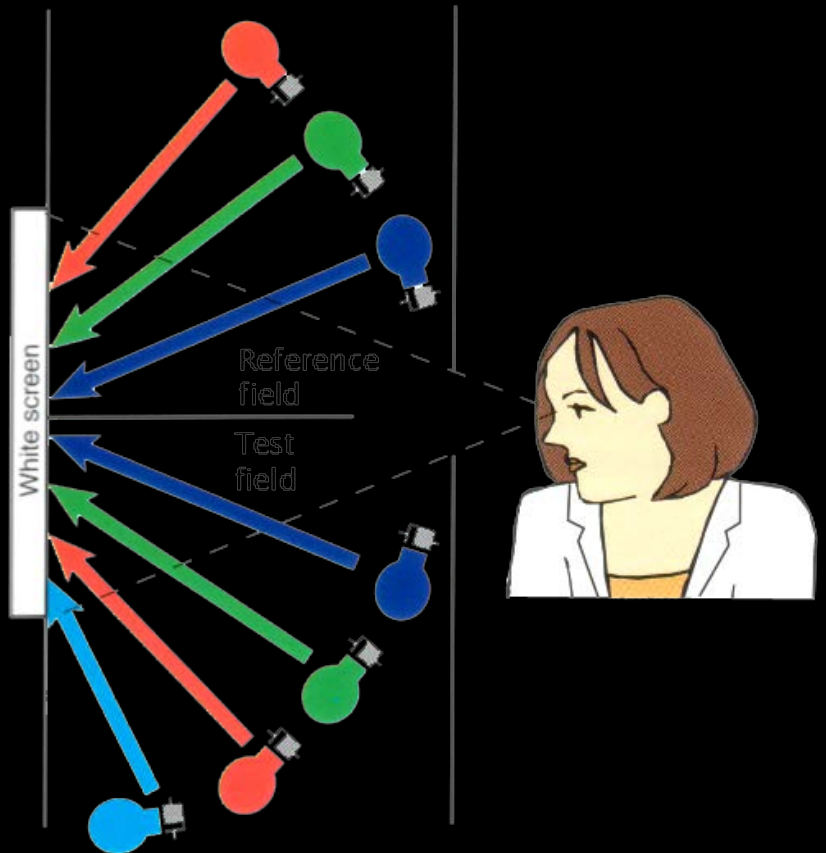
You are...

THE LAB RAT



Color Matching Problem

- Some colors cannot be produced using only positively weighted primaries
- Solution: add light on the other side!

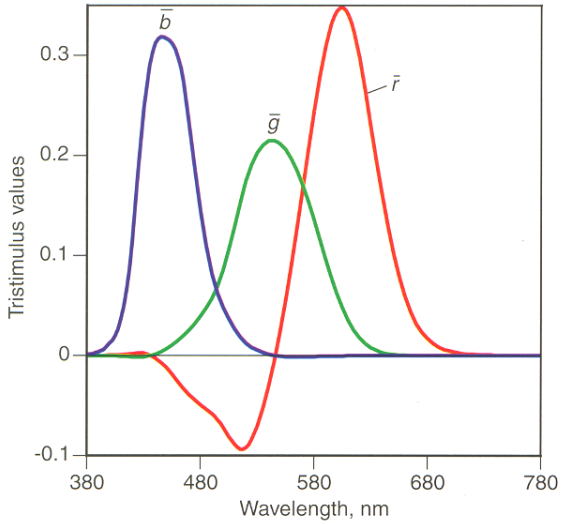


CIE color matching

- Meaning of these curves: a monochromatic wavelength λ can be reproduced with b(λ) amount of the 435.8nm primary, +g(λ) amount of the 546.1 primary, +r(λ) amount of the 700 nm primary

- This fully specifies the color perceived by a human

- Careful: this is not your usual rgb

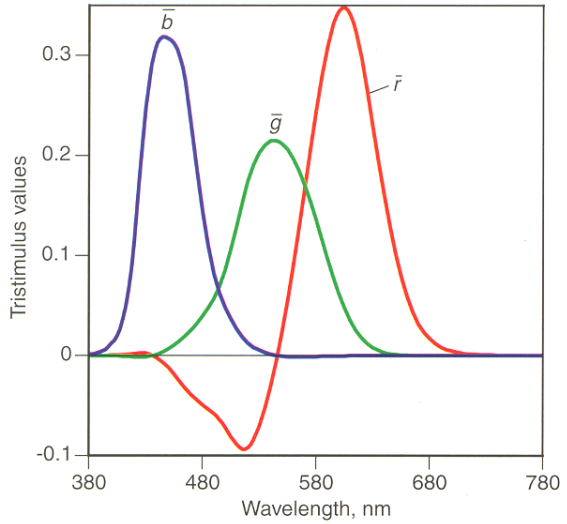


These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

CIE color matching

- Meaning of these curves: a monochromatic wavelength λ can be reproduced with b(λ) amount of the 435.8nm primary, +g(λ) amount of the 546.1 primary, +r(λ) amount of the 700 nm primary

