On the Behavior of Natural Circulation Loops with Phase Change

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Goals

Aim to assess, predict, and physically explain observed two-phase flow behavior in a natural circulation loop

- Using a non-ideal equation of state for water
- Several models for convection and friction
- Multiple risers (non-simple loops)

Leading Questions

- What are two-phase instabilities?

> Transient (possibly oscillatory) thermal hydraulic phenomenon stemming from nonlinear-geometric-multiphase feedback that could lead to system excursions causing dangerous mechanical or thermal damage and possibly human harm.

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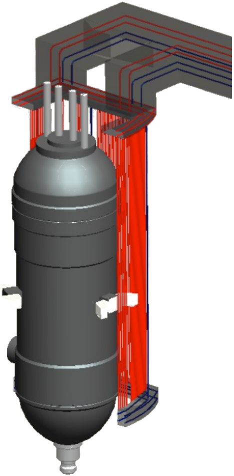
- Applications?
  - Thermosiphon
  - Power cycle loops
  - And...

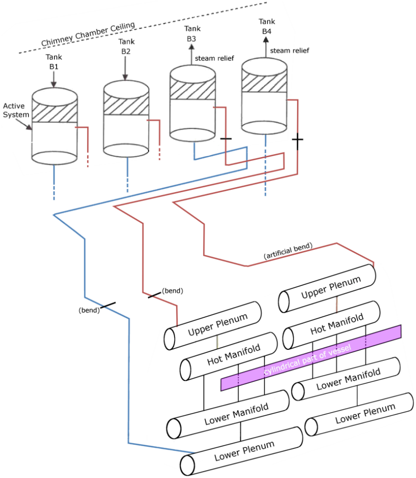
Reactor Cavity Cooling System



Purpose:
Naturally-driven cooling of reactor under accident conditions

Accident conditions:
Loss of onsite, offsite power (station blackout)

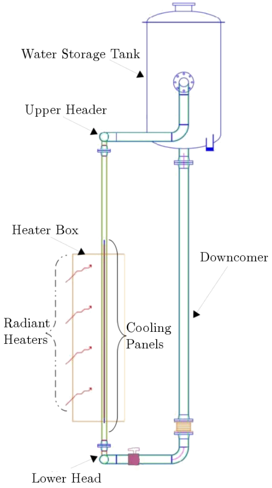
Reactor Cavity Cooling System: Big picture

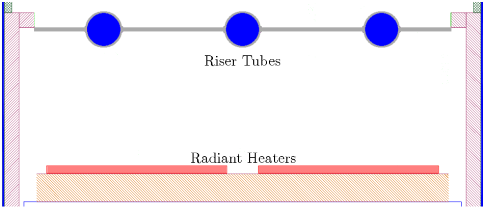


Selected specifications:

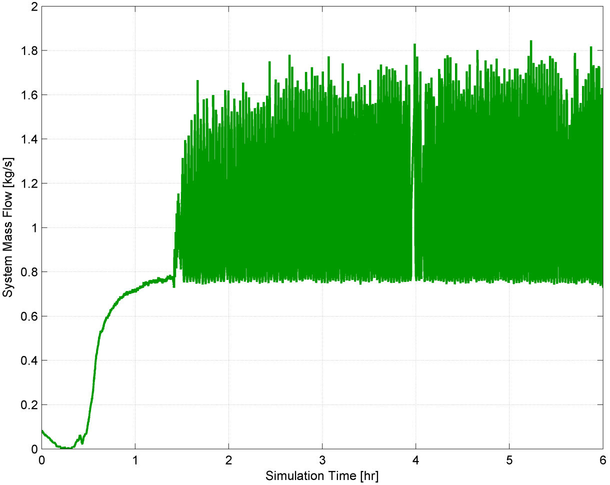
- Trains: 2
- Lines/Tanks: 8
- Elevation change: 35 meters
- Path length: 200 meters
- Heated length: 20 meters
- Risers: 200+
- Heat transfer:
  - Radiation: 80%
  - Convection: 20%

