

# Systems

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## 1 Properties

### 1.1 Linearity

$$f(ax + by) = af(x) + bf(y) \quad (1)$$

### 1.2 Time-Invariance

System is not directly dependent on time:

$$y(t) = F(x(t), t) = F(x(t)) \quad (2)$$

This also means that any shift to time in input translates to the same shift in the output.

$$x(t + \delta) \xrightarrow{F} y(t + \delta) \quad (3)$$

## 2 Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau) d\tau \quad (4)$$

Properties:

- Commutative

## 3 Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \quad (5)$$

## 4 Transfer Functions

$$a_2\ddot{x} + a_1\dot{x} + a_0x = b_2\ddot{y} + b_1\dot{y} + b_0y$$

$$\mathcal{L}\{a_2\ddot{x} + a_1\dot{x} + a_0x\} = \mathcal{L}\{b_2\ddot{y} + b_1\dot{y} + b_0y\}$$

$$a_2s^2X(s) + a_1sX(s) + a_0X(s) = b_2s^2Y(s) + b_1sY(s) + b_0Y(s)$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_2s^2 + b_1s + b_0}{a_2s^2 + a_1s + a_0}$$

Notes:

- All initial conditions are zero
- $X(s) \neq a_2s^2 + a_1s + a_0$
- $Y(s) \neq b_2s^2 + b_1s + b_0$

$$G(s) = \frac{Y(s)}{X(s)}$$

$$Y(s) = G(s)X(s)$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{G(s)X(s)\}$$

$$y(t) = (g * x)(t)$$

$$= \int_{-\infty}^{\infty} g(\tau)x(t - \tau) d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)g(t - \tau) d\tau$$

## 5 Impulse Response

$$Y(s) = G(s)$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{G(s)\}$$

$$y(t) = g(t)$$

Table 1: Laplace Lookup

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
Multiplying by a Constant	$af(at)$	$aF(s)$
Time Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Frequency Scaling	$\frac{1}{a}f\left(\frac{t}{a}\right)$	$F(as)$
Time Shifting	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency Shifting	$e^{at}f(t)$	$F(s-a)$
Convolution	$(f * g)(t)$	$F(s)G(s) = G(s)F(s)$
	1	$\frac{1}{s}$
	$t^n$	$\frac{n!}{s^{n+1}}$
	$\sin at$	$\frac{a}{s^2 + a^2}$
	$\cos at$	$\frac{s}{s^2 + a^2}$
	$\sinh at$	$\frac{a}{s^2 - a^2}$
	$\cosh at$	$\frac{s}{s^2 - a^2}$