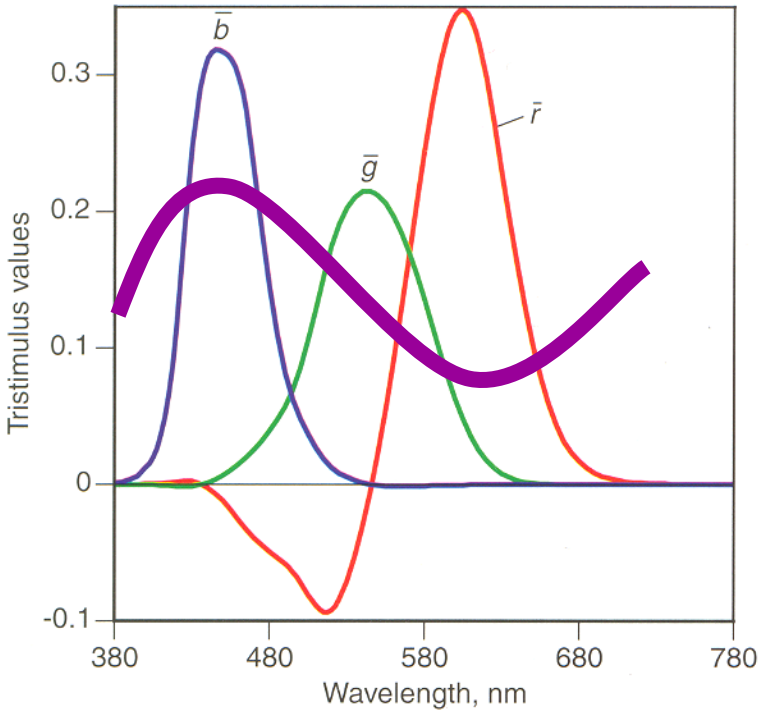
- This fully specifies the color perceived by a human
- However, note that one of the responses can be negative
  - Those colors cannot be reproduced by those 3 primaries.



These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

CIE color matching: what does it mean?

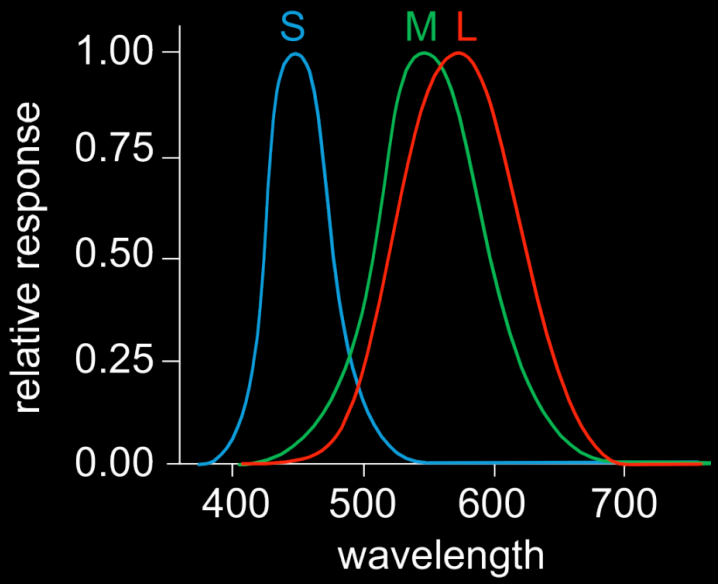
- If I have a given spectrum X
- I compute its response to the 3 matching curves (multiply and integrate)
- I use these 3 responses to scale my 3 primaries (435.8, 546.1 and 700nm)
- I get a metamer of X (perfect color reproduction)

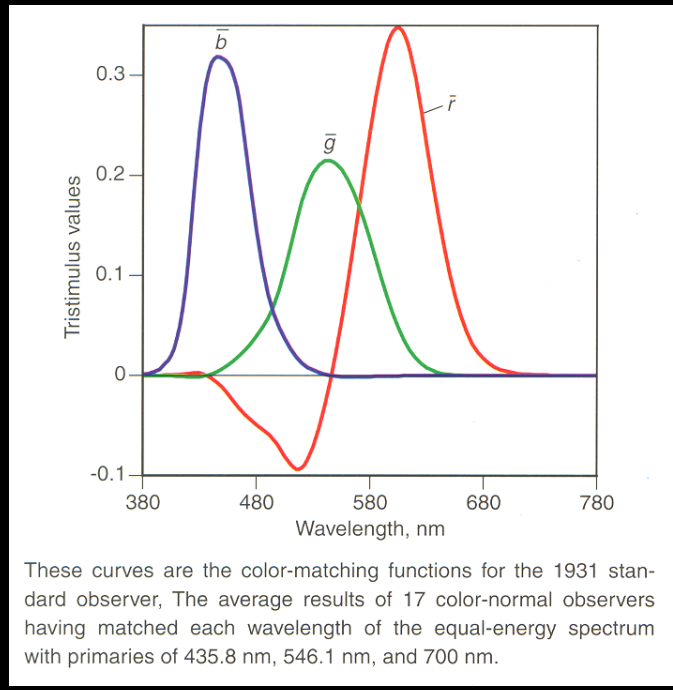


These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

Relation to cone curves

- Project to the same subspace
  - b, g, and r are linear combinations of S, M and L
- Related by 3x3 matrix.
- Unfortunately unknown at that time. This would have made life a lot easier!





