Ajit K. Srivastava, Carroll E. Goering, Roger P. Rohrbach, and Dennis R. Buckmaster. 2006. Agricultural Mechanization and Some Methods of Study. Chapter 1 in Engineering Principles of Agricultural Machines, 2nd ed., 1-14. St. Joseph, Michigan: ASABE, Copyright American Society of Agricultural and Biological Engineers. (Corrected reprint 2012).

# AGRICULTURAL MECHANIZATION AND SOME METHODS OF STUDY

1

## INTRODUCTION

Many factors have contributed to agricultural mechanization. Reducing human drudgery, increasing productivity, improving timeliness of agricultural operations such as planting and harvesting, and reducing peak labor demands are among the most compelling. Farm work is physically demanding and the working conditions are often harsh. It is less strenuous to drive a tractor than to till the soil with a spade all day long. A tractor pulling a plow can cultivate a larger area than a human with a spade in the same amount of time, thereby increasing productivity and timeliness. Timeliness is an important factor in agricultural production. Completing certain farming operations such as planting and harvesting in a timely manner increases yields and improves profitability. Farming operations are seasonal with fluctuating labor demand. More labor is needed during planting and harvesting than during other periods of plant growth. This fluctuation in labor demand creates labor management problems. With mechanization it is possible to reduce peak labor demand and maintain a more stable labor force on the farm.

## 1.1 HISTORY OF MECHANIZED AGRICULTURE

Even though great changes have taken place in the field of agriculture, soil still has to be tilled; seeds still have to be planted in the soil; the growing crop still has to be tended and cared for; and the crops still have to be harvested and threshed. However, the manner in which these operations are performed have changed drastically.

One of the earliest plows used to till soil was a wooden plow pulled either by humans or draft animals. As we learned to work with steel, moldboard plows were developed. The moldboard plow was a major development, since it turned the soil for better weed control and soil aeration. The seeds were planted by broadcasting them by hand. A major development in planting occurred when we learned to plant seeds in rows using dibble sticks in the early stages and later on with planters. Planting in rows had the advantage of controlling the plant population and facilitated better weed control during the plant growth period.

Crop harvesting was done by hand using sickles or scythes. The cut crop was bundled and carried to a central location where it was threshed either by beating it with a stick or by having hoofed animals walk on it. The threshed crop was separated from chaff and straw by winnowing in natural wind. The threshed crop mixture would be slowly dropped from a height and the wind would blow the chaff and small pieces of straw away leaving the clean grains to fall in a pile. The process was repeated until the grain was totally free of chaff and other debris. Later, the grain was cut by mowers that used a reciprocating sicklebar. The crop was still bundled by hand. Reapers combined the cutting and binding process in one machine. The development of steam engines made it possible to develop stationary threshers. Stationary threshers were used to thresh a bundled crop at a central location. The cleaning operation was still done by winnowing but it was done by a fan instead of the natural wind. The development of the internal combustion engine made it possible to combine the cutting, threshing, and cleaning functions. The name "combine" became popular because the machine combined the three operations.

The power for early farming operations was primarily human labor. Later, draft animals were used as the source of power. Horses, water buffalo, oxen, camels, and even elephants were used as power sources. Mechanical power became the primary source with the development of steam engines in 1858. In 1889 the first tractor with an internal combustion engine was built. Tractors powered by internal combustion engines were lighter and more powerful than steam-powered tractors. In the 1930s the high compression diesel engine was adopted for tractors and became very popular. Today's modern tractor is a very sophisticated machine with hydrostatic drive, electrohydraulic servos to control draft force and the operating depth, and an ergonomically designed, climate-controlled operator's station. Developments in technologies such as global positioning systems (GPS) and geospacial information systems (GIS) have led to the development of what is commonly known as *precision agriculture* in which soil variability and fertility data are stored in an on-board computer that controls the application rate of chemicals such as fertilizers, pesticides, and herbicides.

It needs, however, to be pointed out that in many parts of the world, especially the Third World countries, animal and human labor continue to be the major source of power for farming operations. Even in the most advanced countries, manual labor is still used for fresh-market fruit and vegetable harvesting operations because of the delicate nature of the products. The level of mechanization depends upon the availability of human labor and the level of industrialization within each country.

