Your Paper Title Here

Your Name(s)

Date

Abstract

Write your abstract here.

TESTTTING*

Keywords

Keyword 1, *Keyword 2*, *Keyword 3*, *Keyword 4*

Introduction / Background

Provide an introduction with background information.

Statistical Methods

Model

Describe the statistical model used.

Likelihood Inference

Detail the likelihood approach.

#likihood function:

#MLE

#Confidence Interval for Phi

#Large sample confidence interval

#Bootstrap

```
##      2.5%     97.5%
## 0.9150579 0.9808917
```

Bayesian Inference

Detail the Bayesian approach.

#beta binomial model $T_\pi \sim Binom(170, \pi)$ $g(\pi) = Beta(a, b)$

$P(\psi \geq 0.3) = P(\frac{1-2\pi}{1-\pi} \geq 0.3) = P(\pi \leq \frac{7}{17}) = \frac{7}{17}$

#priors We will use the following priors

experimental prior $P(\psi \geq 0.3) \approx 0.57$ $P(\psi \geq 0) = 1$

0.05 prior $P(\psi \geq 0.3) = 0.05$ $P(\psi \geq 0) = 0.5$

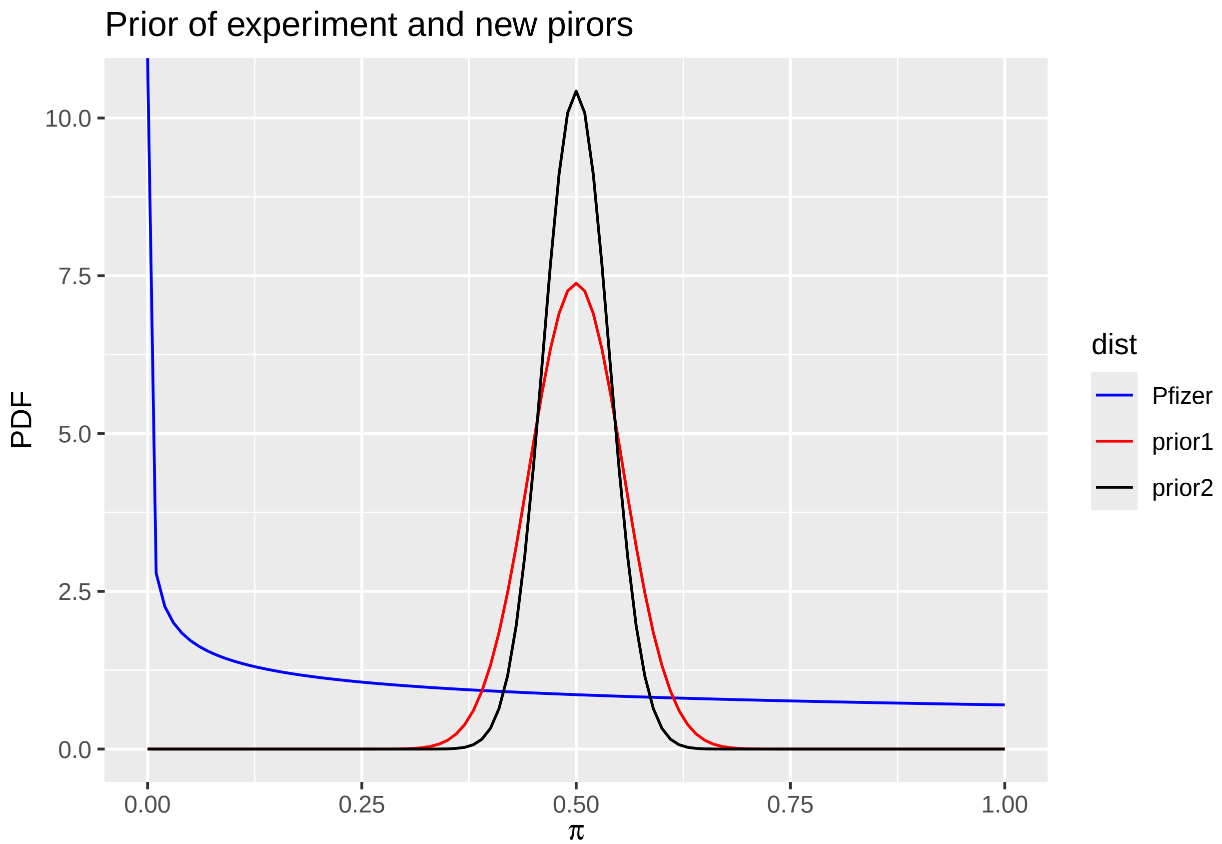
0.01 prior $P(\psi \geq 0.3) = 0.01$ $P(\psi \geq 0) = 0.5$

The experimental prior is the prior that Pfizer used in their study, which gives the vaccine the best chance of passing the tests. The following two priors have a decreasing chance of passing the tests.

