Exercise 04 – October 09, 2024

1. Find the percentage returns from an investment over 5 consecutive years, were:
   Year 1: 10%
   Year 2: 15%
   Year 3: -5%
   Year 4: 8%
   Year 5: 12%

   Solution:
   First, convert the percentages to decimal form and add 1 to each to account for negative growth:
   1.10, 1.15, 0.95, 1.08, 1.12

   Now, apply the geometric mean formula:
   GM = $\sqrt[5]{1.10 \times 1.15 \times 0.95 \times 1.08 \times 1.12}$

   First, multiply the values together:
   $1.10 \times 1.15 \times 0.95 \times 1.08 \times 1.12 = 1.422$

   Now, take the 5th root:
   GM = $\sqrt[5]{1.422} \approx 1.073$

   Convert back to a percentage:
   GM = $(1.073 - 1) \times 100 = 7.3\%$

2. Create a box plot to compare the distribution of data from two different groups, each containing an odd number of data points. Interpret the box plots to compare the central tendency, spread, and potential outliers between the groups.

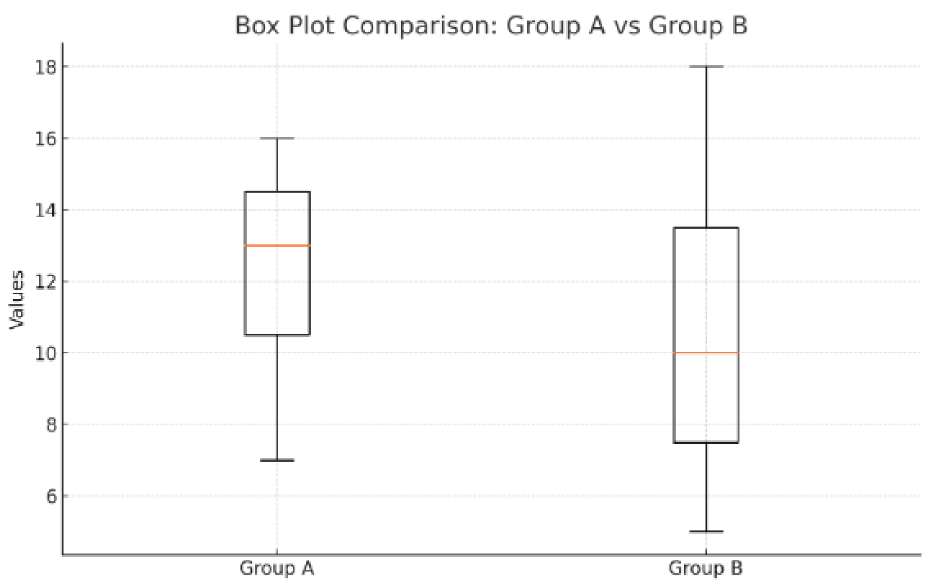
   You are given the following data sets for two groups:
   Group A: 7, 9, 12, 13, 14, 15, 16
   Group B: 5, 7, 8, 10, 12, 15, 18

   Tasks:
   a. Calculate the five-number summary (minimum, 1st quartile Q1, median, 3rd quartile Q3, and maximum) for each group.
   b. Draw the box plots for both groups on the same axis, labeling the minimum, Q1, median, Q3, and maximum values.
   c. Compare the distributions of the two groups based on the box plots:
      - Which group has a higher median?
      - Are there any outliers?

   Solution:
   Group A:
   Minimum: 7
   Q1: Median of the lower half (7,9,12) → Q1 = 9
   Median: Middle value (13)

Q3: Median of the upper half (14,15,16) → Q3 = 15
Maximum: 16
Five-number summary for Group A: 7, 9, 13, 15, 16

Group B:
Minimum: 5
Q1: Median of the lower half (5,7,8) → Q1 = 7
Median: Middle value (10)
Q3: Median of the upper half (12,15,18) → Q3 = 15
Maximum: 18
Five-number summary for Group B: 5, 7, 10, 15, 18



Comparison Between Groups
Median: Group A has a higher median (13) compared to Group B (10).
Outliers: Neither group has extreme outliers based on the data provided.