Mechanization of agriculture was an important factor in reducing labor demands for farming and making it available to develop other industries. In 1900 nearly two-thirds of the U.S. population was engaged in farming. While only 3% of the American population is engaged in production agriculture now, an American farmer produces enough food to feed 60 people and one farm family can manage up to 1200 ha of farmland. Agricultural mechanization has transformed American agriculture from subsistence farming to a major industry. Today, in monetary value, exports from the agriculture sector are second only to the sale of weapons to foreign countries.

Mechanized agriculture is, however, energy and capital intensive. Energy costs and the availability of capital to buy machines determine the level of mechanization in a society. Thus, production agriculture is facing many challenges. Rising energy costs, greater competition in the global marketplace, and the growing concerns for the environment pose new challenges that agricultural engineers must face to keep agriculture productive and affordable. The area of agricultural machines is dynamic and will continue to evolve to meet the changing needs of production agriculture.

# 1.2 FARMING OPERATIONS AND RELATED MACHINES

Plants are the primary production units of agriculture. They receive carbon dioxide from the air through their leaves, and receive water and nutrients from the soil through their roots. Using carbon dioxide, water, nutrients, and solar energy, plants produce seeds, fruits, roots, fibers, and oils that people can use.

The growth of plants happens in nature without any human intervention. However, agriculture arises when people exert control over plant growth. Machines are used as an extension of people's ability to produce and care for plants. This book focuses on many of the machines used by farmers to produce crops in plant agriculture.

A *crop* is a group of similar plants which are growing within the same land area. For example, if a farm produces rice and wheat, that farm is said to produce two crops. A farmer must complete certain operations in order to successfully produce a crop. The first operation is a mechanical stirring of the soil, called *tillage*, to prepare the seed bed. The second operation is called *planting* and it places the seeds in the tilled soil at the correct depth with the appropriate spacing between seeds. When the required soil temperature and soil water content are present, the seeds will germinate and then grow leaves and roots. For some crops the seeds are planted in a small area called a nursery and then the small plants are transplanted to the fields where they will grow to maturity.

As the plants grow the farmer must protect them from pests such as weeds (unwanted plants), insects, other animals, and diseases. Mechanical cultivation (tillage between the plants) is used to control weeds in some cases. Chemicals are frequently used to control weeds, insects, and diseases. Fences and/or noise-making devices may be used for protection from larger animals.

The final crop production operation is the *harvesting* of the plant parts which have economic value for the farmer. In some cases, more than one part of the plant may have economic value. For example, a farmer may use rice straw (stems and leaves) as an energy resource after the rice seeds have been removed from the plants. In other cases, the crop residue (unused plant parts) is stirred into the soil during tillage for the next crop.

The period of time on the calendar which passes from the beginning of the planting operation until the end of the harvest operation is called the *growing season*. The weather in some tropical farming areas is such that the growing season is continuous. In these areas, a crop can be planted any time during the year, and it can be harvested whenever it is mature. In many farming areas, however, the growing season is restricted because of weather conditions. For example, the planting operation may begin during spring when the soil temperature is increasing, and the harvest operation is

**Table 1.1. Example of a crop rotation with four crops.**

| Year | Area 1 | Area 2 | Area 3 | Area 4 |
|---|---|---|---|---|
| 1 | Crop A | Crop B | Crop C | Crop D |
| 2 | Crop B | Crop C | Crop D | Crop A |
| 3 | Crop C | Crop D | Crop A | Crop B |
| 4 | Crop D | Crop A | Crop B | Crop C |

completed during fall before cold weather begins. In other climates, the growing season depends on rainfall patterns with the planting operation done at the beginning of the rainy season so that the plants have adequate water for growth. Some farming areas have weather conditions which cause a short growing season that allows only one crop per calendar year, while other areas have a longer growing season which allows two or more crops each year from a given field. When the growing season is weather dependent, the planting and harvesting operations are very labor intensive in order to complete these operations in a timely way. If planting and harvesting are not completed in a timely way, the crop yield will be lowered.

