University of Victoria

ELEC 250

Linear Circuits: 1

Lab 1 - Circuit Theorems

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Contents

1 Object

To verify and become familiar with the following linear circuit theorems:

- Kirchoff's Current and Voltage Laws
- Linearity and Superposition Theorems
- Thevenin's and Norton's Theorems

2 Results

2.1 Kirchhoff's Current Law

All measured current values in this section of the experiment were consistent with the calculated values within 5% uncertainty. The largest source of uncertainty is the tolerance of the resistors (5%). The calculated and measured quantities are summarized in Figure 2 on the next page. The consistency of the measured values with the calculations confirms that KCL can be applied to this circuit.

Confirmation for KCL at node A is:

$$-I_1 + I_2 + I_3 = 0$$
$$-1.4175mA \quad + \quad 0.0398mA \quad + \quad 1.4571mA \quad = \quad 79.4\mu A \quad \cong \quad 0A$$

(Note: Figure 2.1 in the laboratory manual does not indicate any nodes. Node A has been assumed to be at the position directly above R_2 in Figure 1. However, there are no logical locations for nodes B or C. The upper right corner of Figure 1, where I_3 joins I_L is a trivial node.)

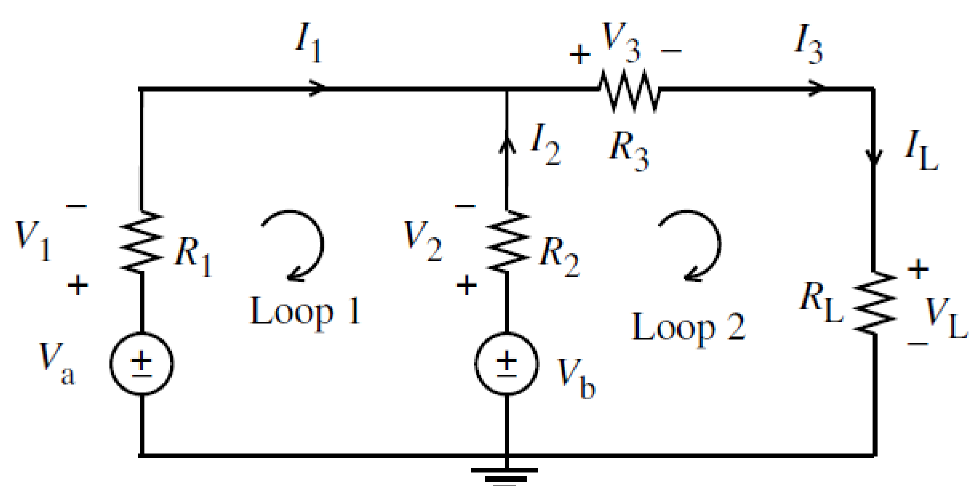

Figure 1: Initial circuit setup

Calculated vs Measured Current				
	I_1	I_2	I_3	I_L
Calculated	1.41 mA	0.04 mA	1.45 mA	1.45 mA
Measured	1.4175 mA	0.0398 mA	1.4571 mA	1.4571 mA

Figure 2: Calculated current and measurement comparison

2.2 Kirchhoff's Voltage Law

Voltage measurements of the circuit used in 2.3.1 were consistent with the calculated values within 5% uncertainty. The calculated and measured quantities are summarized in Figure 3. The consistency of the measured values with the calculations confirms that KVL can be applied to this circuit. Confirmation of KVL in loops 1 and 2 are:

$$-V_a + V_1 - V_2 + V_b = 0 \quad (loop1)$$
$$-6.0306V + 3.1098V - 0.0869V + 2.9839V = -0.0238V \cong 0V$$

$$-V_b + V_2 + V_3 + V_L = 0 \quad (loop2)$$
$$-2.9839V + 0.0869V + 1.4493V + 1.4460V = -0.0017V \cong 0V$$

Calculated vs Measured Voltage				
	V_1	V_2	V_3	V_L
Calculated	3.0957 V	0.0957 V	1.4521 V	1.4521 V
Measured	3.1098 V	0.0869 V	1.4493 V	1.4460 V

Figure 3: Calculated voltage and measurement comparison

2.3 Linearity Theorem

The first step in this section was to disable the voltage source V_b of Figure 1 by replacing it with a short circuit. The input voltage source V_a was then varied to obtain the load current and load voltage plots of Figures 4 and 5 below.

