On the Stability of Natural Circulation Loops with Phase Change

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Outline

1. Introduction
   - Motivation
   - RCCS
   - Literature
2. Thermohydraulic Theory
   - Conservation Laws
   - Numerics
3. Stability Theory
   - Derivation
   - Solutions
4. Current Work
   - Steady-State Solver
5. Proposed Work

Outline

Goals

- Aim to assess, predict, and physically explain observed two-phase instabilities in a natural circulation loop

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- Investigate the effects of different models for multiphase flow

Leading Questions

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- Applications?

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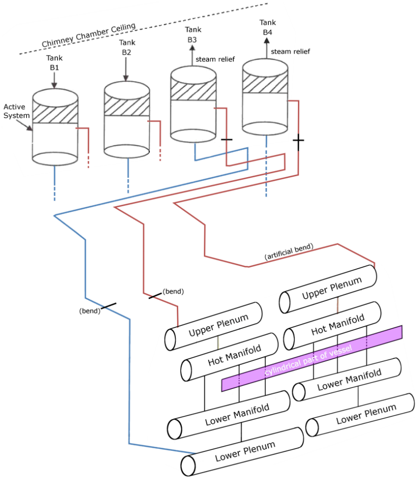
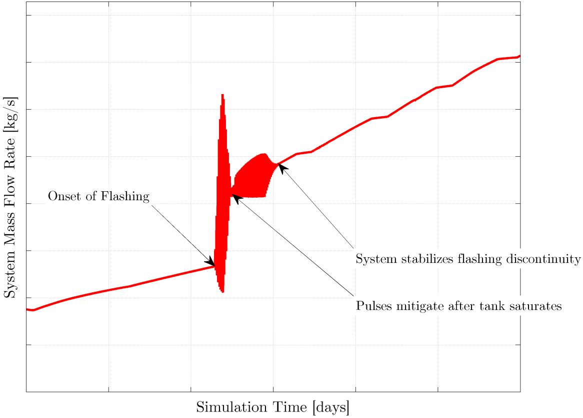
- Applications?
  - Thermosiphon
  - Power cycle loops
  - And...

Outline

RCCS Overview

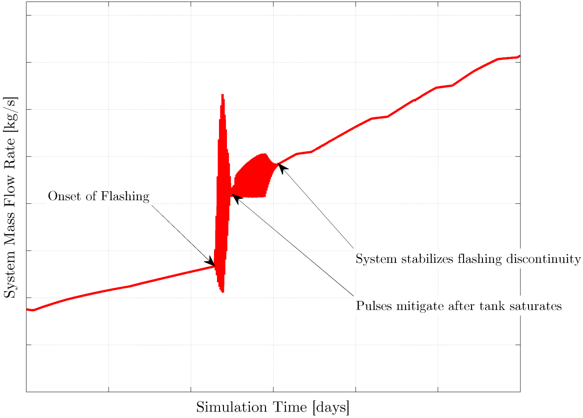


MELCOR Simulations

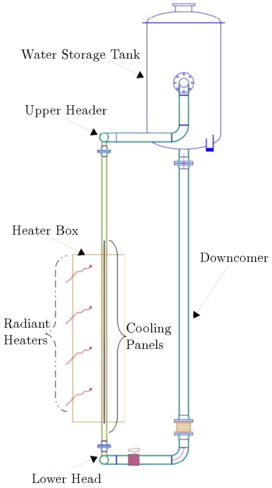


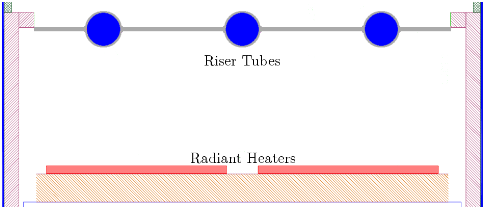
MELCOR Simulations

Is this behavior real?