Agricultural crops such as rice and wheat are *annual plants* which have one harvest after each planting. The annual plants die after they reach maturity and a new crop must be planted before another harvest can be achieved. Crops like hay (used for livestock feed) are *perennial plants* which live for several years and can be harvested several times after a single planting operation.

*Field crops* include grains, hay, and sugar beets, while *horticultural crops* include fruit and vegetables. The crops which farmers choose for their own farm depends on soil type, climate, labor availability, machine availability, profit potential, social customs, government programs, and the farmer's skills.

Many farmers produce more than one type of crop during each calendar year. For example, a farm may be divided into four land areas with a different crop grown on each of the four areas. Alternating these crops in a fixed sequence is called a *crop rotation* and an example is illustrated in Table 1.1. Using a crop rotation spreads the farmer's work load over a longer period of time and reduces the economic risk in case one crop fails. A good crop rotation can also improve crop yield and the soil. Crop rotation affects the set of machines that must be available on the farm. For example, if wheat, corn, and soybeans are all grown, then the farmer needs a grain drill and a row-planter to plant crops, and a grain head and a row crop head as attachments to the combine to harvest crops. A broad selection of machines adds to capital cost and must be taken into account when selecting a crop rotation system.

## 1.3 FUNCTIONAL ANALYSIS OF AGRICULTURAL MACHINES

An agricultural machine has components that work together as a system in order for the machine to perform its intended function. Any machine, however simple, may be divided into many subcomponents. To understand how a machine works, consider the machine as a collection (or system) of several subsystems made up of components and subcomponents. In this section, we will learn how to identify the various systems found in a modern agricultural machine and the functions performed by the subsystems.

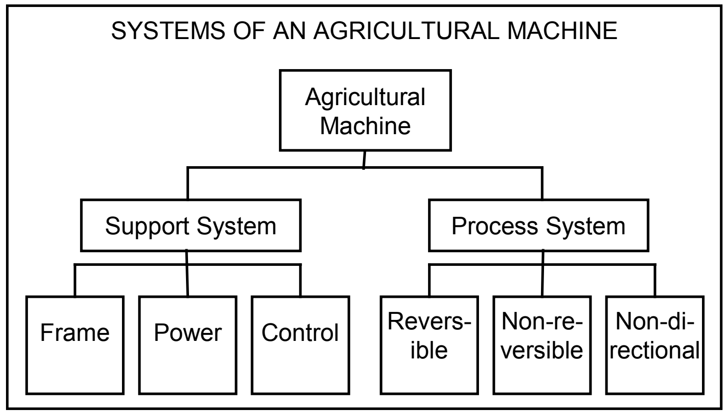


**Figure 1.1 – Systems of agricultural machines.**

It is often useful to look at a complex machine, such as an agricultural machine, as including two kinds of systems: process systems and support systems. The *process systems* are those components of the machine that actually perform the function(s) that the machine is designed to perform, i.e., cut, separate, mix, etc. The *support systems* are the parts that support or aid the process systems in performing their functions.

Process systems may be divided into three types: reversible, non-reversible, and non-directional. Reversible processes include processes such as separation and compaction. Non-reversible processes include cutting and grinding. Examples of non-directional processes are conveying, metering, and storing materials.

Support systems may be divided into three subsystems: the framing, control, and power subsystems. The framing system consists of all structural parts of the machine that hold pieces together so they function properly. The control system provides control over the process system. Controls may be automatic or manual. Power systems supply the power to the process systems. Self-propelled machines contain both the power source (the engine) and the power transmission devices (the drivetrain). Machines that depend on the tractor as a power source contain power transmission devices such as chains, belts, gears, PTO shafts, etc. Together these devices form the power system, which drives the process system.

A breakdown of the types of systems found in an agricultural machine is given in Figure 1.1. This illustration should aid in developing the concept of the agricultural machine as a system.

