CSC311 HW3

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1.

(a)

The dimension of $W^{(1)}$ is $d * d$

The dimension of $W^{(2)}$ is $d * 1$

The dimension of z_1 is $d * 1$

The dimension of z_2 is $d * 1$

(b)

There are a total of $d * d + d$ parameters

(c)

$\overline{y} = \frac{dL}{dy} = y - t$

$\overline{W}^{(2)} = \frac{dL}{dW^{(2)}} = \frac{dL}{dy} * \frac{dy}{dW^{(2)}} = \overline{y} * z_2$

$\overline{z_2} = \frac{dL}{dz_2} = \frac{dL}{dy} * \frac{dy}{dz_2} = \overline{y} * W^{(2)}$

$\overline{h} = \frac{dL}{dh} = \frac{dL}{dz_2} * \frac{dz_2}{dh} = \overline{z_2}$

$\overline{z_1} = \frac{dL}{dz_1} = \frac{dL}{dh} * \frac{dh}{dz_1} = \overline{z_2} * \sigma'(z_1)$

$\overline{W}^{(1)} = \frac{dL}{dW^{(1)}} = \frac{dL}{dz_1} * \frac{dz_1}{dW^{(1)}} = \overline{z_2} * \sigma'(z_1) * x^T$

$\overline{x} = \frac{dL}{dx} = \frac{dL}{dz_1} * \frac{dz_1}{dx} + \frac{dL}{dz_2} * \frac{dz_3}{dx} = \overline{z_2} * \sigma'(z_1) * W^{(1)} + \overline{y} * W^{(2)}$

2.

(a)

Solve $\frac{dy_k}{dz_{k'}}$

Given that $y_k = \frac{exp(z_k)}{\sum_{k'=1}^{K} exp(z_{k'})}$

$(1)\ If\ k' = k$

$$\frac{dy_k}{dz_{k'}} = \frac{exp(z_k)*\sum_{k'=1}^{K} exp(z_{k'}) - exp(z_k)*exp(z_{k'})}{(\sum_{k'=1}^{K} exp(z_{k'}))^2} = (\frac{exp(z_k)}{\sum_{k'=1}^{K} exp(z_{k'})}) * (1 - \frac{exp(z_k)}{\sum_{k'=1}^{K} exp(z_{k'})}) = y_k * (1 - y_k) = y_k - {y_k}^2$$

$(2)\ If\ k' \neq k$

$$\frac{dy_k}{dz_{k'}} = \frac{0*\sum_{k'=1}^{K} exp(z_{k'}) - exp(z_k)*exp(z_{k'})}{(\sum_{k'=1}^{K} exp(z_{k'}))^2} = (\frac{exp(z_k)}{\sum_{k'=1}^{K} exp(z_{k'})}) * (- \frac{exp(z_k')}{\sum_{k'=1}^{K} exp(z_{k'})}) = y_k * (-y_{k'}) = -y_k * y_{k'}$$

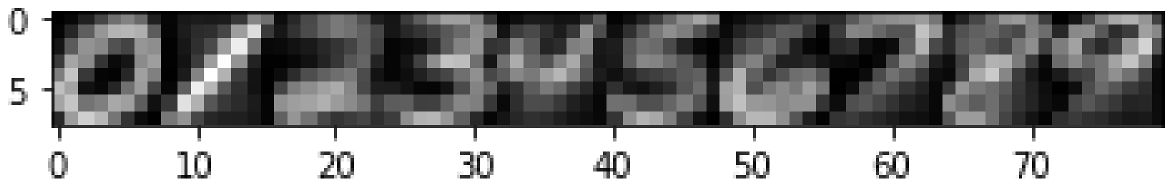
(b)

$$\frac{dL_{CE}}{dw_k} = \frac{dL_{CE}}{dy_k} * \frac{dy_k}{dz_k} * \frac{dz_k}{dw_k}$$

$$\frac{dL_{CE}}{dy_k} = -\sum_{k=1}^{K} t_k * \frac{1}{y_k}$$

$$\frac{dL_{CE}}{dw_k} = ((-\sum_{K'=K} t_k * \frac{1}{y_k}) * (y_k - {y_k}^2) - \sum_{K' \neq K} \frac{t_{k'}}{y_k}(-y_k y_{k'})) * x = (-t_k + y_k \sum_{k'} t_{k'})x = (y_k - t_k)x$$

(3)
0.
Here are 10 plotted mean:

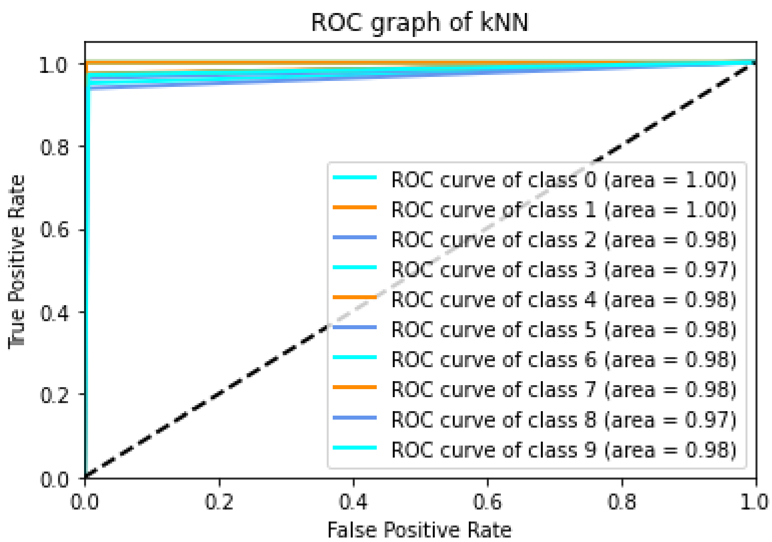


1.
1.
(a)
Test accuracy for k=1
0.96875
Train accuracy for k=1
1.0
(b)
Test accuracy for k=15
0.96
Train accuracy for k=15
0.9612857142857143
2.
Randomly return one from all ties with the use of random.choice(). This is a fair method since all tied labels are equally likely to be the label of the new input. Therefore, I choose to use random in situation of a tie.
3.
For 10 fold
Train accuracy:
k= 1 , accuracy = 0.9651428571428571
k= 2 , accuracy = 0.959
k= 3 , accuracy = 0.9641428571428572
k= 4 , accuracy = 0.9611428571428572
k= 5 , accuracy = 0.9627142857142857
k= 6 , accuracy = 0.9595714285714285
k= 7 , accuracy = 0.9584285714285714
k= 8 , accuracy = 0.9565714285714286
k= 9 , accuracy = 0.9545714285714286
k= 10 , accuracy = 0.9542857142857143
k= 11 , accuracy = 0.9522857142857143
k= 12 , accuracy = 0.9512857142857143
k= 13 , accuracy = 0.9524285714285714
k= 14 , accuracy = 0.9501428571428571
k= 15 , accuracy = 0.9491428571428571
The average accuracy is 0.9567238095238095

Test accuracy:
k= 1 , accuracy = 0.9530000000000001
k= 2 , accuracy = 0.9410000000000001
k= 3 , accuracy = 0.9520000000000002
k= 4 , accuracy = 0.9460000000000001
k= 5 , accuracy = 0.952
k= 6 , accuracy = 0.9465
k= 7 , accuracy = 0.9515
k= 8 , accuracy = 0.9424999999999999
k= 9 , accuracy = 0.9460000000000001
k= 10 , accuracy = 0.94275
k= 11 , accuracy = 0.9424999999999999
k= 12 , accuracy = 0.9400000000000001
k= 13 , accuracy = 0.93575
k= 14 , accuracy = 0.93575
k= 15 , accuracy = 0.9352499999999999
The average is 0.9412238095238095

The best is when k = 1.

3.
kNN:
Accuracy: 0.97025



confusion matrix:

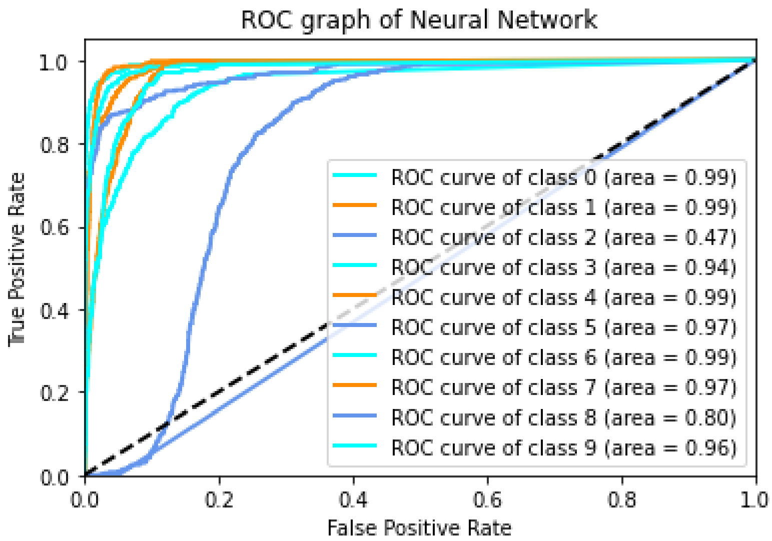
```
[[400   0   0   0   0   0   0   0   0   0]
 [  0 400   0   0   0   0   0   0   0   0]
 [  4   0 388   0   1   1   2   2   1   1]
 [  0   1   3 380   0   8   1   1   5   1]
 [  0   1   0   0 388   0   1   1   0   9]
 [  2   0   1   6   0 385   1   1   4   0]
 [  2   4   1   0   1   1 388   0   3   0]
```

```
 [  0   2   1   0   2   0   0 389   0   6]
 [  1   2   1   1   1  10   1   1 375   7]
 [  0   0   0   1   4   0   0   7   0 388]]
```

Precision: 0.9703887732518048

Recall: 0.9702500000000001

NN:



Test accuracy: 0.86225

Train accuracy: 0.86225

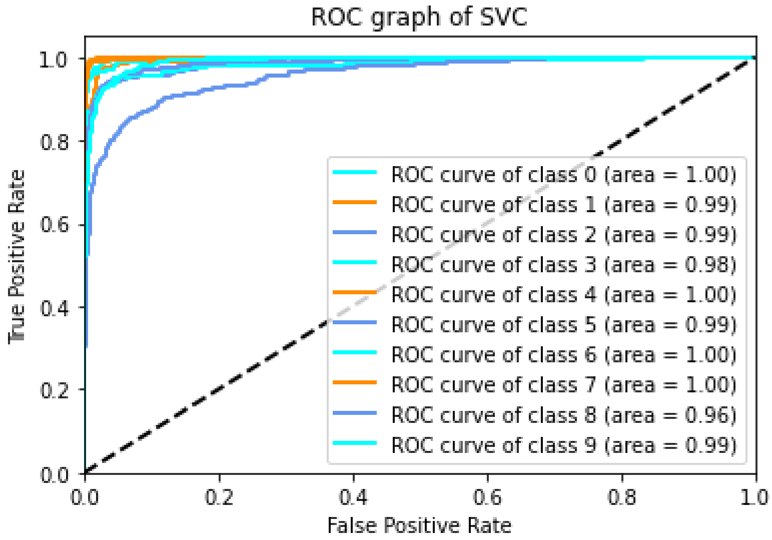
confusion matrix:

```
[[390   3   0   3   0   0   3   0   0   1]
 [  0 391   0   0   1   0   1   0   7   0]
 [ 14   2   0  21   2  17  13   3  78 250]
 [  1   0   0 367   0  11   0   4  14   3]
 [  0   0   0   0 391   0   4   0   0   5]
 [  2   0   0  10   0 379   0   2   5   2]
 [  3   3   0   0   3   0 391   0   0   0]
 [  0   0   0   1   2   0   0 387   2   8]
 [  1   7   0   3   1   5   0   3 377   3]
 [  0   0   0   1   5   2   0  12   4 376]]
```

Precision: 0.7941800408615676

Recall: 0.8622500000000001

SVM:



Accuracy: 0.93225

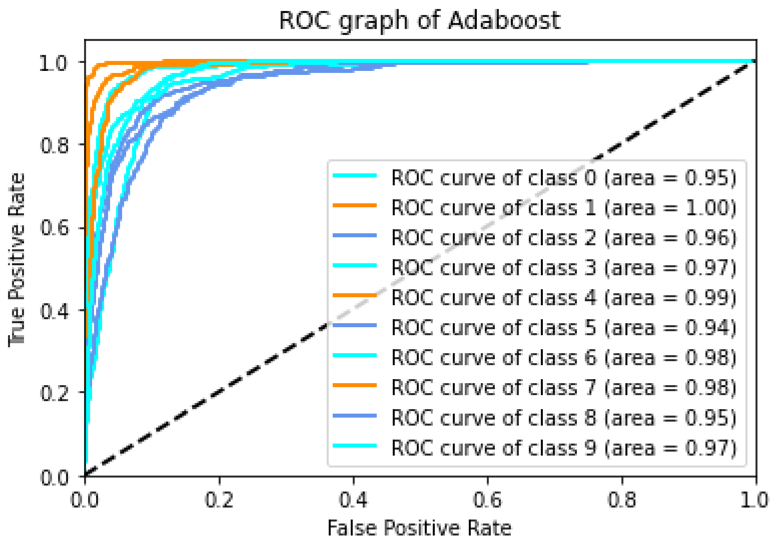
confusion matrix:

```
[[391   0   0   0   3   1   1   1   3   0]
 [  0 386   1   1   4   0   1   0   7   0]
 [  0   4 367   5   2   3   7   1   9   2]
 [  5   1  10 356   0  20   0   2   5   1]
 [  0   5   1   0 388   0   1   0   2   3]
 [  3   2   1  25   0 362   1   2   3   1]
 [  4   7   3   0   5   1 380   0   0   0]
 [  1   2   0   0   1   1   0 390   0   5]
 [  6   7   9  10   1  17   1   1 346   2]
 [  1   6   1   2   5   3   0   9  10 363]]
```

Precision: 0.9323913701166033

Recall: 0.93225

Adaboost:



Accuracy: 0.80975

```
confusion matrix:
[[362   1   1   1   0  25   3   0   7   0]
 [  2 376   0   2  12   0   3   1   4   0]
 [ 19   1 319   8   1  18  14   0  18   2]
 [  1   0  25 342   0  17   1   0  13   1]
 [  3   5   3   0 364   0   8   5   5   7]
 [  4   0   4  64   2 311   1   0  12   2]
 [146   5   6   0   4  39 198   0   2   0]
 [  0   1   1  18   3   1   0 281   5  90]
 [ 18   3   6  11   1  12   0   1 346   2]
 [  3   1   0   2  11   0   0  21  22 340]]
Precision:  0.8233458387457826
Recall:  0.80975
```

kNN performed the best in term of accuracy and overall performance. Adaboost performed the worst.

It is a bit of a surprise but as expected. As we have limited data and a small number of dimensions. Simpler methods like kNN is suitable for this type of data set while more complex models like neural network and adaboost require larger and more complex dataset to show its value to users.