These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

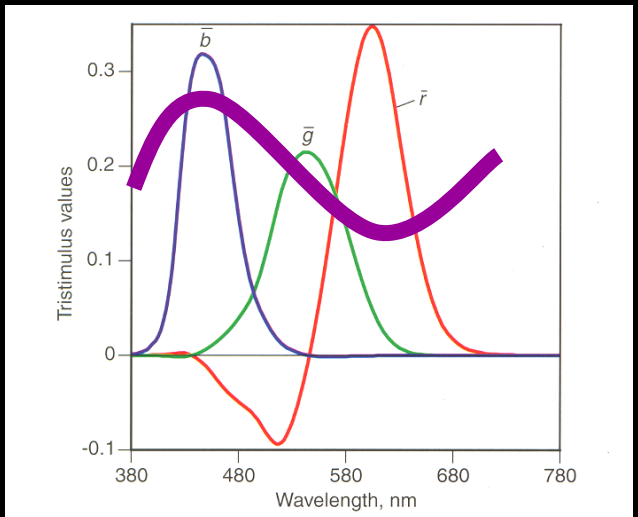
Recap

- Spectra : infinite dimensional
- Cones: 3 spectral responses
- Metamers: spectra that look the same (same projection onto cone responses)
- CIE measured color response:
  - chose 3 primaries
  - tristimulus curves to reproduce any wavelength

- Questions?

How to build a measurement device?

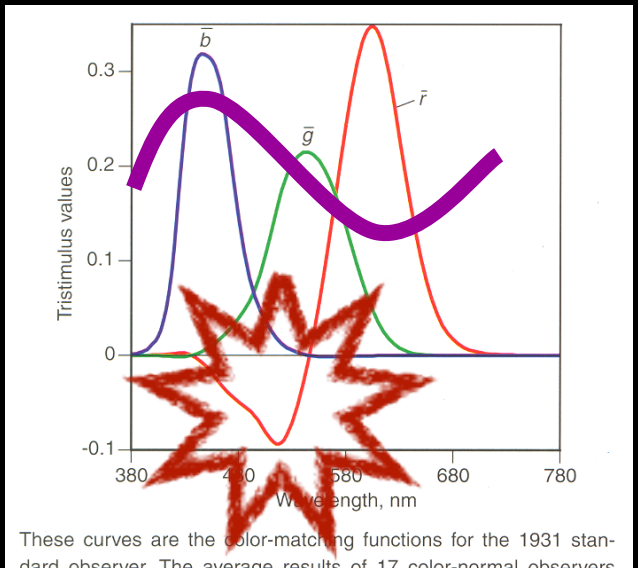
- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra b, r, g
  - measure 3 numbers
- This is pretty much what the eyes do!



These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

CIE's problem

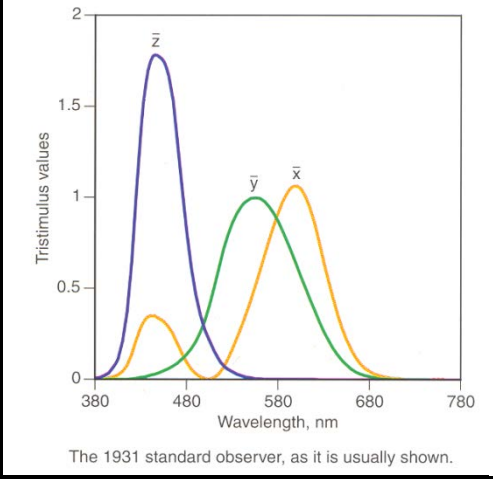
- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra b, r, g
  - measure 3 numbers
- But for those primaries, we need negative spectra



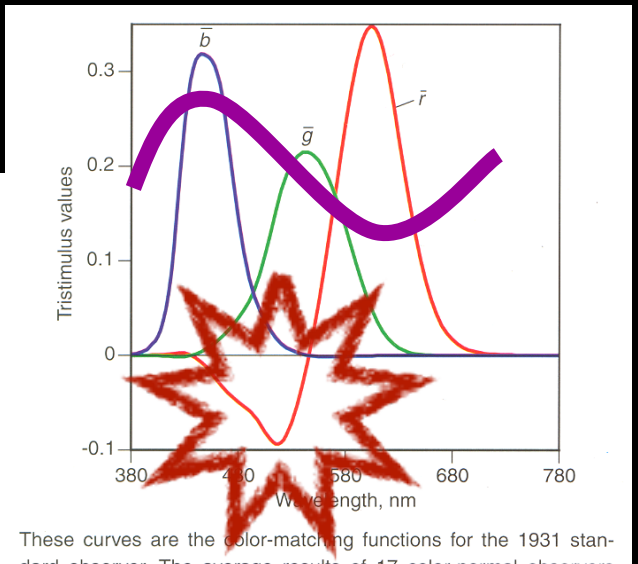
These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

CIE's problem

- Obvious solution:
  use cone response!
  - but unknown at the time
- =>new set of tristimulus curves
  - linear combinations of b, g, r
  - pretty much add enough b and g until r is positive



The 1931 standard observer, as it is usually shown.



