New Results of Optimal Rate in Joint Source-Channel Coding of Correlated Sources

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***Abstract*—This paper proposes a method to improve the computational cost of the calculus the optimal rate in joint source-channel coding of a set of correlated sources. Here is analyzed the case when the sources are so far of the joint decoder, so that the channel capacities are approximately same. We got lower the complexity of calculus of the optimal rate, for M correlated sources, so that the complexity grows linearly with M.**

***Index Terms*—Multiple correlated sources, large scale sensor networks, source-channel coding.**

I. Introduction

In the work seen in [1], they are presented equations to the calculus of two cases the optimal minimum rates in joint source-channel coding of a set of correlated sources; thus, were showed methods for to get the minimal, common rate and sum rate. These rates fulfill the Slepian-Wolf [2] and channel capacity limit [3] theorems. The results show that, the calculus of the optimal rates of M correlated sources grows in complexity exponentially with M.

In this paper, is assumed the same system model and analyzed the same two cases presented in [1], with the additional restriction that the sources are so far of the joint decoder, in comparing with the distance between sensors, so that the channel capacities in all channels are approximately same. By other side, the correlation values between sources are assumed as random or with spatial correlation [4], in contrast with studied in [5], [6], where the correlation values between any source pair is same.

Other restriction, considered in this article it is the use of a specific model of correlated sources; being used a model similar to seen in [5], [6]; where M correlated sources are generates passing a common source across M binary symmetric channels (BSC). Using all these considerations, it can be deduced for the calculus of optimal common rate, a method with a calculus complexity that grows linearly with the number of sources, and for the calculus of optimal sum rate a method with very low complexity when compared of common rate case.

This paper is organized as follows. The model system and some definitions used in this work are presented in Section II, a brief review of the work in [1] joint with a new solution method to get the optimal common rate is presented in Section III, in this line a new method to get the optimal sum rate is described in Section IV. Some demonstrations need to solve the last sections are presented in Section VI and Section V concludes the paper with some final remarks.

Manuscript received XXXX XX, 2014; revised XXXXX XX, 2014.

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II. System Model and Definitions

The Fig. 1 shows the diagram of the transmission model used in this article. In the figure can be seen M correlated binary sources U_m, $\forall m \in \{1, 2, ..., M\}$. In each source a vector with k bits is selected and this is coded with a rate $r_m = k/n_m$, after coding we have a binary vector $X_m^{n_m}$ with n_m bits per vector. These vectors are send across communication channels with capacity C. The informations obtained after channels are used to get approximations $\hat{U}_m$ of U_m.

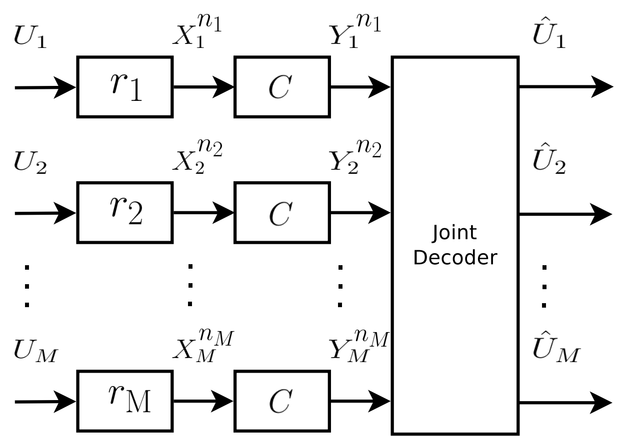

Fig. 1. System Model.

