Team 2

Dina Shalaby
Joseph Binny
Trevor Hanson

code presentation on youtube

column	definition
date	Date in format DD/MM/YYYY
tmax	Maximum temperature of the day in °F
tmin	Minimum temperature of the day in °F
tavg	Average temperature of the day in °F
departure	Departure from normal temperature in °F
HDD	Heating Degree Days
CDD	Cooling Degree Days
precipitation	Precipitation in inches
new_snow	New snowfall in inches
snow_depth	Snowfall depth in inches.

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
import statsmodels.api as sm
from statsmodels.formula.api import ols
import seaborn as sns
from matplotlib.dates import DateFormatter
from statsmodels.graphics.gofplots import ProbPlot
from scipy import stats
import math
from mpl_toolkits import mplot3d
```

1. Data Importing and Pre-processing:

In [2]:

```
df = pd.read_csv("nyc_temperature.csv")

# change the dates to a Timestamp
df["date"] = df["date"].apply(pd.to_datetime)
print(type(df["date"][0]))
# df.loc[df["precipitation"] == "T"].count()
df.head(13)
```

```
<class 'pandas._libs.tslibs.timestamps.Timestamp'>
```

Out[2]:

	date	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
0	2019-01-01	60	40	50.0	13.9	15	0	0.08	0	0
1	2019-02-01	41	35	38.0	2.1	27	0	0	0	0
2	2019-03-01	45	39	42.0	6.3	23	0	T	0	0
3	2019-04-01	47	37	42.0	6.5	23	0	0	0	0
4	2019-05-01	47	42	44.5	9.1	20	0	0.45	0	0
5	2019-06-01	49	32	40.5	5.3	24	0	T	0	0
6	2019-07-01	35	26	30.5	-4.5	34	0	0	0	0
7	2019-08-01	47	35	41.0	6.1	24	0	0.21	0	0
8	2019-09-01	46	35	40.5	5.8	24	0	0.07	0	0
9	2019-10-01	35	30	32.5	-2.1	32	0	0	0	0
10	2019-11-01	32	22	27.0	-7.5	38	0	0	0	0
11	2019-12-01	34	21	27.5	-6.9	37	0	0	0	0
12	2019-01-13	33	25	29.0	-5.2	36	0	T	T	T

In [3]:

```
# We clean out the T values from the last three columns

df["precipitation"] = df["precipitation"].replace({"T": 0})
df["new_snow"] = df["new_snow"].replace({"T": 0})
df["snow_depth"] = df["snow_depth"].replace({"T": 0})
```

In [4]:
```
# just making sure we don't have any T's left -> we don't!

# df.loc[df["precipitaion"] == "T"]
# df.loc[df["new_snow"] == "T"]
# df.loc[df["snow_depth"] == "T"]
```

In [5]:
```
# checking the type of data in the last three columns.. we want them to be floats

print(type(df["precipitation"][0]))
print(type(df["new_snow"][0]))
print(type(df["snow_depth"][0]))
```

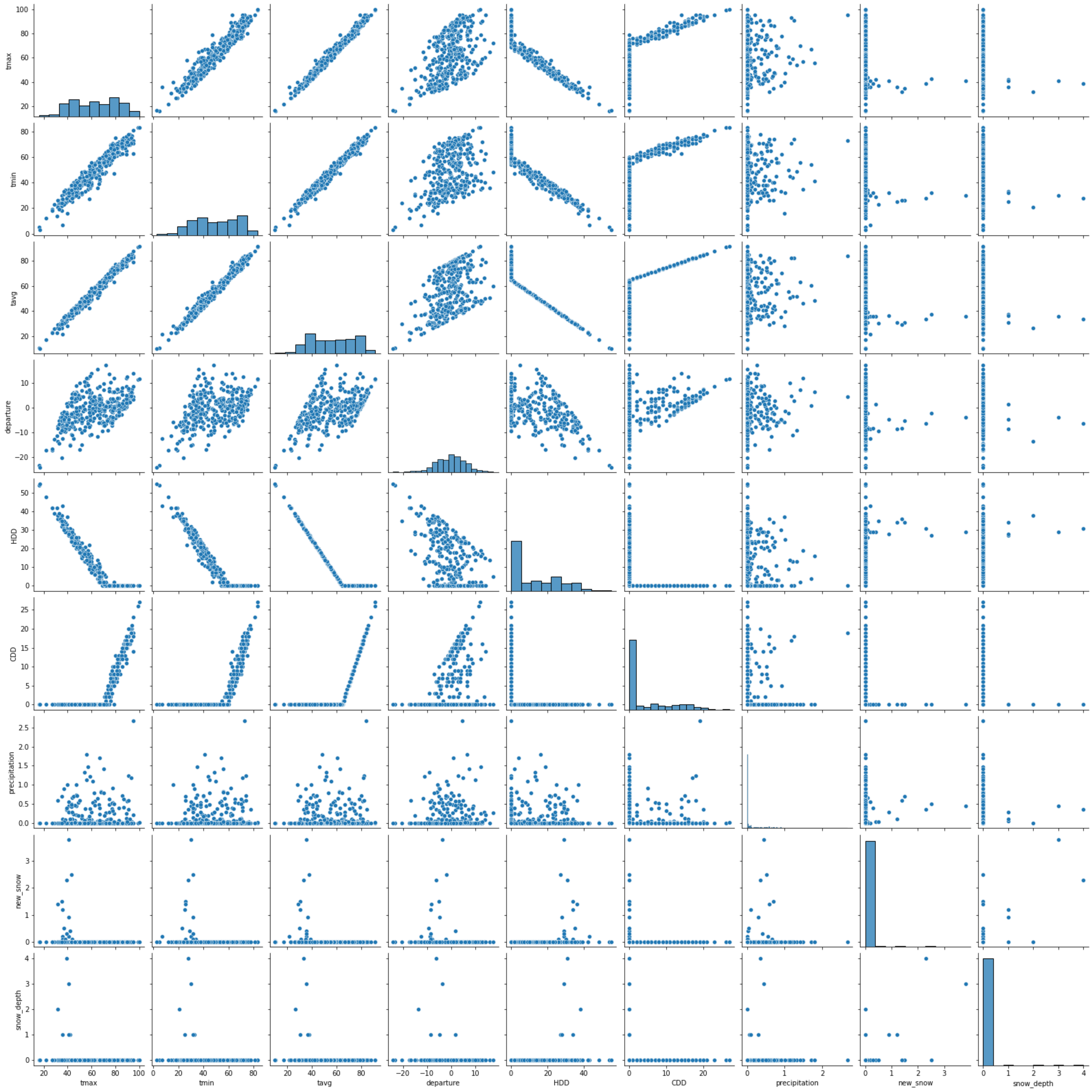
```
<class 'str'>
<class 'str'>
<class 'str'>
```

In [6]:
```
# We transform these columns' data to type float

df["precipitation"] = df["precipitation"].apply(float)
df["new_snow"] = df["new_snow"].apply(float)
df["snow_depth"] = df["snow_depth"].apply(float)
```

2. Statistical Analysis and Visualization:

In [7]:
```
sns.pairplot(df)
```

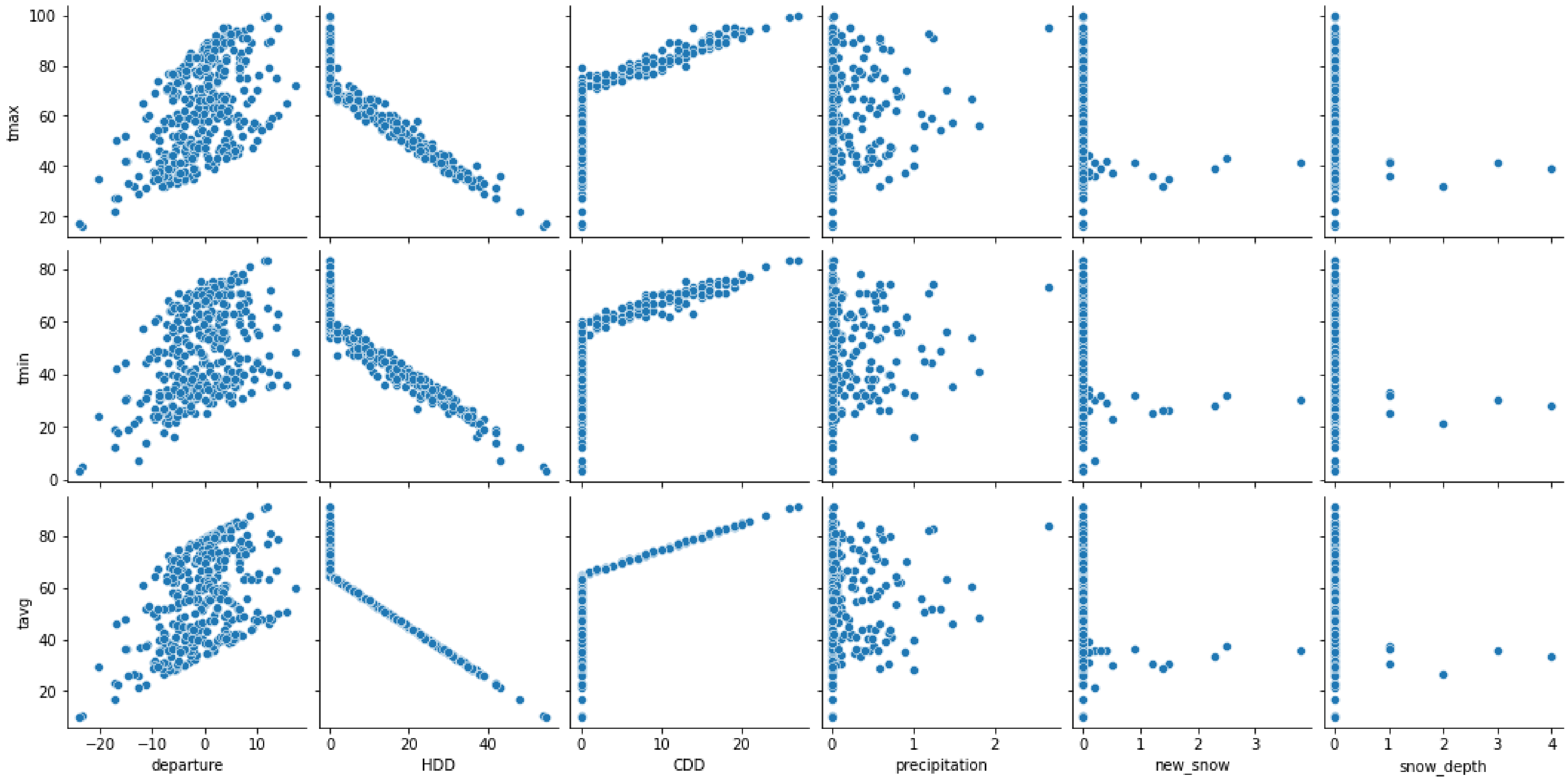
Out[7]:
```
<seaborn.axisgrid.PairGrid at 0x2bc85968970>
```



In [8]:

```
x_vars = ["departure", "HDD", "CDD", "precipitation", "new_snow", "snow_depth"]
y_vars = ["tmax", "tmin", "tavg"]

# sns.pairplot(df, x_vars=x_vars, y_vars=y_vars, kind="reg") #decided not go print with a linear regression line
sns.pairplot(df, x_vars=x_vars, y_vars=y_vars)
```

Out[8]: <seaborn.axisgrid.PairGrid at 0x2bc8a9941c0>



In [9]:

```
df.corr()
```

Out[9]:

	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
tmax	1.000000	0.964680	0.991930	0.504420	-0.948704	0.801030	0.008210	-0.183409	-0.146762
tmin	0.964680	1.000000	0.990294	0.433581	-0.951052	0.792244	0.015270	-0.174763	-0.141384
tavg	0.991930	0.990294	1.000000	0.474830	-0.958306	0.803954	0.011681	-0.180886	-0.145484
departure	0.504420	0.433581	0.474830	1.000000	-0.477517	0.332172	0.033311	-0.104159	-0.100274
HDD	-0.948704	-0.951052	-0.958306	-0.477517	1.000000	-0.600872	-0.025594	0.197466	0.159156
CDD	0.801030	0.792244	0.803954	0.332172	-0.600872	1.000000	-0.020160	-0.091041	-0.071355
precipitation	0.008210	0.015270	0.011681	0.033311	-0.025594	-0.020160	1.000000	0.121013	0.044353
new_snow	-0.183409	-0.174763	-0.180886	-0.104159	0.197466	-0.091041	0.121013	1.000000	0.693532
snow_depth	-0.146762	-0.141384	-0.145484	-0.100274	0.159156	-0.071355	0.044353	0.693532	1.000000

In [10]:

```
df[["tavg","departure","HDD","CDD","precipitation","new_snow","snow_depth"]].corr()
```

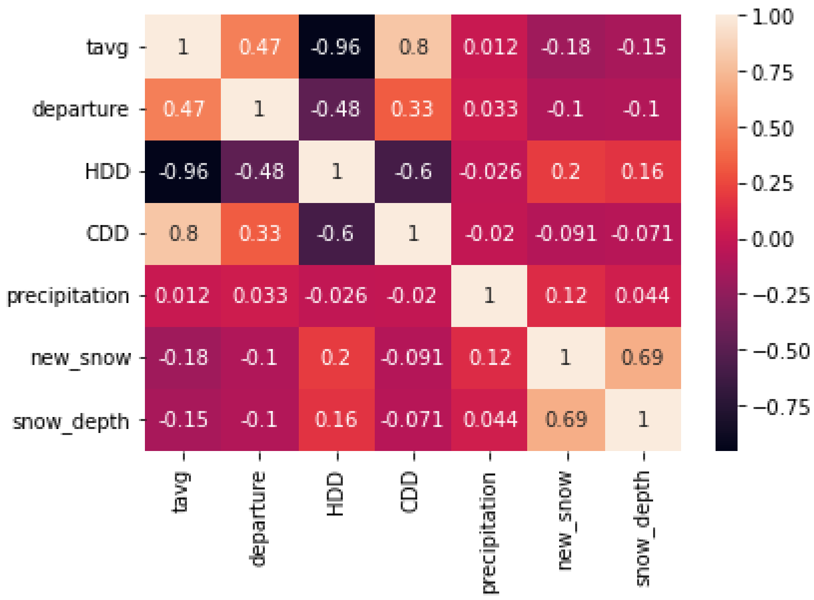
Out[10]:

	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
tavg	1.000000	0.474830	-0.958306	0.803954	0.011681	-0.180886	-0.145484
departure	0.474830	1.000000	-0.477517	0.332172	0.033311	-0.104159	-0.100274
HDD	-0.958306	-0.477517	1.000000	-0.600872	-0.025594	0.197466	0.159156
CDD	0.803954	0.332172	-0.600872	1.000000	-0.020160	-0.091041	-0.071355
precipitation	0.011681	0.033311	-0.025594	-0.020160	1.000000	0.121013	0.044353
new_snow	-0.180886	-0.104159	0.197466	-0.091041	0.121013	1.000000	0.693532
snow_depth	-0.145484	-0.100274	0.159156	-0.071355	0.044353	0.693532	1.000000

In [11]:

```
sns.heatmap(df[["tavg","departure","HDD","CDD","precipitation","new_snow","snow_depth"]].corr(), annot=True)
```

Out[11]: <AxesSubplot:>



`Analying the correlation`

It seems like `departure` and `HDD` and `CDD`
are well correlated with `tavg` but not the other columns
Thus we'll use them in our regression model. We omitted `tmin` and `tmax` as we felt like it'd be too easy.
We will do a multicolliniarity test on them first though to make sure
they're not collinear or multicollinear

In [12]:
```
# calculate_vif was taken from
#https://towardsdatascience.com/statistics-in-python-collinearity-and-multicollinearity-4cc4dcd82b3f

def calculate_vif(df, features):
    vif, tolerance = {}, {}
    # all the features that you want to examine
    for feature in features:
        # extract all the other features you will regress against
        X = [f for f in features if f != feature]
        X, y = df[X], df[feature]
        # extract r-squared from the fit
        r2 = LinearRegression().fit(X, y).score(X, y)

        # calculate tolerance
        tolerance[feature] = 1 - r2
        # calculate VIF
        vif[feature] = 1/(tolerance[feature])
    # return VIF DataFrame
    return pd.DataFrame({'VIF': vif, 'Tolerance': tolerance})
```

In [13]:
```
tavg_df = df[["tavg","departure","HDD","CDD"]]

calculate_vif(tavg_df, features=["departure","HDD","CDD"])
```

Out[13]:

	VIF	Tolerance
departure	1.300773	0.768774
HDD	1.811162	0.552132
CDD	1.571583	0.636301

Interpreting VIF Values

The valid value for VIF ranges from 1 to infinity. A rule of thumb for interpreting VIF values is:

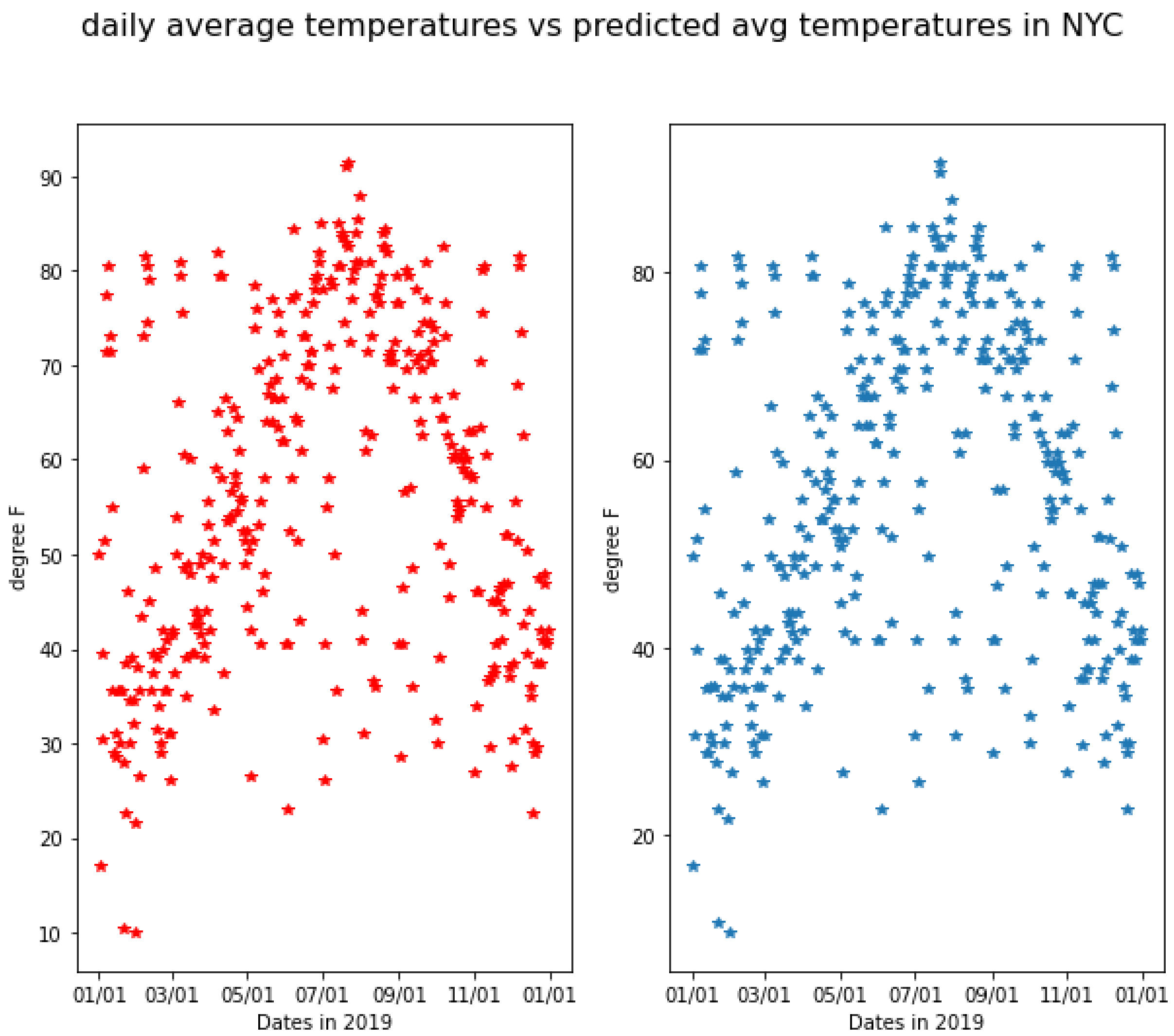
- 1 — features are not correlated
- 1<VIF<5 — features are moderately correlated
- VIF>5 — features are highly correlated
- VIF>10 — high correlation between features and is cause for concern

Since the `variance inflation factor` isn't alarmingly high among the relevant columns, we will not ommit any of these variables from the model.

3. Regression Model

In [14]:
```
X = df[["departure","HDD","CDD"]]
y = df["tavg"]

tavg_reg = LinearRegression().fit(X,y)
tavg_predicted = tavg_reg.predict(X)
```

In [15]:
```
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(10,8))
fig.suptitle("daily average temperatures"+
             " vs predicted avg temperatures in NYC", size=16)
ax1.set(xlabel="Dates in 2019",
       ylabel=" degree F")
ax2.set(xlabel="Dates in 2019",
       ylabel="degree F")
date_form = DateFormatter("%m/%d") #to format the date and omit the year
ax1.xaxis.set_major_formatter(date_form)
ax2.xaxis.set_major_formatter(date_form)
ax1.plot(df["date"],df["tavg"], "r*")
ax2.plot(df["date"],tavg_predicted, "*")
plt.show()
```



In [16]:
```
tavg_residuals = df["tavg"] - tavg_predicted
print(tavg_reg.score(X,y))
```

```
0.9998057764134336
```

$$R^2 = 0.9998$$

In [17]:
```
OLS_model = sm.OLS(y, sm.add_constant(X))
model_fit = OLS_model.fit()
model_fitted_y = model_fit.fittedvalues
model_fit.summary()
```

```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all argum
ents of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

Out[17]:

OLS Regression Results

Dep. Variable:	tavg	**R-squared:**	1.000
Model:	OLS	**Adj. R-squared:**	1.000
Method:	Least Squares	**F-statistic:**	6.194e+05
Date:	Sat, 22 Oct 2022	**Prob (F-statistic):**	0.00
Time:	22:36:07	**Log-Likelihood:**	-9.0216
No. Observations:	365	**AIC:**	26.04
Df Residuals:	361	**BIC:**	41.64
Df Model:	3		

