

# Laplace Transform

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## 1 Definition

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt \quad (1)$$

Table 1: Laplace Lookup

|                           | $f(t) = \mathcal{L}^{-1}\{F(s)\}$      | $F(s) = \mathcal{L}\{f(t)\}$           |
|---------------------------|--|--|
| Multiplying by a Constant | $af(at)$                               | $aF(s)$                                |
| Time Scaling              | $f(at)$                                | $\frac{1}{a}F\left(\frac{s}{a}\right)$ |
| Frequency Scaling         | $\frac{1}{a}f\left(\frac{t}{a}\right)$ | $F(as)$                                |
| Time Shifting             | $f(t-a)u(t-a)$                         | $e^{-as}F(s)$                          |
| Frequency Shifting        | $e^{at}f(t)$                           | $F(s-a)$                               |
| Convolution               | $(f * g)(t)$                           | $F(s)G(s) = G(s)F(s)$                  |
|                           | 1                                      | $\frac{1}{s}$                          |
|                           | $t^n$                                  | $\frac{n!}{s^{n+1}}$                   |
|                           | $\sin at$                              | $\frac{a}{s^2 + a^2}$                  |
|                           | $\cos at$                              | $\frac{s}{s^2 + a^2}$                  |
|                           | $\sinh at$                             | $\frac{a}{s^2 - a^2}$                  |
|                           | $\cosh at$                             | $\frac{s}{s^2 - a^2}$                  |