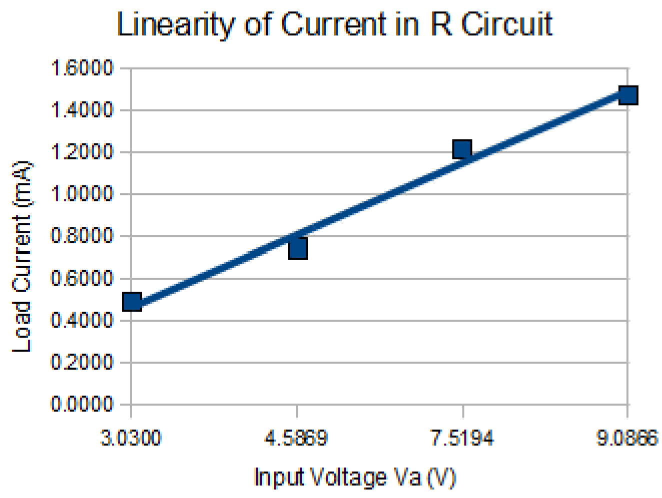

Figure 4: Linearity of Resistive Circuit

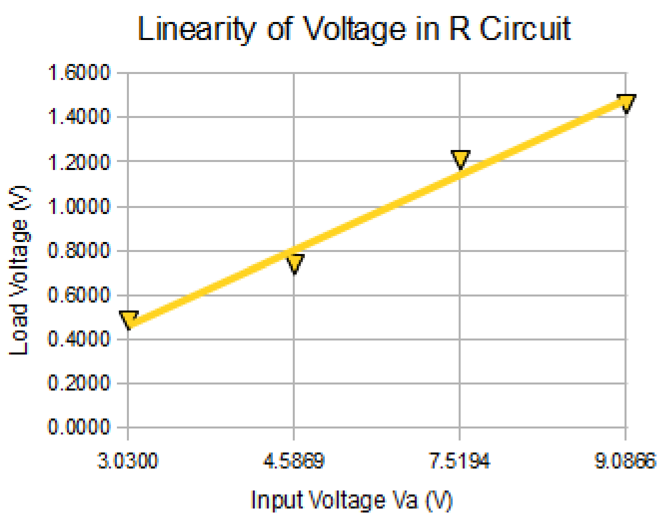

Figure 5: Linearity of Resistive Circuit

The theoretical proportionality constants for this modified circuit were calculated using the following equations:

$$V_L = K_1 V_a$$
$$I_L = K_2 V_a$$

$$\text{Where } K_1 = \tfrac{0.970}{6} = 0.162,$$
$$\text{and } K_2 = \tfrac{0.970}{6} = 0.162$$

The experimental proportionality constants using meter readings were as follows:

$$K_1 = \tfrac{1.4616}{9.0866} = 0.1609,$$
$$\text{and } K_2 = \tfrac{1.4729}{9.0866} = 0.1621$$

Thus, the theoretical and experimental proportionality constants agree to two decimal places.

	Linear Property of the Circuit				
V_a	3.0300	4.5869	6.0306	7.5144	9.0866
I_L (mA)	0.4910	0.7436	0.9774	1.2175	1.4729
V_L	0.48731	0.73790	0.96957	1.2082	1.4616
K_1	0.1608	0.1609	0.1608	0.1608	0.1609
K_2	0.1620	0.1621	0.1621	0.1620	0.1621

Figure 6: Linearity

2.4 Superposition Theorem

In this section, the circuit is still configured as in the previous step with V_a set to 6 V. The load voltage and current were then measured to be 0.96957 V and 0.9774 mA respectively. These figures closely agreed with the theoretically calculated values of 0.97 V and 0.970 mA. The next circuit modification was to disable V_a and reconnect V_b with a voltage of 3 V. This time, the load voltage and current were measured to be 0.4804 V and 0.4841 mA. The theoretical values were 0.484 V and 0.484 mA. Again, the measurements and theoretical values agree to 2 decimal places.
From sections 2.3.1 and 2.3.2 in the lab manual, the values of V_L and I_L were found to be 1.4460 V and 1.4571 mA. Using these values and measurements from this section we were able to verify the superposition theorem which states:

$$V_L = V_{La} + V_{Lb}$$
$$I_L = I_{La} + I_{Lb}$$

From our measurements we obtained:

$$0.96957V + 0.4804V = 1.4500V$$
$$0.9774mA + 0.4841mA = 1.4615mA$$

Both values agree to theoretical values by at least one decimal place and verify the superposition theorem.