RCCS Experiment





Experimental data: flow oscillations during boiling



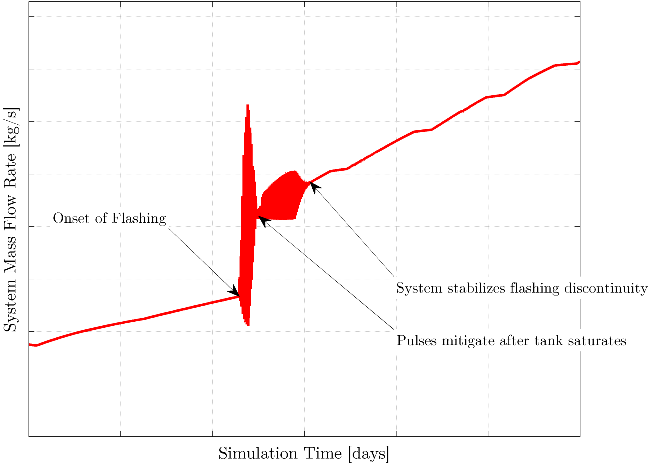
1. Background
   - Motivation
   - RCCS
   - Literature
2. Preliminary Work
   - MELCOR Simulations
   - Thermohydraulics
   - Steady-state Solver
   - Stability
3. Proposed Work

Been done before?

Two-phase, natural circulation stability literature exists. But...

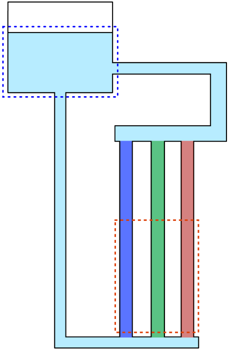
- Almost no analytical work on non-simple, closed loop (multiple riser)
- None had depletion of inventory in two-phase

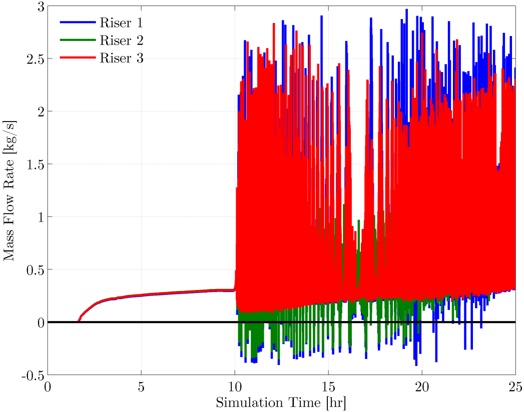
Full-scale model under accident conditions



alternate

RCCS Experiment: Model Simulation





General Conservation Law (CLaw)

Conservation laws balance a vector of conserved variables $\mathbf{q}$ over a control volume.

Nonlinear form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial z} = \mathbf{S}(\mathbf{q}, z, t) \tag{1}$$

Quasilinear form:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial \mathbf{q}} \frac{\partial \mathbf{q}(z, t)}{\partial z} = \mathbf{S}(\mathbf{q}, z, t) \tag{2}$$

Characteristic speeds:

$$\Lambda = \mathrm{Eig}\left[\frac{\partial \mathbf{F}(\mathbf{q}; z, t)}{\partial \mathbf{q}}\right] \tag{3}$$

Homogeneous Equilibrium Model (HEM)

Nonlinear form:

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ \rho i\end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix}\rho u \\ u\,\rho u + P(\rho, i) \\ u[\rho i + P(\rho, i)]\end{bmatrix} = \begin{bmatrix}0 \\ \rho g(z) - \frac{K_{\text{eff}}(\mathbf{q})}{2}|\rho u|u \\ \dot{Q}_{\text{add}}(\mathbf{q}, z, t)\end{bmatrix} \tag{4}$$

HEM Speeds

Flux Jacobian:

$$\mathbb{J}_\mathsf{F} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{dP}{d\rho} - u^2 & 2\,u & \dfrac{1}{\rho}\dfrac{\partial P}{\partial i} \\ u\left(\dfrac{dP}{d\rho} - h\right) & h & u\left(1 + \dfrac{1}{\rho}\dfrac{\partial P}{\partial i}\right) \end{bmatrix} \tag{5}$$