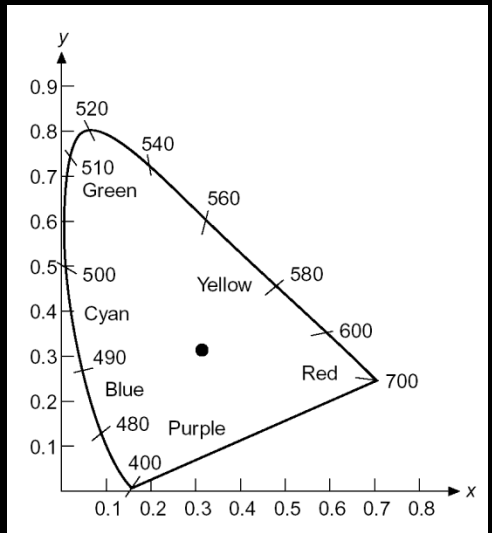
These curves are the color-matching functions for the 1931 standard observer, The average results of 17 color-normal observers having matched each wavelength of the equal-energy spectrum with primaries of 435.8 nm, 546.1 nm, and 700 nm.

Chromaticity diagrams

- 3D space are tough to visualize
- Usually project to 2D for clarity
- Chromaticity diagram:
  - normalize against X + Y + Z:

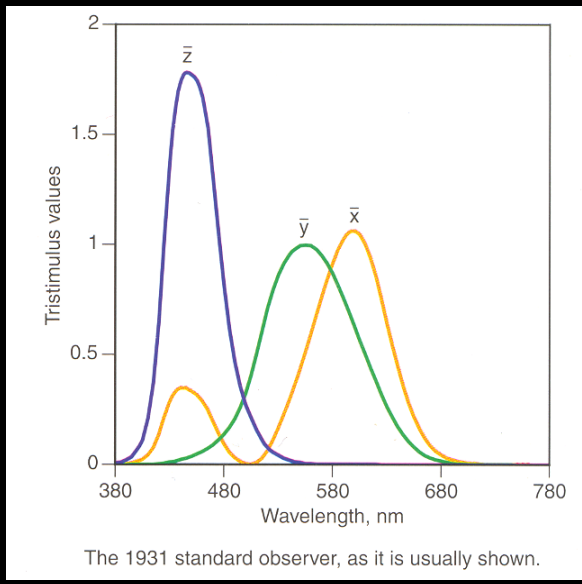
$$x = \frac{X}{X+Y+Z}; \quad y = \frac{Y}{X+Y+Z}; \quad z = \frac{Z}{X+Y+Z}$$

  - Perspective projection to plane X+Y+Z=1



CIE XYZ -recap

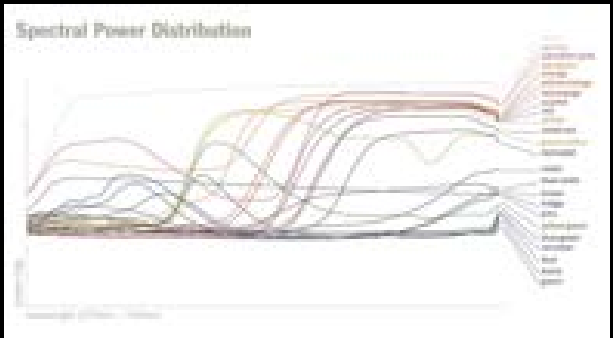
- THE standard for color specification
- Lots of legacy decision - I wish it were LMS
- Based on color matching
  - 3 monochromatic primaries
  - Subjects matched every wavelength
  - Tricks to avoid negative numbers
  - These 3 values “measure” or describe a perceived color.

