Problem Set #3
Business 34902
Winter 2025

The due date for this assignment is Friday Feb 7 8:30am (upload on canvas)

Write up the solution clearly, using tables and graphs where appropriate, preferably using LaTeX. In addition to showing the results, you must also explain and describe your results in words, as you would do if you wrote a paper. Carry out your analysis in Matlab, python, or R, and attach to your solution the code that you wrote. Structure your code carefully and insert some explanatory comments so that it is transparent to an outsider what your code does.

Data: On canvas you can find a data set: `yields.csv`. The data set contains monthly series of zero-coupon yields of German Government Bonds with maturities $1, 2, ..., 10$ years (the Bundesbank extracts those from the market prices of German government coupon bonds).

As a first step in preparing the data, convert the percentage yields in the data set into log yields. Below I refer to the log yields just as "yields."

Your task is to fit a three-factor multi-factor term structure model based on these yields. Use a simplified version of the affine multi-factor model that we discussed in class: Assume that $\mathbf{\Lambda}_1 = 0$. Other than this additional parameter restriction, follow the approach we discussed in class (including the JSZ normalization). Assume that the level, slope, and curvature factors of these yields are measured without error. The weights of these factors are the eigenvectors associated with the three highest eigenvalues of the covariance matrix of yield changes (i.e., first differences).

1. Make a plot of the eigenvectors associated with the three highest eigenvalues of the covariance matrix of yield changes

2. Form yield factors using these three eigenvectors as weights. Report the covariance matrix of changes in these yield factors. (Use this covariance matrix $\mathbf{\Sigma}_{PC}$ to get $\mathbf{\Sigma}$ as we discussed in class).

3. Report your point estimates of δ_0, $\mathbf{\Phi}^*$, and the implied $\mathbf{\Phi}^*_{PC}$ for the risk-neutral dynamics of the PC factors. (No need to report standard errors. You could get them in the usual GMM way, but no need to do this here. Just focus on point estimates.) Compare $\mathbf{\Phi}^*_{PC}$ to $\mathbf{\Phi}_{PC}$. Does this comparison tell you anything about possible dimensions of misspecification of the model?

4. To visualize the model fit plot the actual yields and the fitted yields implied by the estimated model in months 100, 200, 300, and 400 (all in the same plot). What's your judgement of the model fit based on these plots?

5. Report the prices of risk of the PC factors, $\boldsymbol{\lambda}_0$ and the *annualized* Sharpe ratios of the PC factors. Looking at the Sharpe ratios you obtained, is this what you would have expected in terms of which of these three factors should be earning the highest risk premium based on economic considerations?

Some hints for the estimation:

- Choose reasonable values as initial values for the numerical minimization of the sum of squared errors. For example, δ_0 close to zero and diagonal elements of $\boldsymbol{\Phi}^*$ equal to $\boldsymbol{\Phi}_{PC}$ obtained from a VAR of the PC factors may be good starting values (although obviously not necessarily very close to the values that minimize the sum of squared errors of the model).

- Once the optimizer has declared success, try different initial values to see whether you get the same results.

- Before you accept the output for the optimizer as estimates, check that the optimizer actually improved the estimation objective substantially relative to the value of the objective based on the initial values. If it didn't, you probably have a bug somewhere in your code that lead to an erroneous calculation of the model-implied yields with weird behavior that leads the optimizer to get stuck.