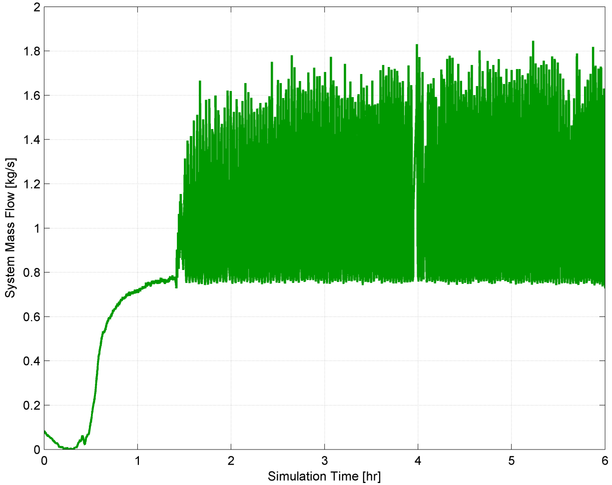


RCCS Experiment Overview

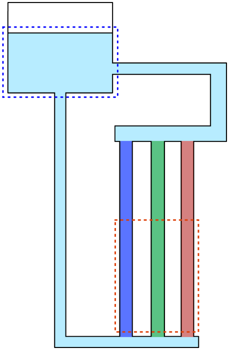


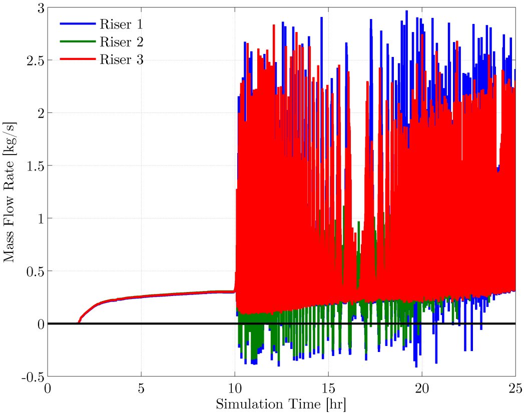


RCCS Experiment's Mass Flow Rate



RCCS Experiment Simulation





Outline

Been done before?

Yes, two-phase, natural circulation stability has been done (see report).
But...

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- Extremely limited analytical work on non-simple, closed loop (multiple riser)
- None had depletion of inventory in two-phase

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General Conservation Law (CLaw)

Conservation laws balance a vector of conserved variables $\mathbf{q}$ over a control volume.

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Nonlinear form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial z} = \mathbf{S}(\mathbf{q}, z, t) \tag{1}$$

Quasilinear form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial \mathbf{q}} \frac{\partial \mathbf{q}(z, t)}{\partial z} = \mathbf{S}(\mathbf{q}, z, t) \tag{2}$$

Characteristic speeds:

$$\Lambda = \mathrm{Eig}\left[\frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial \mathbf{q}}\right] \tag{3}$$

Homogenous Equilibrium Model (HEM)

Nonlinear form:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ \rho i\end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix}\rho u \\ \frac{\rho u^2}{\rho} + P\left(\rho, \frac{\rho i}{\rho}\right) \\ \frac{\rho u}{\rho}\left[\rho i + P\left(\rho, \frac{\rho i}{\rho}\right)\right]\end{bmatrix} = \begin{bmatrix}0 \\ \rho g(z) - \frac{K_{\text{eff}}(\mathbf{q})}{2}\frac{\rho u|\rho u|}{\rho} \\ \dot{Q}_{\text{add}}(\mathbf{q}, z, t)\end{bmatrix} \tag{4}$$

Homogenous Equilibrium Model (HEM)

Flux Jacobian:

$$\mathbb{J}_{\mathsf{F}} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{dP}{d\rho} - u^2 & 2\,u & \dfrac{1}{\rho}\dfrac{\partial P}{\partial i} \\ u\left(\dfrac{dP}{d\rho} - h\right) & h & u\left(1 + \dfrac{1}{\rho}\dfrac{\partial P}{\partial i}\right) \end{bmatrix} \tag{5}$$

Characteristic speeds:

$$\lambda_{\mathsf{HEM}} = \begin{bmatrix} u \\ \left(1 + \dfrac{1}{2\rho}\dfrac{\partial P}{\partial i}\right) u \pm \dfrac{1}{2\rho}\sqrt{4P(\rho, i)\dfrac{\partial P}{\partial i} + \left(u\dfrac{\partial P}{\partial i}\right)^2 + 4\rho^2\dfrac{\partial P}{\partial \rho}} \end{bmatrix} \tag{6}$$

Multiphase Model

$$\frac{\partial}{\partial t}\begin{bmatrix} \alpha\rho_\phi \\ \alpha\rho u_\phi \\ \alpha\rho i_\phi \end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix} \alpha\rho u_\phi \\ u_\phi\,\alpha\rho u_\phi + P(\rho_\phi, i_\phi) \\ u_\phi[\alpha\rho i_\phi + P(\rho_\phi, i_\phi)] \end{bmatrix} = \tag{7}$$

$$\begin{bmatrix} \mathbb{M}_\phi \\ \alpha\rho_\phi g(z) - \frac{K_{\text{eff},\phi}(\mathbf{q})}{2} u_\phi\,|\alpha\rho u_\phi| + \mathbb{P}_\phi \\ \dot{Q}_{\text{add},\phi}(\mathbf{q}, z, t) + \mathbb{E}_\phi \end{bmatrix}$$

