Exercise 03 – October 02, 2024

1. You have 8 people, and you need to select and arrange 4 of them in a row for a photo. How many different ways can you arrange them?

   Solution:
   Here, n = 8 (total people) and r = 4 (people to arrange).
   We apply the permutation formula:

   $$P(8,4) = \frac{8!}{(8-4)!} = \frac{8!}{4!}$$

   First, calculate 8! and 4!:
         8! = 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 40,320
         4! = 4 × 3 × 2 × 1 = 24

   Now, calculate P(8, 4):

   $$P(8,4) = \frac{40,320}{24} = 1,680$$

   So, there are 1,680 ways to arrange 4 people out of 8 in a row.

2. You have 7 books, and you want to choose 4 to take on a trip. How many different ways can you select the books?

   Solution:
   This is a combination problem where n = 7 and r = 4:

   $$C(7,4) = \frac{7!}{4!(7-4)!} = \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1} = \frac{840}{24} = 35$$

   So, there are 35 ways to choose 4 books from a set of 7.

3. A bag contains 10 red balls and 15 blue balls. If you randomly select 5 balls without replacement, what is the probability that exactly 3 of the selected balls are red?

   Solution:

   N = 25 (total balls/ population),
   k = 10 (total red balls),
   n = 5 (balls selected/ sample),
   x = 3 (we want to find the probability of selecting 3 red balls).

   Using the hypergeometric formula:

   $$p = \frac{{}_kC_x \; {}_{(N-k)}C_{(n-x)}}{{}_NC_n}$$

   Remember Combination formula:

   $$_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$p = \frac{_{10}C_3 \;\; _{(25-10)}C_{(5-3)}}{_{25}C_5}$$

$$_{10}C_3 = \frac{10!}{7!\,3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

$$_{15}C_2 = \frac{15!}{13!\,2!} = \frac{15 \times 14}{2 \times 1} = 105$$

$$_{25}C_5 = \frac{25!}{20!\,5!} = \frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1} = 53{,}130$$

Now, calculate the probability:

$$p = \frac{120 \times 105}{53{,}130} = \frac{12{,}600}{53{,}130} \approx 0.2372$$

So, the probability of drawing exactly 3 red balls is approximately 0.2372 or 23.72%.