Calculated vs Measured Single Source Data				
	V_{La}	I_{La}	V_{Lb}	I_{Lb}
Measured	0.9695 V	0.9774 mA	0.4804 V	0.4841 mA
Calculated	0.9678 V	0.9678 mA	0.484 V	0.484 mA

V_a = 6.0306 V V_b = 2.9839 V

Figure 7: Superposition Measurements

2.5 Thevenin's Theorem

To calculate Thevenin's equivalent, the circuit was configured as in Figure 1 with the exception of resistor RL which was disconnected. The potential difference across AB (V_{ab}) was measured to be V_{ab}= 4.5051 V, where $V_{ab} = V_T$ and V_T is the Thevenin equivalent voltage.

With resistor R_L still disconnected and V_a and V_b disabled, the equivalent circuit resistance was measured to be $R_T = 2.0847\text{k}\Omega$, where R_T is the Thevenin equivalent resistor. The measured R_T was within the 5% uncertainty of the calculated 2.11 kΩ.

The circuit was then reconfigured to replicate Figure 8 with $V_T = 4.5051$ V, and $R_T =$ 2.0707 kΩ $+/-$ 5%. Two 1 kΩ and one 100 Ω resistors in series were chosen to replicate this circuit. In this configuration, $V_L\prime$ and $I_L\prime$ were measured to be 1.4486 V and 1.4613 mA respectively, which were within 5% uncertainty of the V_L and I_L (1.4460 V and 1.4571 mA) measured in section 2.3.1 and 2.3.2.

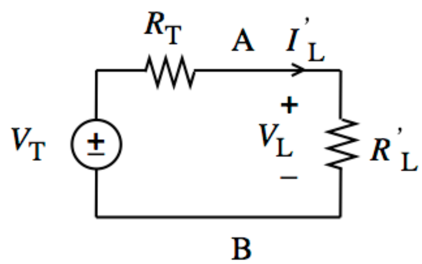

Figure 8: Thevenin Equivalent

Calculated vs Measured Thevenin Equivalent					
	V_T	R_T	V_L	V_T	I_L
Calculated	4.5 V	2.11 kΩ	1.4521 V	N/A	1.45 mA
Measured	4.45051 V	2.0847 kΩ	1.4486 V	4.4975 V	1.4613 mA

Figure 9: Thevenin Measurements

2.6 Norton's Theorem

To calculate Norton's equivalent, the circuit was configured as in Figure 1 with the exception of R_L being replaced with a short-circuit. The current I_N (Norton equivalent) across the short-circuit was measured to be 2.1511 mA. The circuit was then reconfigured to replicate Figure 10 with $I_N = 2.1511$mA and $R_T = 2.0707$ kΩ +/- 5% (from 2.3.5). In this circuit configuration, V_L'' and I_L'' were measured to be 1.4747 V and 1.4767 mA respectively, which is within 5% uncertainty of the V_L and R_L (1.4460 V and 1.4571 mA) measured in section 2.3.1 and 2.3.2.

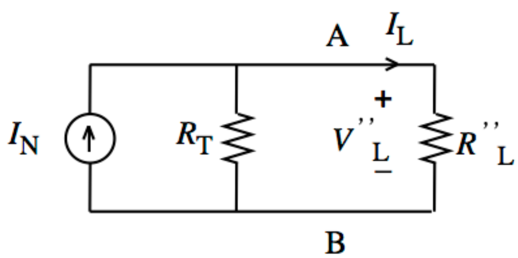

Figure 10: Norton Equivalent

Norton's theorem, which states that a single current source and resistor in parallel can represent a more complex circuit, has been verified experimentally. The circuits from Figure 1 and Figure 10 are completely different, yet they yield the same results regarding voltage and current across the load resistor (R_L).

Calculated vs Measured Norton Equivalent				
	I_N	V_L	I_L	I_N
Calculated	2.1429 mA	1.4521 V	1.45 mA	N/A
Measured	2.1511 mA	1.4747 V	1.4767 mA	2.1694 mA

Figure 11: Norton Measurements

3 Attachments

3.1 Measurements

3.2 Prelab Work