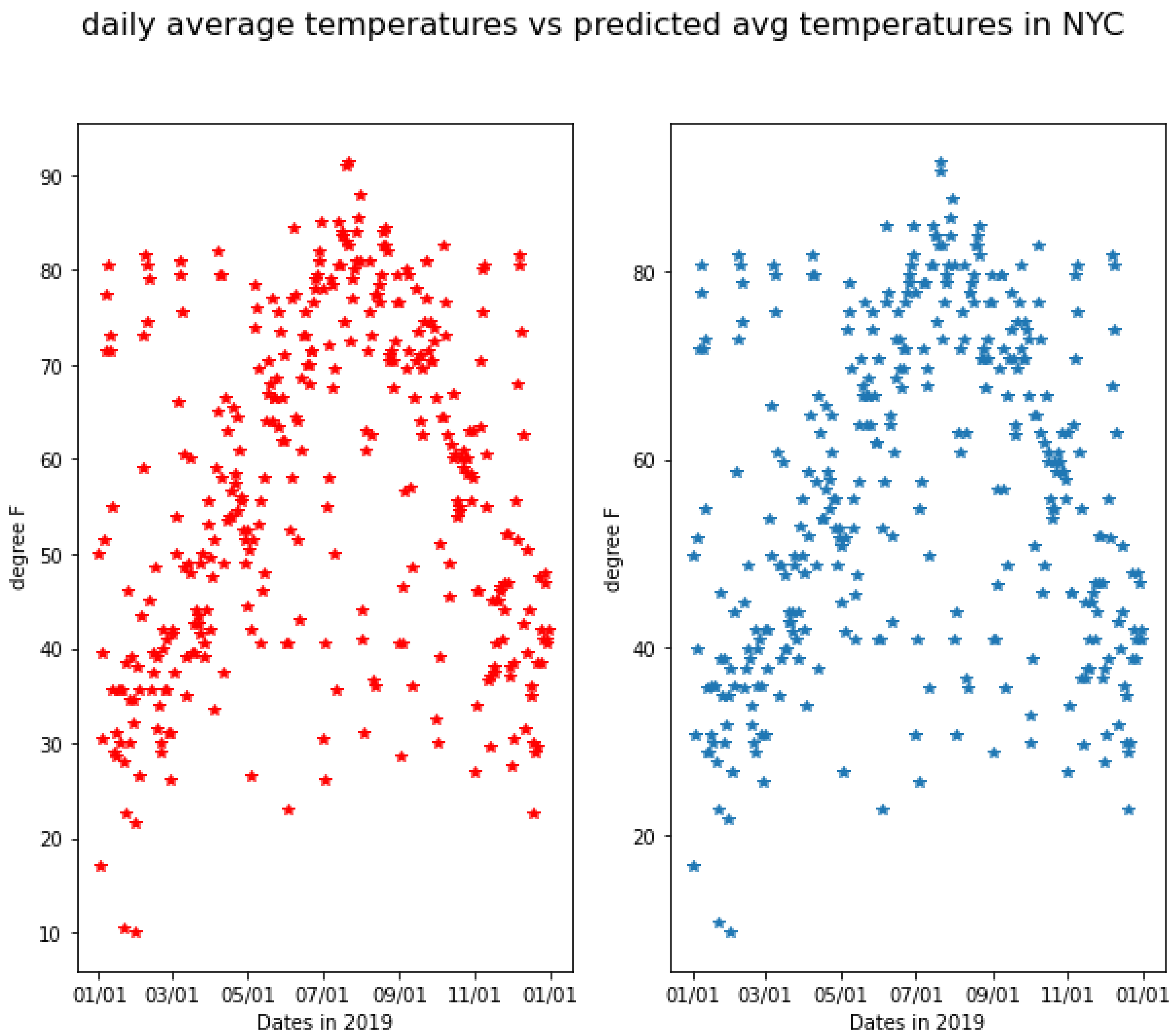
Covariance Type:	nonrobust

	coef	std err	t	P>|t|	[0.025	0.975]
const	64.7296	0.027	2403.285	0.000	64.677	64.783
departure	0.0039	0.002	1.660	0.098	-0.001	0.008
HDD	-0.9984	0.001	-752.849	0.000	-1.001	-0.996
CDD	0.9979	0.003	388.184	0.000	0.993	1.003

Omnibus:	1690.482	**Durbin-Watson:**	1.829
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	57.560
Skew:	0.090	**Prob(JB):**	3.17e-13
Kurtosis:	1.063	**Cond. No.**	38.1

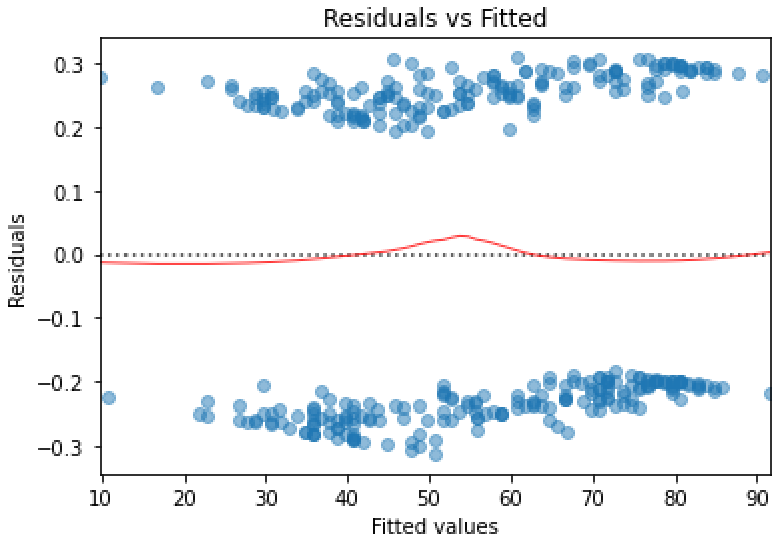
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [18]:
```
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(10,8))
fig.suptitle("daily average temperatures"+
             " vs predicted avg temperatures in NYC", size=16)
ax1.set(xlabel="Dates in 2019",
       ylabel=" degree F")
ax2.set(xlabel="Dates in 2019",
       ylabel="degree F")
date_form = DateFormatter("%m/%d") #to format the date and omit the year
ax1.xaxis.set_major_formatter(date_form)
ax2.xaxis.set_major_formatter(date_form)
ax1.plot(df["date"],df["tavg"], "r*")
ax2.plot(df["date"],model_fitted_y, "*")
plt.show()
```



In [19]:
```
desired_df = pd.concat([X, y], axis=1) # creating a df from X, y for easier plot handling

model_residuals = model_fit.resid
model_norm_residuals = model_fit.get_influence().resid_studentized_internal # normalize residuals

model_norm_residuals_abs_sqrt = np.sqrt(np.abs(model_norm_residuals)) # absoulte squared normalized residuals
model_abs_resid = np.abs(model_residuals) # absolute residuals

model_leverage = model_fit.get_influence().cooks_distance[0]

plot_lm_1 = plt.figure()
plot_lm_1.axes[0] = sns.residplot(model_fitted_y, desired_df.columns[-1], data=desired_df,
                                  lowess=True,
                                  scatter_kws={'alpha': 0.5},
                                  line_kws={"color":"red", "lw":1, "alpha":0.8})
plot_lm_1.axes[0].set_title('Residuals vs Fitted')
plot_lm_1.axes[0].set_xlabel('Fitted values')
plot_lm_1.axes[0].set_ylabel('Residuals');
```

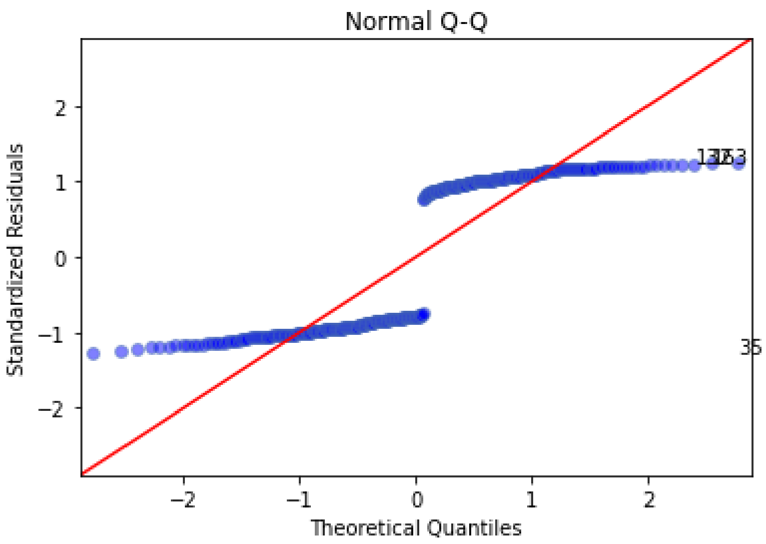
```
C:\Users\yousi\anaconda3\lib\site-packages\seaborn\_decorators.py:36: FutureWarning: Pass the following variables as keyword args: x, y. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keyword will result in an error or misinterpretation.
  warnings.warn(
```



In [20]:

```
QQ = ProbPlot(model_norm_residuals)
plot_lm_2 = QQ.qqplot(line='45', alpha=0.5, color='#4C72B0', lw=1)
plot_lm_2.axes[0].set_title('Normal Q-Q')
plot_lm_2.axes[0].set_xlabel('Theoretical Quantiles')
plot_lm_2.axes[0].set_ylabel('Standardized Residuals');
# annotations
abs_norm_resid = np.flip(np.argsort(np.abs(model_norm_residuals)), 0)
abs_norm_resid_top_3 = abs_norm_resid[:3]
for r, i in enumerate(abs_norm_resid_top_3):
    plot_lm_2.axes[0].annotate(i,
                               xy=(np.flip(QQ.theoretical_quantiles, 0)[r],
                                   model_norm_residuals[i]));
```

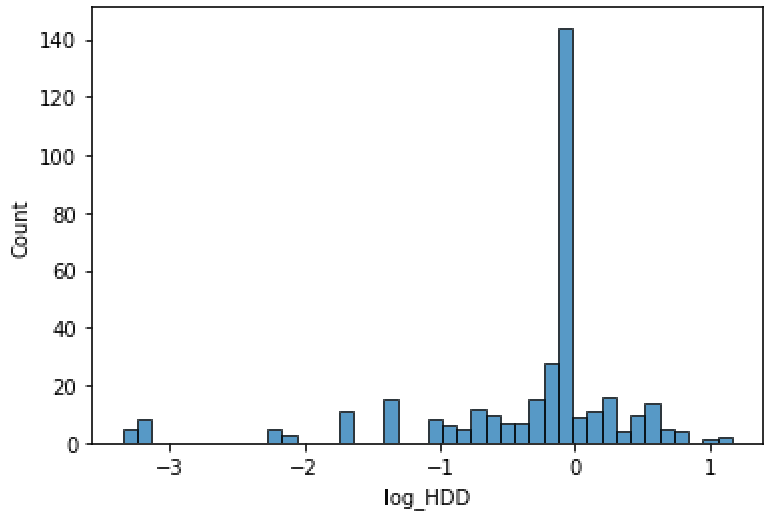
```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: marker is redundantly defined by the 'marker' keyword argument and the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: color is redundantly defined by the 'color' keyword argument and the fmt string "bo" (-> color='b'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



In [21]:

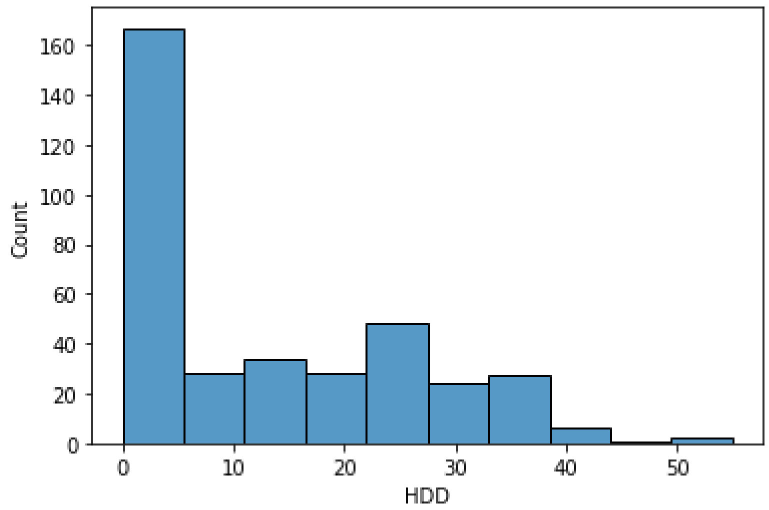
```
df["log_HDD"] = (df["HDD"]-np.mean(df["HDD"]))/np.sqrt(np.var(df["HDD"]))
df["log_HDD"] = np.log(np.abs(df["log_HDD"]))

#The following code made the model fit way worse and was therefore emitted
# df["log_CDD"] = (df["CDD"]-np.mean(df["CDD"]))/np.sqrt(np.var(df["CDD"]))
# df["log_CDD"] = np.sqrt(np.abs(df["log_CDD"]))
# df["departure_transformed"] = (df["departure"]-np.mean(df["departure"]))/np.sqrt(np.var(df["departure"]))

sns.histplot(df["log_HDD"])

# The following code was used to evaluate transformations
# sns.histplot(df["departure_transformed"])
# sns.histplot(df["departure"])
# df.loc[df["precipitation"] == np.inf]
```

Out[21]: <AxesSubplot:xlabel='log_HDD', ylabel='Count'>



In [22]:
```
sns.histplot(df["HDD"])
```

Out[22]:
```
<AxesSubplot:xlabel='HDD', ylabel='Count'>
```



In [23]:
```
tavg_df = df[["tavg","departure","log_HDD","CDD"]]

calculate_vif(tavg_df, features=["departure","log_HDD","CDD"])
```

Out[23]:

	VIF	Tolerance
departure	1.229530	0.813319
log_HDD	1.150942	0.868854
CDD	1.244294	0.803669

In [24]:
```
X = df[["departure","log_HDD","CDD"]]
y = df["tavg"]
OLS_model = sm.OLS(y, sm.add_constant(X))
model_fit = OLS_model.fit()
model_fitted_y = model_fit.fittedvalues
model_fit.summary()
```