#betas

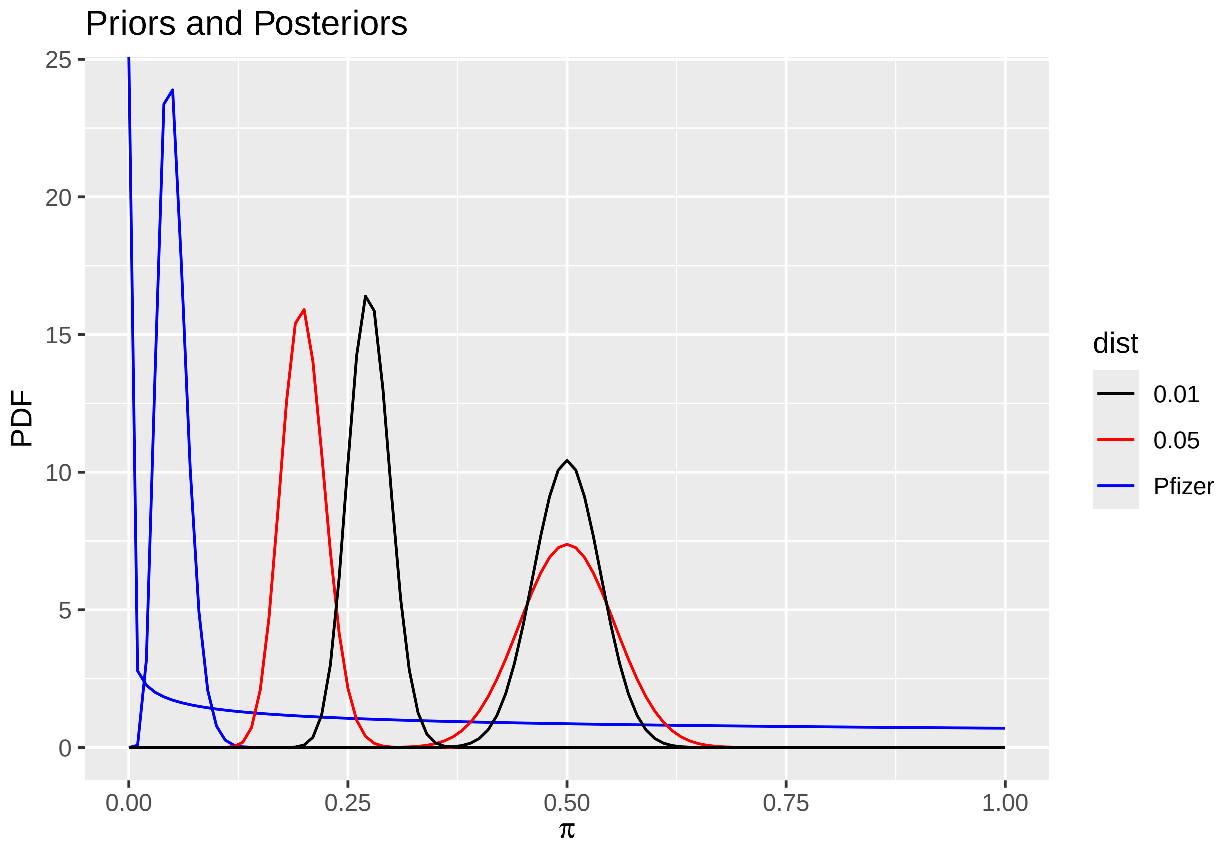
The Pfizer study used $Beta(0.700102, 1)$ as their prior Betas. We will be trying $Beta(43.03, 43.03)$ and $Beta(85.63, 85.63)$ as our prior Beta values.

We can visualize how much more likely their prior is than ours



So, the posterior distributions are as follows: Pfizer: $h(\pi|t) = Beta($ 8.700102, 163) 0.05 posterior: $h(\pi|t) = Beta($ 51.03, 205.03) 0.01 posterior: $h(\pi|t) = Beta($ 93.63, 247.63)

We can visualize the comparison between all of the priors and posteriors:



summarizing the posterior distribution of π

Now we can do some confidence tests to see the significance of our estimates.

