

In[ ]:=

```
(*  
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NOTE: This implementation of the Charges in a Cubic Conductor (originally produced by Larry Engelhardt in python) contains the basic structure for calculating the dynamical motion of charges that interact via the Coulomb interaction, using a typical Molecular Dynamics pair potential approach. It does not attempt to animate the motion of the charges during the calculation

(although there are several ways to accomplish this in Mathematica), but only plots the final position of the charges (including an image of the cubic conductor) in a 3D scatter plot after the program has looped through the specified number of time steps.

```
*)  
ClearAll["Global`*"]  
  
(*set physical parameter values*)  
ncharges = 100; (*number of individual charges*)  
Q =  $5.0 \times 10^{-6}$ ; (*net charge in Coulombs*)  
m =  $1.0 \times 10^{-3}$ ; (*mass of each charge in kg*)  
L = 0.2; (*side length of conducting cube in meters*)  
k =  $8.99 \times 10^9$ ; (*Coulomb constant*)  
qi = Q/ncharges; (*charge on each individual charge*)
```

(\* Coordinates of field point P-- Electric field components due to all the charges will be calculated at this point\*)

```
Px = -0.05;  
Py = 0.043;  
Pz = -0.01;
```

(\*algorithmic parameters\*)

```
 $\Delta t$  = 0.001; (*time step in seconds*)  
tsteps = 100; (*total number of time steps*)
```

(\*create charges at random initial positions (all initially at rest)\*)

```
For[i = 1, i ≤ ncharges, i++,  
  x[i] = (-1)^IntegerPart[2 * RandomReal[]] * L/2 * RandomReal[];  
  y[i] = (-1)^IntegerPart[2 * RandomReal[]] * L/2 * RandomReal[];  
  z[i] = (-1)^IntegerPart[2 * RandomReal[]] * L/2 * RandomReal[];  
  vx[i] = 0;  
  vy[i] = 0;  
  vz[i] = 0;  
]
```

(\*Initialize electric field components for each time step to zero\*)

```
For[n = 1, n ≤ tsteps, n++,  
  Ex[n] = 0;  
  Ey[n] = 0;
```

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    Ez[n] = 0;
]
(*Main loop*)
For[n = 1, n ≤ tsteps, n++,
    time[n] = n * Δt;
    (* reset net force in x,y,z directions to zero for all charges *)
    For[i = 1, i ≤ ncharges, i++,
        fx[i] = 0;
        fy[i] = 0;
        fz[i] = 0;
    ]
    (*Calculate forces on each pair of charges by considering
       i-j pairs.% The structure of the nested for loops prevents double counting of
       pairs*) ×
    For[i = 1, i ≤ ncharges - 1, i++,
        For[j = i + 1, j ≤ ncharges, j++,
            dx = x[i] - x[j];
            dy = y[i] - y[j];
            dz = z[i] - z[j];
            rmag = Sqrt[dx * dx + dy * dy + dz * dz];
            (*Magnitude of mutual Coulomb force between charges i and j*)
            fij = k * qi * qj / (rmag * rmag);
            (*The three spatial components of the force between charges
               i and j*)
            fijx = fij * dx / rmag;
            fijy = fij * dy / rmag;
            fijz = fij * dz / rmag;
            (*Accumulation of the components of the net force acting on
               charges i and j*)
            fx[i] += fijx;
            fy[i] += fijy;
            fz[i] += fijz;
            fx[j] += -fijx;
            fy[j] += -fijy;
            fz[j] += -fijz;
        ]
    ]
    (* positions updated for all charges
       For the algorithm for updating positions, use the large friction
       limit wherein the force is replaced by displacement. This approach
       is the simplest stable algorithm for this geometry; thus, this
       simulation emphasizes the resultant configuration of excess charge
       after equilibration, rather than the correct physics governing the
       dynamical motion of the system.*) ×
    For[i = 1, i ≤ ncharges, i++,
        (* With force representing displacement, rescale displacement
           in the case of a large calculated displacement *)
        magf = Sqrt[fx[i] * fx[i] + fy[i] * fy[i] + fz[i] * fz[i]];
        If[magf > L / 100,
            fx[i] = (L / 100) * fx[i] / magf;

```

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    fy[i] = (L/100) * fy[i] / magf;
    fz[i] = (L/100) * fz[i] / magf;
  ];
  (* update positions with large friction approximation*)
    x[i] += fx[i];
    y[i] += fy[i];
    z[i] += fz[i];
  (* if charge moved beyond surface of conductor,bring it back to surface *)
  If[x[i] > L/2, x[i] = L/2];
  If[x[i] < -L/2, x[i] = -L/2];
  If[y[i] > L/2, y[i] = L/2];
  If[y[i] < -L/2, y[i] = -L/2];
  If[z[i] > L/2, z[i] = L/2];
  If[z[i] < -L/2, z[i] = -L/2];
  (* Calculation of Electric Field components at point P*)
  dxP = Px - x[i];
  dyP = Py - y[i];
  dzP = Pz - z[i];
  rvector = Sqrt[dxP^2 + dyP^2 + dzP^2];
  (* Each E-field component, as well as the magnitude of the
  electric field vector is calculated. Units of micro-N/C for
  the electric field are used for ease in plotting.*)
  Ex[n] = Ex[n] + 10^-6 * k * qi / rvector^2 * (dxP / rvector);
  Ey[n] = Ey[n] + 10^-6 * k * qi / rvector^2 * (dyP / rvector);
  Ez[n] = Ez[n] + 10^-6 * k * qi / rvector^2 * (dzP / rvector);
  Emag[n] = Sqrt[Ex[n]^2 + Ey[n]^2 + Ez[n]^2];
]
] (* end of main loop *)
(*Plotting commands*)
tx = Table[{time[i], Ex[i]}, {i, 1, tsteps}];
ty = Table[{time[i], Ey[i]}, {i, 1, tsteps}];
tz = Table[{time[i], Ez[i]}, {i, 1, tsteps}];
tmag = Table[{time[i], Emag[i]}, {i, 1, tsteps}];
ListLinePlot[{tx, ty, tz, tmag}, ImageSize -> Large, PlotRange -> All,
  AxesLabel -> {HoldForm[time[s]], HoldForm[Electric Field [ $\mu$  N / C]]},
  PlotLabel -> None, PlotLegends -> {"Ex", "Ey", "Ez", "Emag"},
  LabelStyle -> {FontFamily -> "Arial", 18, GrayLevel[0]}]
plot = Table[{x[i], y[i], z[i]}, {i, 1, ncharges}];
Graphics3D[{Opacity[0.5],
  {Glow[LightBlue], Black, Cuboid[{-L/2, -L/2, -L/2}, {L/2, L/2, L/2}]},
  Red, PointSize[0.015], Point[plot], PlotMarkers -> {"Sphere", Large},
  Axes -> True, BoxRatios -> 1, Boxed -> False}, ImageSize -> Large,
  PlotRange -> {{-L/2, L/2}, {-L/2, L/2}, {-L/2, L/2}}, Lighting -> Automatic]

```