```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all argum
ents of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

Out[24]:

OLS Regression Results

Dep. Variable:	tavg	**R-squared:**	0.735
Model:	OLS	**Adj. R-squared:**	0.733
Method:	Least Squares	**F-statistic:**	334.0
Date:	Sat, 22 Oct 2022	**Prob (F-statistic):**	9.33e-104
Time:	22:36:08	**Log-Likelihood:**	-1326.3
No. Observations:	365	**AIC:**	2661.
Df Residuals:	361	**BIC:**	2676.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	45.9164	0.682	67.359	0.000	44.576	47.257
departure	0.4673	0.083	5.606	0.000	0.303	0.631
log_HDD	-4.7265	0.638	-7.409	0.000	-5.981	-3.472
CDD	2.2252	0.084	26.344	0.000	2.059	2.391

Omnibus:	33.380	**Durbin-Watson:**	0.432

Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	20.901
Skew:	0.451	**Prob(JB):**	2.89e-05
Kurtosis:	2.252	**Cond. No.**	13.4

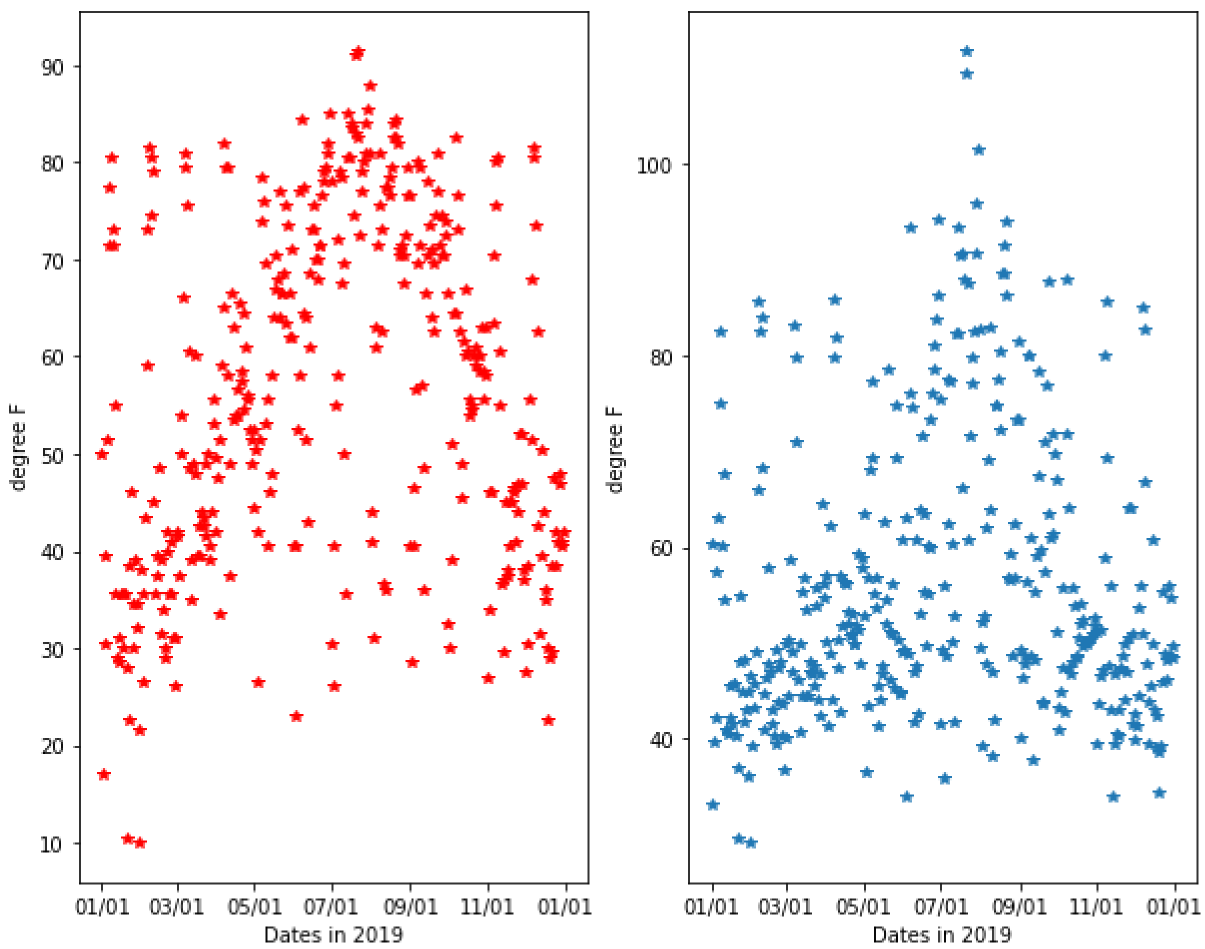
Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [25]:

```
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(10,8))
fig.suptitle("daily average temperatures"+
             " vs predicted avg temperatures in NYC", size=16)
ax1.set(xlabel="Dates in 2019",
       ylabel=" degree F")
ax2.set(xlabel="Dates in 2019",
       ylabel="degree F")
date_form = DateFormatter("%m/%d") #to format the date and omit the year
ax1.xaxis.set_major_formatter(date_form)
ax2.xaxis.set_major_formatter(date_form)
ax1.plot(df["date"],df["tavg"], "r*")
ax2.plot(df["date"],model_fitted_y, "*")
plt.show()
y_hat_tavg = model_fitted_y.copy()
```

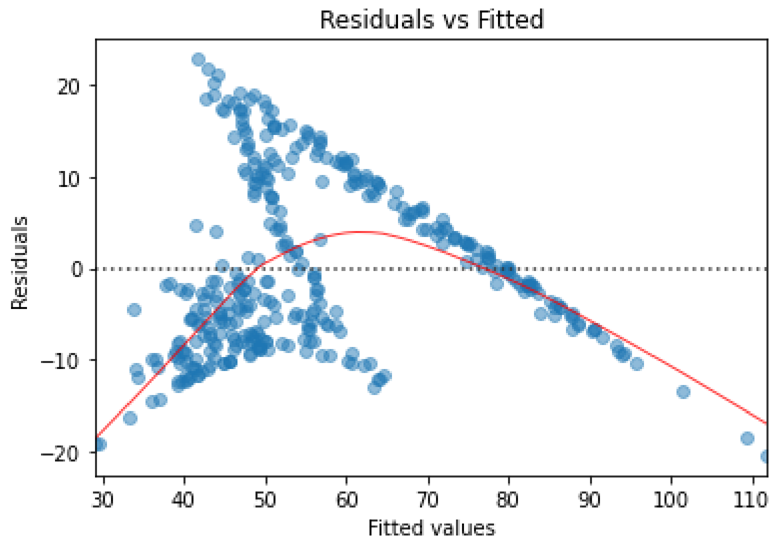
daily average temperatures vs predicted avg temperatures in NYC



In [26]:

```
desired_df = pd.concat([X, y], axis=1) # creating a df from X, y for easier plot handling

model_residuals = model_fit.resid
model_norm_residuals = model_fit.get_influence().resid_studentized_internal # normalize residuals

model_norm_residuals_abs_sqrt = np.sqrt(np.abs(model_norm_residuals)) # absoulte squared normalized residuals
model_abs_resid = np.abs(model_residuals) # absolute residuals

model_leverage = model_fit.get_influence().cooks_distance[0]

plot_lm_1 = plt.figure()
plot_lm_1.axes[0] = sns.residplot(model_fitted_y, desired_df.columns[-1], data=desired_df,
                                  lowess=True,
                                  scatter_kws={'alpha': 0.5},
                                  line_kws={"color":"red", "lw":1, "alpha":0.8})
plot_lm_1.axes[0].set_title('Residuals vs Fitted')
plot_lm_1.axes[0].set_xlabel('Fitted values')
plot_lm_1.axes[0].set_ylabel('Residuals');
```

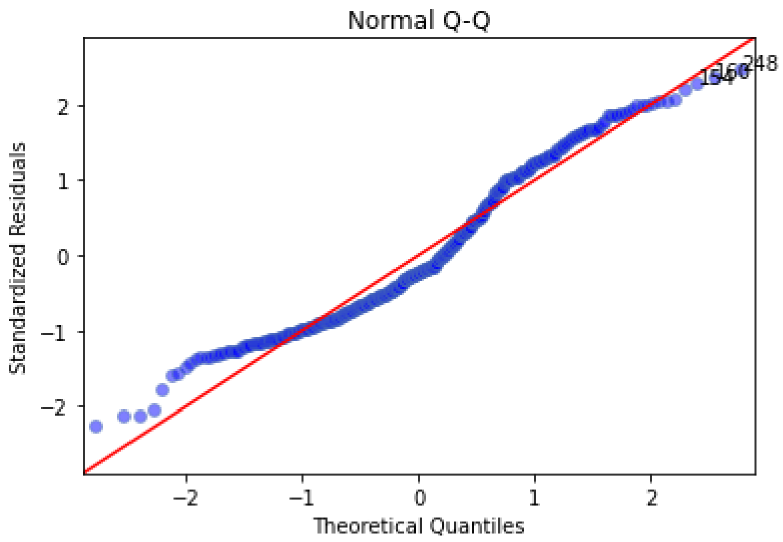
```
C:\Users\yousi\anaconda3\lib\site-packages\seaborn\_decorators.py:36: FutureWarning: Pass the following variables as keyword args:
x, y. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keywor
d will result in an error or misinterpretation.
  warnings.warn(
```



In [27]:

```
QQ = ProbPlot(model_norm_residuals)
plot_lm_2 = QQ.qqplot(line='45', alpha=0.5, color='#4C72B0', lw=1)
plot_lm_2.axes[0].set_title('Normal Q-Q')
plot_lm_2.axes[0].set_xlabel('Theoretical Quantiles')
plot_lm_2.axes[0].set_ylabel('Standardized Residuals');
# annotations
abs_norm_resid = np.flip(np.argsort(np.abs(model_norm_residuals)), 0)
abs_norm_resid_top_3 = abs_norm_resid[:3]
for r, i in enumerate(abs_norm_resid_top_3):
    plot_lm_2.axes[0].annotate(i,
                               xy=(np.flip(QQ.theoretical_quantiles, 0)[r],
                                   model_norm_residuals[i]));
```

```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: marker is redundantly defined by the 'marker' keyword argument and the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: color is redundantly defined by the 'color' keyword argument and the fmt string "bo" (-> color='b'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



In [28]:

```
X = df[["tavg"]]
y = df["tmax"]
OLS_model = sm.OLS(y, sm.add_constant(X))
model_fit = OLS_model.fit()
model_fitted_y = model_fit.fittedvalues
model_fit.summary()
```

```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all arguments of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

Out[28]:

OLS Regression Results

Dep. Variable:	tmax	**R-squared:**	0.984
Model:	OLS	**Adj. R-squared:**	0.984
Method:	Least Squares	**F-statistic:**	2.222e+04
Date:	Sat, 22 Oct 2022	**Prob (F-statistic):**	0.00
Time:	22:36:09	**Log-Likelihood:**	-834.57
No. Observations:	365	**AIC:**	1673.
Df Residuals:	363	**BIC:**	1681.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	4.1951	0.415	10.111	0.000	3.379	5.011
tavg	1.0467	0.007	149.060	0.000	1.033	1.061