Definition 1 *It is assumed that the source U_m, $\forall\ m \in \{1, 2, ..., M\}$, it is created passing a source U_0, with $Pr(U_0 = 1) = 0.5$, across a BSC channel with error probability $Pr(U_m \neq U_0|U_0) = p_m$.*

Definition 2 *Let $S \subseteq \{1, 2, \ldots, M\}$ be an index subset and S^c your complement. Then, $U(S) \equiv \{U_i : i \in S\}$ is the set the sources indexed by S, and $U(S^c)$ is the set of sources indexed by S^c.*

Lemma 3 *It is well known [1] that for transmit information without error across channels with capacity C, it is necessary fulfill the next relation between the codification rate r_m and the channel capacity C,*

$$H(U(S)|U(S^c)) - \sum_{i \in S} \frac{C}{r_i} \leq 0. \quad (1)$$

Definition 4 *Let Ω_m, $\forall m \in \{1, 2, ..., M\}$, be a set of the m first correlated sources, so that*

$$\Omega_m \equiv \{U_i : i \in \mathbf{Z}^+, 1 \leq i \leq m\}; \tag{2}$$

with the especial case where Ω_0 equal to null, so that $H(\Omega_0) = 0$.

Definition 5 *Let S_m be anything set of m sources in Ω_M, with the especial case of S_0 equal to null, so that $H(S_0) = 0$. Also it is defined $\tilde{S}_m$ with $M - m$ sources, as the complement of S_m, so that $S_m \cup \tilde{S}_m = \Omega_M$.*

Definition 6 *The binary entropy is defined as $h_b(\rho)$, so that*

$$h_b(\rho) = -\rho\ log_2(\rho) - (1-\rho)\ log_2(1-\rho). \tag{3}$$

If the probability p_m was evaluated, then is defined

$$h_m \equiv h_b(p_m). \tag{4}$$

III. Optimal Common Rate

As is seen in [1] and (1), the calculus of optimal codification rate for the case where $r_m = r$ and $C_m = C$, $\forall m \in \{1, 2, ..., M\}$, it is obtained finding the maximum value of r in the next relation, that it is represent by a set of inequalities,

$$r \leq \frac{\sum_{i \in S} C}{H(U(S)|U(S^c))}. \tag{5}$$

Analyzing these inequalities is easy to see that for M sources is necessary do comparisons with $2^M - 1$ inequalities (number of no-null subsets in S), so that this number of comparisons grows exponentially with M and it is not manageable. In the next subsection will be show how simplify and reduce the equation (5) to have an expression that grow linearly with M.

A. Optimizing the Calculus of Optimal Common Rate

For to simplify the equation (5) is necessary to choose each index m of U_m, in ascending entropy of $h_b(p_m) \equiv H(U_m|U_0)$, so that $h_b(p_i) \leq h_b(p_{i+1})$, $\forall i \in \{1, 2, ..., M-1\}$. The Table I show this relation.

TABLE I
Correlated sources in ascending entropy $h_b(p_m)$

	U_1	U_2	U_3	$\ldots$	U_M
p_m	p_1	p_2	p_3	$\ldots$	p_M
$h_b(p_m)$	h_1	h_2	h_3	$\ldots$	h_M

The equation (5) can be rewrite then as

$$r \leq \frac{mC}{H(S_m|\tilde{S}_m)}. \tag{6}$$

Each value m, $\forall m \in \{1, 2, ..., M\}$, in (6) represent a group of $\binom{M}{m}$ inequalities. As in the calculus of r is indifferent the order in that the subsets S_m are compared, the equation (6) can be rewrite as

$$r \leq \frac{mC}{H(\Omega_M) - H(S_{M-m})}. \tag{7}$$

Using the Lemma 14 of Section VI, we know that of all the values of $H(S_{M-m})$ in (7), we found the minimum value to $H(\Omega_{M-m})$, so that we rewrite the equation (7) as (8),

$$r \leq \frac{mC}{H(\Omega_M) - H(\Omega_{M-m})}. \tag{8}$$

Thus, we just need M inequalities for calculate the optimal common rate of M sources. This imply that is necessary the calculus of $H(\Omega_m)$, $\forall m \in \{1, 2, ..., M\}$, such that now; excluding the work of calculates $H(\Omega_m)$; the complexity is linear.