Outline

Collocated Nodal Method (steady-state over simple, closed loop)

Simplest to derive; hardest to solve.

$$\frac{\partial \mathbf{F}(\mathbf{q}; z)}{\partial z} = \mathbf{S}(\mathbf{q}, z) \tag{8}$$

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Integrating from z_i to z_{i+1}:

$$\mathbf{F}(\mathbf{q}; z_{i+1}) - \mathbf{F}(\mathbf{q}; z_i) = \beta_i \mathbf{S}(\mathbf{q}, z_i) + \beta_{i+1} \mathbf{S}(\mathbf{q}, z_{i+1}) + \mathcal{O}(|z_{i+1} - z_i|^2) \quad (9)$$

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Residual to drive to 0:

$$\mathbf{R}_i(\mathbf{q}) = [\mathbf{F}(\mathbf{q}; z_{i+1}) - \mathbf{F}(\mathbf{q}; z_i)] - [\beta_i \mathbf{S}(\mathbf{q}, z_i) + \beta_{i+1} \mathbf{S}(\mathbf{q}, z_{i+1})] \tag{10}$$

Collocated Nodal Method (steady-state over simple, closed loop)

Simple form for arbitrary node count:

$$\mathbf{R} = \mathbb{C}_F \mathbf{F} - \mathbb{C}_S \mathbf{S} \tag{11}$$

Connectivity matrices:

$$\mathbb{C}_* = \begin{bmatrix} \mathbf{C}_* & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_* & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_* \end{bmatrix} \tag{12}$$

$$\mathbf{C}_F = \begin{bmatrix} -1 & +1 & & & \\ & -1 & +1 & & \\ & & \ddots & \ddots & \\ & & & -1 & +1 \\ 1 & & & & -1 \end{bmatrix} \qquad \mathbf{C}_S = \begin{bmatrix} \beta_1 & \beta_2 & & & \\ & \beta_2 & \beta_3 & & \\ & & \ddots & \ddots & \\ & & & \beta_{N-1} & \beta_N \\ \beta_1 & & & & \beta_N \end{bmatrix}$$

Collocated Nodal Method (steady-state over simple, closed loop)

Matrices of this form are singular!

Sweeping approach used for steady-state solver:

- Tank thermodynamic state is known.
- Assume a momentum.
- Sweep through system back to the tank.
- Two possibilities
  - Integrated pressure is less than the tank's: reduce momentum
  - Integrated pressure is greater than the tank's: increase momentum

Momentum corrections are made through a secant method update using the pressure difference as an abscissa.

Staggered Finite Volume Method

This is the method to be used for transient solution.

"Staggered" means that the mass and energy equations are integrated over a different space than the momentum equations

- Avoids velocity-pressure decoupling
- Allows a much more flexible interpretation of velocities: information propagators

Time integration method to be used on semi-discrete form (next slide) is implicit Euler

- Completely overcomes acoustic limitation on the time step value
- Unconditionally TVD (very diffusive)

Staggered Finite Volume Method

$$\frac{\partial \rho_{\mathrm{k}}}{\partial t} = -\frac{1}{\Omega_{\mathrm{k}}} \sum \rho u_{\mathrm{k}} A_{\mathrm{k}} \tag{13a}$$

$$\frac{\partial \rho i_{\mathrm{k}}}{\partial t} = -\frac{1}{\Omega_{\mathrm{k}}} \sum u_d [\rho i_d + P(\rho_d, i_d)] A_{\mathrm{k}} + \dot{Q}_{\mathrm{add,k}}(z,t) \tag{13b}$$

$$\frac{\partial \rho u_{\mathrm{m}}}{\partial t} = -\frac{1}{\Omega_{\mathrm{m}}} \left[u\, \rho u|_{\mathrm{from}}^{\mathrm{to}} + P_{\mathrm{to}} - P_{\mathrm{from}} \right] A_{\mathrm{m}} + \tag{13c}$$

$$\frac{A_{\mathrm{m}}}{\Omega_{\mathrm{m}}} \int_{\mathrm{from}}^{\mathrm{to}} \left[\rho g(z) - \frac{K_{\mathrm{eff}}(\mathbf{q})}{2} u \left| \rho u \right| \right] \mathrm{d}s \tag{13d}$$

