Your Paper Title Here

Your Name(s)

Date

Abstract

Write your abstract here.

TESTTTING*

Keywords

Keyword 1, *Keyword 2*, *Keyword 3*, *Keyword 4*

Introduction / Background

Provide an introduction with background information.

Statistical Methods

Model

Describe the statistical model used.

Likelihood Inference

Detail the likelihood approach.

#likihood function:

#MLE

#Confidence Interval for Phi

#Large sample confidence interval

#Bootstrap

```
##   [1]  6 10  7 11 12  3  8 11  8  7 12  7  9  8  4 11  6  3  6 12 11  9  8 15  8
##  [26]  9  8  8  6  5 13 11  9 10  3  7  9  5  6  5  5  7  7  6  5  5  6  7  6 10
##  [51]  3  7 10  4  8  5  5  9 11  7  9  4  7  6 10  7 10 10 10  7  9  8  9  1  7
##  [76]  5  7  8  6  4  6  9  7 10  4  7 14 11 11  5  5  8  6  8  6  5 10  4  7  7
```

```
## [1] 0.9503042
```

```
##  [1] 0.9610390 0.9333333 0.9542484 0.9261745 0.9189189 0.9808917 0.9473684
##  [8] 0.9261745 0.9473684 0.9542484 0.9189189 0.9542484 0.9403974 0.9473684
## [15] 0.9743590 0.9261745 0.9610390 0.9808917 0.9610390 0.9189189 0.9261745
## [22] 0.9403974 0.9473684 0.8965517 0.9473684 0.9403974 0.9473684 0.9473684
```

```
##  [29] 0.9610390 0.9677419 0.9115646 0.9261745 0.9403974 0.9333333 0.9808917
##  [36] 0.9542484 0.9403974 0.9677419 0.9610390 0.9677419 0.9677419 0.9542484
##  [43] 0.9542484 0.9610390 0.9677419 0.9677419 0.9610390 0.9542484 0.9610390
##  [50] 0.9333333 0.9808917 0.9542484 0.9333333 0.9743590 0.9473684 0.9677419
##  [57] 0.9677419 0.9403974 0.9261745 0.9542484 0.9403974 0.9743590 0.9542484
##  [64] 0.9610390 0.9333333 0.9542484 0.9333333 0.9333333 0.9333333 0.9542484
##  [71] 0.9403974 0.9473684 0.9403974 0.9937107 0.9542484 0.9677419 0.9542484
##  [78] 0.9473684 0.9610390 0.9743590 0.9610390 0.9403974 0.9542484 0.9333333
##  [85] 0.9743590 0.9542484 0.9041096 0.9261745 0.9261745 0.9677419 0.9677419
##  [92] 0.9473684 0.9610390 0.9473684 0.9610390 0.9677419 0.9333333 0.9743590
##  [99] 0.9542484 0.9542484
```

```
##      2.5%     97.5%
## 0.9150579 0.9808917
```

Bayesian Inference

Detail the Bayesian approach.

#beta binomial model $T_\pi \sim Binom(170, \pi)$ $g(\pi) = Beta(a, b)$

$P(\psi \geq 0.3) = P(\frac{1-2\pi}{1-\pi} \geq 0.3) = P(\pi \leq \frac{7}{17}) = \frac{7}{17}$

#priors We will use the following priors

experimental prior $P(\psi \geq 0.3) = 0.700102$ $P(\psi \geq 0) = 1$

0.05 prior $P(\psi \geq 0.3) = 0.05$ $P(\psi \geq 0) = 0.5$

0.01 prior $P(\psi \geq 0.3) = 0.01$ $P(\psi \geq 0) = 0.5$

The experimental prior is the prior that Pfizer used in their study, which gives the vaccine the best chance of passing the tests. The following two priors have a decreasing chance of passing the tests.

We can visualize how much more likely their prior is than ours

#betas

The Pfizer study used $Beta(0.700102, 1)$ as their prior Betas. We will be trying \$Beta(43.03, 43.03)\$ and \$Beta(85.63, 85.63)\$ as our prior Beta values. So, the posterior distributions are as follows: Pfizer: $h(\pi|t) = Beta($ 8.700102, 162.700102) 0.05 posterior: $h(\pi|t) = Beta($ 51.03, 205.03) 0.01 posterior: $h(\pi|t) = Beta($ 93.63, 247.63)

summarizing the posterior distribution of π

```
## posterior median 0.3877
```

```
## [1] 0.3394897 0.4374038
```

```
## Warning: package 'HDInterval' was built under R version 4.4.3
```

```
##     lower     upper
## 0.3391011 0.4370029
## attr(,"credMass")
## [1] 0.95
```

Results

Present your findings.

Discussion / Conclusion

Discuss / conclude here.

Bibliography

Brown, B. (2024). *Lecture Title.* Lecture slides, Course Name, University Name.

Doe, J. (2020). Title of the Paper. *Journal Name*, 12(3), 45-67.

Last, F., & Last, F. (2025). *Book Title.* Publisher.

Smith, A., & Johnson, C. (2023). *Title of the Online Article*. Retrieved from https://www.example.com.

Appendix

Code

```
# You can reference your code in the appendix (sample here).
```

Proofs

If applicable, include detailed mathematical derivations or additional theoretical explanations.

Proof of the Maximum Likelihood Estimator for $\widehat{\psi}_0^{mle}$

We can begin with the Likelihood function. Recall that the design of our experiment is binomial; we observe 170 infections and try to make inference on π, the proportion of infections from those who have the vaccine. Assuming each observation is independent (which is necessary for a binomial model), we can write our likelihood function in terms of π:

$$L(\pi) = \binom{170}{x}(\pi)^x(1-\pi)^{170-x}$$

Where x is the observed number of infections where the patient had the vaccine. We observe $x = 8$, so we can rewrite our likelihood function:

$$L(\pi; x = 8) = \binom{170}{8}(\pi)^8(1-\pi)^{162}$$

Notably, however, we are not hoping to draw inference on π, but rather on ψ, the efficacy of the vaccine. We can write this parameter in terms of π.

$$\psi = \frac{1-2\pi}{1-\pi}$$

In order to write our Likelihood function in terms of ψ, we can rewrite ψ in terms of π:

$$\pi = \frac{\psi - 1}{\psi - 2}$$

Now we rewrite our likelihood function.

$$L^*(\psi) = \binom{170}{8} \left(\frac{\psi - 1}{\psi - 2}\right)^8 \left(\frac{-1}{\psi - 2}\right)^{162}$$

Now that we have written our full likelihood function in terms of ψ, we can