University of Victoria

ELEC 250

Linear Circuits I

Lab 4 - Resonance and Power

Instructor:
Dr. Nikitas Dimopoulos

Teaching Assistant:
Zhen Liu

Clayton Kihn V00794569
Yves Senechal V00213837
Tyler Stephen V00812021
A01 - B01

November 25, 2014

1 Object

This lab will study series resonance as well as the measurement of power in a circuit using a wattmeter.

2 Series Resonance

2.1 Procedure

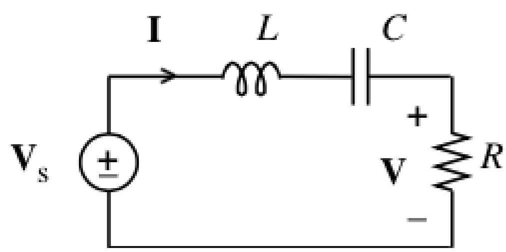

Figure 1: Circuit diagram of resonant RLC circuit
$L = 1.00$ mH, $C = 22$ nF, $R = 43\ \Omega$

The circuit was assembled as shown in Figure 1. The value of R required to give the circuit a quality factor, Q, of 5 was found by

$$R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{5}\sqrt{\frac{1.0\text{ mH}}{22\text{ nF}}} = 43\ \Omega$$

The source voltage and voltage across R were displayed on the oscilloscope. The frequency of the signal generator was increased from 10→50 kHz, with extra measurements taken around the resonant frequency.

The values of R, L, and C in this circuit determine several characteristics.

The resonant frequency, f_0, is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.0\text{ mH})(22\text{ nF})}} = 33.932\text{ kHz}$$

The 3dB frequencies were calculated via the next two equations

$$f_1 = \frac{R}{4\pi L}\left[\sqrt{1+\frac{4L}{R^2C}} - 1\right] = 30.682\text{ kHz}$$

$$f_2 = \frac{R}{4\pi L}\left[\sqrt{1+\frac{4L}{R^2C}}+1\right] = 37.526 \text{ kHz}$$

Giving a bandwidth B of

$$B = f_2 - f_1 = 6.844 \text{ kHz}$$

2.2 Results

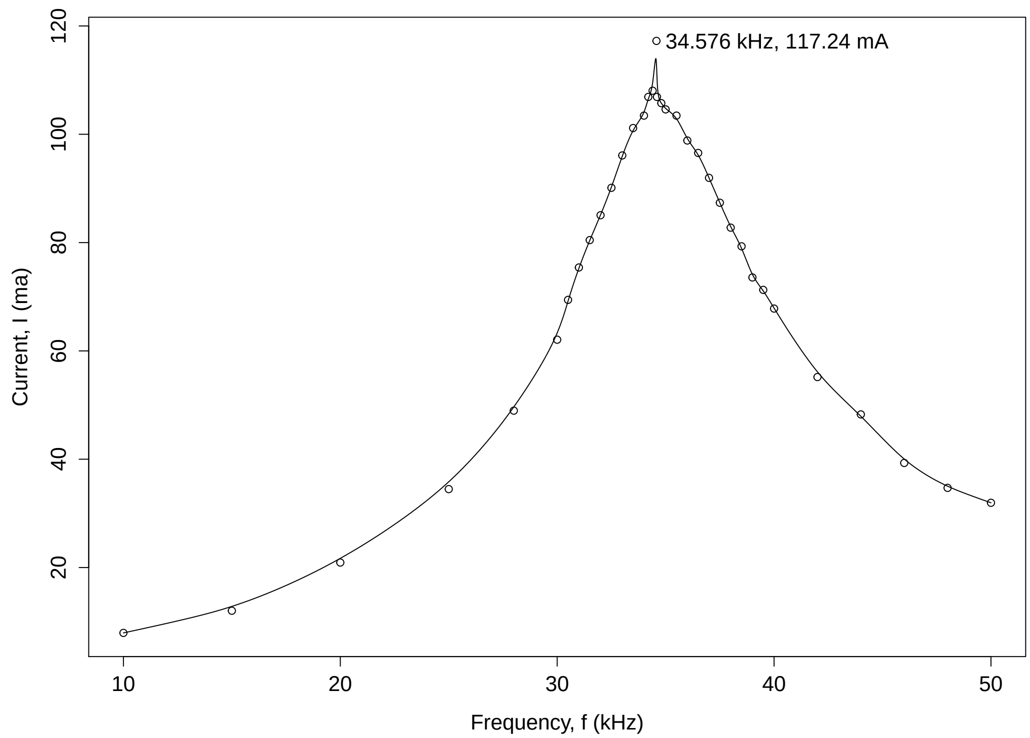

Figure 2: Current through RLC circuit as frequency passes through resonance

Figure 2 shows the response of the current in the circuit to a change in the source frequency. The peak of the graph indicates the resonant frequency as 34.576 kHz. By using Figure 2 and the associated table of values (Table 2) the 3 dB frequencies f_1 and f_2 can be interpolated. The 3dB frequencies occur when $\theta = \pm 45°$. This method gives approximations for the frequencies and bandwidth as:

$$f_1 \approx 30.75 \text{ kHz}$$
$$f_2 \approx 38.75 \text{ kHz}$$
$$B \approx 8.00 \text{ kHz}$$

Using these values, the Q value of the circuit becomes

$$Q = \frac{f_0}{B} = 4.322$$

2.3 Discussion

The measured value of the resonant frequency f_0 is very consistent, with the calculated value being 98% of the measured value.

