Operations Research for Supply Chains I

ISyE 6333, Fall 2023

Supermarket Sweep Project, Due: December 3, 2023

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Project Rules:

1. Each team should hand in one set of solutions and one slide kit on Canvas.
2. You may *not* interact with fellow students outside of your team when conducting your project.
3. The solutions and Python codes should be submitted electronically on Canvas prior to the deadline. Any handwritten solutions should be scanned so that they are easily read by the TA, and then submitted electronically. Typed solutions are preferred.
4. When formulating optimization problems, clearly state the parameters, decision variables, constraints, and objective function of your formulation. If you are using compact notations, i.e., $\sum$, $\prod$, $\forall$, state the ranges of indices for each of them.
5. Show your work. Answers requiring a justification will not receive full grades if the justification is not provided. Similarly, answers requiring intermediate computations will not receive full grades if those computations are not shown in your solutions.

Problem Background

Supermarket Sweep was an American television game show that first ran in the 1960s. It was later brought back in the 1990s. And it was brought back again in the early 2000s. There are even plans to bring it back in the next few years. The concept of this game was simple: contestants had a specified amount of time to run through a supermarket and pick up items; upon returning to the starting point, they would then receive a cash prize based on the total monetary value of the goods they brought back.

For most contestants on the show, strategies were simple. Some would run straight for the most expensive items; others would choose a path that allowed them to collect as many items as possible, regardless of value. For this project, you will model the problem of optimally filling up two baskets for a team of two contestants. Then you will solve the problem using Gurobi Solver.

1. You are provided with an original data file containing information about 56 different grocery items. This information includes each item's name, monetary value, and X and Y coordinates within the store aisles.
2. A team of two shoppers has a 60-second window of time to collect items and return to the start point location. They begin at a specified start point location in the store, and must return to that point before the time limit is up.
3. Each shopper in the team can include at most 10 items in their respective cart.

4. Assume that each shopper moves at a speed of 10 feet per second, and takes 2 seconds to pick up an item and place it in their cart.
5. For simplicity, an item can only be grabbed once (by either of the shoppers).
6. Each shopper must take one continuous path through the store – no teleporting allowed!
7. The goal of the team is to maximize the total value of the items they bring back.

Supermarket Layout:

- For more clarity about the composition of the supermarket, please refer to Figure 1.
- In this layout, there are 11 vertical aisles.
- The shoppers start at the X at the bottom left corner of the layout.
- Each aisle is 100ft long and there are 10ft between the walkable areas of the aisles.
- The paths the shoppers can take are shown by the red dashed lines. Assume all items are reachable from the red dashed line.

