Department of Electrical and Computer Engineering

University of Victoria

ELEC 300 - Linear Circuits II

Laboratory Report

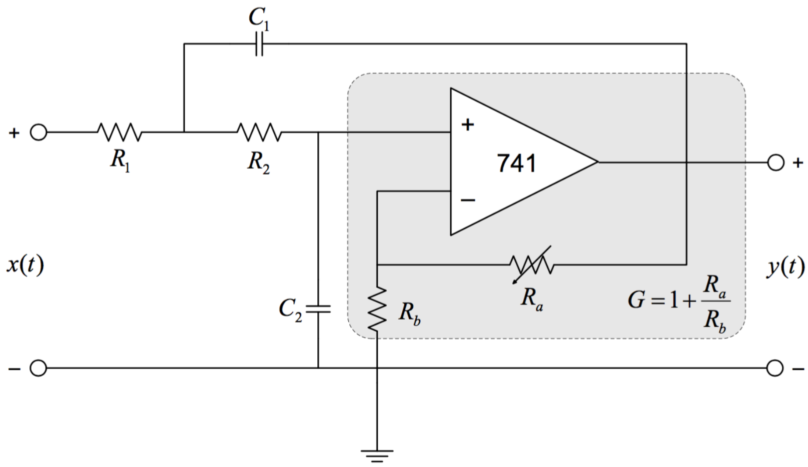
Experiment No.:	3
Title:	Time Domain Responses
Date of experiment:	4 March, 2016
Report submitted on:	11 March, 2016
To:	H. Singh, B07
Names:	M. Drinnan (V00755525)
	T. Mulligan (V00819591)
	T. Stephen (V00812021)

1 Objective

This lab will create an active realization of a second order system. The output of the circuit will be used to verify the system parameters.

2 Introduction

Inductors in second order systems can be replaced by amplifiers and capacitors. Fig. 1 shows an active second order system with a gain of $G = 1 + \frac{R_a}{R_b}$.



$$C_1 = C_2 = 16\,\mathrm{nF} = C, \quad R_1 = R_2 = 10\,\mathrm{k\Omega} = R, \quad R_b = 39\,\mathrm{k\Omega}, \quad R_a = 78(1-\zeta)\,\mathrm{k\Omega}$$

Figure 1: Active realization of a second order system

The transfer function of this system is:

$$H(s) = \frac{G\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \tag{1}$$

where

$$\omega_0 = \frac{1}{RC} \tag{2}$$

and

$$\zeta = 1 - \frac{R_a}{2R_b}. \tag{3}$$

When the unit step function is applied to Fig. 1 it will produce an output similar to Fig. 2.

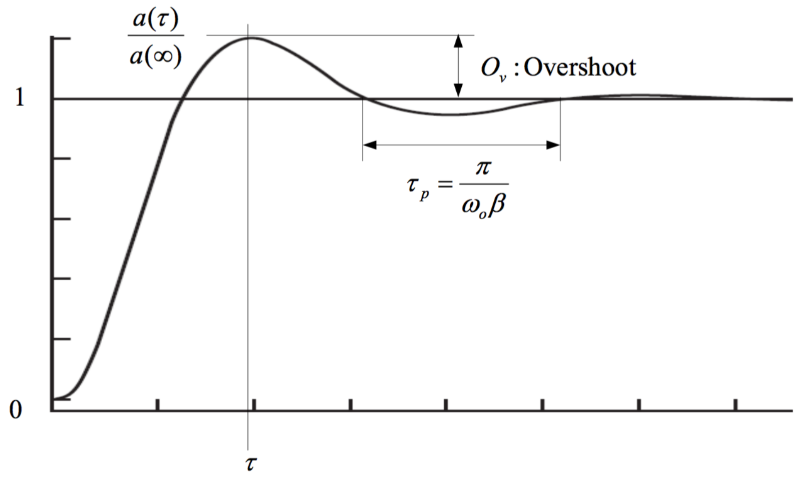


Figure 2: Step response of second order system

The system parameters ζ and ω_0 can be obtained from this graph since:

$$O_v = \exp\left(\frac{-\zeta\pi}{\beta}\right) \tag{4}$$

$$T_p = \frac{\pi}{\omega_0\beta}. \tag{5}$$

O_v can be obtained from Fig. 2 as:

$$O_v = \frac{V_{peak} - V_{steady}}{V_{steady}}.$$

3 Results and Discussion

Fig. 1 was realized with damping factor $\zeta = 0.1$. Using (3), $R_a = 70.2\,\mathrm{k}\Omega$. The opamp supply power was set at $\pm 15\,\mathrm{V}$. An input signal of 1 $\mathrm{V_{pp}}$ at 50 Hz square wave was applied to the circuit. One pulse of the square wave will behave locally like a unit step function.

