Quantum Computing: An Applied Approach

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Chapter 2 Problems: A Brief History of Quantum Computing

1. Do the following physical systems satisfy DiVincenzo's criteria? For each system state which of the criteria the system violates, if any.

   (a) A set of five trapped ion qubits that can be initialized to the $|0\rangle$ state and measured in the Z basis with the ability to implement arbitrary single qubit rotations.

   This violates the criteria that a universal set of gates must exist. If we can only use single qubit rotations no bell states can be created, for example.

   (b) A system with 5,000 qubits that can find the ground state, but the individual qubits are not addressable.

This violates the criteria that a qubit-specific measurement capability must exist.

2. If we are operating on 3 -vectors, how are we able to use 2x2 matrices? If your first answer is that we are using complex numbers as entries in the matrix then that would lead to operating effectively on 4 -vectors as $\mathbb{C}^2$ is isomorphic to $\mathbb{R}^4$.

   Are there dimensional shenanigans going on here? A priori, a state vector is a two-complex-dimensional vector and a two-complex-dimensional vector is in fact a four-real-dimensional vector. So, how is it that a state vector could be reasonably represented as a point on a sphere, which lives in three-realdimensional space? Secondly, if we believe that these state vectors are fairly described as elements of a sphere in three-real-dimensional space, why then is it fair to act on them with two-complex-dimensional matrices?

It makes sense to use a 3-dimentional sphere since we can use the normalization condition (sum of squares of amplitudes equals 1) and calculate θ and ϕ, the polar and azimuthal angles. We can use 2-dimentional matricies because a qubit can be represented as a 2-dimentional vector of its amplitude measured in a given computational basis.

3. For the CZ gate, does it matter which qubit is the control qubit and which is the target?

It does not matter which qubit is the control or target since CZ is symmetric.

$$\text{The CZ matrix: } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$