Characteristic speeds (plot):

$$\lambda_\mathsf{HEM} = \begin{bmatrix} u \\ \left(1 + \dfrac{1}{2\rho}\dfrac{\partial P}{\partial i}\right) u \pm \dfrac{1}{2\rho}\sqrt{4P(\rho, i)\dfrac{\partial P}{\partial i} + \left(u\dfrac{\partial P}{\partial i}\right)^2 + 4\rho^2\dfrac{\partial P}{\partial \rho}} \end{bmatrix} \tag{6}$$

Equation of State

- IAPWS-95 non-ideal equation of state for water
- Magnificently huge curve fit of Helmholtz free energy potential
- Natural variables are ρ and T
- Back calculate T from ρ and i (plot)

Collocated Nodal Method (steady-state over simple, closed loop)

Simplest to derive; hardest to solve.

$$\frac{\partial \mathbf{F}(\mathbf{q}; z)}{\partial z} = \mathbf{S}(\mathbf{q}, z) \tag{7}$$

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Integrating from z_i to z_{i+1}:

$$\mathbf{F}(\mathbf{q}; z_{i+1}) - \mathbf{F}(\mathbf{q}; z_i) = \beta_i \mathbf{S}(\mathbf{q}, z_i) + \beta_{i+1} \mathbf{S}(\mathbf{q}, z_{i+1}) + \mathcal{O}(|z_{i+1} - z_i|^2) \quad (8)$$

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Residual to drive to 0:

$$\mathbf{R}_i(\mathbf{q}) = [\mathbf{F}(\mathbf{q}; z_{i+1}) - \mathbf{F}(\mathbf{q}; z_i)] - [\beta_i \mathbf{S}(\mathbf{q}, z_i) + \beta_{i+1}\mathbf{S}(\mathbf{q}, z_{i+1})] \tag{9}$$

Collocated Nodal Method (steady-state over simple, closed loop)

Simple form for arbitrary node count:

$$\mathbf{R} = \mathbb{C}_F \mathbf{F} - \mathbb{C}_S \mathbf{S} \tag{10}$$

Connectivity matrices:

$$\mathbb{C}_* = \begin{bmatrix} \mathbf{C}_* & & \\ & \mathbf{C}_* & \\ & & \mathbf{C}_* \end{bmatrix} \tag{11}$$

$$\mathbf{C}_F = \begin{bmatrix} -1 & +1 & & & \\ & -1 & +1 & & \\ & & \ddots & \ddots & \\ & & & -1 & +1 \\ 1 & & & & -1 \end{bmatrix} \qquad \mathbf{C}_S = \begin{bmatrix} \beta_1 & \beta_2 & & & \\ & \beta_2 & \beta_3 & & \\ & & \ddots & \ddots & \\ & & & \beta_{N-1} & \beta_N \\ \beta_1 & & & & \beta_N \end{bmatrix}$$

Collocated Nodal Method (steady-state over simple, closed loop)

Matrices of this form are singular!

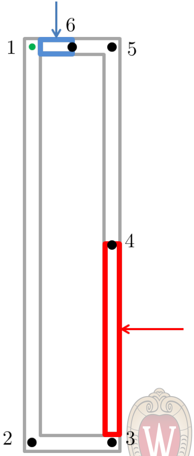
Sweeping approach used for steady-state solver:

- Tank state is known.
- Assume a momentum.
- Sweep through system back to the tank (nonlinear solver over each control volume).
- Two possibilities
  - Integrated pressure is less than the tank's: reduce momentum
  - Integrated pressure is greater than the tank's: increase momentum

Momentum corrections are made through a secant update using the pressure difference as the minimizer.