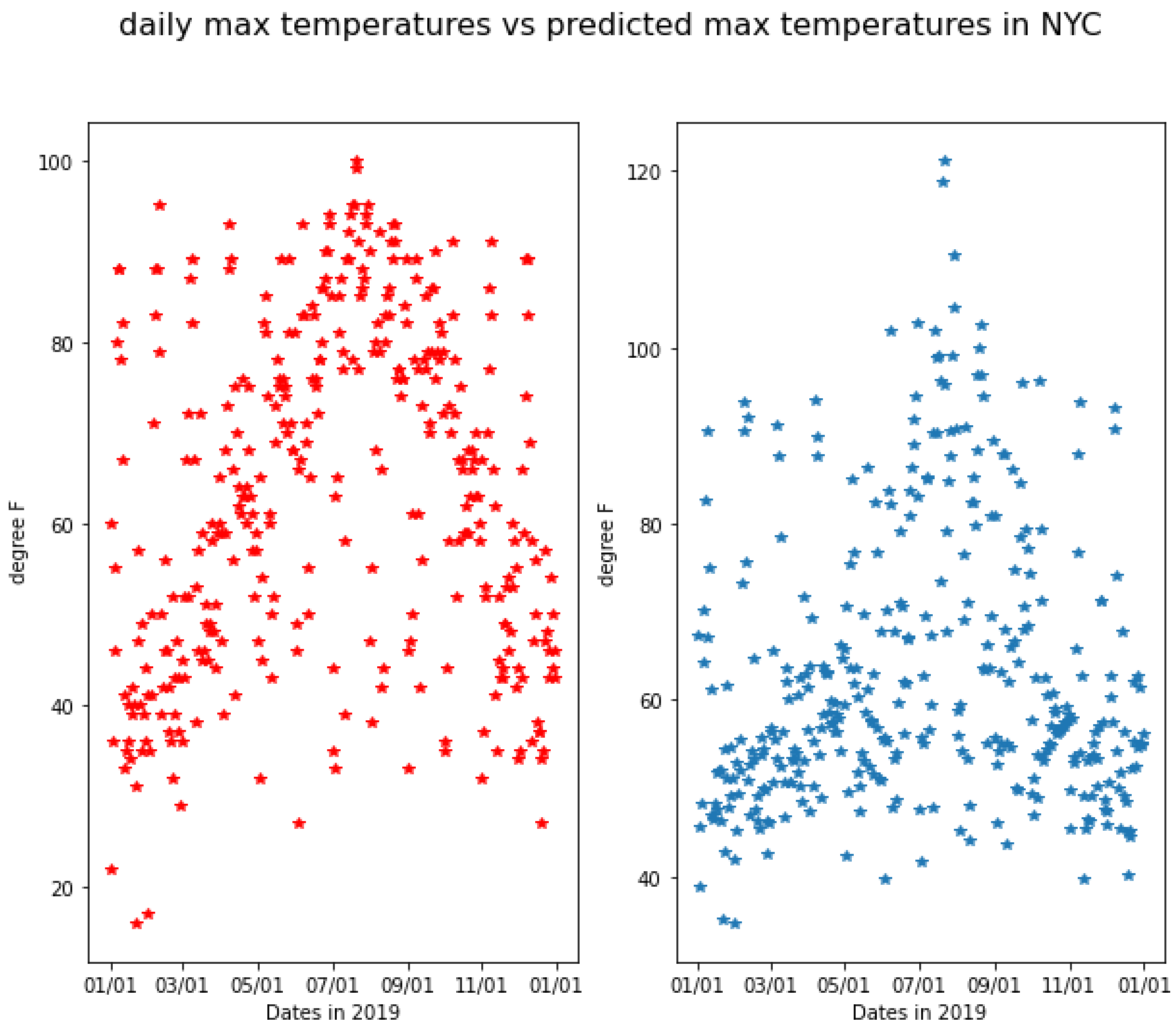
Omnibus:	51.383	**Durbin-Watson:**	1.791

Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	83.652
Skew:	0.851	**Prob(JB):**	6.84e-19
Kurtosis:	4.613	**Cond. No.**	196.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

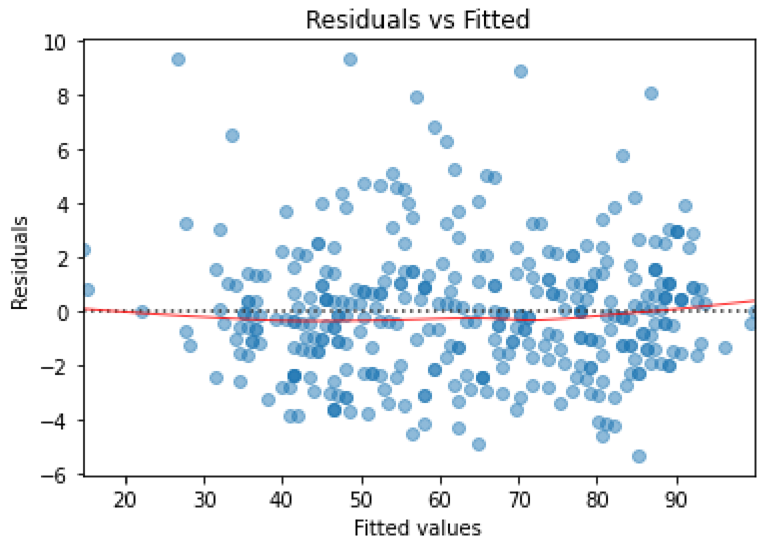
In [29]:
```
fig, (ax1, ax2) = plt.subplots(1,2, figsize=(10,8))
fig.suptitle("daily max temperatures"+
             " vs predicted max temperatures in NYC", size=16)
ax1.set(xlabel="Dates in 2019",
       ylabel=" degree F")
ax2.set(xlabel="Dates in 2019",
       ylabel="degree F")
date_form = DateFormatter("%m/%d") #to format the date and omit the year
ax1.xaxis.set_major_formatter(date_form)
ax2.xaxis.set_major_formatter(date_form)
ax1.plot(df["date"],df["tmax"], "r*")
y_hat_max = model_fit.predict(sm.add_constant(y_hat_tavg))
ax2.plot(df["date"],y_hat_max, "*")
plt.show()
```

```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: In a future version of pandas all argum
ents of concat except for the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```



In [30]:
```
desired_df = pd.concat([X, y], axis=1) # creating a df from X, y for easier plot handling

model_residuals = model_fit.resid
model_norm_residuals = model_fit.get_influence().resid_studentized_internal # normalize residuals

model_norm_residuals_abs_sqrt = np.sqrt(np.abs(model_norm_residuals)) # absoulte squared normalized residuals
model_abs_resid = np.abs(model_residuals) # absolute residuals

model_leverage = model_fit.get_influence().cooks_distance[0]

plot_lm_1 = plt.figure()
plot_lm_1.axes[0] = sns.residplot(model_fitted_y, desired_df.columns[-1], data=desired_df,
                                  lowess=True,
                                  scatter_kws={'alpha': 0.5},
                                  line_kws={"color":"red", "lw":1, "alpha":0.8})
plot_lm_1.axes[0].set_title('Residuals vs Fitted')
plot_lm_1.axes[0].set_xlabel('Fitted values')
plot_lm_1.axes[0].set_ylabel('Residuals');
```

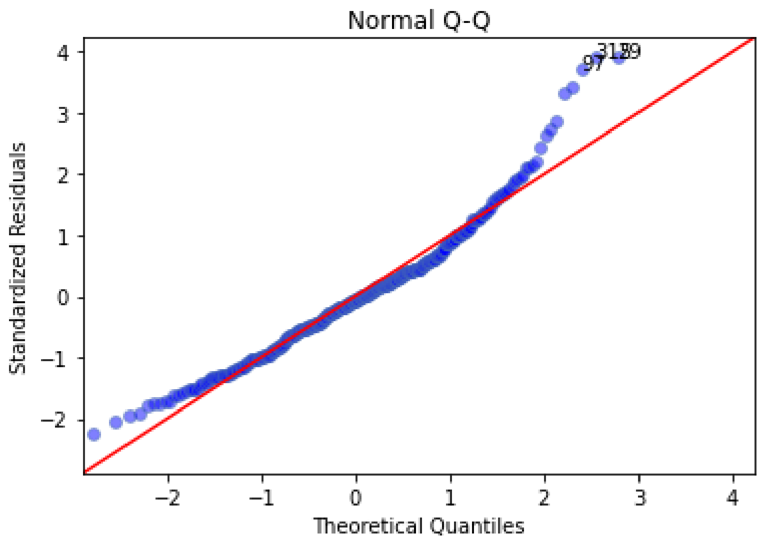
```
C:\Users\yousi\anaconda3\lib\site-packages\seaborn\_decorators.py:36: FutureWarning: Pass the following variables as keyword args:
x, y. From version 0.12, the only valid positional argument will be `data`, and passing other arguments without an explicit keywor
d will result in an error or misinterpretation.
  warnings.warn(
```



In [31]:

```
QQ = ProbPlot(model_norm_residuals)
plot_lm_2 = QQ.qqplot(line='45', alpha=0.5, color='#4C72B0', lw=1)
plot_lm_2.axes[0].set_title('Normal Q-Q')
plot_lm_2.axes[0].set_xlabel('Theoretical Quantiles')
plot_lm_2.axes[0].set_ylabel('Standardized Residuals');
# annotations
abs_norm_resid = np.flip(np.argsort(np.abs(model_norm_residuals)), 0)
abs_norm_resid_top_3 = abs_norm_resid[:3]
for r, i in enumerate(abs_norm_resid_top_3):
    plot_lm_2.axes[0].annotate(i,
                               xy=(np.flip(QQ.theoretical_quantiles, 0)[r],
                                   model_norm_residuals[i]));
```

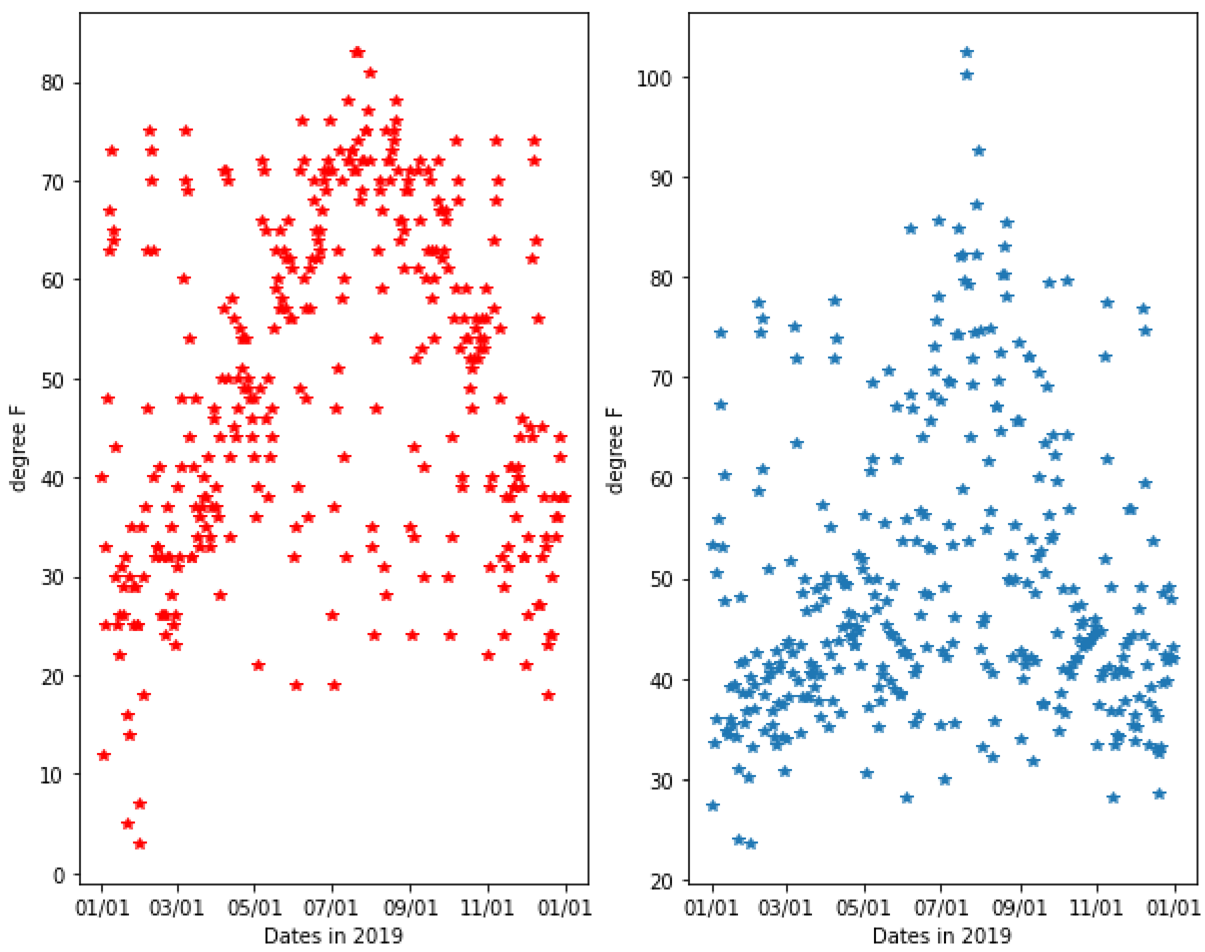
```
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: marker is redundantly defined by the
'marker' keyword argument and the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
C:\Users\yousi\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993: UserWarning: color is redundantly defined by the
'color' keyword argument and the fmt string "bo" (-> color='b'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