### 1.3.1 Basic processes of agricultural machines

In this book we will concentrate on process systems of agricultural machines. The process systems of a machine include all parts that perform reversible, non-reversible, or non-directional processes, whereas these processes are the functions the machine was designed to perform. For example, the hay baler was designed to package hay material in the form of a bale so it can be transported and stored for later feeding to animals. In order to perform this task, several processes must be performed on the hay

**Table 1.2. Basic processes of agricultural machines.**

| Reversible Processes | | Non-Reversible Processes | Non-Directional Processes |
|---|---|---|---|
| Mix | Separate | Dissociate | Convey |
| Fluff | Pack | Cut | Meter |
| Pickup | Deposit | Crush | Store |
| Scatter | Position | Grind | |

material. They include non-reversible processes such as cutting, reversible processes such as pickup and compaction, and non-directional processes such as conveying and metering of hay. Table 1.2 lists the processes commonly found in various agricultural machines. The reversible processes are listed in opposing pairs under the appropriate category in the table. The list is not comprehensive, but it includes most commonly found processes in modern agricultural machines.

## 1.3.2 Process diagrams

An exercise that can be helpful in understanding the operation of an agricultural machine is to draw a diagram of the processes that occur in the machine. The diagram is formed by following the flow of material through the machine and listing the processes in order. The processes can be connected with lines to indicate the flow of the material through the machine.

Any of the processes can occur either totally within the machine or with machine mobility as part of the process. For example, the forward motion of a baler is essential to pick up hay. However, after hay is picked up, it will be baled regardless of the forward motion of the machine. When machine mobility is a part of the process, the process is, in this book, enclosed in a box. A process occurring totally within the machine is enclosed in a circle or an oval.

A few examples should be helpful in understanding the concept of process diagramming. A good first example is the moldboard plow. The first step is to determine what processes occur as the plow moves through the soil. As the plow moves forward, the soil is cut, picked up, positioned, and deposited. The second step is to determine whether the processes are dependent upon forward motion. In the case of a moldboard plow, all functions would cease as soon as the plow is stopped. The process diagram for the moldboard plow is given in Figure 1.2. The processes of picking up and positioning occur simultaneously and, therefore, are diagrammed as a pair.

A more complex machine to diagram is the conventional hay baler. The processes that occur in the machine are pickup, convey, meter, cut, pack, bind, convey, and deposit. The process which is dependent upon forward motion of the baler is pickup. The process diagram is given in Figure 1.3.

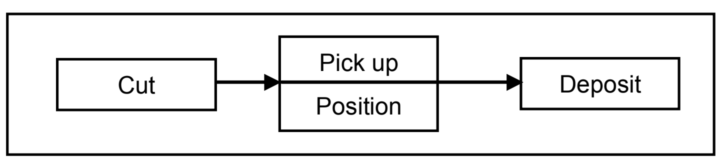


**Figure 1.2 – Process diagram for a moldboard plow.**

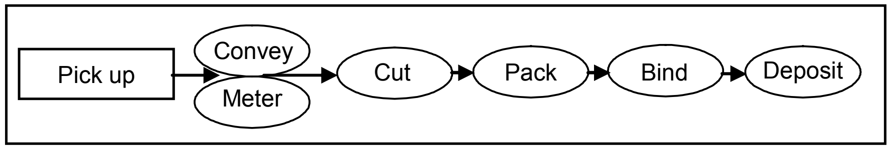


**Figure 1.3 – Process diagram of for a hay baler.**

The concepts of machine systems and process diagramming are introduced here as tools to aid students in learning more about the makeup and operation of agricultural machines. It is hoped that these concepts will provide a new and more interesting way to study agricultural machines, or any machine for that matter.

# 1.4 DIMENSIONAL ANALYSIS

Engineers like to develop predictive models to study a process or a phenomenon. Ideally we would like to develop a model that is based on the natural laws that govern the process. For example, to predict the droplet size and its distribution in a sprayer nozzle we need to understand basic fluid mechanics and the physics of a jet breakup in another fluid such as air. However, this can be a very complex process and does not lend itself to easy modeling. In cases like these another technique which is often useful is *dimensional analysis*. In dimensional analysis we need only to identify all pertinent physical quantities that influence the process. We then combine these quantities in groups so that each group is dimensionless. Experiments are then carried out to develop a power law model to relate the dependent dimensionless group to the independent ones. Dimensional analysis can be applied to highly complex processes to develop a prediction equation; however, the basic underlying natural laws are not necessarily revealed. It is, however, better than regression models in that the number of variables that must be studied are reduced substantially. Below is an abbreviated discussion of dimensional analysis.