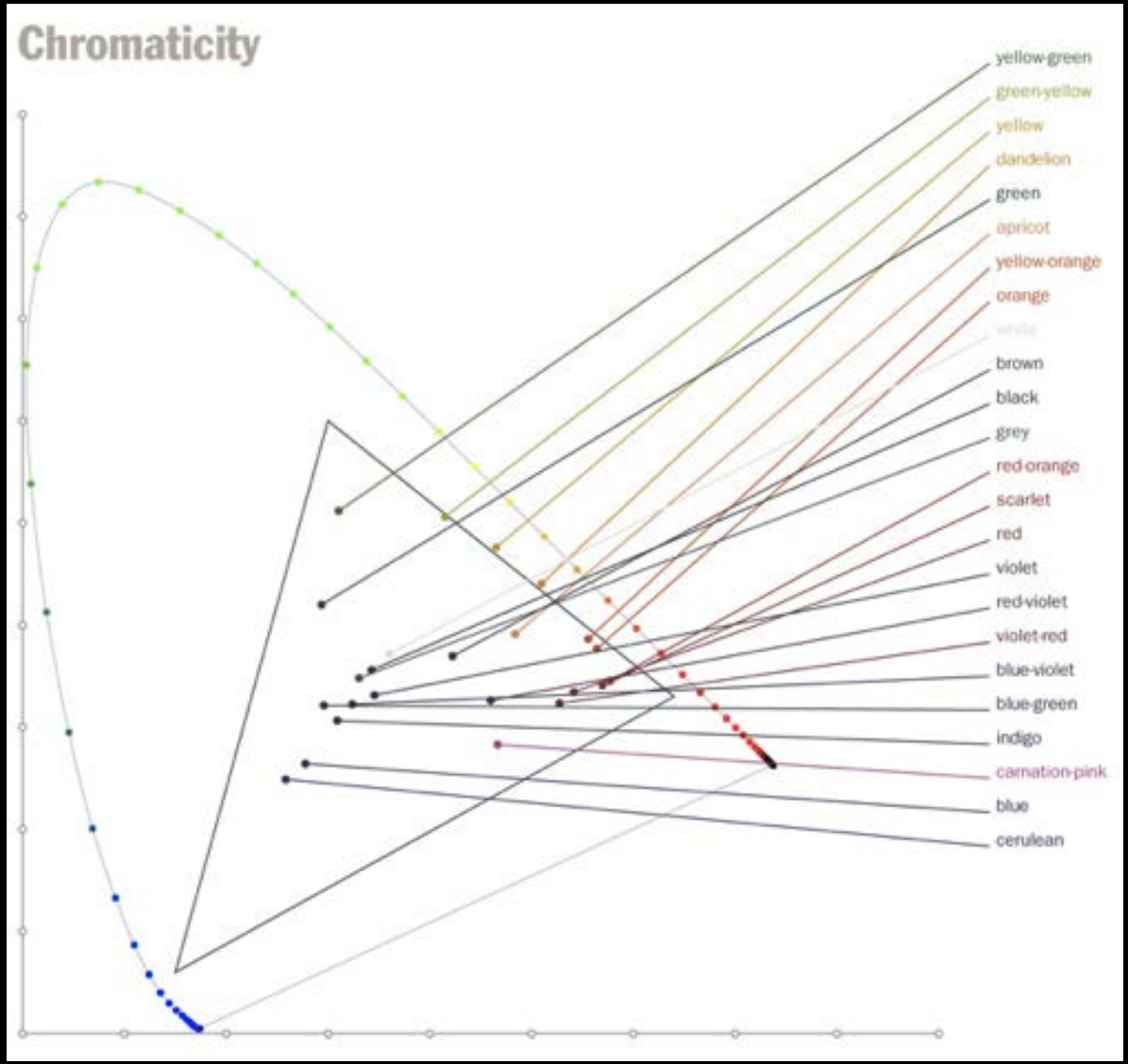


The 1931 standard observer, as it is usually shown.

Crayons

- http://www.photo-mark.com/notes/2011/sep/20/crayon-colors/





Questions?

Other primaries

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching,
  can be obtained from XYZ by a 3x3 matrix

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

one example RGB space

Other primaries

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching,
  can be obtained from XYZ by a 3x3 matrix
- This matrix tells us how to match the 3 primary spectra from XYZ using the new 3 primaries

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

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one example RGB space

XYZ to RGB & back

- e.g. http://www.brucelindbloom.com/index.html?Eqn_RGB_XYZ_Matrix.html
- sRGB to XYZ

0.412424	0.212656	0.0193324
0.357579	0.715158	0.119193
0.180464	0.0721856	0.950444

XYZ to sRGB

3.24071	-0.969258	0.0556352
-1.53726	1.87599	-0.203996
0.498571	0.0415557	1.05707

- Adobe RGB to XYZ

0.576700	0.297361	0.0270328
0.185556	0.627355	0.0706879
0.188212	0.0752847	0.991248

XYZ to Adobe RGB

2.04148	-0.969258	0.0134455
-0.564977	1.87599	-0.118373
-0.344713	0.0415557	1.01527