IV. Optimal Sum Rate

Remark 7 *Na quinta linha do artigo [1] diz: "we chose to formulate it as the following equivalent problem;" e tem que dizer "we chose formulate instead it the following similar problem;" debido a que a solucao $\mathbf{n}^*$ que minimiza $\sum_i n_i^* = S$ contem a solucao $\mathbf{n^a}$ que maximiza $\sum_i \frac{1}{n_i^a} = T$, mas nao todos os $\mathbf{n}^*$ encontrados maximizam $\mathbf{n^a}$. Exemplo:$\mathbf{n}^* = (3, 3) \rightarrow S = 6$ e $\mathbf{n}^* = (2, 4) \rightarrow S = 6$; agora los valores inversos serian $\mathbf{1}/\mathbf{n}^* = (1/3, 1/3) \rightarrow T = \frac{3}{2} = 0.6667$ e $\mathbf{1}/\mathbf{n}^* = (1/2, 1/4) \rightarrow T = \frac{3}{4} = 0.75$, respectivamente.*

Remark 8 *En el ejemplo 4 depois da equacao (30) diz" represents the set of solutions that minimize the sum" e deve dizer "contents the set of solutions that minimize the sum".*

Pois, falta a restrio de que $n_i^* \geq k$ and consequently $1/k \geq 1/n_i^*$ e outras restricoes do slepian wolf theorem

In [1] can be seen that, the problem of maximize $\sum_{m=1}^{M} r_m$ is equivalent to minimize the sum $N = \sum_{m=1}^{M} n_m$. Follow (1) and [1], we know that for minimize N we are restricted to that the lengths n_m (of rates $r_m = k/n_m$, $\forall m \in \{1, 2, ..., M\}$), fulfill the next inequalities,

$$\sum_{i \in S} n_i \geq \frac{k}{C} H(U(S)|U(S^c)). \tag{9}$$

The M-tuples that fulfill the last equation are represent for $\mathbf{n} = (n_1, n_2, ..., n_M)$. Thus, is necessary to get

$$\mathbf{n}^* = \arg \min_{\{n_m\}} N, \tag{10}$$

restrict to (9), where $\mathbf{n}^* = (n_1^*, n_2^*, ..., n_M^*)$ represents the M-tuples of $\mathbf{n}$ that minimize N. In [1] can be seen that for to get these values is necessary to solve a matrix inequation with M columns a $2^M - 1$ rows.

A. Calculus of Optimal Sum Rate

If we see the equation (9), it is easy to perceive that the bounded region of this equation is proportional to the bounded region of Slepian-Wolf theorem [2], [3]. Thus, the possibles values of M-tuple $\mathbf{n}$ are equals to $\mathbf{n} = \frac{k}{C}\mathbf{R}$, where the M-tuple $\mathbf{R} = (R_1, R_2, ..., R_M)$ represent the possibles informations rates in the bounded region of Slepian-Wolf Theorem. Known this, (10) can be rewrite as (11),

$$\mathbf{n}^* = \frac{k}{C} \arg \min_{\{R_m\}} \sum_{m=1}^{M} R_m, \tag{11}$$

so that

$$H(U(S)|U(S^c)) \leq \sum_{i \in S} R_i. \tag{12}$$

By other side, in [7], [8], [9] we can see that, the corner points $\mathbf{R}^* = (R_1^*, R_2^*, ..., R_M^*)$ that pertain to $\mathbf{R}$, and also delimit with segments a hyperplane portion ℓ_0 inside $\ell : \sum_{m=1}^{M} R_m = H(U_1 U_2 ... U_M)$, they represent solutions that minimize the sum $\sum_{m=1}^{M} R_m$ conditioned to (12), being that these solutions are not uniques and any point inside ℓ_0 also minimize the sum.