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Derivation of perturbation equations

Assumed that the true solution is a summation of a steady-state and a transient

$$\mathbf{q}(z,t) = \mathbf{q}^0(z) + \widehat{\mathbf{q}}(z,t). \tag{14}$$

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General nonlinear perturbation equation:

$$\frac{\partial \widehat{\mathbf{q}}(z,t)}{\partial t} + \frac{\partial}{\partial z}\left[\mathbf{F}\left(\mathbf{q}^0 + \widehat{\mathbf{q}}; z, t\right)\right] = \mathbf{S}\left(\mathbf{q}^0 + \widehat{\mathbf{q}}, z, t\right) \tag{15}$$

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Taylor expansion about perturbation (neglecting H.O.T.) yields general linear perturbation equation:

$$\frac{\partial \widehat{\mathbf{q}}(z,t)}{\partial t} + \frac{\partial}{\partial z}\left[\frac{\partial \mathbf{F}}{\partial \mathbf{q}^0}\widehat{\mathbf{q}}\right] = \frac{\partial \mathbf{S}}{\partial \mathbf{q}^0}\widehat{\mathbf{q}} \tag{16}$$

Outline

Solution methods of linear equation

Wave form ansatz:

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Laplace transform (zeros of transfer function):

$$s\breve{\widehat{\mathbf{q}}} - \widehat{\mathbf{q}}(z,0) + \frac{\partial}{\partial z}\left[\frac{\partial \mathbf{F}}{\partial \mathbf{q}^0}\breve{\widehat{\mathbf{q}}}\right] = \frac{\partial \mathbf{S}}{\partial \mathbf{q}^0}\breve{\widehat{\mathbf{q}}} \tag{19}$$

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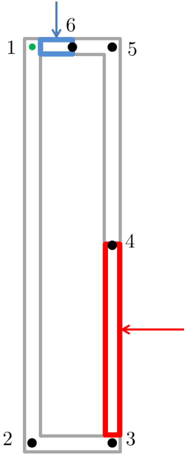
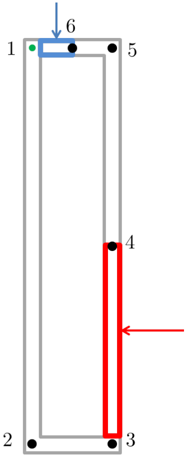
Test Problem

- Single phase
- Constant friction factor
- 10 kW load
- State:
  - $P = 101325$ Pa
  - $T = 300$ K



Test Problem

Measure	Pressure [kPa]		Temperature [K]		Density [kg/m^3]	
Point	**Solver**	**Hand**	**Solver**	**Hand**	**Solver**	**Hand**
1	101325	101325	300.0	300	996.6	996.6
2	150192	150178	300.0	300	996.6	996.6
3	150190	150175	300.0	300	996.6	996.6
4	130640	130631	302.9	302.9	995.7	995.7
5	101327	101328	302.9	302.9	995.7	995.7
6	101326	101326	302.9	302.9	995.7	995.7



Outline

Path Goals

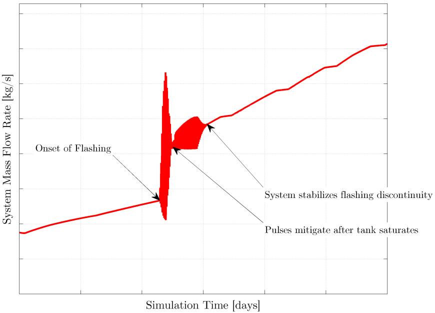
Path Forward:

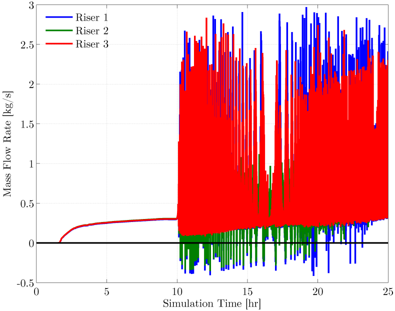
- Complete transient, HEM solver
- Look at HEM boiling behavior
- Attain non-simple, closed-loop steady-state calculations

End Goal

Assess, model, and physically explain the observed instabilities

Path Goals ... with Pictures!





End

"We will do what is hard. We will achieve what is great."