In [32]:

```
tmin_predicted = y_hat_tavg * 2 - y_hat_max

fig, (ax1, ax2) = plt.subplots(1,2, figsize=(10,8))
fig.suptitle("daily min temperatures"+
             " vs predicted min temperatures in NYC", size=16)
ax1.set(xlabel="Dates in 2019",
       ylabel=" degree F")
ax2.set(xlabel="Dates in 2019",
       ylabel="degree F")
date_form = DateFormatter("%m/%d") #to format the date and omit the year
ax1.xaxis.set_major_formatter(date_form)
ax2.xaxis.set_major_formatter(date_form)
ax1.plot(df["date"],df["tmin"], "r*")
ax2.plot(df["date"],tmin_predicted, "*")
plt.show()
```

daily min temperatures vs predicted min temperatures in NYC



In [ ]:

In [ ]:

In [ ]:

HDD as target variable

1. Data Importing and Pre-processing:

In [33]:

```
 # import the csv data as a dataframe
data = pd.read_csv("nyc_temperature.csv")
data_5= data.head()
# print(data_5)
# print(data_5.dtypes)
```

In [34]:

```
# will convert all T values to 0
data['CDD'] = data['CDD'].replace(['T'],'0')
data['precipitation'] = data['precipitation'].replace(['T'],'0')
data['new_snow'] = data['new_snow'].replace(['T'],'0')
data['snow_depth'] = data['snow_depth'].replace(['T'],'0')
data
```

Out[34]:

	date	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
0	1/1/19	60	40	50.0	13.9	15	0	0.08	0	0
1	2/1/19	41	35	38.0	2.1	27	0	0	0	0
2	3/1/19	45	39	42.0	6.3	23	0	0	0	0
3	4/1/19	47	37	42.0	6.5	23	0	0	0	0
4	5/1/19	47	42	44.5	9.1	20	0	0.45	0	0
...	...	...	...	...	...	...	...	...	...	...
360	27/12/19	54	42	48.0	10.8	17	0	0	0	0
361	28/12/19	50	44	47.0	10.0	18	0	0	0	0
362	29/12/19	44	38	41.0	4.3	24	0	0.29	0	0
363	30/12/19	43	38	40.5	4.0	24	0	0.49	0	0
364	31/12/19	46	38	42.0	5.7	23	0	0.01	0	0

365 rows × 10 columns

In [35]:

```python
# convert all values into float
data['tmax'] = data['tmax'].astype(float)
data['tmin'] = data['tmin'].astype(float)
data['tavg'] = data['tavg'].astype(float)
data['departure'] = data['departure'].astype(float)
data['HDD'] = data['HDD'].astype(float)
data['CDD'] = data['CDD'].astype(float)
data['precipitation'] = data['precipitation'].astype(float)
data['new_snow'] = data['new_snow'].astype(float)
data['snow_depth'] = data['snow_depth'].astype(float)
print(data.dtypes)
```

```
date              object
tmax             float64
tmin             float64
tavg             float64
departure        float64
HDD              float64
CDD              float64
precipitation    float64
new_snow         float64
snow_depth       float64
dtype: object
```

2. Statistical Analysis and Visualization:

In [36]:

```python
#Now the variables needed for the statistical analysis of the data frame are all numerical
data[["tmax", "tmin", "tavg", "departure", "HDD", "CDD", "precipitation", "new_snow", "snow_depth"]].describe()
```

Out[36]:

	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
count	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000
mean	63.169863	49.512329	56.341096	-0.527945	12.463014	4.065753	0.139342	0.042466	0.032877
std	18.806232	17.154853	17.821404	6.421460	13.264920	6.374835	0.321168	0.298309	0.294666
min	16.000000	3.000000	10.000000	-24.100000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	47.000000	35.000000	41.000000	-4.700000	0.000000	0.000000	0.000000	0.000000	0.000000
50%	63.000000	49.000000	56.000000	0.100000	9.000000	0.000000	0.000000	0.000000	0.000000
75%	79.000000	65.000000	72.000000	3.700000	24.000000	7.000000	0.080000	0.000000	0.000000
max	100.000000	83.000000	91.500000	17.400000	55.000000	27.000000	2.670000	3.800000	4.000000

In [37]:

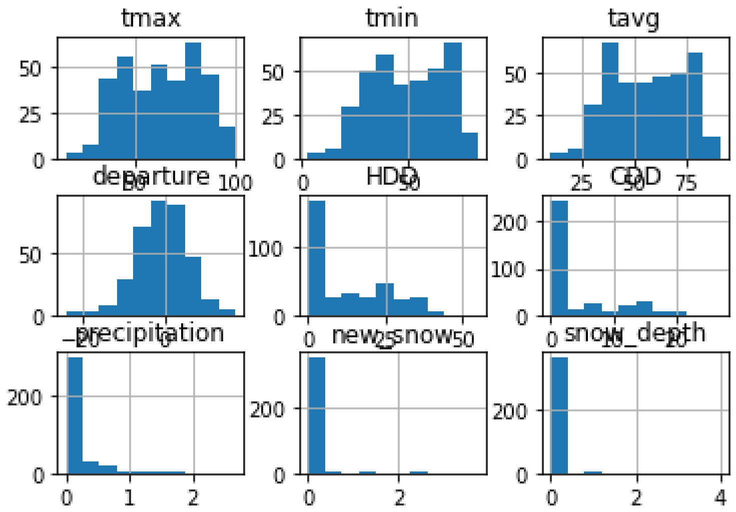

```python
#Now the variables needed for the statistical analysis of the data frame are all numerical
data[["tmax", "tmin", "tavg", "departure", "HDD", "CDD", "precipitation", "new_snow", "snow_depth"]].describe()
```


Out[37]:

	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
count	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000	365.000000
mean	63.169863	49.512329	56.341096	-0.527945	12.463014	4.065753	0.139342	0.042466	0.032877
std	18.806232	17.154853	17.821404	6.421460	13.264920	6.374835	0.321168	0.298309	0.294666
min	16.000000	3.000000	10.000000	-24.100000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	47.000000	35.000000	41.000000	-4.700000	0.000000	0.000000	0.000000	0.000000	0.000000
50%	63.000000	49.000000	56.000000	0.100000	9.000000	0.000000	0.000000	0.000000	0.000000
75%	79.000000	65.000000	72.000000	3.700000	24.000000	7.000000	0.080000	0.000000	0.000000
max	100.000000	83.000000	91.500000	17.400000	55.000000	27.000000	2.670000	3.800000	4.000000

In [38]:

```python
# will calculate and plot histograms for the different variables
data.hist()
plt.show()
```



In [39]:

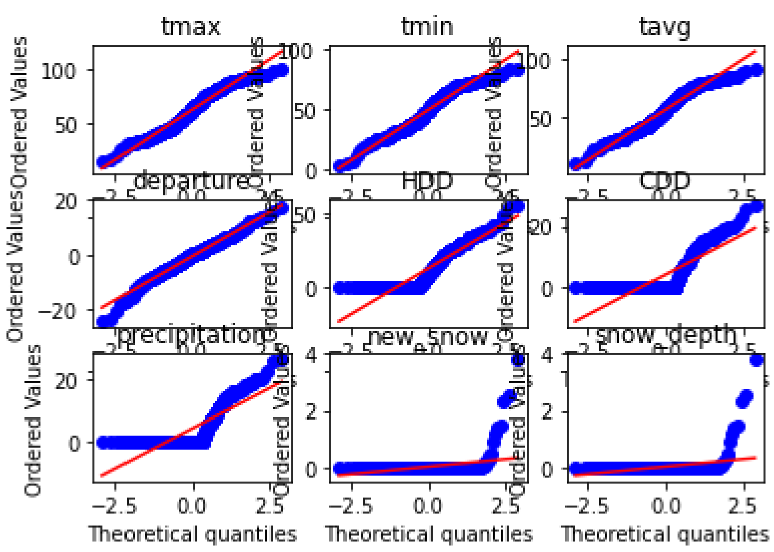
```python
# convert the columns in the dataframe to np arrays
# used np.asarray to convert to array
HDD_array = np.asarray(data["HDD"].to_numpy())
tmax_array = np.asarray(data["tmax"].to_numpy())
tmax_array=tmax_array.reshape(-1,1)
tmin_array = np.asarray(data["tmin"].to_numpy())
tmin_array=tmin_array.reshape(-1,1)
tavg_array = np.asarray(data["tavg"].to_numpy())
tavg_array=tavg_array.reshape(-1,1)
departure_array=np.asarray(data["departure"].to_numpy())
departure_array=departure_array.reshape(-1,1)
HDD_array=np.asarray(data["HDD"].to_numpy())
HDD_array=HDD_array.reshape(-1,1)
CDD_array=np.asarray(data["CDD"].to_numpy())
CDD_array=CDD_array.reshape(-1,1)
precipitation_array=np.asarray(data["precipitation"].to_numpy())
precipitation_array=precipitation_array.reshape(-1,1)
new_snow_array=np.asarray(data["new_snow"].to_numpy())
new_snow_array=new_snow_array.reshape(-1,1)
snow_depth_array=np.asarray(data["snow_depth"].to_numpy())
snow_depth_array=snow_depth_array.reshape(-1,1)
```

In [40]:

```python
#from scipy import stats
fig= plt.figure()
tmax_plt=fig.add_subplot(3,3,1)
tmax_1d=tmax_array.reshape(365,)
tmax_norm=stats.probplot(tmax_1d,plot=plt)
plt.title("tmax")

tmin_plt=fig.add_subplot(3,3,2)
tmin_1d=tmin_array.reshape(365,)
tmin_norm=stats.probplot(tmin_1d,plot=plt)
plt.title("tmin")

tavg_plt=fig.add_subplot(3,3,3)
tavg_1d=tavg_array.reshape(365,)
tavg_norm=stats.probplot(tavg_1d,plot=plt)
plt.title("tavg")

departure_plt=fig.add_subplot(3,3,4)
departure_1d=departure_array.reshape(365,)
departure_norm=stats.probplot(departure_1d,plot=plt)
plt.title("departure")

HDD_plt=fig.add_subplot(3,3,5)
HDD_1d=HDD_array.reshape(365,)
HDD_norm=stats.probplot(HDD_1d,plot=plt)
plt.title("HDD")

CDD_plt=fig.add_subplot(3,3,6)
CDD_1d=CDD_array.reshape(365,)
CDD_norm=stats.probplot(CDD_1d,plot=plt)
plt.title("CDD")

precipitation_plt=fig.add_subplot(3,3,7)
precipitation_1d=CDD_array.reshape(365,)
precipitation_norm=stats.probplot(precipitation_1d,plot=plt)
plt.title("precipitation")

new_snow_plt=fig.add_subplot(3,3,8)
new_snow_1d=new_snow_array.reshape(365,)
new_snow_norm=stats.probplot(new_snow_1d,plot=plt)
plt.title("new_snow")

snow_depth_plt=fig.add_subplot(3,3,9)
snow_depth_1d=new_snow_array.reshape(365,)
snow_depth_norm=stats.probplot(snow_depth_1d,plot=plt)
plt.title("snow_depth")
```