Consequently, the corner points $\mathbf{R}^*$ describe the outline of solution region ℓ_0, these M-tuples can be obtained by expanding $H(U_1 U_2 ... U_M)$, by successive applications of the chain rule in any order, and assigning to each rate R_m^* the term in the expansion that correspond to the analyzed source in the entropy function [7]. By example, in the bi-dimensional case with a $H(U_2 U_1)$, the corner points (R_1^*, R_2^*) of ℓ_0 are in $(H(U_1|U_2), H(U_2))$ and $(H(U_1), H(U_2|U_1))$; by other side, to the multidimensional case one corner point can be

$$\begin{aligned} R_i^* &= H(U_i|U_{i-1}U_{i-2}...U_1) \\ &= H(\Omega_i) - H(\Omega_{i-1}), \end{aligned} \tag{13}$$

for $i \in \{1, 2, ..., M-1\}$ with $R_1^* = H(U_1)$.

By (13), we know that the equation (10) has one solution in

$$\mathbf{n}^* = \frac{k}{C} \begin{pmatrix} H(U_1) \\ H(\Omega_2) - H(U_1) \\ \vdots \\ H(\Omega_i) - H(\Omega_{i-1}) \\ \vdots \\ H(\Omega_M) - H(\Omega_{M-1}) \end{pmatrix}^T . \tag{14}$$

Thus, for calculate the optimal sum rate of M sources imply that is necessary the calculus of $H(\Omega_m)$, $\forall m \in \{1, 2, ..., M\}$.

V. Final Remarks and Conclusions

In this letter, we considered joint source-channel coding of correlated sources transmitted over orthogonal channels with the same channel capacity, and were solved two optimization problems focusing on the computation low complexity of optimal rates calculus. It is therefore expected that the optimal rates derived in this letter can serve as guidance to practical coding schemes.

But remain open the search of a method of calculus of the joint entropy $H(\Omega_m)$. This entropy yet need much calculus time. Analyzing this problem is recommended for futures works that the problem be addressed as an approximate calculation.

Acknowledgment

H(Omega) no lo calcules !!!simulalo!!!

VI. Appendix

Lemma 9 *Known a set of m correlated sources Ω_m. Then, is true that*

$$Pr(\Omega_m = \mathbf{a}) = \frac{\Psi(\mathbf{a}) + \Psi(\bar{\mathbf{a}})}{2}, \tag{15}$$

$$\Psi(\mathbf{a}) \equiv \prod_{i=1}^{m} Pr(U_i = a_i | U_0 = 0). \tag{16}$$

being $\mathbf{a} = \{a_1, a_2, ..., a_m\}$ and $\bar{\mathbf{a}}$ two binary vectors, both with m elements, where $\bar{\mathbf{a}} \oplus \mathbf{a} = \mathbf{0}$.

Proof.

$$\begin{aligned} Pr(\Omega_m = \mathbf{a}) &= Pr(\Omega_m = \mathbf{a}|U_0 = 0)Pr(U_0 = 0) \\ &+ Pr(\Omega_m = \mathbf{a}|U_0 = 1)Pr(U_0 = 1), \end{aligned} \tag{17}$$

when U_0 is known the probabilities of sources in Ω_m are independents,

$$\begin{aligned} Pr(\Omega_m = \mathbf{a}) &= (1/2) \prod_{U_i \in \Omega_m} Pr(U_i = a_i | U_0 = 0) \\ &+ (1/2) \prod_{U_i \in \Omega_m} Pr(U_i = a_i | U_0 = 1), \end{aligned} \tag{18}$$

■

Lemma 10 *The value of entropy function $H(\Omega_m)$ is the same for a set of sources U_i with $Pr(U_i \neq U_0 | U_0)$ equal to p_i or $1 - p_i$*

Proof. Without loss of generality, we assume that need demonstrate $H(U_i \Omega_m)$ is the same for $Pr(U_i \neq U_0|U_0)$ equal to p_i or $1 - p_i$.

$$\begin{aligned} & H(U_i \Omega_m) \\ &= \sum_{U_i, \Omega_m} Pr(U_i \Omega_m) log_2(Pr(U_i \Omega_m)) \\ &= \sum_{\mathbf{a}} Pr(U_i = 1, \Omega_m = \mathbf{a}) log_2(Pr(U_i = 1, \Omega_m = \mathbf{a})) \\ &+ \sum_{\mathbf{a}} Pr(U_i = 0, \Omega_m = \mathbf{a}) log_2(Pr(U_i = 0, \Omega_m = \mathbf{a})) \end{aligned} \tag{19}$$

