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(*  
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NOTE:This implementation of the Charges in a Spherical Conductor  
(originally produced by Larry Engelhardt in python) contains the basic  
structure for calculating the dynamical motion of charges that interact  
via the Coulomb interaction, using a typical Molecular Dynamics pair  
potential approach. It does not attempt to animate the motion of the  
charges during the calculation
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(although there are several ways to accomplish this in Mathematica),  
but only plots the final position of the  
charges (including an image of the spherical conductor)  
in a 3D scatter plot after the program has looped through the  
specified number of time steps.
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ClearAll["Global`*"]
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(*set physical parameter values*)  
ncharges = 100; (*number of individual charges*)  
Q =  $5.0 \times 10^{-6}$ ; (*net excess charge in Coulombs*)  
m =  $1.0 \times 10^{-3}$ ; (*mass of each charge in kg*)  
R = 0.1; (*radius of conducting sphere in meters*)  
k =  $8.99 \times 10^9$ ; (*Coulomb constant*)  
qi = Q/ncharges; (*charge on each individual charge*)
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(* Coordinates of field point P -- Electric field components due  
to all the charges will be calculated at this point*)
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Px = -0.05;  
Py = 0.043;  
Pz = -0.01;
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(*algorithmic parameters*)  
 $\Delta t$  = 0.001; (*time step in seconds*)  
tsteps = 100; (*total number of time steps*)
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(*Create charges with random initial positions (all initially at rest). The  
center of the spherical conductor of radius R is at the origin.*)
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For[i = 1, i ≤ ncharges, i++,  
  x[i] = (-1)^IntegerPart[2 * RandomReal[]] * R/Sqrt[3] * RandomReal[];  
  y[i] = (-1)^IntegerPart[2 * RandomReal[]] * R/Sqrt[3] * RandomReal[];  
  z[i] = (-1)^IntegerPart[2 * RandomReal[]] * R/Sqrt[3] * RandomReal[];  
  vx[i] = 0;  
  vy[i] = 0;  
  vz[i] = 0;  
]
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(*Initialize electric field components for each time step to zero*)
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For[n = 1, n ≤ tsteps, n++,
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Ex[n] = 0;
Ey[n] = 0;
Ez[n] = 0;
]
(*Main loop*)
For[n = 1, n ≤ tsteps, n++,
time[n] = n * Δt;
(* reset net force in x,y,z directions to zero for all charges *)
For[i = 1, i ≤ ncharges, i++,
fx[i] = 0;
fy[i] = 0;
fz[i] = 0;
]
(*Calculate forces on each pair of charges by considering
i-j pairs. The structure of the nested for loops prevents double counting of
pairs.*) ×
For[i = 1, i ≤ ncharges - 1, i++,
For[j = i + 1, j ≤ ncharges, j++,
dx = x[i] - x[j];
dy = y[i] - y[j];
dz = z[i] - z[j];
rmag = Sqrt[dx * dx + dy * dy + dz * dz];
(*Magnitude of mutual Coulomb force between charges i and j*)
fij = k * qi * qj / (rmag * rmag);
(*The three spatial components of the force between charges
i and j*)
fijx = fij * dx / rmag;
fijy = fij * dy / rmag;
fijz = fij * dz / rmag;
(*Accumulation of the componenets of the net force acting on
charges i and j*)
fx[i] += fijx;
fy[i] += fijy;
fz[i] += fijz;
fx[j] += -fijx;
fy[j] += -fijy;
fz[j] += -fijz;
]
]
(* velocities and positions updated for all charges *) ×
For[i = 1, i ≤ ncharges, i++,
(* rescale acceleration in case of huge value encountered *)
xaccel[i] = fx[i] / m;
yaccel[i] = fy[i] / m;
zaccel[i] = fz[i] / m;
maga = Sqrt[xaccel[i] * xaccel[i] + yaccel[i] * yaccel[i] + zaccel[i] * zaccel[i]];
If[maga > 1000,
xaccel[i] = 1000 * xaccel[i] / maga;
yaccel[i] = 1000 * yaccel[i] / maga;
zaccel[i] = 1000 * zaccel[i] / maga;
]
]

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];
(* Euler-Cromer algorithm for updataing velocities and positions *)
vx[i] += xaccel[i] * Δt;
vy[i] += yaccel[i] * Δt;
vz[i] += zaccel[i] * Δt;
x[i] += vx[i] * Δt;
y[i] += vy[i] * Δt;
z[i] += vz[i] * Δt;
(* d is the distance from center of sphere to charge*)
d = Sqrt[x[i] * x[i] + y[i] * y[i] + z[i] * z[i]];
(*if charge moved beyond surface of conuctor,bring it back to surface*)
If[d > R,
x[i] = x[i] * R / d;
y[i] = y[i] * R / d;
z[i] = z[i] * R / d;
];
(* Calculation of Electric Field components at point P*)
(*-----*)
-
- THE CALCULATION OF THE ELECTRIC FIELD COMPONENTS AND MAGNITUDE
- GOES HERE
-
-----*)
]
] (* end of main loop *)
(*Plotting commands*)
tx = Table[{time[i], Ex[i]}, {i, 1, tsteps}];
ty = Table[{time[i], Ey[i]}, {i, 1, tsteps}];
tz = Table[{time[i], Ez[i]}, {i, 1, tsteps}];
tmag = Table[{time[i], Emag[i]}, {i, 1, tsteps}];
ListLinePlot[{tx, ty, tz, tmag}, ImageSize → Large, PlotRange → All,
  AxesLabel → {HoldForm[time[s]], HoldForm[Electric Field [ $\mu$  N / C]]},
  PlotLabel → None, PlotLegends → {"Ex", "Ey", "Ez", "Emag"},
  LabelStyle → {FontFamily → "Arial", 18, GrayLevel[0]}]
plot = Table[{x[i], y[i], z[i]}, {i, 1, ncharges}];
Graphics3D[{Opacity[0.5], Sphere[{0, 0, 0}, R], Blue, PointSize[0.015],
  Point[plot], PlotMarkers → {"Sphere", Medium}, Axes → True, BoxRatios → 1},
  ImageSize → Large, PlotRange → {{-R, R}, {-R, R}, {-R, R}}, Lighting → Automatic]

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