Out[40]:

```
Text(0.5, 1.0, 'snow_depth')
```

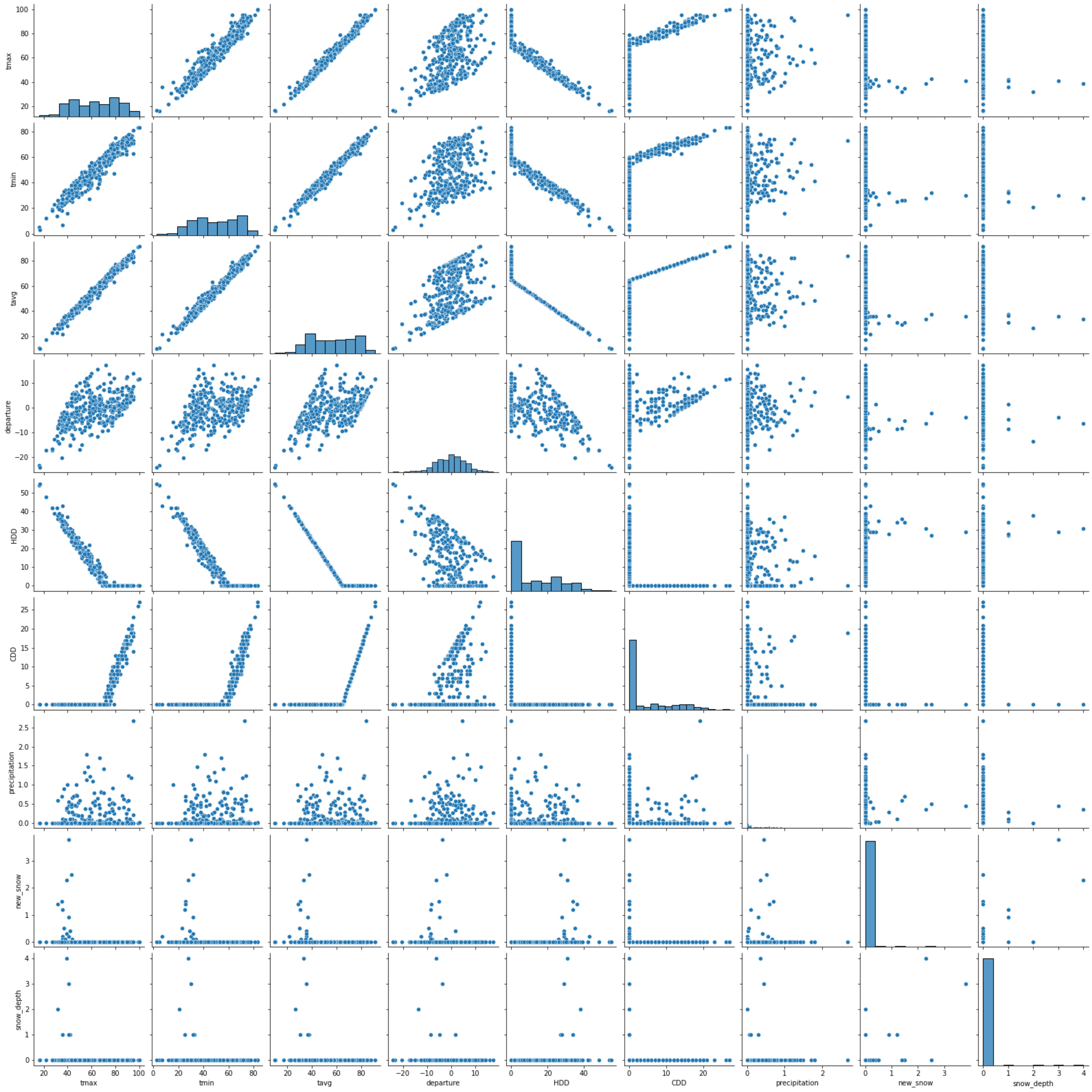


Based on the plotted histograms the departure seems like the best normally distributed variable
the snow_depth and new_snow seem to be the least normally distributed

this can be due to a few number of days of snow meaning few samples
However, according to the normal probability plots above tmax,tmin, tavg and departure seem to be a Gaussian distribution

In [41]:

```
sns.pairplot(data)
plt.show()
```



Based on the visualizations above, it looks like HDD has negative correlation with tmin, tmax, and taverage
we can confirm this visual correlaion by calculating the correlation coeffients
Other examples that show strong correlation are the following:(tmax, taverage), (tmax,tmin), (tmin, taverage)
Examples that show no good correlations are the following:(precipitation, tmax), (precipitation, HDD), (tmin, precipitation)

In [42]:

```
# calculate correlation between the columns
data.corr()
```

Out[42]:

	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
tmax	1.000000	0.964680	0.991930	0.504420	-0.948704	0.801030	0.008210	-0.183409	-0.146762
tmin	0.964680	1.000000	0.990294	0.433581	-0.951052	0.792244	0.015270	-0.174763	-0.141384
tavg	0.991930	0.990294	1.000000	0.474830	-0.958306	0.803954	0.011681	-0.180886	-0.145484
departure	0.504420	0.433581	0.474830	1.000000	-0.477517	0.332172	0.033311	-0.104159	-0.100274
HDD	-0.948704	-0.951052	-0.958306	-0.477517	1.000000	-0.600872	-0.025594	0.197466	0.159156
CDD	0.801030	0.792244	0.803954	0.332172	-0.600872	1.000000	-0.020160	-0.091041	-0.071355
precipitation	0.008210	0.015270	0.011681	0.033311	-0.025594	-0.020160	1.000000	0.121013	0.044353
new_snow	-0.183409	-0.174763	-0.180886	-0.104159	0.197466	-0.091041	0.121013	1.000000	0.693532

	tmax	tmin	tavg	departure	HDD	CDD	precipitation	new_snow	snow_depth
snow_depth	-0.146762	-0.141384	-0.145484	-0.100274	0.159156	-0.071355	0.044353	0.693532	1.000000

The correlation between HDD and tmax= -0.948 The correlation between HDD and tmin= -0.951 The correlation between HDD and taverage= -0.958

3. Regression Model

Based on the correlation table above we will develop regression models to predict HDD from tmax, tmin, taverage and departure
will develop five regression models:
(HDD a function of tmax)
(HDD a function of tmin)
(HDD a function of taverage)
(HDD a function of both tmax and tmin)
(HDD a function of tmax, tmin and depature)
for each one of these regression models will calculate the mean square error (MSE) and will choose the model with the least value of MSE

In [43]:

```
# first regression model_1 of HDD as a function of tmax
model_1=LinearRegression().fit(tmax_array, HDD_array )
print(model_1)
b_0=model_1.intercept_
print("b_0=", model_1.intercept_)
b_1=model_1.coef_
print("b_1=", model_1.coef_[0])
```

```
LinearRegression()
b_0= [54.73412354]
b_1= [-0.66916577]
```

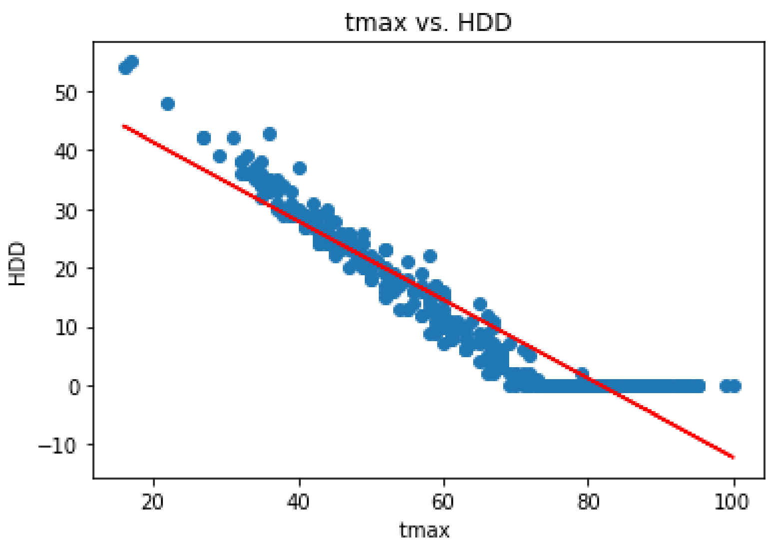
In [44]:

```
#find Y_hat
Y_hat=b_0+b_1*tmax_array
#print("Y_hat=", Y_hat)
Y_hat=Y_hat.reshape(-1,1)
```

fit a regression line of model_1 to the scatter plot of tmax vs HDD

In [45]:

```
plt.scatter(tmax_array, HDD_array)
plt.title("tmax vs. HDD")
plt.xlabel("tmax ")
plt.ylabel("HDD")
plt.plot(tmax_array, Y_hat, color='red')
plt.show()
```



In [46]:

```
# Calculate MSE=SSE/(n-2) for the regression model_1
#first calculate SSE
n=365
E_1=(HDD_array-Y_hat)**2
#print(E_1)
#print(E_1.shape)
SSE_1=np.sum(E_1)
print("SSE_1=", SSE_1)
MSE_1=SSE_1/(n-2)
print("MSE_tmax_HDD=", MSE_1)
```

```
SSE_1= 6402.323519271758
MSE_tmax_HDD= 17.637254874026883
```

second regression model_2 of HDD as a function of tmin

In [47]:

```
# second regression model_2 of HDD as a function of tmin
model_2=LinearRegression().fit(tmin_array, HDD_array )
print(model_2)
b_0=model_2.intercept_
print("b_0=", model_2.intercept_)
b_1=model_2.coef_
print("b_1=", model_2.coef_[0])
```

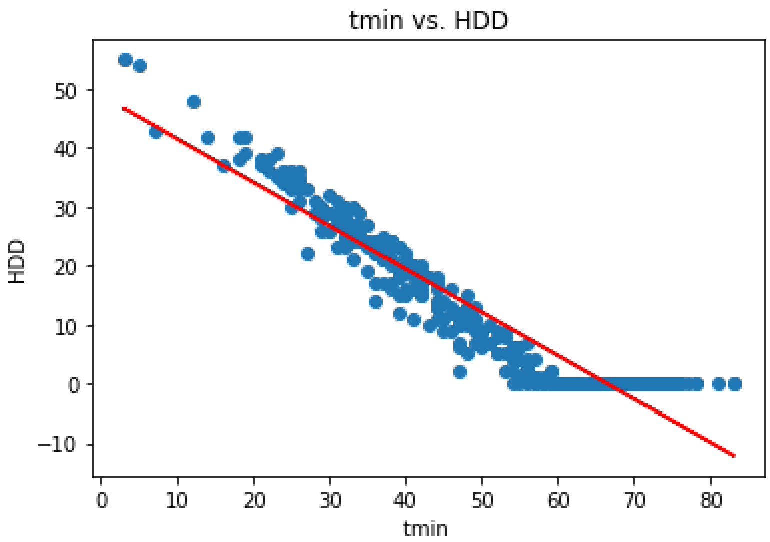
```
LinearRegression()
b_0= [48.87422179]
b_1= [-0.7353968]
```

In [48]:

```
#find Y_hat
Y_hat=b_0+b_1*tmin_array
#print("Y_hat_=", Y_hat)
Y_hat=Y_hat.reshape(-1, 1)
```

In [49]:

```
#fit a regression line of model_2 to the scatter plot of tmin vs HDD

plt.scatter(tmin_array, HDD_array)
plt.title("tmin vs. HDD")
plt.xlabel("tmin ")
plt.ylabel("HDD")
plt.plot(tmin_array, Y_hat, color='red')
plt.show()
```



In [50]:

```
# Calculate MSE=SSE/(n-2) for the regression model_2
#first calculate SSE
E_2=(HDD_array-Y_hat)**2
SSE_2=np.sum(E_2)
MSE_2=SSE_2/(n-2)
print("MSE_tmin_HDD=", MSE_2)
```

```
MSE_tmin_HDD= 16.850422294766958
```

third regression model_3 of HDD as a function of tavg

In [51]:

```
model_3=LinearRegression().fit(tavg_array, HDD_array )
print(model_3)
b_0=model_3.intercept_
print("b_0=", model_3.intercept_)
b_1=model_3.coef_
print("b_1=", model_3.coef_[0])
```

