Performance Lower Limit in an Asymmetric Binary CEO Problem

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***Abstract*—This paper proposes a method for the calculus of performance lower limit in decoding of binary CEO problem. Here is analyzed the case when, over a set of M correlated observed sources, the probabilities between these and the unknown source are different.**

I. Introduction

- the Chief Executive Officer(CEO) problem is defined in [1].
- An optimal, maximum a posteriori (MAP), algorithm is defined in [2] for a binary CEO problem. Data fusion algorithm.
- In [3], [4] one theoretical limit is presented for the case of symmetric binary CEO problem

II. System Model and Definitions

The Fig. 1 show the diagram of transmission model used in this article. In the figure can be seen a binary source U_0, $Pr(U_0 = 1) = 0.5$, that transmit your information across BSC channels, with error probability $Pr(U_m \neq U_0|U_0) = p_m$, $\forall m \in \{1,\ 2,\ ...,\ M\}$. In the out of these channels we obtain M correlated binary sources U_m. Each source U_m, is a noise version of U_0. Thus, is possible to get an approximate version of U_0, called $\hat{U}_0$, using the data of U_m. This procedure is called data fusion.

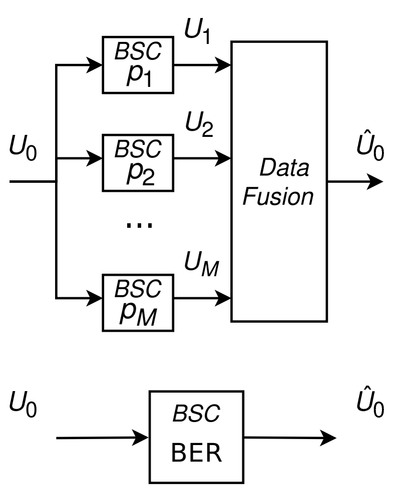

Fig. 1. System Model.

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Definition 1 *Let Ω_m, $\forall m \in \{1,\ 2,\ ...,\ M\}$, be a set of correlated sources as*

$$\Omega_m \equiv \{U_i : i \in \mathbf{Z}^+, 1 \leq i \leq m\}, \tag{1}$$

Definition 2 *The binary entropy is defined as $h_b(\rho)$, so that*

$$h_b(\rho) = -\rho\ log_2(\rho) - (1-\rho)\ log_2(1-\rho). \tag{2}$$

If the probability p_m is used, then is defined

$$h_m \equiv h_b(p_m). \tag{3}$$

III. Optimal Data fusion

As is seen in [2], an optimal decision for to get an estimation $\hat{U}^0$ in the model of the Fig. 1, is to use a criterion MAP, making the quotient

$$L(\mathbf{a}) = \frac{P(U^0 = 1|\Omega^M = \mathbf{a})}{P(U^0 = 0|\Omega^M = \mathbf{a})}, \tag{4}$$

where $\mathbf{a} = \{a_1, a_2, ..., a_M\}$ is a m-dimensional binary vector, that represent a set of observed values of U_m, $\forall m \in \{1,\ 2,\ ...,\ M\}$. The approximation $\hat{U}^0$ of U^0, is obtained with

$$\hat{U}^0 = \begin{cases} 1 & \text{if } L(\mathbf{a}) > 1 \\ 0 & \text{if } L(\mathbf{a}) \leq 1 \end{cases}. \tag{5}$$

Applying logarithm in equations (4) and (5), these can be replaced for

$$\phi(\mathbf{a}) = \sum_{m=1}^{M} B_m, \tag{6}$$

with $B_m = (2a_m - 1)log(1 - p_m/p_m)$. $\hat{U}^0$ is obtained with

$$\hat{U}^0 = \begin{cases} 1 & \text{if } \phi(\mathbf{a}) > 0 \\ 0 & \text{if } \phi(\mathbf{a}) \leq 0 \end{cases}. \tag{7}$$

A. Performance Lower Limit in Optimal Asymmetric Data Fusion

How can be seen in [3], [4] a theoretical performance lower limit for the case of symmetric binary CEO problem is presented, where $Pr(U_m \neq U_0|U_0) = \rho$, obtaining

$$Pr(\hat{U}_0 \neq U_0) = \begin{cases} \frac{1}{2}\binom{M}{\frac{M}{2}}\rho^{\frac{M}{2}}(1-\rho)^{\frac{M}{2}} + \sum\limits_{k=\frac{M}{2}+1}^{M} \binom{M}{k}\rho^k(1-\rho)^{M-k} & \text{if } M \text{ even} \\ \sum\limits_{k=\frac{M+1}{2}}^{M} \binom{M}{k}\rho^k(1-\rho)^{M-k} & \text{if } M \text{ odd} \end{cases}. \tag{8}$$

