On the Behavior of Natural Circulation Loops with Phase Change

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Outline

1. Background
   - Motivation
   - RCCS
   - Literature
2. Thermohydraulics
3. Discretization of Conservation Equations
4. JFNK
5. Stability
6. Results

Outline

Goals

Simulate a simple closed-loop with naturally circulating water in both single and two-phase regimes using

- Non-ideal equation of state for water
- Simple but accurate models for frictional pressure losses
- Modern, nonlinear solver with complete residual convergence
- Rigorous discretization that allows for exact integration of conservation equations over physical domain

And ultimately determine the linear stability of the system under consideration.

Stability

What is stability?

- Often used term
- Used with many definitions (both implicit and explicit)

Stability

One definition:

Transient (possibly oscillatory) thermal hydraulic phenomenon stemming from nonlinear-geometric-multiphase feedback that could lead to system excursions causing dangerous mechanical or thermal damage and possibly human harm.

Stability

Another definition:

A system of the form

$$\partial_t q = f(t, q) \tag{1}$$

is stable if $q \in [q_{\text{low}}, q_{\text{hi}}]$ for all time.

Stability

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Definition for here:

A system of the form

$$\partial_t q = Aq \tag{2}$$

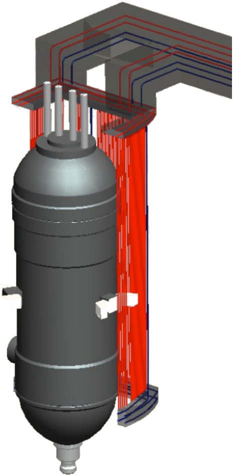
is stable if all eigenvalues of A are less than or equal to zero.

Stability

Applications?

- Thermosiphon
- Power cycle loops
- And...

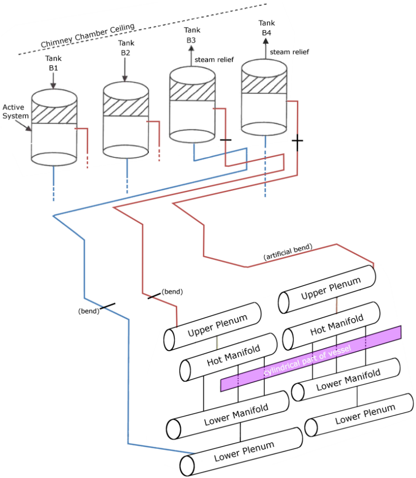
Reactor Cavity Cooling System



Purpose:
Naturally-driven cooling of reactor under accident conditions

Accident conditions:
Loss of onsite, offsite power (station blackout)

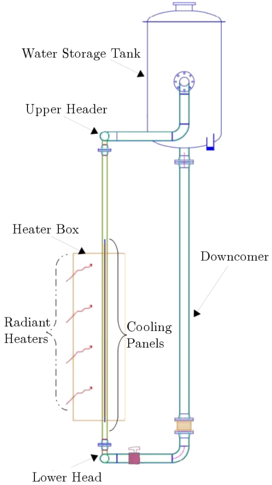
Reactor Cavity Cooling System: Big picture

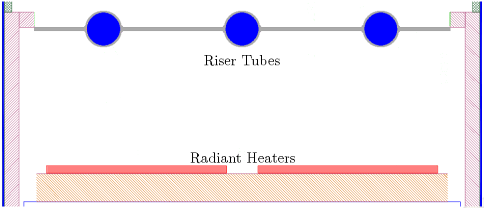


Selected specifications:

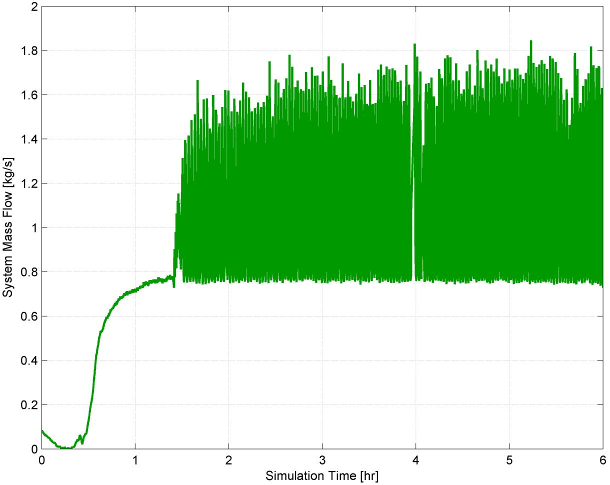
- Trains: 2
- Lines/Tanks: 8
- Elevation change: 35 meters
- Path length: 200 meters
- Heated length: 20 meters
- Risers: 200+
- Heat transfer:
  - Radiation: 80%
  - Convection: 20%

RCCS Experiment





Experimental data: flow oscillations during boiling



Been done before?

