Linear Algebra

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1 Standard Matrix Representation

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \tag{1}$$

2 Identity Matrix

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{2}$$

I_n is $n \times n$.

3 Selection Matrix

$$S_{ik} = \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 \end{bmatrix} \tag{3}$$

$$S_{ik}A = \begin{bmatrix} a_{i1} & a_{i2} & a_{i3} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{k1} & a_{k2} & a_{k3} & \cdots & a_{kn} \end{bmatrix} \tag{4}$$

$$AS_{jl}^T = \begin{bmatrix} a_{1j} & \cdots & a_{1l} \\ a_{2j} & \cdots & a_{2l} \\ a_{3j} & \cdots & a_{3l} \\ \vdots & \ddots & \vdots \\ a_{mj} & \cdots & a_{ml} \end{bmatrix} \tag{5}$$

4 Minor Matrix

The elements of the minor matrix M are given by:

$$M_{ij} = \begin{vmatrix} a_{11} & \cdots & a_{1(j-1)} & a_{1(j+1)} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{(i-1)1} & \cdots & a_{(i-1)(j-1)} & a_{(i-1)(j+1)} & \cdots & a_{(i-1)n} \\ a_{(i+1)1} & \cdots & a_{(i+1)(j-1)} & a_{(i+1)(j+1)} & \cdots & a_{(i+1)n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{1(j-1)} & a_{1(j+1)} & \cdots & a_{mn} \end{vmatrix} \tag{6}$$

5 Co-Factor Matrix

The elements of the co-factor matrix C are given by:

$$C_{ij} = (-1)^{i+j} M_{ij} \tag{7}$$

$(-1)^{i+j}$ has the effect of alternating negative sign both vertically and horizontally, e.g.:

$$C = \begin{bmatrix} a & -b & c \\ -d & e & -f \\ g & -h & i \end{bmatrix}$$

6 Adjugate Matrix

$$\operatorname{adj} A = C^{\mathrm{T}} \tag{8}$$

7 Determinant

Recursive definition for any $n \times n$ matrix:

$$\det A = \begin{cases} \sum_{k=1}^{n} a_{1k} C_{1k} & n > 0 \\ 1 & n = 0 \end{cases} \tag{9}$$