## 1.4.1 Scope

Dimensional analysis is a method by which we deduce information about a phenomenon from the single premise that the phenomenon can be described by a dimensionally correct equation among pertinent variables.

The result of a dimensional analysis of a problem is a reduction in the number of variables in the problem. This results in a considerable savings in both cost and labor during the experimental determination of the function.

## 1.4.2 Physical dimensions

Scientific reasoning is based on such abstract entities such as force, mass, length, time, accelerations, velocity, temperature, specific heat, and electric charge. Each of these entities is assigned a unit of measurement. Of these entities, mass, length, time, temperature, and electric charge are in a sense independent and their units of measurement are specified by international standards. Furthermore, specified units of these entities determine the units of all other entities. There is, however, nothing fundamental in the set of entities, mass, length, time, temperature, and electric charge. A great many possibilities exist for choosing five mutually independent entities. Frequently, unit of force is prescribed, rather than the unit of mass. The unit of mass is determined by Newton's law, F = ma. In this case the system of measurement is called a *force system*. Dimensions are a code for telling us how the numerical value of a quantity changes when the basic units of measurement are subjected to prescribed changes. The symbols [F], [M], [L], [T], and [θ] have been employed to denote dimensions of force, mass, length, time, and temperature, respectively, and any entity that has no units is denoted by [1] (Table 1.3).

**Table 1.3. Dimensions of entities.**

| | Mass System | Force System |
|---|---|---|
| Length | L | L |
| Time | T | T |
| Temperature | $\theta$ | $\theta$ |
| Force | $MLT^{-2}$ | F |
| Mass | M | $FL^{-1}T^{-2}$ |
| Mass density | ML | $LF^{-4}T^{2}$ |
| Pressure and stress | $ML^{-1}T^{-2}$ | $LF^{-2}$ |
| Energy, work | $ML^{2}T^{-2}$ | FL |
| Viscosity | $ML^{-1}T^{-1}$ | $FL^{-2}T$ |
| Mass movement of inertia | $ML^{2}$ | FLT |
| Surface tension | $MT^{-2}$ | $FL^{-1}$ |
| Strain | 1 | 1 |
| Poisson's ratio[a] | 1 | 1 |

[a] Any ratio of like-dimensioned quantities (i.e., unitless) has the dimension of one.

## 1.4.3 Units of measurement

**CGS (Centimeter Gram Second) System**

Force, measured in dynes, is defined as the force required to accelerate a 1 gram mass with 1 cm/s$^2$ acceleration. Thus, the weight of a gram mass is:

$$
\begin{aligned}
W &= mg \\
&= (1\ \text{g})\ (981\ \text{cm/s}^2) \\
&= 981\ (\text{g}\cdot\text{cm/s}^2) \\
&= 981\ \text{dynes}
\end{aligned}
$$

**SI (International) System**

Force, measured in Newtons, is defined as the force required to accelerate a 1 kg mass with 1 m/s$^2$ acceleration. Thus, the weight of a kilogram mass is:

$$
\begin{aligned}
W &= mg \\
&= (1\text{kg})\ (9.81\ \text{m/s}^2) \\
&= 9.81\ \text{kg m/s}^2 \\
&= 9.81\ \text{Newtons}
\end{aligned}
$$

**U.S. Customary System**

Force = pound (lb)
Length = foot (ft)
Time = second (s)
Mass, measured in slugs, is defined as that mass which will require a 1 lb force in order to accelerate with 1 ft/s$^2$ acceleration. Thus, the weight of 1 slug is:

$$
\begin{aligned}
W &= mg \\
&= (1\ \text{slug})\ (32.2\ \text{ft/s}^2) \\
&= 32.2\ (\text{slug ft/s}^2) = 32.2\ \text{lb}
\end{aligned}
$$

**Conversion Factors**

1 m = 3.281 ft
1 ft = 0.0348 m
1 kg = 0.06852 slug
1 slug = 14.594 kg
1 Newton = 0.2248 lb
1 lb = 4.448 Newtons
1º C = 1.9º F