Using the Lemma 9 we know that for the case of $Pr(U_i \neq U_0|U_0) = p_i$

$$Pr(U_i = 1, \Omega_m = \mathbf{a}) = \frac{p_i \Psi(\mathbf{a}) + (1 - p_i)\Psi(\bar{\mathbf{a}})}{2}, \tag{20}$$

$$Pr(U_i = 0, \Omega_m = \mathbf{a}) = \frac{(1 - p_i)\Psi(\mathbf{a}) + p_i \Psi(\bar{\mathbf{a}})}{2}, \tag{21}$$

and for the case of $Pr(U_i \neq U_0|U_0) = 1 - p_i$

$$Pr(U_i = 1, \Omega_m = \mathbf{a}) = \frac{(1 - p_i)\Psi(\mathbf{a}) + p_i \Psi(\bar{\mathbf{a}})}{2}, \tag{22}$$

$$Pr(U_i = 0, \Omega_m = \mathbf{a}) = \frac{p_i \Psi(\mathbf{a}) + (1 - p_i)\Psi(\bar{\mathbf{a}})}{2}. \tag{23}$$

Using this results is easy see that for both case we obtain the same value of $H(U_i \Omega_m)$. ■

Corollary 11 *Follow the Lemma 10, for calculate the joint entropy $H(\Omega_m)$ we can assume that all probabilities $p_i \leq 1/2$. Thus, exist a bijective function that link the probability p_i an the binary entropy h_i. Therefore the joint entropy $H(\Omega_m)$ is a function that depend of $\mathbf{h} = \{h_1, h_2, ..., h_m\}$.*

Lemma 12 *Known a set of m correlated sources Ω_m. Then, is true that the value of entropy $H(\Omega_m)$ grow in relation growing of $h_b(p_i)$, $\forall i \in \{1,\ 2,\ ...,\ m\}$, so that*

$$\frac{\partial H(\Omega_m)}{\partial h_i} \geq 0 \tag{24}$$

Proof. For the Corollary 11 we know that $H(\Omega_m)$ depend of h_i and this depend of p_i. Without loss of generality, we assume that need demonstrate $(\partial H(U_j\Omega_m)/\partial p_j)(\partial p_j/\partial h_j) \geq 0$, being that $U_j \notin \Omega_m$. Thus,

$$\frac{\partial H(U_j\Omega_m)}{\partial h_j} = \frac{-\partial \sum\limits_{U_j,\Omega_m} Pr(U_j\Omega_m)log_2(Pr(U_j\Omega_m))}{\partial p_j}\frac{\partial p_j}{\partial h_j} \quad (25)$$

$$\frac{\partial H(U_j\Omega_m)}{\partial h_j} = -\frac{\partial \sum\limits_{\mathbf{a}} Pr(U_j=1,\Omega_m=\mathbf{a})log_2(Pr(U_j=1,\Omega_m=\mathbf{a}))}{\partial p_j}\frac{\partial p_j}{\partial h_j} -\frac{\partial \sum\limits_{\mathbf{a}} Pr(U_j=0,\Omega_m=\mathbf{a})log_2(Pr(U_j=0,\Omega_m=\mathbf{a}))}{\partial p_j}\frac{\partial p_j}{\partial h_j} \quad (26)$$

and using the Lemma 9 we obtain

$$Pr(U_j = 1, \Omega_m = \mathbf{a}) = \frac{p_j\Psi(\mathbf{a}) + (1-p_j)\Psi(\bar{\mathbf{a}})}{2}, \quad (27)$$

$$Pr(U_j = 0, \Omega_m = \mathbf{a}) = \frac{(1-p_j)\Psi(\mathbf{a}) + p_j\Psi(\bar{\mathbf{a}})}{2}, \quad (28)$$