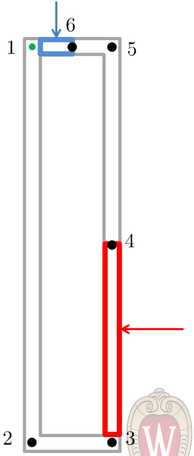
Test Problem

- Single phase
- Constant friction factor
- 10 kW load
- State:
  - $P = 101325$ Pa
  - $T = 300$ K



Test Problem

Measure	Pressure [kPa]		Temperature [K]		Density [kg/m^3]	
Point	**Solver**	**Hand**	**Solver**	**Hand**	**Solver**	**Hand**
1	101325	101325	300.0	300	996.6	996.6
2	150192	150178	300.0	300	996.6	996.6
3	150190	150175	300.0	300	996.6	996.6
4	130640	130631	302.9	302.9	995.7	995.7
5	101327	101328	302.9	302.9	995.7	995.7
6	101326	101326	302.9	302.9	995.7	995.7



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Perturbation equations

Assumed that the true solution is a summation of a steady-state and a transient

$$\mathbf{q}(z,t) = \mathbf{q}^0(z) + \widehat{\mathbf{q}}(z,t). \tag{12}$$

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General nonlinear perturbation equation (diagrams):

$$\frac{\partial \widehat{\mathbf{q}}(z,t)}{\partial t} + \frac{\partial}{\partial z}\left[\mathbf{F}\left(\mathbf{q}^0 + \widehat{\mathbf{q}}; z, t\right)\right] = \mathbf{S}\left(\mathbf{q}^0 + \widehat{\mathbf{q}}, z, t\right) \tag{13}$$

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Taylor expansion about perturbation (neglecting H.O.T.) yields general linear perturbation equation:

$$\frac{\partial \widehat{\mathbf{q}}(z,t)}{\partial t} + \frac{\partial}{\partial z}\left[\frac{\partial \mathbf{F}}{\partial \mathbf{q}^0}\widehat{\mathbf{q}}\right] = \frac{\partial \mathbf{S}}{\partial \mathbf{q}^0}\widehat{\mathbf{q}} \tag{14}$$

Solution methods of linear equation

Wave form ansatz:

$$\widetilde{\widehat{\mathbf{q}}} = \widehat{\mathbf{q}}^0 \,\mathrm{Exp}[j(\kappa z + \omega t)] \tag{15}$$

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Eigenvalues of dynamical system (piecewise integration around loop):

$$\frac{\partial \overline{\widehat{\mathbf{q}}}}{\partial t} = \mathbb{A}(\mathbf{q}^0, t)\overline{\widehat{\mathbf{q}}} \tag{16}$$

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Laplace transform (zeros of transfer function):

$$s\breve{\widehat{\mathbf{q}}} - \widehat{\mathbf{q}}(z,0) + \frac{\partial}{\partial z}\left[\frac{\partial \mathbf{F}}{\partial \mathbf{q}^0}\breve{\widehat{\mathbf{q}}}\right] = \frac{\partial \mathbf{S}}{\partial \mathbf{q}^0}\breve{\widehat{\mathbf{q}}} \tag{17}$$

Is the Test Problem Stable?

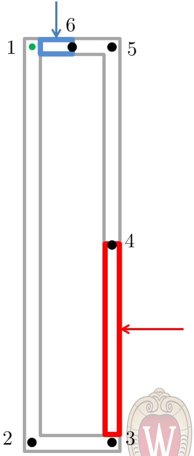
Eigenvalue approach

Loop Integrated:

$$\Re(\lambda) = \begin{bmatrix} -0.0531 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

4-partition Piecewise Integration:

$$\Re(\lambda) = \begin{bmatrix} \pm 1510 \\ \pm(\pm 346) \\ \pm 56.2 \\ -0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (19)$$



Current Work

"Staggered": mass and energy equations are integrated over a different space than the momentum equations

- Avoids velocity-pressure decoupling
- Flexible interpretation of momentums: information propagators
- Allows Non-rigorous grids

Time integration: Implicit Euler

- Unconditionally stable and TVD
- Very diffusive

Semidiscrete, Donor-Cell equations

Momentum cells have a "from" and "to" for handedness. Donor quantities depend on sign of momentum.

$$\frac{\partial \rho_\mathrm{k}}{\partial t} = -\frac{1}{\Omega_\mathrm{k}} \sum \rho u_\mathrm{d} A_\mathrm{d} \tag{20a}$$

$$\frac{\partial \rho i_\mathrm{k}}{\partial t} = -\frac{1}{\Omega_\mathrm{k}} \sum u_d [\rho i_d + P(\rho_d, i_d)] A_\mathrm{k} + \dot{Q}_{\mathrm{add,k}}(\mathbf{q}, z, t) \tag{20b}$$

$$\frac{\partial \rho u_\mathrm{m}}{\partial t} = -\frac{1}{\Omega_\mathrm{m}} \left[u\, \rho u|_\mathrm{from}^\mathrm{to} + P_\mathrm{to} - P_\mathrm{from} \right] A_\mathrm{m} + \frac{A_\mathrm{m}}{\Omega_\mathrm{m}} \int_\mathrm{from}^\mathrm{to} \left[\rho g(z) - \frac{K_\mathrm{eff}(\mathbf{q})}{2} u \left| \rho u \right| \right] \mathrm{d}s \tag{20c}$$

Goals

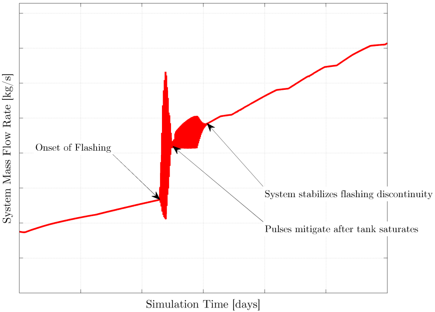
Path Forward:

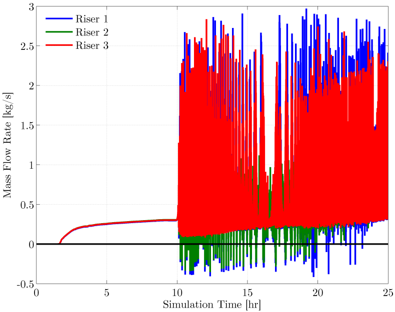
- Complete transient, HEM solver
- Look at HEM boiling behavior
- Attain true non-simple, closed-loop steady-state calculations

Outcomes

- Look at
- Examine the quasi-steady stability of the RCCS experiment
- Stability maps of the three-riser system, in general, for various parameters
- Analytical and physical understanding of how boil-off affects flow behavior

Path Goals ... with Pictures!





Acknowledgements

NEUP Program and NRC Fellowship.

Questions

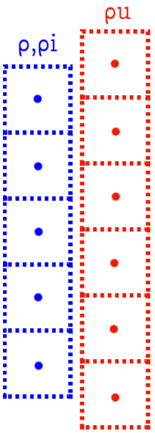
"The key to wisdom is this: constant and frequent questioning. For by doubting we are led to question, and by questioning we arrive at the truth."

— Peter Abelard

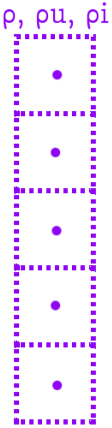
4 Supplements

Staggered/Collocated

Staggered mesh:

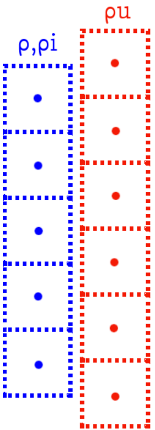


Collocated mesh:

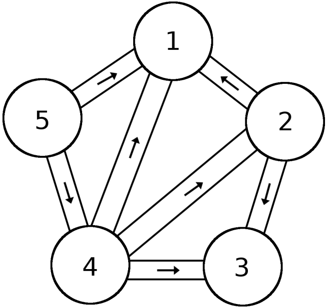


Rigorous vs. Non-Rigorous

Rigorous staggered mesh (CFD):



Non-rigorous staggered mesh (System codes):