The measured bandwidth was larger than expected. An explanation for this discrepancy may be obtained from the equations for f_1 and f_2. If the resistance in the circuit is increased then the 3dB frequencies spread out from the resonant frequency. Additional resistance in the circuit is the likely cause of this discrepancy. Experiment 2 indicated that the non-ideal nature of the circuit components will have a measurable effect on the experiment. This discrepancy resulted in a measured Q of 4.322 which is only 86% of the desired Q of 5.

An interesting aspect observed from taking measurements over a large frequency range was that the phase between the source and resistor voltages. The phase difference changed from a ninety degree negative shift, to zero at the resonant frequency, then up to a ninety degree positive shift above f_0. Another important observation was that the phase shift was forty five degrees at f_1 and f_2, the 3dB down frequencies.

3 Power Measurement

3.1 Procedure

The power factor pf is the ratio of true versus apparent power. A leading pf is achieved in an RC circuit, while a lagging pf is the result of a RL circuit. In this section, two experiments were performed where a leading and lagging pf were obtained separately. The block diagram in Figure 3 represents the physical setup of the experiment that includes:

- high-wattage component board that carries a 200 Ω resistor, a 300 mH inductor, and a 10 μF capacitor
- variac with an isolation transformer used to reduce the voltage to 50V_{rms}
- a wattmeter between the isolation transformer and the component board to measure real power (shown in Figure 4).

Figure 4 represents the wiring diagram for the RC circuit; the RL circuit is similar with the substitution of and inductor L in place of the capacitor C.

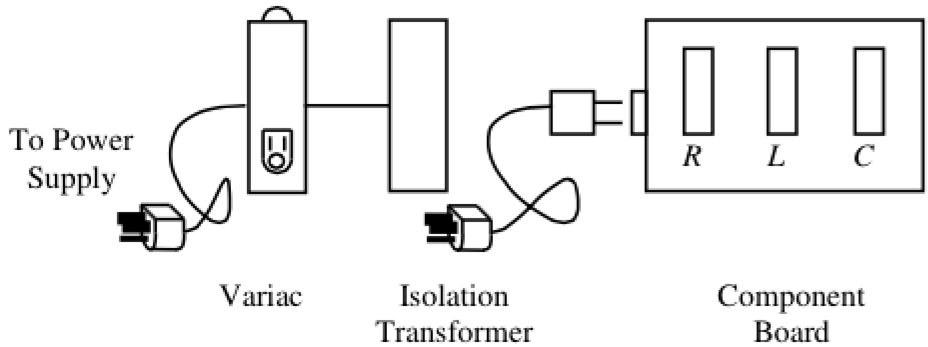

Figure 3: Block diagram of circuit components
$R = 200\ \Omega$, $L = 300\ m\text{H}$, $C = 10\ \mu\text{F}$

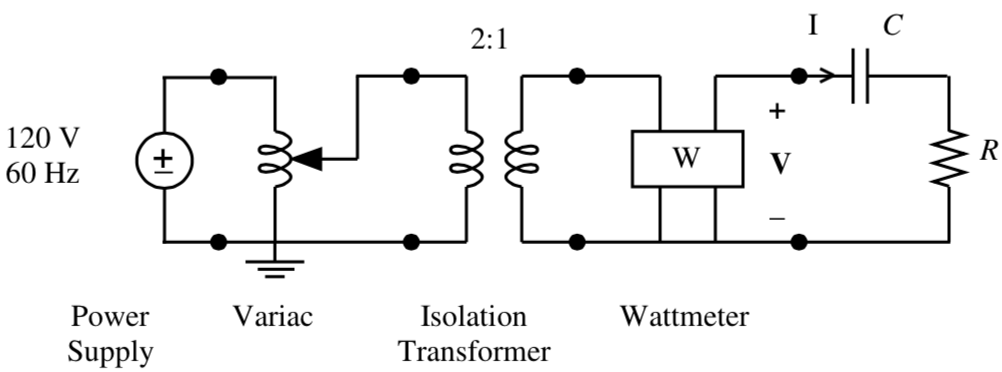

Figure 4: RC pf measurement circuit setup

3.2 Results

Table 1 is a summary of the values obtained in this section.