Differently of seen in [3], [4], here $Pr(U_m \neq U_0|U_0) = p_m$

IV. Final Remarks and Conclusions

In this letter, we considered

V. Appendix

Lemma 3 *Known a set of m correlated sources Ω_m. Then, is true that*

$$Pr(\Omega_m = \mathbf{a}) = \frac{\Psi(\mathbf{a}) + \Psi(\bar{\mathbf{a}})}{2}, \tag{9}$$

$$\Psi(\mathbf{a}) \equiv \prod_{i=1}^{m} Pr(U_i = a_i | U_0 = 0). \tag{10}$$

being $\mathbf{a} = \{a_1, a_2, ..., a_m\}$ and $\bar{\mathbf{a}}$ two binary vectors, both with m elements, where $\bar{\mathbf{a}} \oplus \mathbf{a} = \mathbf{0}$.

Proof.

$$\begin{aligned} Pr(\Omega_m = \mathbf{a}) &= Pr(\Omega_m = \mathbf{a}|U_0 = 0)Pr(U_0 = 0) \\ &+ Pr(\Omega_m = \mathbf{a}|U_0 = 1)Pr(U_0 = 1), \end{aligned} \tag{11}$$

when U_0 is known the probabilities of sources in Ω_m are independents,

$$\begin{aligned} Pr(\Omega_m = \mathbf{a}) &= (1/2) \prod_{U_i \in \Omega_m} Pr(U_i = a_i|U_0 = 0) \\ &+ (1/2) \prod_{U_i \in \Omega_m} Pr(U_i = a_i|U_0 = 1), \end{aligned} \tag{12}$$

■

Lemma 4 *The value of entropy function $H(\Omega_m)$ is the same for a set of sources U_i with $Pr(U_i \neq U_0|U_0)$ equal to p_i or $1 - p_i$*

Proof. Without loss of generality, we assume that need demonstrate $H(U_i\Omega_m)$ is the same for $Pr(U_i \neq U_0|U_0)$ equal to p_i or $1 - p_i$.

$$\begin{aligned} & H(U_i\Omega_m) \\ &= \sum_{U_i,\Omega_m} Pr(U_i\Omega_m) log_2(Pr(U_i\Omega_m)) \\ &= \sum_{\mathbf{a}} Pr(U_i = 1, \Omega_m = \mathbf{a}) log_2(Pr(U_i = 1, \Omega_m = \mathbf{a})) \\ &+ \sum_{\mathbf{a}} Pr(U_i = 0, \Omega_m = \mathbf{a}) log_2(Pr(U_i = 0, \Omega_m = \mathbf{a})) \end{aligned} \tag{13}$$

Using the Lemma 3 we know that for the case of $Pr(U_i \neq U_0|U_0) = p_i$

$$Pr(U_i = 1, \Omega_m = \mathbf{a}) = \frac{p_i\Psi(\mathbf{a}) + (1 - p_i)\Psi(\bar{\mathbf{a}})}{2}, \tag{14}$$

$$Pr(U_i = 0, \Omega_m = \mathbf{a}) = \frac{(1 - p_i)\Psi(\mathbf{a}) + p_i\Psi(\bar{\mathbf{a}})}{2}, \tag{15}$$

and for the case of $Pr(U_i \neq U_0|U_0) = 1 - p_i$

$$Pr(U_i = 1, \Omega_m = \mathbf{a}) = \frac{(1 - p_i)\Psi(\mathbf{a}) + p_i\Psi(\bar{\mathbf{a}})}{2}, \tag{16}$$

$$Pr(U_i = 0, \Omega_m = \mathbf{a}) = \frac{p_i\Psi(\mathbf{a}) + (1 - p_i)\Psi(\bar{\mathbf{a}})}{2}. \tag{17}$$

Using this results is easy see that for both case we obtain the same value of $H(U_i\Omega_m)$. ■

Corollary 5 *Follow the Lemma 4, for calculate the joint entropy $H(\Omega_m)$ we can assume that all probabilities $p_i \leq 1/2$. Thus, exist a bijective function that link the probability p_i an the binary entropy h_i. Therefore the joint entropy $H(\Omega_m)$ is a function that depend of $\mathbf{h} = \{h_1, h_2, ..., h_m\}$.*

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