3. A card is drawn from a standard deck of 52 cards, and then a coin is flipped. What is the probability of drawing a "King" from the deck and flipping a "Tail"?

Solution:
Probability of drawing a "King" from a deck of cards:
There are 4 Kings in a deck of 52 cards, so:
P(King) = $\frac{4}{52} = \frac{1}{13}$

Probability of flipping a "Tail":
P(Tail) = $\frac{1}{2}$

Since the two events are independent, the probability of both events happening together is:

$P(\text{King and Tail}) = P(\text{King}) \times P(\text{Tail}) = \frac{1}{13} \times \frac{1}{2} = \frac{1}{26}$

4. Two departments at a company recorded the number of sales made by their top 10 salespeople in a month. The number of sales made are as follows:
   Department X Sales: 12, 14, 17, 19, 21, 24, 26, 28, 30, 32
   Department Y Sales: 13, 16, 18, 20, 23, 25, 27, 29, 31, 33

   Please, construct a back-to-back stem-and-leaf display for the two departments' sales data.

   Solution:
   Back-to-Back Stem-and-Leaf Display:

Department X (Leaf)	Stem	Department Y (Leaf)
2 4 7 9	1	3 6 8
1 4 6 8	2	0 3 5 7 9
0 2	3	1 3

5. Calculate the probability of getting exactly 3 heads when flipping a fair coin 5 times (where getting heads is considered a success)

   Solution:
   N = 5 (number of trials)
   x = 3 (number of successes)
   π = 0.5 (probability of success, i.e., getting heads)

   Using the formula:

   $P(x = 3) = \frac{N!}{x!(N-x)!} \pi^x (1 - \pi)^{N-x}$

   $P(x = 3) = \frac{5!}{3!\ 2!} 0.5^3\ 0.5^2$

   $P(x = 3) = \frac{5 \times 4}{2 \times 1} 0.5^5 = 10 \times \frac{1}{32} = \frac{10}{32} = \frac{5}{16}$

   Thus, the probability of getting exactly 3 heads in 5 flips of a fair coin is $\frac{5}{16}$ or approximately 0.3125

6. In a basketball game, a player has a free throw success rate of 80%. If the player takes 15 free throws, what is the probability that they make at least 12 successful free throws?

   Solution:
   To find the probability of making at least 12 successful free throws, we need to calculate

   $P(x \geq 12) = P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$

   N = 15 (number of trials)
   π = 0.8 (probability of success)

For x = 12

$P(x = 12) = \frac{15!}{12!\ 3!} 0.8^{12}\ 0.2^{3} \approx 0.227$

For x = 13

$P(x = 13) = \frac{15!}{13!\ 2!} 0.8^{13}\ 0.2^{2} \approx 0.236$

For x = 14

$P(x = 14) = \frac{15!}{14!\ 1!} 0.8^{14}\ 0.2^{1} \approx 0.137$

For x = 15

$P(x = 15) = \frac{15!}{15!} 0.8^{15}\ 0.2^{0} \approx 0.035$

So, the probability that the player makes at least 12 successful free throws is approximately 0.227 + 0.236 + 0.137 + 0.035 = 0.635.

7. A biologist studies the relationship between the number of hours of sunlight a plant receives and its height. The following data shows the hours of sunlight and the corresponding heights of 5 plants:

Hours of Sunlight (X)	**Height (cm) (Y)**
2	10
4	15
6	20
8	25
10	30

Calculate the Pearson correlation coefficient.

Solution:

	X	**Y**	**x**	**y**	**xy**	**x^2**	**y^2**
	2	10	-4	-10	40	16	100
	4	15	-2	-5	10	4	25
	6	20	0	0	0	0	0
	8	25	2	5	10	4	25
	10	30	4	10	40	16	100
Total	30	100	0	0	100	40	250
Mean	6	20	0	0			

Calculating

$r = (\sum xy)/\sqrt{}(\sum x^2 \sum y^2) = 100/\sqrt{(40 + 250)}$

$r = 100/100 = 1$