Control Theory

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1 Systems

1.1 Linearity

$$f(ax + by) = af(x) + bf(y) \tag{1}$$

1.2 Time-Invariance

Transfer function is not directly dependent on time:

$$y(t) = F(x(t), t) = F(x(t)) \tag{2}$$

This also means that any shift to time in input translates to the same shift in the output.

$$x(t + \delta) \xrightarrow{F} y(t + \delta) \tag{3}$$

2 Transfer Functions

$$a_2\ddot{x} + a_1\dot{x} + a_0x = b_2\ddot{y} + b_1\dot{y} + b_0y$$
$$\mathscr{L}\{a_2\ddot{x} + a_1\dot{x} + a_0x\} = \mathscr{L}\{b_2\ddot{y} + b_1\dot{y} + b_0y\}$$
$$a_2s^2X(s) + a_1sX(s) + a_0X(s) = b_2s^2Y(s) + b_1sY(S) + b_0Y(s)$$
$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_2s^2 + b_1s + b_0}{a_2s^2 + a_1s + a_0}$$