$$\frac{\partial Pr(U_j = 1, \Omega_m = \mathbf{a})}{\partial p_j} = \frac{\Psi(\mathbf{a}) - \Psi(\bar{\mathbf{a}})}{2}, \quad (29)$$

$$\frac{\partial Pr(U_j = 0, \Omega_m = \mathbf{a})}{\partial p_j} = -\frac{\Psi(\mathbf{a}) - \Psi(\bar{\mathbf{a}})}{2}, \quad (30)$$

$$\frac{\partial h_j}{\partial p_j} = log_2\left(\frac{1-p_j}{p_j}\right), \quad (31)$$

using (27), (28), (29), (30) and (31) in (26)

$$\frac{\partial H(U_j\Omega_m)}{\partial h_j} = \sum_{\mathbf{a}} f(p_j, \mathbf{a}) \quad (32)$$

$$f(p_j, \mathbf{a}) = \frac{\Psi(\mathbf{a}) - \Psi(\bar{\mathbf{a}})}{2\ log_2\left(\frac{1-p_j}{p_j}\right)} log_2\left(\frac{(1-p_j)\Psi(\mathbf{a}) + p_j\Psi(\bar{\mathbf{a}})}{p_j\Psi(\mathbf{a}) + (1-p_j)\Psi(\bar{\mathbf{a}})}\right) \quad (33)$$

if $p_j = 1/2$, then

$$f(1/2, \mathbf{a}) = \frac{1}{2}\frac{(\Psi(\mathbf{a}) - \Psi(\bar{\mathbf{a}}))^2}{\Psi(\mathbf{a}) + \Psi(\bar{\mathbf{a}})} \geq 0. \quad (34)$$

Analyzing the equation (33) for $p_j < 1/2$, $p_j > 1/2$, $\Psi(\mathbf{a}) \leq \Psi(\bar{\mathbf{a}})$ and $\Psi(\mathbf{a}) > \Psi(\bar{\mathbf{a}})$, can be seen that $f(p_j, \mathbf{a})$ is ever positive, thus the Lemma 12 is proved. ∎

Corollary 13 *We know that $H(\Omega_m)$ is in function of $\mathbf{h} = \{h_1, h_2, ..., h_m\}$, thus $H(\Omega_m) = \phi(\mathbf{h})$, where $\phi(.)$ is a m-dimensional function. Follow the Lemma 12 we know that*

$$\phi(\mathbf{h}) \leq \phi(\mathbf{h} + \triangle\mathbf{h}) \quad (35)$$

when the elements of $\triangle\mathbf{h}$ are higher or equal to zero.

Lemma 14 *Known M correlated sources with indexes chosen in ascending entropy $h_b(p_m)$, $\forall m \in \{1, 2, ..., M\}$, as in the Table I. If we select two set of m sources, Ω_m and S_m, then*

$$H(\Omega_m) \leq H(S_m), \quad (36)$$

Proof. The joint entropy $H(S_m)$ is in function of a set probabilities $p_i = P(U_i \neq U_0|U_0)$, $\forall i : U_i \in S_m$. For the symmetry of entropy function, the order of p_i is irrelevant for the calculus of $H(S_m)$. We define $\tilde{\mathbf{h}}$ as a set of binary entropies, in ascending order, of all probabilities p_i linked to S_m. Knowing this, we can say that the entropies $H(\Omega_m)$ and $H(S_m)$ are in function of two sets of m entropies $\mathbf{h} = \{h_1, h_2, ..., h_m\}$ and $\tilde{\mathbf{h}} = \{\tilde{h}_1, \tilde{h}_2, ..., \tilde{h}_m\}$ respectively, being that both are write in ascending entropy. By definition of problem $\mathbf{h}$ have a set of entropies with lower values, the other hand $\tilde{\mathbf{h}}$ is a set of m anybody entropies h_i in the Table I. Given that element to element $\tilde{\mathbf{h}}$ ever will be major than $\mathbf{h}$ and using the Corollary 13 we know that $\phi(\mathbf{h}) \leq \phi(\tilde{\mathbf{h}})$ and the Lemma 14 is proved. ∎

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