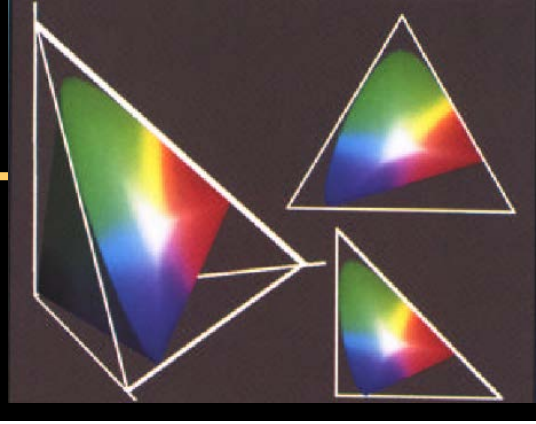
- NTSC RGB to XYZ

0.606734	0.298839	0.000000
0.173564	0.586811	0.0661196
0.200112	0.114350	1.11491

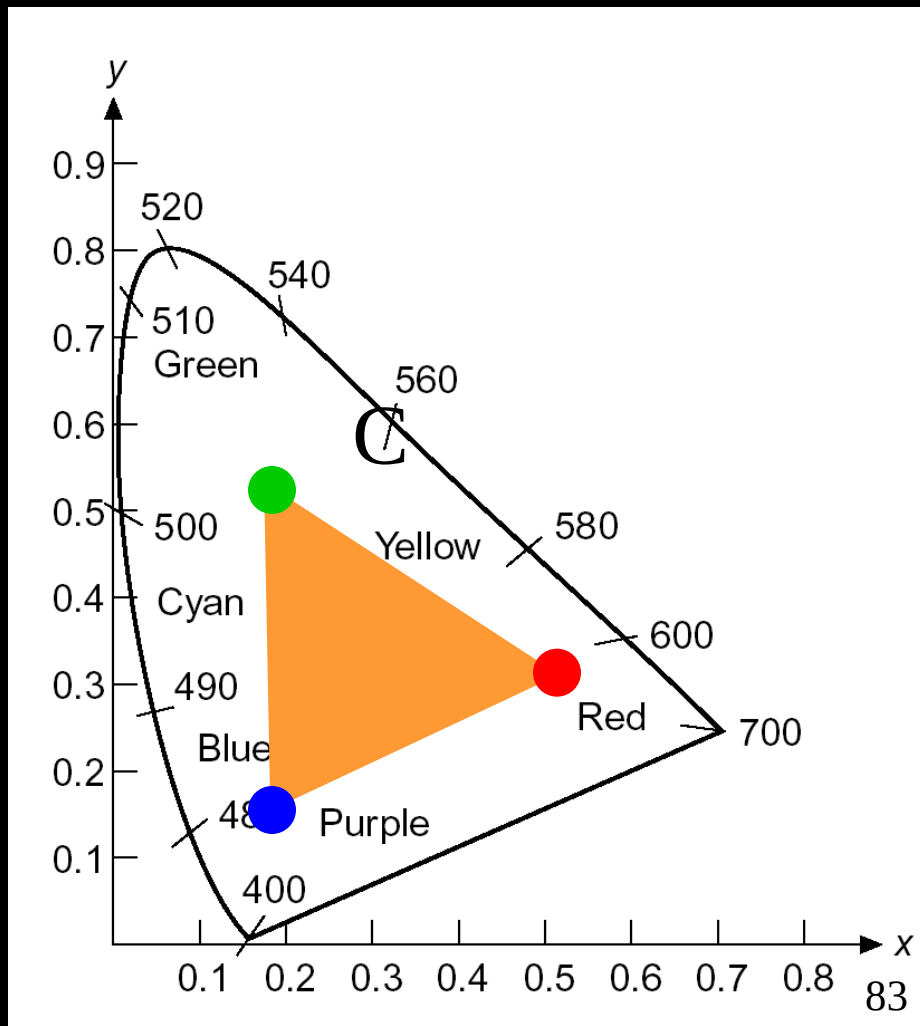
XYZ to NTSC RGB

1.91049	-0.984310	0.0583744
-0.532592	1.99845	-0.118518
-0.288284	-0.0282980	0.898611