### 1.4.4 Developing a prediction equation

A critical step in dimensional analysis is to decide what physical quantities enter the problem. It is important that there be no redundancy and that no pertinent quantities are left out. To list pertinent variables, it is useful to develop an understanding of the basic phenomena or laws that affect the system. For example, let us consider that we want to develop an equation to predict the period of oscillation of a simple pendulum, that is, a mass is attached to one end of a string while the other end is attached to a support in a way such that the mass is allowed to swing with no friction. We will also neglect the aerodynamic effects. An equation of the following form may be written:

$$T = C_\alpha\, l^a\, m^b\, g^c \tag{1.1}$$

where T = period, a time entity denoted by dimension [T]

$C_\alpha$ = a dimensional coefficient denoted by dimension [1]

l = string length, a length entity denoted by dimension [L]

m = mass, an entity denoted by dimension [M]

g = acceleration due to gravity, denoted by dimension $[LT^{-2}]$

a, b, and c = dimensionless exponents

Substituting the dimension of each physical quantity in Equation 1.1 we get:

$$[T] = [1]\,[L]^a\,[M]^b\,[LT^{-2}]^c \tag{1.2}$$

It may clarify the next step to place the [L] and [M] dimensions on both sides of the equation, each with a zero exponent:

$$[M]^0\,[L]^0\,[T] = [1]\,[L]^a\,[M]^b\,[LT^{-2}]^c$$

Then, collecting and equating the exponents of the above equation we get:

for [M]: 0 = b, because the [M] exponent on the left is 0 and the [M] exponent on the right is b;

for [L]: 0 = a + c, thus a = – c, because the [L] exponent on the left is 0 and on the right the collected [L] exponents are a + c; and similarly,

for [T]: 1 = – 2c

c = – 1/2

a = 1/2

Substituting the values of a, b, and c in Equation 1.1 we get

$$T = C_\alpha\, l^{1/2}\, m^0\, g^{-1/2}$$

or

$$T = C_\alpha \sqrt{l/g}$$

or

$$\frac{T}{\sqrt{l/g}} = C_\alpha \tag{1.3}$$

Note that the quantity on the left hand side of Equation 1.3 is a dimensionless group. Also note that mass, m, dropped off. This is true since we know that the period of oscillation does not depend on mass as heavier objects do not fall faster. The coefficient $C_\alpha$ needs to be determined experimentally. We know from mechanics that the value of the constant is $2\pi$. Also note that we began with four physical quantities and we reduced the equation by three (a number equal to the number of basic dimensions in the problem) to one dimensionless term in Equation 1.3.

### 1.4.5 Buckingham's Theorem

Buckingham's Theorem states that "If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products."

Suppose that we are interested in the drag force, F, acting on a sphere of diameter, D, submerged in a fluid with an average velocity, V, and having density, ρ, and viscosity, μ. Consider tentatively the relationship:

$$F = C_\alpha \, V^a D^b \rho^c \mu^d \tag{1.4}$$

where $C_\alpha$ = dimensionless coefficient; and a, b, c, d = dimensionless exponents.

In order for the equation to be *dimensionally homogeneous* both sides of the equation should have the same dimensions. This is similar to checking your units in a complicated equation; they must be the same on each side. This is accomplished by replacing the variables by their dimensions (Table 1.3) in the above equations. (Note that we use the force system since our objective is to develop a prediction equation for drag force. This can be done in the mass system, but the result will not be intuitive since force will need to be expressed as mass times the acceleration.) Replacing the variables by their dimensions results in:

$$[F] = [1]\,[LT^{-1}]^a\,[L]^b\,[FL^{-4}T^2]^c\,[FL^{-2}T]^d \tag{1.5}$$

for [F]: $1 = c + d$
for [L]: $0 = a + b - 4c - 2d$
for [T]: $0 = -a + 2c + d$
$a = 2 - d$
$b = 2 - d$
$c = 1 - d$

Substituting the values in Equation 1.4 we get:

$$F = C_\alpha \, V^2 D^2 \left(\frac{\mu}{VDP}\right)^d \tag{1.6}$$

rearranging,

$$\left(\frac{F}{PV^2D^2}\right) = C_\alpha \left(\frac{PVD}{\mu}\right)^\eta \tag{1.7}$$

Note that the pressure coefficient, $\overline{P} = \left(\frac{F}{PV^2D^2}\right)$, and the Reynolds number, $N_{Re} = \left(\frac{PVD}{\mu}\right)$, can be substituted into the rearranged equation, which then simplifies to:

$$\overline{P} = f(N_{Re}) \tag{1.8}$$

where *f* is any general function.