Two-phase, natural circulation stability in the literature exists (see thesis). But...

- Analytical work limited to simple equations-of-state
- Large volumes with piece-wise properties (non-dimensional numbers abound)
- System code usage hindered by simple discretization scheme and confounded by complicated models

Outline

General Conservation Law (CLaw)

Conservation laws balance a vector of conserved variables **q**over a control volume.

Nonlinear form:

$$\frac{\partial q_i}{\partial t} + \frac{\partial f_i(q_i; x_i, t)}{\partial x_i} = s_i(q_i, z, t) \quad (1)$$

Quasilinear form:

$$\frac{\partial q_i}{\partial t} + \frac{\partial f_i(q_i; z, t)}{\partial q_i} \frac{\partial q_i(x_i, t)}{\partial z} = s_i(q_i, x_i, t) \quad (2)$$

Characteristic speeds:

$$\Lambda = \mathrm{Eig}\left[\frac{\partial f_i(q_i; z, t)}{\partial q_i} \right] \quad (3)$$

Conservation of Mass

Integral form:

$$\partial_t \int_\Omega \rho \, \mathrm{d}\Omega = \int_\Gamma -u_j \rho n_j \, \mathrm{d}\Gamma + \int_\Omega s^\rho \, \mathrm{d}\Omega \tag{4}$$

Differential form:

$$\partial_t \rho + \partial_j (u_j \rho) = s^\rho \tag{5}$$

Conservation of Momentum

Integral form:

$$\partial_t \int_\Omega \rho u_i \,\mathrm{d}\Omega = \int_\Gamma (-u_j \rho u_i) n_j \,\mathrm{d}\Gamma + \int_\Gamma (-\delta_{ij} P + \tau_{ij}) n_j \,\mathrm{d}\Gamma + \int_\Omega \rho g_i + s^u \,\mathrm{d}\Omega \qquad (6)$$

Differential form:

$$\partial_t (\rho u_i) + \partial_j (u_j \rho u_i) = -\partial_i P + \partial_i \tau_{ij} + \rho g_i + s^u \qquad (7)$$

Conservation of Energy

Integral form:

$$\partial_t \int_\Omega \rho e \,\mathrm{d}\Omega = \int_\Gamma [-(\rho e + P)u_j + u_i \tau_{ij} - q_j] n_j \,\mathrm{d}\Gamma + \int_\Omega \rho g_j u_j + s^e \,\mathrm{d}\Omega \tag{8}$$

Differential form:

$$\partial_t(\rho e) + \partial_j[(\rho e + P)u_j] = \partial_j(u_i \tau_{ij} - q_j) + \rho g_j u_j + s^e \tag{9}$$

Conservation of Bulk Momentum

Only want to track one momentum per cell.

Dot the Conservation of Momentum equation with bulk flow direction z_i:

$$\partial_t \int_\Omega \rho u_i z_i \, \mathrm{d}\Omega = \int_\Gamma (-u_j \rho u_i) n_j z_i \, \mathrm{d}\Gamma + \int_\Gamma (-\delta_{ij} P + \tau_{ij}) n_j z_i \, \mathrm{d}\Gamma + \int_\Omega \rho g_i z_i + s_z^u \, \mathrm{d}\Omega \quad (10)$$

Let $u_z = u_i z_i$:

$$\partial_t \int_\Omega \rho u_z \, \mathrm{d}\Omega = \int_\Gamma (-u_j \rho u_z) n_j \, \mathrm{d}\Gamma + \int_\Gamma (-\delta_{ij} P + \tau_{ij}) n_j z_i \, \mathrm{d}\Gamma + \int_\Omega \rho g_i z_i + s_z^u \, \mathrm{d}\Omega \quad (11)$$

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Channel flow conservation:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_\Omega \rho u_z \,\mathrm{d}\Omega = \int_{\Gamma_1+\Gamma_2} \pm (P + u_z \rho u_z) \,\mathrm{d}\Gamma + \int_{\Gamma_w} \tau_{ij} z_i n_j \,\mathrm{d}\Gamma + \int_\Omega \rho g \ \mathrm{Cos}(\theta) + s_z^u \,\mathrm{d}\Omega, \quad (12)$$