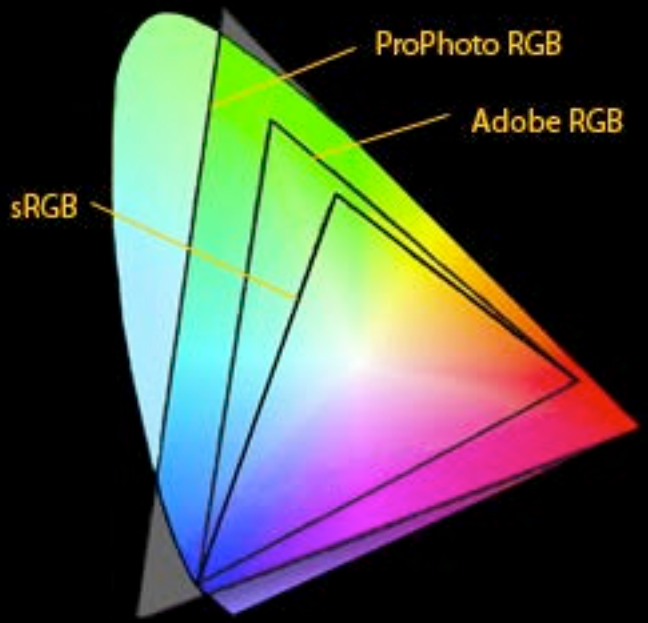
Color gamut

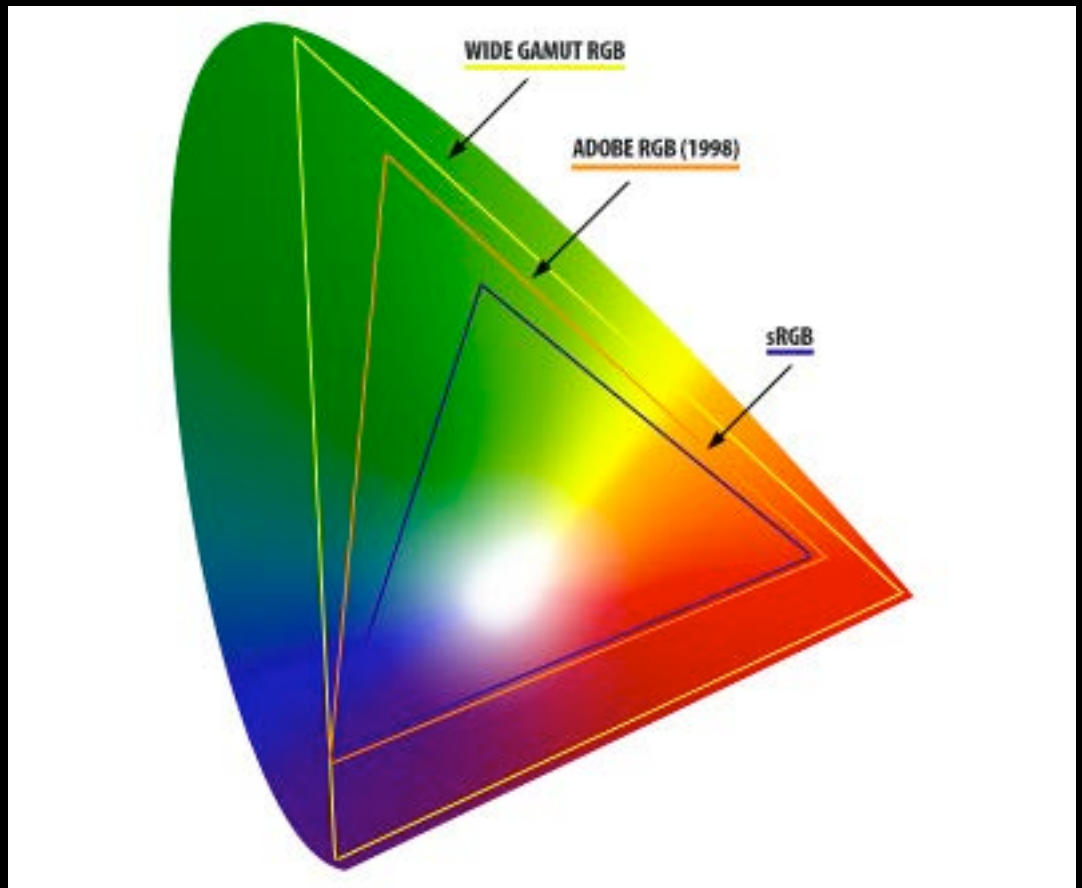


- Given 3 primaries
- The realizable chromaticities lay in the triangle in xy chromaticity diagram
- Because we can only add light, no negative light



- http://dba.med.sc.edu/price/irf/Adobe_tg/manage/images/gamuts.jpg
- http://www.petrvodnakphotography.com/Articles/ColorSpace.htm





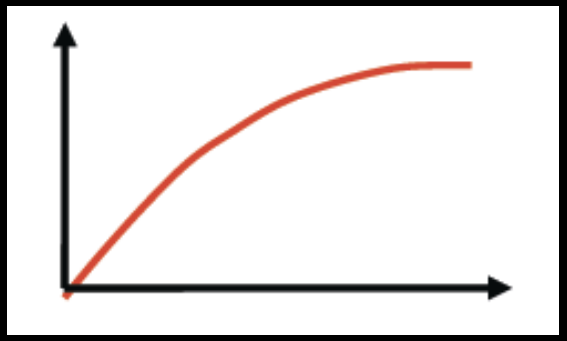
In summary

- It’s all about linear algebra
  - Projection from infinite-dimensional spectrum to a 3D response
  - Then any space based on color matching and metamerism can be converted by 3x3 matrix
- Complicated because
  - Projection from infinite-dimensional space
  - Non-orthogonal basis (cone responses overlap)
  - No negative light
- XYZ is the most standard color space
- RGB has many flavors

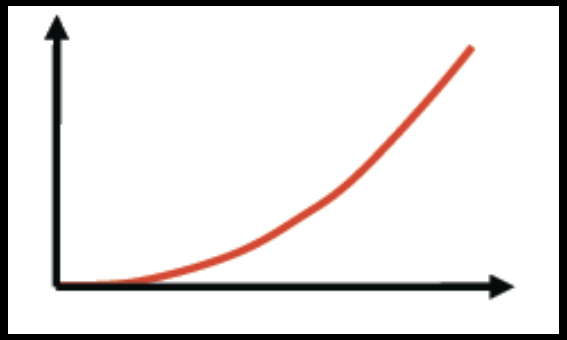
Questions?

Gamma encoding overview

- Digital images are usually not encoded linearly
- Instead, the value $\mathbf{X}^{1/\gamma}$ is stored



- Need to be decoded if we want linear values



Color quantization gamma

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- Ideal encoding?

  Log
- Problems with log?

  Gets crazy around zero

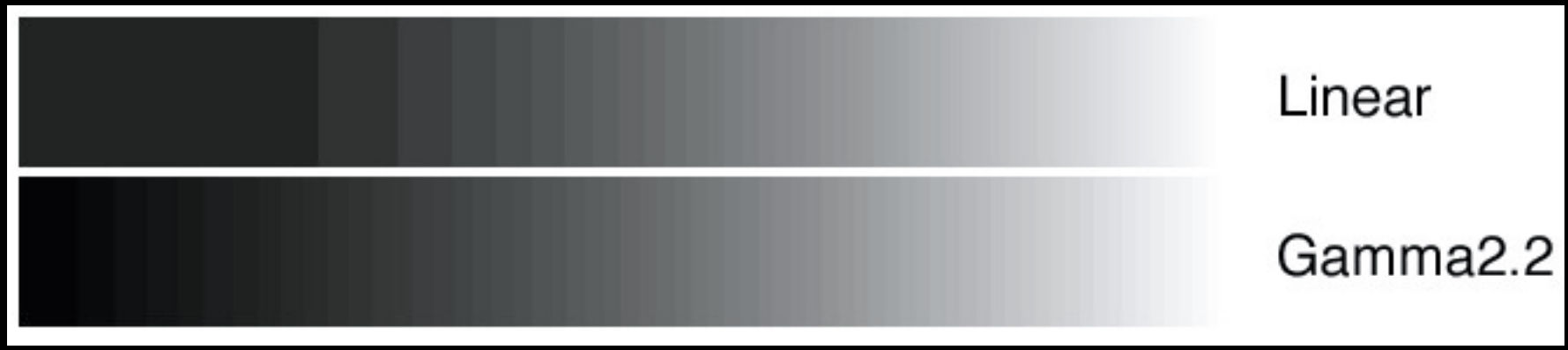
Solution: gamma

Color quantization gamma

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- This is why a non-linear gamma remapping of about 2.0 is applied before encoding
- True also of analog imaging to optimize signal-noise ratio

Gamma encoding

- From Greg Ward
- Only 6 bits for emphasis



Important Message

- Digital images are usually gamma encoded
  - Often $\gamma = 2.2$ (but 1.8 for Profoto RGB)
- To get linear values, you must decode
  - apply $x \Rightarrow x^{\gamma}$

Questions?