Both $\overline{P}$, the pressure coefficient, and $N_{Re}$, Reynolds number, are dimensionless quantities. In general, a dimensional equation can be reduced to dimensionless quantities (call the *pi-terms*) related by a general function *f*. Notice that there are only two terms in the dimensionless form of the equation (Equation 1.8) whereas there are five variables in the dimensional form (Equation 1.7).

Stated generally, Buckingham's Theorem allows us to conclude that if n variables are connected by an unknown dimensionally homogeneous equation, it can be expressed in the form of n – r dimensionless products, where r is the number of basic dimensions.

We follow up with Equation 1.7 while noting that the projected area of a sphere is A = (1/4) π $D^2$. Substituting, we obtain:

$$\left(\frac{F}{PV^2A}\right) = \frac{1}{2}\frac{8}{\pi} f\left(N_{Re}\right) \tag{1.9}$$

The term $\frac{8}{\pi} f\left(N_{Re}\right)$ is called the drag coefficient, $C_D$. Thus, the equation for drag on a sphere can be written as:

$$F = \frac{1}{2} C_D PV^2 A \tag{1.10}$$

where $C_D$ is a function of $N_{Re}$. It is plotted in Figure 1.4. The figure is an experimental graph for smooth spherical bodies. It gives complete information concerning the drag forces on smooth spherical bodies of all sizes in an incompressible fluid with any speed of flow. To provide the same information without using dimensional analysis would require about 25 graphs that would show separately the effects of each of the variables V, D, ρ, and μ.

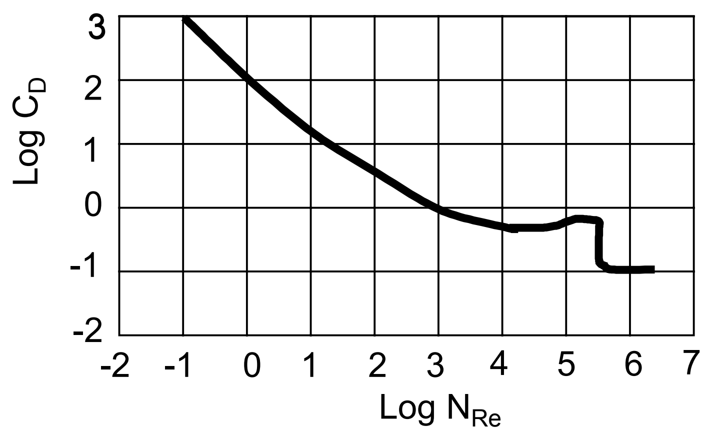


**Figure 1.4 – Drag coefficient as a function of Reynolds number for smooth spherical bodies.**

### 1.4.6 Systematic calculation of the dimensionless products

Consider the problem of computing dimensionless products of variables P, Q, R, S, T, U, V, whose dimensional matrix is given below:

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ |
|---|---|---|---|---|---|---|---|
| | P | Q | R | S | T | U | V |
| M | 2 | –1 | 3 | 0 | 0 | –2 | 1 |
| L | 1 | 0 | –1 | 0 | 2 | 1 | 2 |
| T | 0 | 1 | 0 | 3 | 1 | –1 | 2 |

The first step is to calculate the r, rank of the matrix. The determinant to the right hand side of the matrix is:

$$\begin{vmatrix} 0 & -2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 1 \tag{1.11}$$