The output of this circuit is shown in Fig. 3. The large overshoot corresponds to significant underdamping, which is consistent with $\zeta = 0.1$.

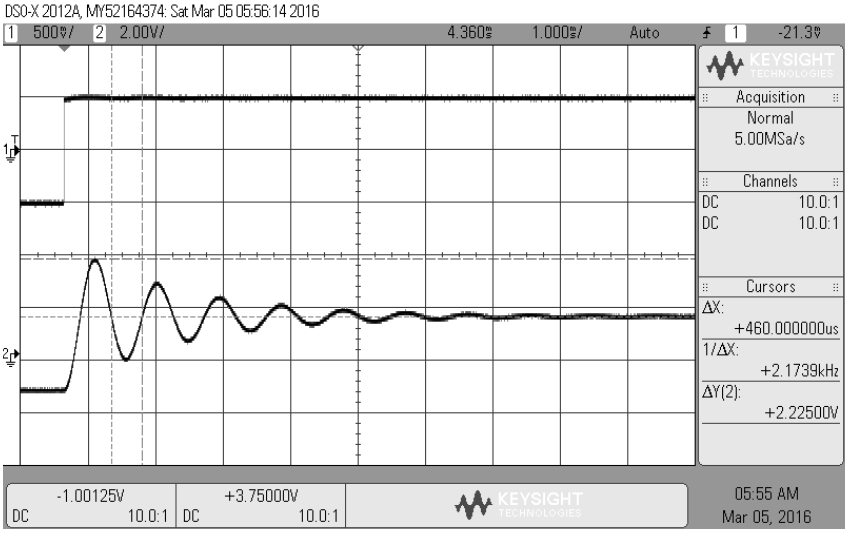


Figure 3: Measured response of second order system

The measured values are $O_v = \frac{2.225\,\mathrm{V}}{2.8\,\mathrm{V}} = 0.7946$ and $T_p = 460\,\mu\mathrm{s}$. As the transient response is damped, the value of the output settles to 2.8 V. This is consistent with $G = 2.8$ for an input of 1 V.

Using equation (4) and Fig. 3, Zeta could be solved for.

$$\frac{\ln(O_v)}{\pi} = \frac{\zeta}{\sqrt{1-\zeta^2}} \tag{6}$$

The experimentally determined value of ζ was found to be $\zeta = 0.07$. Next, ω_0 was solved for by rearranging (5).

$$\omega_0 = \frac{\pi}{T_p \beta} \tag{7}$$

Solving equation (7) gives an angular velocity of $\omega_0 = 6845\ \mathrm{rad\,s^{-1}}$.

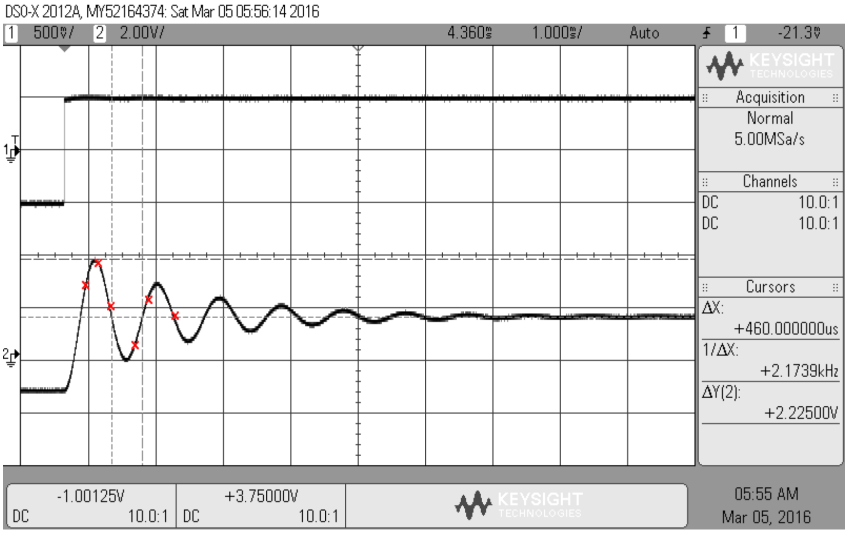


Figure 4: Some simulated time domain response values over the measured response of the second order system

Using the determined values for ζ and ω_0, several simulated time domain response points were generated using equation 3.12a in the lab manual. The equation and resulting figure can be seen below.

$$a(t) = G(1 - \frac{1}{\beta} exp(-t\zeta\omega_0) sin(t\omega_0\beta + \theta)) \tag{8}$$

4 Conclusion

We realized a second-order voltage amplifier, and found the response of the system using a unit-step input. Our results confirmed our calculated values of ζ and ω, and resembled the general response of a second order system.