Other Assumptions

- Heat conduction and viscous heating is negligible compared to enthalpy flow
- Fluid-fluid friction is negligible compared to fluid-wall friction and form losses
- Time-rate of change of potential and kinetic energy is negligible to the thermal energy change

Results to equations

$$\rho e \approx \rho i \quad (13)$$

$$q_j \approx 0 \quad (14)$$

$$u_i \tau_{ij} \approx 0 \quad (15)$$

$$\tau_{ij} z_i n_j \approx \frac{1}{2} f_{\mathrm{darcy}} \frac{L_{\mathrm{char}}}{D_h} \mathrm{Abs}(\rho u_z)\, u_z \quad (16)$$

Equation of State

- IAPWS-95 non-ideal equation of state for water
- Magnificently huge curve fit of Helmholtz free energy potential
- Natural variables are ρ and T
- Back calculate T from ρ and i (plot)

Outline

Purpose

Derive quasi-two-dimensional thermohydraulic equations to enable adequate modeling of a branched system.

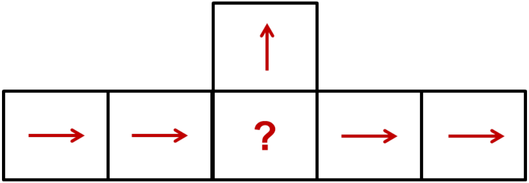
Consider only conservation of mass, momentum, and energy.

Methodology

- Consider a collection of control volumes and momentum cells.
- Information is exchanged through surface fluxes.
- Control volumes and momentum cells cover same physical space; but are off-set.

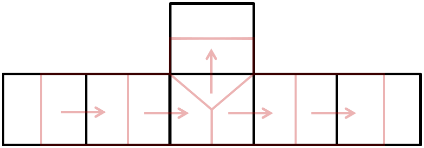
Coincident Spatial Grid

All equations solved on same grid:

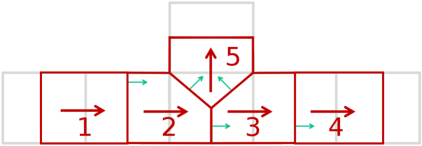


Staggered Spatial Grid

Mass and energy grid:



Momentum grid:



Semi-Discretized Control Volume Equations

Mass and energy for control k:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}\rho_k \,\mathrm{d}\Omega = s_k^{\rho} V_k + \sum_{n=1}^{N} u_n \rho_{d,n} A_n \tag{17}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}\rho i \,\mathrm{d}\Omega = s_k^{e} V_k + \sum_{n=1}^{N} u_n \rho h_{d,n} A_n \tag{18}$$

Semi-Discretized Momentum Cell Equation

Momentum for momentum cell k:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}\rho u_k\,\mathrm{d}\Omega = \left(\rho_k g_k + s_k^u\right) V_k - \sum_{n=1}^{N}\left(P_n z_n + u_{\mathrm{I},n}\rho u_{d,n}\right)A_n - \frac{1}{2} f_{\mathrm{D},\mathrm{k}} \frac{L_{\mathrm{char,k}}}{D_{\mathrm{eff,k}}}\,\mathrm{Abs}(\rho u_k) u_k A_k \tag{19}$$

Time Stepping

Semi-discrete equations are now of the form:

$$\partial_t q_i = D_i(q_i) \tag{20}$$

Various choices of stepping over a time step p:

$$q_i^p - q_i^{p-1} = \Delta t \, D_i(q_i^{p-1}) \tag{21}$$

$$q_i^p - q_i^{p-1} = \Delta t \, D_i(q_i^p) \tag{22}$$

$$q_i^p - q_i^{p-1} = \frac{1}{2}\Delta t \left[D_i(q_i^{p-1}) + D_i(q_i^p) \right] \tag{23}$$

Outline

Full discretized equations

Consider the Implicit Euler full discretization:

$$q_i^p - q_i^{p-1} = \Delta t \, D_i(q_i^p) \tag{24}$$

How do you find q_i^p to satisfy that equations assuming D_i is nonlinear?

Newton-Raphson: Procedure

Put the equation into "residual" form

$$r(q_i^p) = q_i^p - q_i^{p-1} - \Delta t\, D_i(q_i^p) \tag{25}$$

and search for the vector q_i^p that makes $r(q_i^p)$ equal to 0 (or close enough).