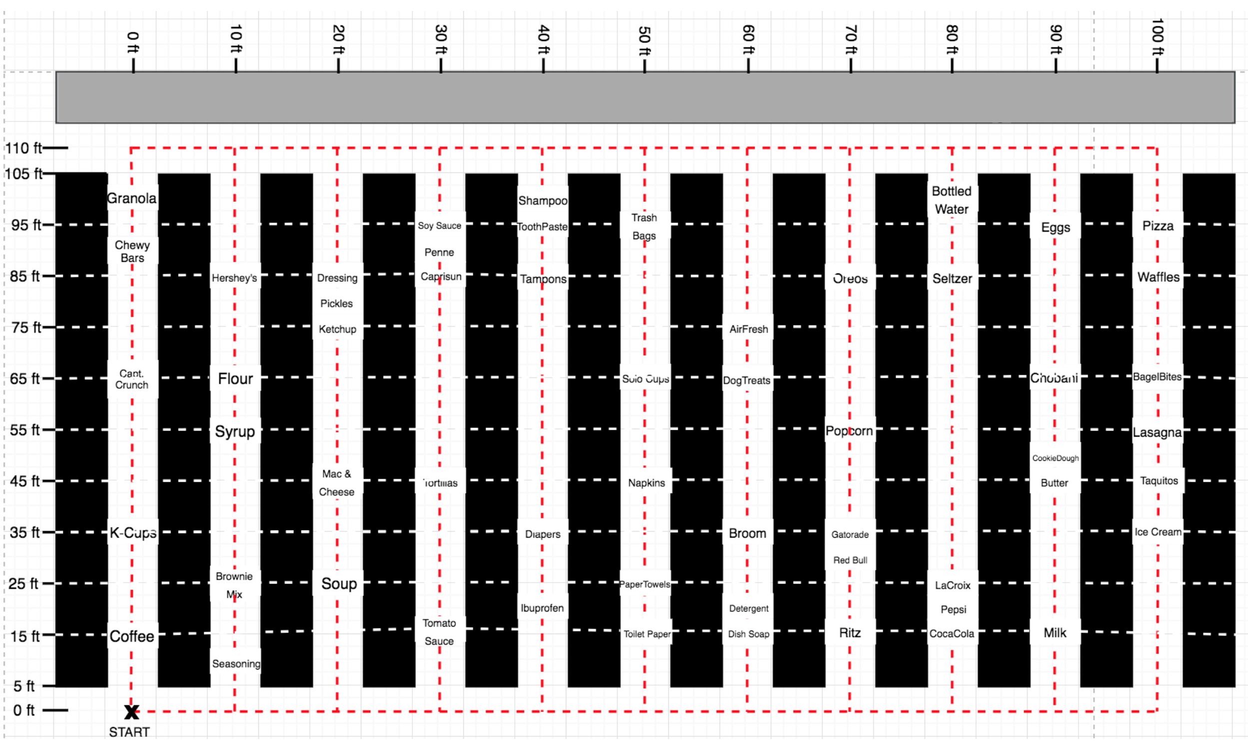

Figure 1: Supermarket layout.

Questions:

(a) (5 points) For each pair of items $i, j \in [\![1, 56]\!]$, compute the shortest time d_{ij} to go from item i to item j, and store these values. You may not use any optimization software to answer this question. Compute also the shortest time to travel between the start/end location and any item.

(b) (20 points) Formulate the Supermarket Sweep problem as a Mixed-Integer Program (optimization model with both continuous and integer variables).

   – Use a similar formulation to the TSP model with the x, y, and t decision variables. You may need to consider additional decision variables.
   – Unlike the TSP, you have two people traveling and you don't need to go to every item.
   – Use the shortest times d_{ij} computed in (a) as parameters.

(c) (10 points) Code and solve your optimization model. What are the optimal paths? Which items are picked? What is the total value of these items?

   – Tip: It will be helpful to first code and solve the problem with much fewer items to make sure that your code is correct.

(d) (10 points) How does the optimal value of the problem vary with respect to the total amount of time allowed (initially set to 60 sec)? Answer this question by plotting and analyzing the optimal value of the problem with different values of the total amount of time that is allowed. This is up to you to consider enough values to create a meaningful plot.

(e) (35 points) We next consider another approach for obtaining a solution to the Supermarket Sweep problem. In this new approach, we will compute the shoppers' paths sequentially: First, we will compute the path for the first shopper to maximize the total value of the items in their own cart. Then, we will compute the path for the second shopper to maximize the total value of the items in their own cart, while ensuring that the second shopper does not pick the items already selected by the first shopper. We assume the time window is still 60 seconds and each cart can carry at most 10 items.

   (i) (15 points) Formulate the MIP to determine the path for the first shopper.
   (ii) (5 points) Code and solve this new MIP. What is the path for the first shopper? What is the value of the items picked by the first shopper?
   (ii) (5 points) Let $i_1, \ldots, i_m$ be the items picked by the first shopper. Formulate constraints to add to the MIP formulated in part (e.i) to ensure the second shopper does not pick items already picked by the first shopper.
   (iii) (5 points) Solve the MIP from part (e.i) with the new constraints. What is the path for the second shopper? What is the value of the items picked by the second shopper?
   (iv) (5 points) What is the total value of the items picked by both shoppers? Comment on your findings.

(f) (20 points) Present your project. Your presentation should introduce the problem (for the students who have not worked on this project), and describe your results to the questions in a clean and organized manner.