Systems

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1 Properties

1.1 Linearity

$$f(ax + by) = af(x) + bf(y) \tag{1}$$

1.2 Time-Invariance

System is not directly dependent on time:

$$y(t) = F(x(t), t) = F(x(t)) \tag{2}$$

This also means that any shift to time in input translates to the same shift in the output.

$$x(t + \delta) \xrightarrow{F} y(t + \delta) \tag{3}$$

2 Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)\ d\tau \tag{4}$$

Properties:

- Commutative

3 Transfer Functions

$$\begin{gathered}
a_2\ddot{x} + a_1\dot{x} + a_0x = b_2\ddot{y} + b_1\dot{y} + b_0y \\
\mathscr{L}\{a_2\ddot{x} + a_1\dot{x} + a_0x\} = \mathscr{L}\{b_2\ddot{y} + b_1\dot{y} + b_0y\} \\
a_2s^2X(s) + a_1sX(s) + a_0X(s) = b_2s^2Y(s) + b_1sY(S) + b_0Y(s) \\
G(s) = \frac{Y(s)}{X(s)} = \frac{b_2s^2 + b_1s + b_0}{a_2s^2 + a_1s + a_0}
\end{gathered}$$

Notes:

- All initial conditions are zero
- $X(s) \neq a_2 s^2 + a_1 s + a_0$
- $Y(s) \neq b_2 s^2 + b_1 s + b_0$

$$\begin{aligned}
G(s) &= \frac{Y(s)}{X(s)} \\
Y(s) &= G(s)X(s) \\
\mathscr{L}^{-1}\{Y(s)\} &= \mathscr{L}^{-1}\{G(s)X(s)\} \\
y(t) &= (g * x)(t) \\
&= \int_{-\infty}^{\infty} g(\tau)x(t-\tau)\ d\tau \\
&= \int_{-\infty}^{\infty} x(\tau)g(t-\tau)\ d\tau
\end{aligned}$$

4 Impulse Response

$$\begin{aligned}
Y(s) &= G(s) \\
\mathscr{L}^{-1}\{Y(s)\} &= \mathscr{L}^{-1}\{G(s)\} \\
y(t) &= g(t)
\end{aligned}$$