Common search technique is Newton-Raphson method:

$$\text{Solve } \partial_{q_i} r(q_i^p)\, \Delta q_i^p = -r(q_i^p) \tag{26}$$

$$q_i^p = q_i^p + \Delta q_i^p \tag{27}$$

Newton-Raphson: Problems

- Calculating the Jacobian $\partial_{q_i} r(q_i^p)$ can be time and memory intensive.
- Solving the linear system is likewise difficult

JFNK

JFNK: Jacobian-Free Newton-Krylov method.

Krylov method

A particular way of solving the linear system $Ax = b$:

1. Compute a search direction z_n:

$$z_n = \begin{cases} r_{n-1} & \text{if } r_{n-1} < r_{n-2} \\ v_{n-1} & \text{otherwise} \end{cases} \tag{28}$$

2. Update a QR factorization:

$$[A\,z_1, A\,z_2, ..., A\,z_n] = V_n R_n \tag{29}$$

3. Update residual:

$$r_n = r_{n-1} - v_n^{\mathsf{T}} r_{n-1} v_n \tag{30}$$

4. Solve the system

$$R_n w_n = [v_1^{\mathsf{T}} r_1, ..., v_n^{\mathsf{T}} r_{n-1}]^{\mathsf{T}}; \quad x_n = x_0 + [z_1, ..., z_n] w_n \tag{31}$$

Approximate Jacobian

Important part to notice

$$[A\,z_1, A\,z_2, ..., A\,z_n] \tag{32}$$

The only new computation every iteration is Az_n (matrix-vector product).

Jacobian-Free method uses the following finite difference relation:

$$\partial_{q_i} r(q_i^p) z_n \approx \frac{r(q_i^p + \varepsilon z_n) - r(q_i^p)}{\varepsilon} \tag{33}$$

Instead of creating the Jacobian, approximate its existence using this formula (Jacobian-free) in a Krylov Method.

Jacobian-Free Newton-Krylov

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Jacobian-Free Newton-Krylov

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2. Update a QR factorization:

$$[z_1, \frac{r(q_i^p + \varepsilon z_2) - r(q_i^p)}{\varepsilon}, \dots, \frac{r(q_i^p + \varepsilon z_n) - r(q_i^p)}{\varepsilon}] = V_n R_n \tag{35}$$

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Outline

Perturbation equations

Assumed that the true solution is a summation of a steady-state and a transient

$$q_i(x_i, t) = q_i^{\mathrm{ss}}(x_i) + \delta q_i(x_i, t). \tag{38}$$

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General nonlinear perturbation equation (diagrams):

$$\frac{\partial \delta q_i}{\partial t} + \frac{\partial}{\partial x_j}[f_{ij}(q_i^{\mathrm{ss}} + \delta q_i)] = s_i(q_i^{\mathrm{ss}} + \delta q_i) \tag{39}$$

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Taylor expansion about perturbation (neglecting H.O.T.) yields general linear perturbation equation:

$$\frac{\partial \delta q_i}{\partial t} + \frac{\partial}{\partial x_j}\left[\frac{\partial f_{ij}}{\partial q_k^{ss}} \delta q_k\right] = \frac{\partial s_i}{\partial q_k^{ss}} \delta q_k \tag{40}$$

Solution method of general linear equations

Perturbations still part of spatially varying PDE.

Discretizing like full transient equations will yield spurious, positive eigenvalues from mass/energy advection.

Solution: integration over entire system and isolate global time-evolution on left-hand side:

$$\partial_t \delta q_i(t) = \frac{1}{\int_\Omega \mathrm{d}\Omega} \left[\int_\Omega \frac{\partial s_i}{\partial q_k^{\mathrm{ss}}} \, \mathrm{d}\Omega - \int_\Gamma \frac{\partial f_{ij}}{\partial q_k^{\mathrm{ss}}} n_j \, \mathrm{d}\Gamma \right] \delta q_k(t) \tag{41}$$