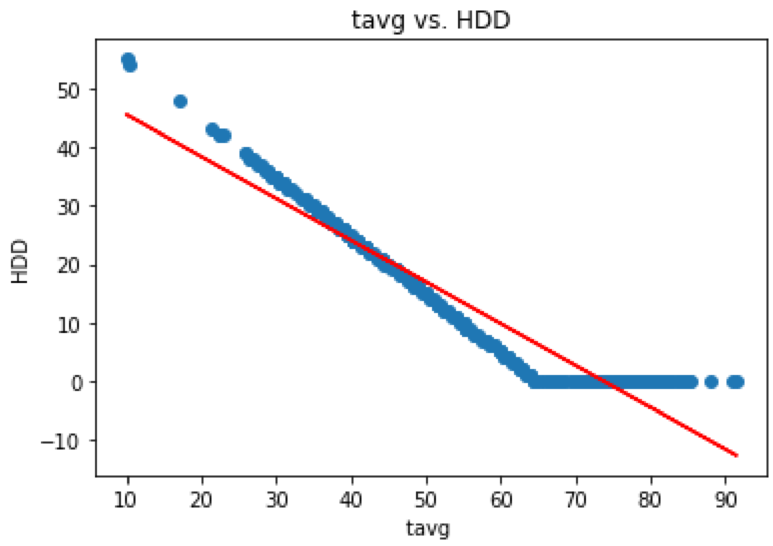
```
LinearRegression()
b_0= [52.65061535]
b_1= [-0.71329109]
```

In [52]:

```
#find Y_hat
Y_hat=b_0+b_1*tavg_array
#print("Y_hat_=", Y_hat)
Y_hat=Y_hat.reshape(-1, 1)
```

In [53]:

```
#fit a regression line of model_3 to the scatter plot of tavg vs HDD

plt.scatter(tavg_array, HDD_array)
plt.title("tavg vs. HDD")
plt.xlabel("tavg ")
plt.ylabel("HDD")
plt.plot(tavg_array, Y_hat, color='red')
plt.show()
```



In [54]:

```
# Calculate MSE=SSE/(n-2) for the regression model_3
#first calculate SSE
E_3=(HDD_array-Y_hat)**2
SSE_3=np.sum(E_3)
MSE_3=SSE_3/(n-2)
print("MSE_tavg_HDD=", MSE_3)
```

```
MSE_tavg_HDD= 14.406594939745002
```

Fourth regression model_4 is a multi regression linear model of HDD as a function of (tmax and tmin)

In [55]:

```
tmax_min=np.concatenate((tmax_array, tmin_array), axis=1)
#print(tmax_min.shape)
model_4=LinearRegression().fit(tmax_min, HDD_array )
print(model_4)
b_0=model_4.intercept_
print("b_0=", model_4.intercept_)
b_1=model_4.coef_
print("b_1=", model_4.coef_[0])
```

```
LinearRegression()
b_0= [52.30678213]
b_1= [-0.31758083 -0.39954151]
```

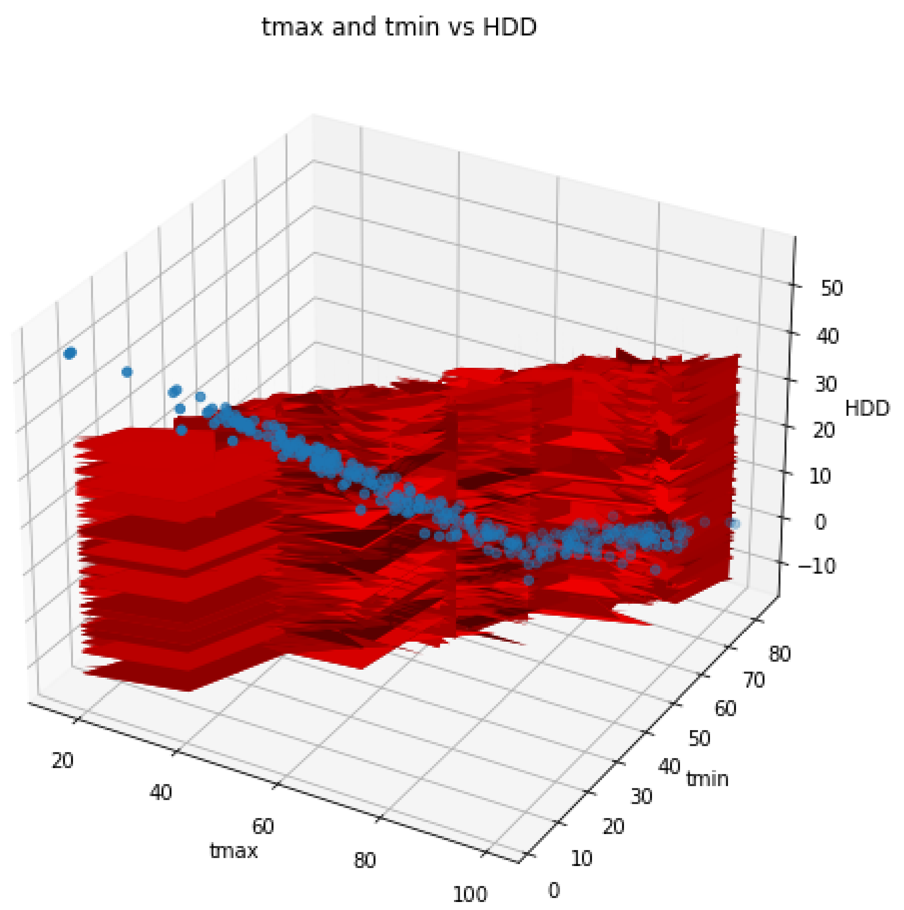
In [56]:

```
#find Y_hat
Y_hat=b_0+b_1[0,0]*tmax_array+b_1[0,1]*tmin_array
#print("Y_hat_=", Y_hat)
Y_hat=Y_hat.reshape(-1, 1)
```

In [57]:

```
#fit a regression line of model_4 to the scatter plot of tmax_min vs HDD
fig=plt.figure(figsize=(8,8))
ax=plt.axes(projection="3d")
ax.scatter(tmax_array.reshape(365,),tmin_array.reshape(365,), HDD_array.reshape(365,))
ax.set_xlabel("tmax")
ax.set_ylabel("tmin")
ax.set_zlabel("HDD")
ax.set_title("tmax and tmin vs HDD")
ax.plot_surface(tmax_array.reshape(365,),tmin_array.reshape(365,), Y_hat.reshape(365,1),color="red")

fig.show()
```

```
C:\Users\yousi\AppData\Local\Temp/ipykernel_14552/1425713347.py:11: UserWarning: Matplotlib is currently using module://matplotlib
_inline.backend_inline, which is a non-GUI backend, so cannot show the figure.
  fig.show()
```



In [58]:

```
# Calculate MSE=SSE/(n-2) for the regression model_4
#first calculate SSE
E_4=(HDD_array-Y_hat)**2
SSE_4=np.sum(E_4)
MSE_4=SSE_4/(n-2)
print("MSE_tmax_tmin=", MSE_4)
```

```
MSE_tmax_tmin= 14.3682986973997
```

Fifth regression model_5 is a multi regression linear model of HDD as a function of (tmax, tmin and depature)

In [59]:

```
tmax_min_dep=np.concatenate((tmax_array, tmin_array, departure_array), axis=1)
#print(tmax_min_dep.shape)
model_5=LinearRegression().fit(tmax_min_dep, HDD_array )
print(model_5)
b_0=model_5.intercept_
print("b_0=", model_5.intercept_)
b_1=model_5.coef_
print("b_1=", model_5.coef_[0])
```

```
LinearRegression()
b_0= [51.29119268]
b_1= [-0.28483031 -0.4216379  -0.07725642]
```

In [60]:

```
#find Y_hat
Y_hat=b_0+b_1[0,0]*tmax_array+b_1[0,1]*tmin_array+b_1[0,2]*departure_array
#print("Y_hat_=", Y_hat)
Y_hat=Y_hat.reshape(-1, 1)
```

In [61]:

```
# Calculate MSE=SSE/(n-2) for the regression model_5
#first calculate SSE
E_5=(HDD_array-Y_hat)**2
SSE_5=np.sum(E_5)
MSE_5=SSE_5/(n-2)
print("MSE_tmax_tmin_dep=", MSE_5)
```

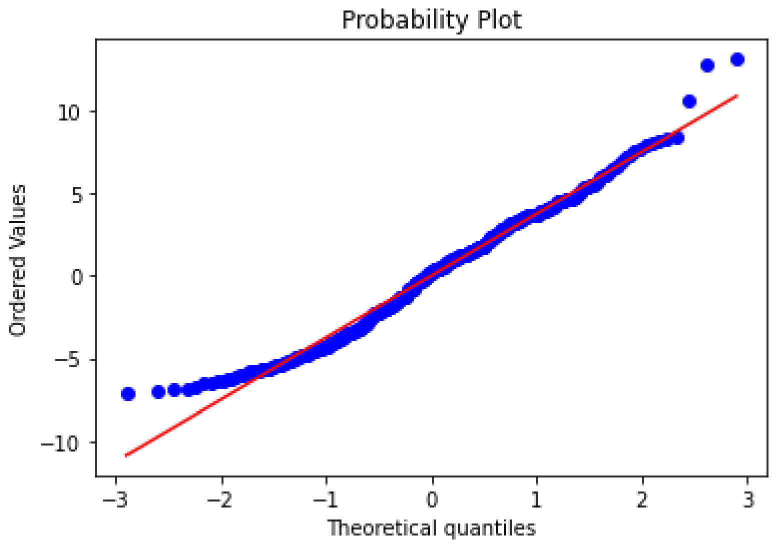
```
MSE_tmax_tmin_dep= 14.194298417750032
```

Comparison of the five regession models

we calculated the mean square errors for the five models and compared them
$\mathbf{Model5}$ which is the multiple regression model of HDD as a function of `tmax`, `tmin` and `departure`, has the least mean square error MSE of 14.19429 compared to the other four models which have MSEs: MSE_tmax_HDD $= 17.637254$, MSE_tmin_HDD= 16.85042,MSE_tavg_HDD $= 14.40659$, MSE_tmax_tmin $= 14.36829$
Therefore this model results in the best predictions
Next we will evaluate the assumptions of the regression model: linearity and normal distribution

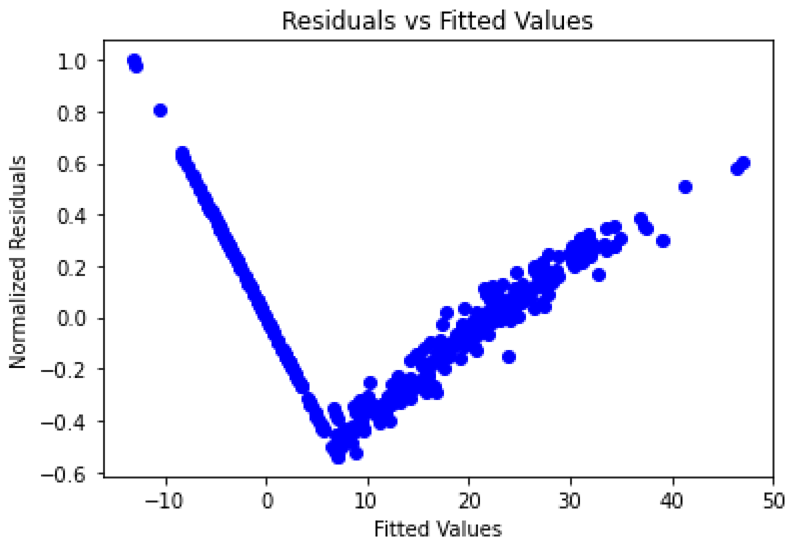
In [62]:

```
#testing the normality assumption for the residuals
residuals=HDD_array-Y_hat
stats.probplot(residuals.reshape(365,),plot=plt)
plt.show()
```



the results show that the residuals are almost normally distributed

In [63]:
```
plt.title("Residuals vs Fitted Values")
plt.xlabel("Fitted Values")
plt.ylabel("Normalized Residuals")

plt.scatter(Y_hat, (residuals/max(np.abs(residuals))),color='blue')
plt.show()
```



Since the residuals have higher values for low and high fitted values, and smaller values in the middle therefore a linear model doesnt seem the best fit

In [ ]: