Laplace Transform

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1 Definition

$$F(s) = \mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt \tag{1}$$

1.1 Inverse

$$f(t) = \mathscr{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} e^{st} F(s)\ ds \tag{2}$$

$$t > 0$$

2 properties

- Linear
-

3 Solving Laplace Equations

4 Solving Inverse Laplace Equations

Table 1: Laplace Transform Operations

	$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$
Multiplying by a Constant	$af(at)$	$aF(s)$
Time Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Frequency Scaling	$\frac{1}{a}f\left(\frac{t}{a}\right)$	$F(as)$
Time Shifting	$f(t-a)u(t-a)$	$e^{-at}F(s)$
Frequency Shifting	$e^{at}f(t)$	$F(s-a)$
Convolution	$(f*g)(t)$	$F(s)G(s)$

Table 2: Laplace Transform Base Functions

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$

Table 3: Laplace Transform Calculus

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$
$f^{(n)}(t)$	$s^n F(s) - \sum_{k=0}^{n-1} s^{n-k} f^{(k)}(0)$
$\int_0^t f(\tau)\ d\tau$	$\frac{1}{s}F(s)$