Solution method of thermohydraulic system

Apply to mass, energy, momentum system:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\delta\rho\\ \delta\rho i\\ \delta\rho u_z\end{bmatrix} = -\partial_j\left(\begin{bmatrix}\partial_{q_k}(\rho u_j)\\ \\ \partial_{q_k}[(\rho i + P)u_j]\\ \\ \partial_{q_k}(u_j\rho u_z + P\delta_{ij}z_i - \tau_{ij}z_i)\end{bmatrix}\delta q_k\right) + \left(\begin{bmatrix}\partial_{q_k}(s^\rho)\\ \partial_{q_k}(s^e)\\ \partial_{q_k}(\rho g_z + s_z^u)\end{bmatrix}\delta q_k\right) \tag{42}$$

Solution method of thermohydraulic system

Integrating to the skin of the system and eliminating terms that vanish:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\delta\rho\\ \delta\rho i\\ \delta\rho u_z\end{bmatrix} = \frac{1}{V_{\mathsf{sys}}}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ \alpha_\rho & \alpha_{\rho i} & \alpha_{\rho u_z}\end{bmatrix}\begin{bmatrix}\delta\rho\\ \delta\rho i\\ \delta\rho u_z\end{bmatrix}, \tag{42}$$

where $\alpha_* = -\int_\Gamma \partial_* \Delta P_{\mathsf{dar}}\,\mathrm{d}\Gamma$

Solution method of thermohydraulic system

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where $\alpha_* = -\int_\Gamma \partial_* \Delta P_{\text{dar}}\,\mathrm{d}\Gamma$

With the solution

$$\begin{bmatrix}\delta\rho(t)\\ \delta\rho i(t)\\ \delta\rho u_z(t)\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ \frac{\alpha_\rho}{\alpha_{\rho u_z}}\left(e^{\tilde{\alpha}t}-1\right) & \frac{\alpha_{\rho i}}{\alpha_{\rho u_z}}\left(e^{\tilde{\alpha}t}-1\right) & e^{\tilde{\alpha}t}\end{bmatrix}\begin{bmatrix}\delta\rho(0)\\ \delta\rho i(0)\\ \delta\rho u_z(0)\end{bmatrix}, \tag{43}$$

where $\tilde{\alpha} = \alpha_{\rho u_z}/V_{\text{sys}}$.

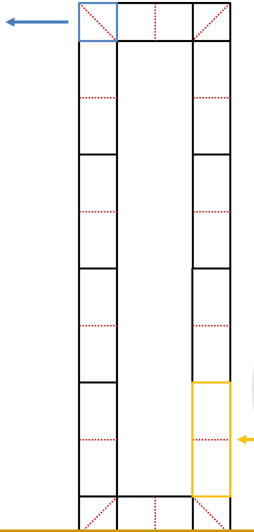
Outline

Methodology

1. All closed-loop systems exhibit a singular steady-state solution.
2. Solution: perform a transient calculation that provided diagonal regularity and run transient to a stationary-state using all previous equations and tools.
3. Once the steady-state is attained, computer the eigenvalues and examine.

Primary Test Loop

1. 14 control volume / momentum cells
2. 1.4 meters high, 0.4 meters wide (3.5 aspect ratio)
3. 0.1 meter hydraulic diameter



Single Phase Results

Two-Phase Results

Two-Riser Results

Two Riser Results?

Questions

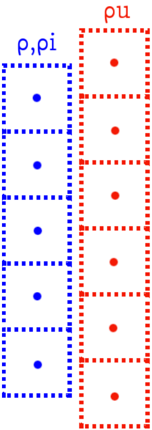
"The key to wisdom is this: constant and frequent questioning. For by doubting we are led to question, and by questioning we arrive at the truth."

— Peter Abelard

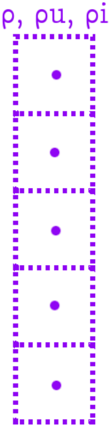
Outline

Staggered/Collocated

Staggered mesh:

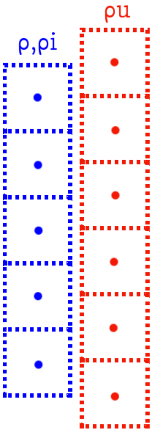


Collocated mesh:

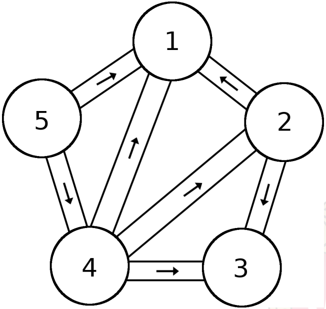


Rigorous vs. Non-Rigorous

Rigorous staggered mesh (CFD):



Non-rigorous staggered mesh (System codes):