Selected Bibliography

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MIT Press; ISBN: 0262161834
760 pages (May 7, 1999)

Billmeyer and Saltzman's Principles of Color Technology, 3rd Edition
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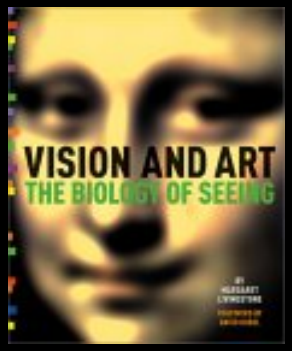

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208 pages (May 2002)

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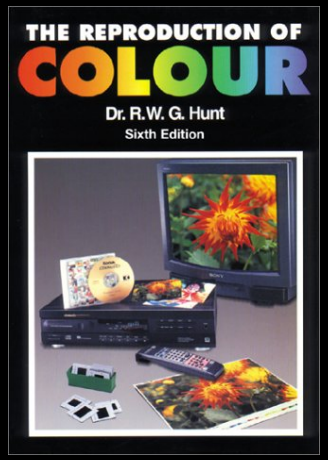
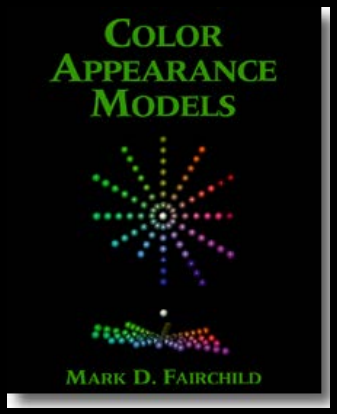


The Reproduction of Color
by R. W. G. Hunt
Fountain Press, 1995



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by Mark Fairchild
Addison Wesley, 1998

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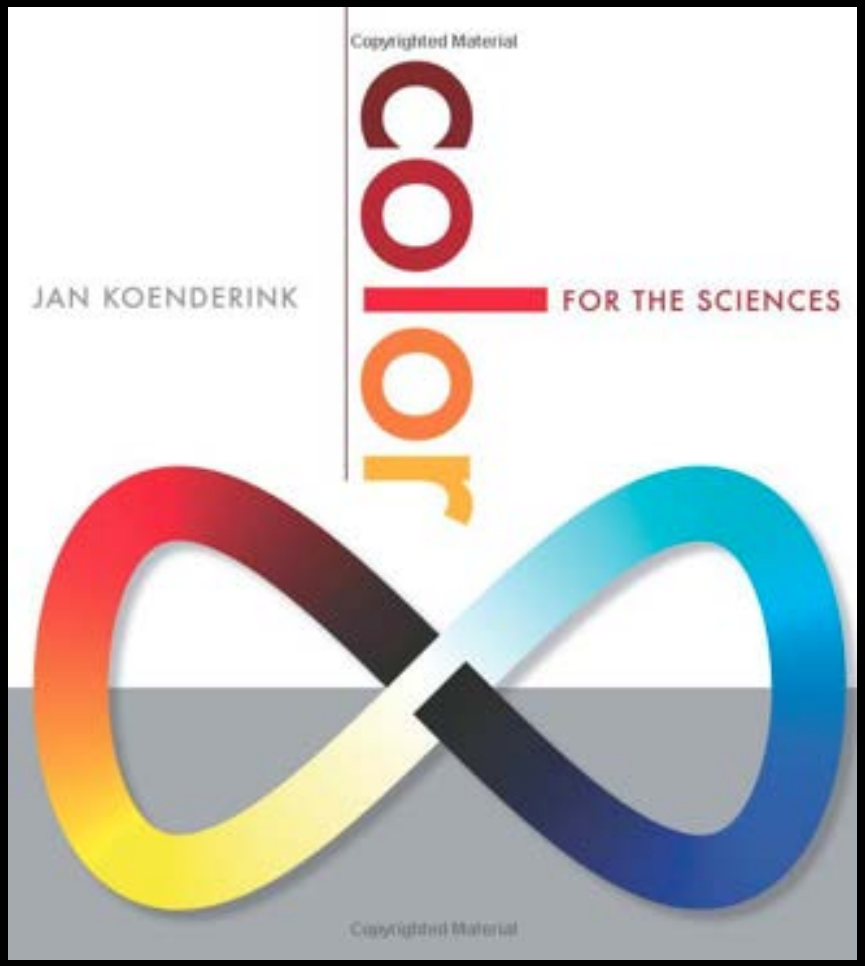

Questions?

VIII. Philipp Otto Runge, *Colour Sphere*, 1809, Hamburg Kunsthalle.