First, power consumption was measured in an RC circuit. With a 200Ω resistor, a 10μF capacitor, and the voltage set to 49.94V$_{rms}$, the wattmeter measured a current of 151.0mA with real power consumption of 4.55W. Next, a digital multimeter (DMM) placed across the resistor indicated V$_R$ = 30.06V$_{rms}$. Using the equation

$$P = \frac{V_R^2}{R} \tag{1}$$

the true power P can be calculated to be 4.52 W. Similarly, the apparent power S can be calculated using the equation,

$$S = V_{rms} I_{rms} \tag{2}$$

which results in $S = 7.54$ VA. The pf can be estimated using

$$pf = \frac{P}{S} \tag{3}$$

which yeilds a result of 0.6034. The pf was then be measured to be 0.6019 by the following relationship.

$$pf = \frac{P}{V_{rms} I_{rms}} \tag{4}$$

The relationship of the reactive power Q, true power P, and apparent power S is represented by the following equation, which results in $Q = 6.014$ VAR.

$$Q = \sqrt{S^2 - P^2} \tag{5}$$

This equation can be visualized by a right-angle triangle where S is the hypothenuse.

Second, an RL circuit was constructed with a 200 Ω resistor, a 300 mH inductor, and the voltage set to 50.09 V_{rms}. A digital multimeter (DMM) placed across the resistor indicated $V_R = 36.10$ V_{rms}. The wattmeter in turn measured a current of 181.5 mA with true power consumption of 7.16 W; however, using equation 1, the true power P was expected to be 6.52 W.

With the relationship of equation 2, the apparent power results in $S = 9.09$ VA.

A pf of 0.7876 is estimated and expected using equation 3; nevertheless, the value of the pf measured to be 0.7207 using the relationship of equation 4.

Using equation 5, there appears to be 5.602 VAR of reactive power Q in the circuit.

Circuit	V_{rms} (V)	I_{rms} (mA)	V_R (V)	Q (VAR)	P (W) *measured*	$\frac{V_R^2}{R}$	pf $\frac{P}{S}$	$\frac{V_R}{V_{rms}}$
RC	49.94	151.0	30.06	6.014	4.55	4.54	0.6034	0.6019
RL	50.09	181.5	36.10	5.602	7.16	6.55	0.7876	0.7207

Table 1: Power measurements in RC and RL circuits

3.3 Discussion

The RC circuit yielded similar results to those expected. In this circuit the difference between measured and calculated real power P and power factor pf was 0.664% and 0.249% respectively, which is negligible and expected when using non-ideal components.

The RL circuit, however, showed larger discrepancies. The difference between measured and calculated P and pf was 8.94% and 8.49% respectively. From past experiments, these results can be expected due to the nature of inductors. Heat, a by-product of current and resistance, will increase internal resistance of the inductor which ultimately skews the results.

4 Conclusion

The investigation of the RLC circuit confirmed that the current through the circuit is at a maximum when it is excited by a source at its resonant frequency. With consideration given to the non-ideal nature of the components, the choice of a 43 Ω resistor yielded a quality factor of approximately 5.

In the power measurement experiment, the wattmeter was confirmed to indicate the true power consumed by the circuit. Excess current may enter and circulate the circuit due to capacitors and inductors, but it is not consumed or transformed; it simply is the result of a current that leads or lags the voltage.

Relating this experiment to the big picture, we can justify power companies and big industries that strive for near unity power factors where all, or at least most, of the current required for circuits is consumed. An increase in current requires an inflated infrastructure.

Frequency f (kHz)	Resistor Voltage V_r (V)	Current I (mA)	Phase Shift θ (°)
10.000	0.345	7.931	−85.0
15.000	0.523	12.023	−82.0
20.000	0.910	20.920	−76.0
25.000	1.500	34.483	−69.0
28.000	2.130	48.966	−61.0
30.000	2.700	62.069	−51.5
30.500	3.020	69.425	−47.0
31.000	3.280	75.402	−43.5
31.500	3.500	80.460	−38.0
32.000	3.700	85.057	−34.5
32.500	3.920	90.115	−28.0
33.000	4.180	96.092	−22.0
33.500	4.400	101.149	−15.0
34.000	4.500	103.448	−8.0
34.200	4.650	106.897	−5.0
34.400	4.700	108.046	−2.3
34.576	5.100	117.241	−0.1
34.600	4.650	106.897	0.8
34.800	4.600	105.747	3.8
35.000	4.550	104.598	6.8
35.500	4.500	103.448	13.5
36.000	4.300	98.851	20.0
36.500	4.200	96.552	25.8
37.000	4.000	91.954	31.5
37.500	3.800	87.356	35.7
38.000	3.600	82.759	40.0
38.500	3.450	79.310	43.5
39.000	3.200	73.563	47.2
39.500	3.100	71.264	50.0
40.000	2.950	67.816	53.0
42.000	2.400	55.172	61.0
44.000	2.100	48.276	66.0
46.000	1.710	39.310	69.5
48.000	1.510	34.713	72.0
50.000	1.390	31.954	74.7

Table 2: Change in current through resistor in RLC circuit as source frequency passes through resonance