# Department of Electrical and Computer Engineering University of Victoria

## ELEC 300 - Linear Circuits II

### LABORATORY REPORT

Experiment No.: 3

Title: Time Domain Responses

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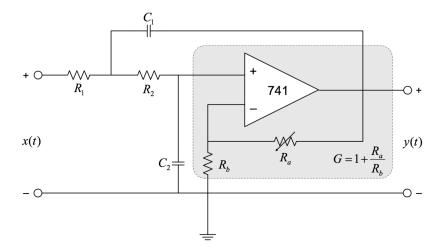
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### 1 Objective

This lab will create an active realization of a second order system. The output of the circuit will be used to verify the system parameters.

#### 2 Introduction

Inductors in second order systems can be replaced by amplifiers and capacitors. Fig. 1 shows an active second order system with a gain of  $G = 1 + \frac{R_a}{R_b}$ .



$$C_1 = C_2 = 16 \text{ nF} = C$$
,  $R_1 = R_2 = 10 \text{ k}\Omega = R$ ,  $R_b = 39 \text{ k}\Omega$ ,  $R_a = 78(1 - \zeta) \text{ k}\Omega$ 

Figure 1: Active realization of a second order system

The transfer function of this system is:

$$H(s) = \frac{G\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{1}$$

where

$$\omega_0 = \frac{1}{RC} \tag{2}$$

and

$$\zeta = 1 - \frac{R_a}{2R_b}. (3)$$

When the unit step function is applied to Fig. 1 it will produce an output similar to Fig. 2.

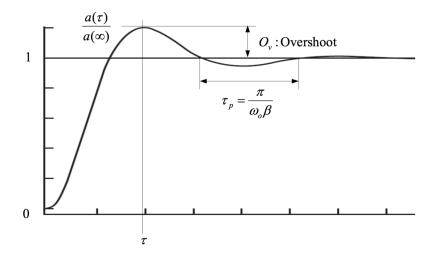


Figure 2: Step response of second order system

The system parameters  $\zeta$  and  $\omega_0$  can be obtained from this graph since:

$$O_v = \exp\left(\frac{-\zeta\pi}{\beta}\right) \tag{4}$$

$$T_p = \frac{\pi}{\omega_0 \beta}.\tag{5}$$

 $O_v$  can be obtained from Fig. 2 as:

$$O_v = \frac{V_{peak} - V_{steady}}{V_{steady}}.$$

#### 3 Results and Discussion

Fig. 1 was realized with damping factor  $\zeta = 0.1$ . Using (3),  $R_a = 70.2 \,\mathrm{k}\Omega$ . The opamp supply power was set at  $\pm 15 \,\mathrm{V}$ . An input signal of 1 V<sub>pp</sub> at 50 Hz square wave was applied to the circuit. One pulse of the square wave will behave locally like a unit step function.

The output of this circuit is shown in Fig. 3. The large overshoot corresponds to significant underdamping, which is consistent with  $\zeta = 0.1$ .

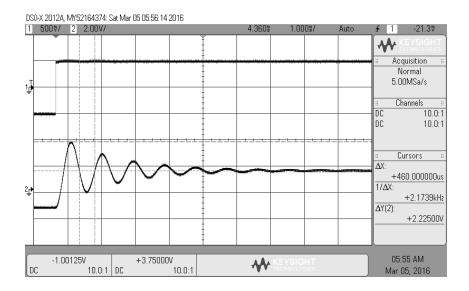


Figure 3: Measured response of second order system

The measured values are  $O_v = \frac{2.225\,\mathrm{V}}{2.8\,\mathrm{V}} = 0.7946$  and  $T_p = 460\,\mathrm{\mu s}$ . As the transient response is damped, the value of the output settles to 2.8 V. This is consistent with G = 2.8 for an input of 1 V.

Using equation (4) and Fig. 3, Zeta could be solved for.

$$\frac{\ln(O_v)}{\pi} = \frac{\zeta}{\sqrt{1-\zeta^2}} \tag{6}$$

The experimentally determined value of  $\zeta$  was found to be  $\zeta = 0.07$ . Next,  $\omega_0$  was solved for by rearranging (5).

$$\omega_0 = \frac{\pi}{T_p \beta} \tag{7}$$

Solving equation (7) gives an angular velocity of  $\omega_0 = 6845 \text{ rad s}^{-1}$ .

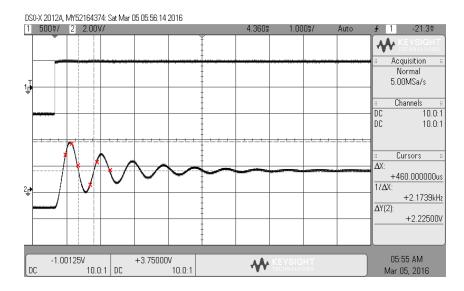


Figure 4: Some simulated time domain response values over the measured response of the second order system

Using the determined values for  $\zeta$  and  $\omega_0$ , several simulated time domain response points were generated using equation 3.12a in the lab manual. The equation and resulting figure can be seen below.

$$a(t) = G(1 - \frac{1}{\beta}exp(-t\zeta\omega_0)sin(t\omega_0\beta + \theta))$$
(8)

#### 4 Conclusion

Justify conclusions and results.