Since the determinant is not zero, r = 3. The number of dimensionless groups is the number of variables minus the rank of the dimensional matrix, i.e., the number of dimensionless groups, or 7 – 3 = 4. The corresponding algebraic equations are:

$$2k_1 - k_2 + 3k_3 - 2k_6 + k_7 = 0$$

$$k_1 - k_3 + 2k_5 + k_6 + 2k_7 = 0$$

$$k_2 + 3k_4 + k_5 - k_6 + 2k_7 = 0$$

There are seven variables in the above three equations. This implies that four variables may be assigned any arbitrary values and the other three may be solved using the above equation. Since the value of the determinant as computed above corresponds to $k_5$, $k_6$, and $k_7$, is non-zero, we will use these as dependent variables. In other words, $k_1$, $k_2$, $k_3$, and $k_4$ may be assigned arbitrary values and $k_5$, $k_6$, and $k_7$ may be solved explicitly. While any value may be assigned to $k_1$ through $k_4$, it is prudent to select a set of values that results in simplicity in calculations.

Let $k_1 = 1$ and $k_2 = k_3 = k_4 = 0$ and find $k_5 = -11$, $k_6 = 5$, and $k_7 = 8$. Similarly, let $k_2 = 1$ and $k_1 = k_3 = k_4 = 0$ and find $k_5 = 9$, $k_6 = -4$, and $k_7 = -7$.

The above procedure can be repeated and the solutions arranged as follows:

| | Solution Matrix | | | | | | |
|---|---|---|---|---|---|---|---|
| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | $k_7$ |
| | P | Q | R | S | T | U | V |
| $\pi_1$ | 1 | 0 | 0 | 0 | – 11 | 5 | 8 |
| $\pi_2$ | 0 | 1 | 0 | 0 | 9 | – 4 | – 7 |
| $\pi_3$ | 0 | 0 | 1 | 0 | – 9 | 5 | 7 |
| $\pi_4$ | 0 | 0 | 0 | 1 | 15 | – 6 | – 12 |

From the above matrix the dimensionless products can be written as follows:

$$\pi_1 = \frac{PU^5V^8}{T^{11}} \qquad \pi_2 = \frac{QT^9}{U^4V^7}$$

$$\pi_3 = \frac{RU^5V^7}{T^9} \qquad \pi_4 = \frac{ST^{15}}{U^6V^{12}}$$

These products are linearly independent of each other. Using these dimensionless terms the following prediction equation can be written:

$$\pi_1 = C_\alpha \quad \pi_2^a \quad \pi_3^b \quad \pi_4^c \tag{1.12}$$

### 1.4.7 Transformation of dimensionless products

New dimensionless products can be determined by forming the products of powers of the old terms. For example, the following set of dimensionless products may be transformed if necessary:

$$\pi_1 = \frac{PF}{\mu^2} \qquad \pi_2 = V^3\sqrt{\frac{P}{\mu g}} \qquad \pi_3 = L^3\sqrt{\frac{P^2 g}{\mu^2}}$$

Suppose we have determined that $\mu$ is not important. Even so, we cannot drop all terms containing $\mu$. Instead, we transform the existing set in such a way that $\mu$ appears only in one group, which can then be discarded if necessary. This is done as follows:

$$\pi_1^* = \frac{\pi_1}{\pi^2\pi_3^2} = \frac{F}{PV^2L^2}$$

$$\pi_2^* = \pi_2\pi_3 = \frac{VLP}{\mu}$$

$$\pi_3^* = \frac{\pi_2^2}{\pi_3} = \frac{V^2}{\mu}$$

where $\pi^*$ denotes the transformed dimensionless products. Now $\mu$ appears only in one term, which we may decide to disregard in order to simplify the investigation.

# PROBLEMS

1.1 Show by dimensional analysis that the centrifugal force of a particle is proportional to its mass, proportional to the square of its velocity, and inversely proportional to radius of curvature of its path.

1.2 Complete a dimensional analysis to predict the traction force of a wheel on soil. With the help of your instructor identify soil properties that should be included in dimensional analysis. Express the prediction equation as a function of dimensionless groups.

1.3 Suppose it is desired to obtain an expression of the draft force of a tillage tool operating in soil. List all variables that affect the draft force and complete a dimensional analysis of the problem suitable for plotting data from experimental tests.

1.4 An agricultural spray nozzle is used to atomize fluid in air. Complete a dimensional analysis to predict the droplet mean diameter of the spray.