

Combinatorics

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$n, r \in \mathbb{W}$.

Permutations:

$$P(n, r) = (n)_r = \frac{n!}{(n-r)!}, r \leq n \quad (1)$$

Combinations:

$$C(n, r) = \binom{n}{r} = \frac{P(n, r)}{r!} = \frac{n!}{r!(n-r)!} \quad (2)$$

Permutations w/ replacement (repeats in output):

$$P_R(n, r) = n^r, P_R(0, 0) = 1 \quad (3)$$

Combinations w/ replacement (repeats in output):

$$C_R(n, r) = C(n+r-1, r) \quad (4)$$

Permutations of multisets (repeats in input):

$$P_M(n, x, y, z) = \binom{n}{x, y, z} = \frac{n!}{x!y!z!} \quad (5)$$

Circular permutations:

$$P_C(n) = (n-1)! \quad (6)$$

$$P(n, 0) = 1$$

$$P(n, 1) = n$$

$$P(n, n) = n!$$

$$P(n, n-1) = P(n, n)$$

$$C(n, n-r) = C(n, r)$$

k consecutive items:

$$P(n - k + 1, r) \tag{7}$$

Count n minus k consecutive items, plus 1 representing the items as a group.
 k items can't be together:

$$P(n - k, r) \cdot P(n - k + 1, k) \tag{8}$$

Count all $n - k$ items without restriction. Then place k items in $n - k$ spaces
 after of each item, and 1 space before first item. Then the items

Table 1: Combinatorial Operations

Operation	Meaning
$+n$	Add n choices to an event
$-n$	Remove n choices from an event
$\times n$	Add an event with n choices
$\div n$	Remove an event with n choices