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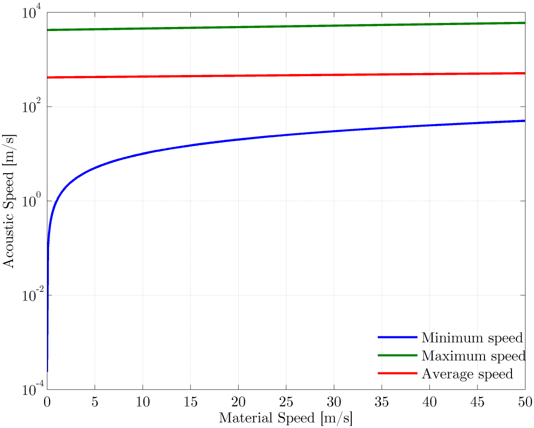
Acoustic Speeds



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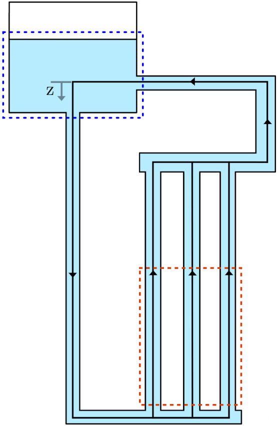
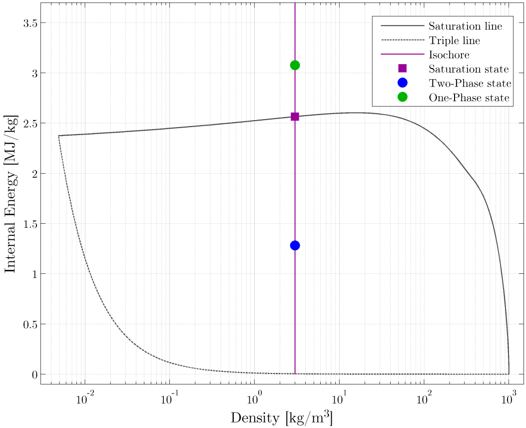
Non-simple, closed loop



i-ρ Diagram



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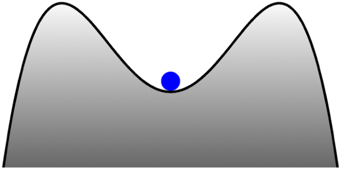
Stability Diagrams

Linearly stable, nonlinearly unstable:



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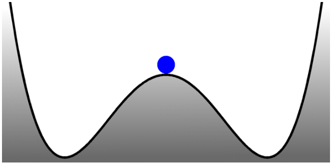

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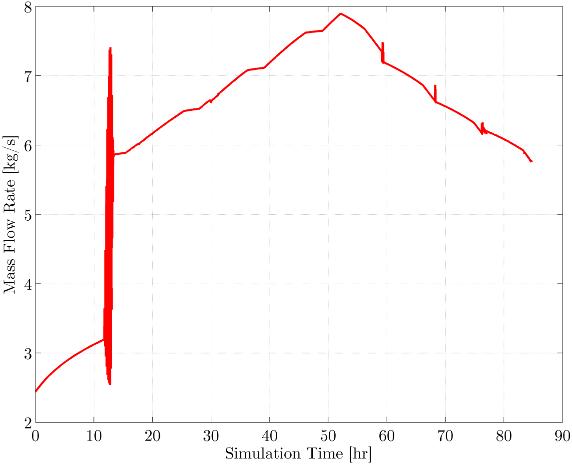
System Mass flow: 4 days



return

Multiphase Model

$$\frac{\partial}{\partial t}\begin{bmatrix}\alpha\rho_\phi \\ \alpha\rho u_\phi \\ \alpha\rho i_\phi\end{bmatrix} + \frac{\partial}{\partial z}\begin{bmatrix}\alpha\rho u_\phi \\ u_\phi\,\alpha\rho u_\phi + P(\rho_\phi, i_\phi) \\ u_\phi[\alpha\rho i_\phi + P(\rho_\phi, i_\phi)]\end{bmatrix} = \tag{21}$$

$$\begin{bmatrix}\mathbb{M}_\phi \\ \alpha\rho_\phi g(z) - \dfrac{K_{\mathrm{eff},\phi}(\mathbf{q})}{2} u_\phi\,|\alpha\rho u_\phi| + \mathbb{P}_\phi \\ \dot{Q}_{\mathrm{add},\phi}(\mathbf{q}, z, t) + \mathbb{E}_\phi\end{bmatrix}$$