Significance Tests and Posterior Distributions

The Pfizer study used a $\text{Beta}(0.700102, 1)$ prior. In our analysis, we compare this with two alternative prior distributions:

$$\text{Beta}\Big(\text{Prior 1: Beta}\Big(43.03,\, 43.03\Big)\Big) \quad \text{and} \quad \text{Beta}\Big(\text{Prior 2: Beta}\Big(85.63,\, 85.63\Big)\Big).$$

Given the data (with 8 successes and 162 failures), the posterior distributions are updated as follows:

- **Pfizer Posterior:**

$$h(\pi \mid t) = \text{Beta}\Big(0.700102 + 8,\, 162 + 1\Big)$$

- **0.05 Prior Posterior:**

$$h(\pi \mid t) = \text{Beta}\Big(43.03 + 8,\ 162 + 43.03\Big)$$

- **0.01 Prior Posterior:**

$$h(\pi \mid t) = \text{Beta}\Big(85.63 + 8,\ 162 + 85.63\Big)$$

Below we compute the posterior medians and approximate 95% credible intervals (using the 2.5th and 97.5th percentiles):

- **Pfizer Posterior:**

  Posterior median: ≈ 0.0489

  95% CI: $\Big[0.0232,\ 0.088\Big]$

- **0.05 Prior Posterior:**

  Posterior median: ≈ 0.1985

  95% CI: $\Big[0.1528,\ 0.2503\Big]$

- **0.01 Prior Posterior:**

  Posterior median: ≈ 0.2739

  95% CI: $\Big[0.2284,\ 0.3228\Big]$

Hypothesis Tests

We test the hypothesis for vaccine effectiveness:

$$H_0 : \pi \geq 0.5 \quad \text{vs} \quad H_1 : \pi < 0.5.$$

For each posterior distribution, the p-value is computed as the probability of observing $\pi \geq 0.5$

under the posterior, i.e.,

$$p\text{-value} = 1 - F(0.5),$$

where $F(\beta)$ is the cumulative distribution function of the corresponding Beta posterior.

Pfizer Posterior:

$$p\text{-value} \approx 0.$$

0.05 Prior Posterior:

$$p\text{-value} \approx 0.$$

0.01 Prior Posterior:

$$p\text{-value} \approx 0.$$

Results

Present your findings.

Discussion / Conclusion

Discuss / conclude here.

Bibliography

Brown, B. (2024). *Lecture Title.* Lecture slides, Course Name, University Name.

Doe, J. (2020). Title of the Paper. *Journal Name*, 12(3), 45-67.

Last, F., & Last, F. (2025). *Book Title.* Publisher.

Smith, A., & Johnson, C. (2023). *Title of the Online Article.* Retrieved from https://www.example.com.

Appendix

Code

Proofs

If applicable, include detailed mathematical derivations or additional theoretical explanations.

Proof of the Maximum Likelihood Estimator for $\widehat{\psi}_0^{mle}$:

We can begin with the Likelihood function. Recall that the design of our experiment is binomial; we observe 170 infections and try to make inference on π, the proportion of infections from those who have the vaccine. Assuming each observation is independent (which is necessary for a binomial model), we can write our likelihood function in terms of π:

$$L(\pi) = \binom{170}{x}(\pi)^x(1-\pi)^{170-x}$$

Where x is the observed number of infections where the patient had the vaccine. We observe $x = 8$, so we can rewrite our likelihood function:

$$L(\pi; x = 8) = \binom{170}{8}(\pi)^8(1-\pi)^{162}$$

Notably, however, we are not hoping to draw inference on π, but rather on ψ, the efficacy of the vaccine. We can write this parameter in terms of π.

$$\psi = \frac{1-2\pi}{1-\pi}$$

In order to write our Likelihood function in terms of ψ, we can rewrite ψ in terms of π:

$$\pi = \frac{\psi-1}{\psi-2}$$

Now we rewrite our likelihood function.

$$L^*(\psi) = \binom{170}{8}\left(\frac{\psi-1}{\psi-2}\right)^8\left(\frac{-1}{\psi-2}\right)^{162}$$

Now that we have written our full likelihood function in terms of ψ, we can write the log likelihood function.

$$\ell^*(\psi) = \log \binom{170}{8} + 8\log(\psi - 1) - 8\log(\psi - 2) - 162\log(\psi - 2) + 162\log(-1)$$

We have $162\log(-1) = \log((-1)^{162}) = \log(1) = 0$:

$$\ell^*(\psi) = \log \binom{170}{8} + 8\log(\psi - 1) - 170\log(\psi - 2)$$

To maximize this equation, we take its derivative with respect to ψ:

$$\frac{d}{d\psi}\ell^*(\psi) = \frac{8}{\psi - 1} - \frac{170}{\psi - 2}$$

Setting this derivative finds the maximum:

$$0 = \frac{8}{\hat{\psi}_0^{mle} - 1} - \frac{170}{\hat{\psi}_0^{mle} - 2}$$

$$\frac{170}{\hat{\psi}_0^{mle} - 2} = \frac{8}{\hat{\psi}_0^{mle} - 1}$$

$$170\hat{\psi}_0^{mle} - 170 = 8\hat{\psi}_0^{mle} - 16$$

$$\hat{\psi}_0^{mle} = \frac{154}{162}$$