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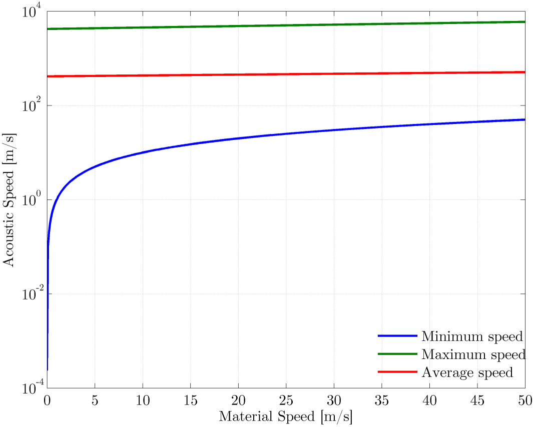
Acoustic Speeds



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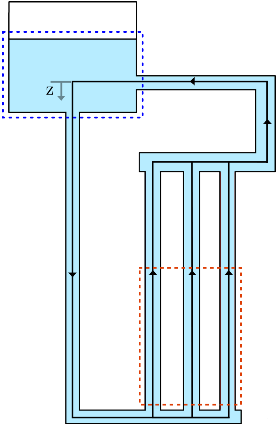
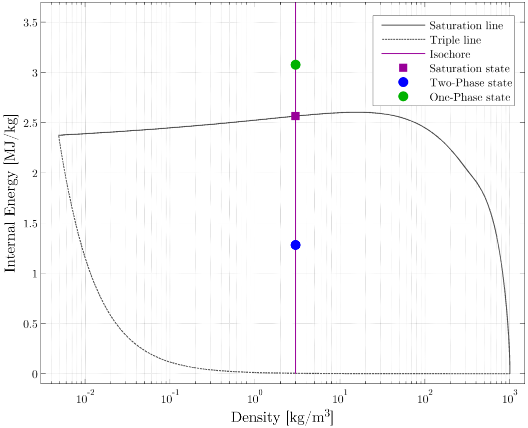
Non-simple, closed loop



i-ρ Diagram



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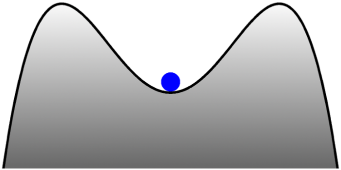
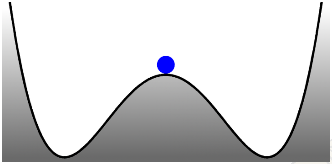
Stability Diagrams

Linearly stable, nonlinearly unstable:



Linearly unstable, nonlinearly stable:



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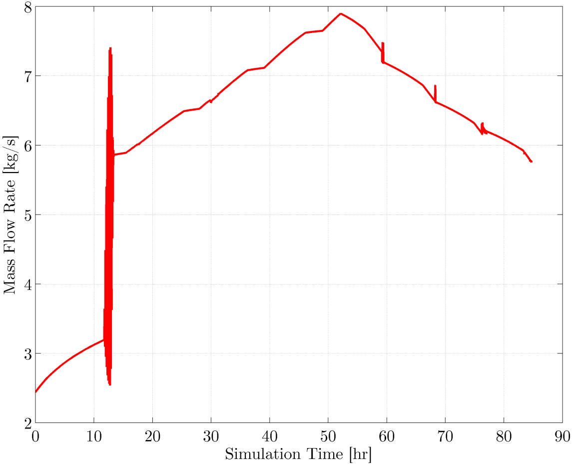
System Mass flow: 4 days



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Multiphase Model

$$\frac{\partial}{\partial t}\begin{bmatrix}\alpha\rho_\phi \\ \alpha\rho u_\phi \\ \alpha\rho i_\phi\end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix}\alpha\rho u_\phi \\ u_\phi\,\alpha\rho u_\phi + P(\rho_\phi, i_\phi) \\ u_\phi[\alpha\rho i_\phi + P(\rho_\phi, i_\phi)]\end{bmatrix} = \begin{bmatrix}\mathbb{M}_\phi \\ \alpha\rho_\phi g(z) - \dfrac{K_{\text{eff},\phi}(\mathbf{q})}{2} u_\phi\,|\alpha\rho u_\phi| + \mathbb{P}_\phi \\ \dot{Q}_{\text{add},\phi}(\mathbf{q}, z, t) + \mathbb{E}_\phi\end{bmatrix} \tag{44}$$