Performance Limit in Iterative Joint Decoding of Systematic LDGM Codes Over Distributed Sources

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Abstract—**This paper proposes a method for calculating the performance limit in iterative joint decoding of systematic LDGM codes over distributed sources.**

Index Terms—**Multiple correlated sources, large scale sensor networks, source-channel coding.**

I. Introduction

- In the references [1], [2] a model with M correlated sources that are generates passing a unknown source across M binary symmetric channels (BSC) are presented. This model is adopted in this paper.
- In this paper is assumed that the sources are so far of the joint decoder that the channel capacity in all channels are approximately same.
- Here is used a systematic Low Density Generator Matrix (LDGM).
- The information is transmit over Binary Input Additive White Gaussian Noise (BI-AWGN) channel with hard decision output.
- The objective of this work is the calculus of lower bound limit for Bit Error Rate (BER) over BI-AWGN channels with systematic LDGM codes.

II. System Model

The Fig. 1 show the diagram of transmission model used in this article. In the figure can be see M correlated binary sources U_m, $\forall m \in \{1, 2, 3, ..., M\}$. The sources U_m are generates passing a binary source U_0, with $P(U_0 = 1) = 0.5$, through of BSC channels with error probability $P(U_0 \neq U_m|U_0) = p_m$. In each source a vector with K bit is select and this information is coded in N bits with a systematic LDGM matrix $G = [I \ P]$ of rate of $r = K/N$, where I is a binary identity matrix of size $K \times K$, lines and columns respectively, and P is a regular sparse matrix of size $K \times (N - K)$ with X ones per line and Y ones per column. After coding we have a binary vector X_m^N with N bits per vector. This vectors are send across BI-AWGN communication channels with capacity C.

Fig. 1. System Model.

The informations obtained after channels are used for to calculate the approximations $\hat{U}_m$ of U_m. Here is assumed that the decoding algorithms used have binary inputs, thus the BI-AWGN channels can be seen as BSC channels with error probability p_c [3], the relation of the energy for sending symbol E_s and the one-sided noise spectral density N_0, in a BI-AWGN channel with hard decision output, with the bit error probability p_c, in a equivalent BSC channel, is giving for

$$p_c = Q(\sqrt{2Es/N_0}), \tag{1}$$

where $Q(.)$ is the q-function defined as

$$Q(x) \equiv \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{a^2}{2}\right) da, \tag{2}$$

Lemma 1 *Known two independent binary random variables A and B with probabilities $Pr(A = 1) = p_A$ and $Pr(B = 1) = p_B$ respectably, the probability*

$$Pr((A \oplus B) = 1) = p_A || p_B, \tag{3}$$

where

$$p_A || p_B \equiv p_A + p_B - 2p_A p_B, \tag{4}$$

$$||^Y p_A \equiv \overbrace{p_A || p_A || ... || p_A}^{Y}. \tag{5}$$

Proof:

$$\begin{array}{rcc} Pr((A \oplus B) = 1) & = & Pr(A = 1, B = 0) \\ & + & Pr(A = 0, B = 1) \\ & = & p_A(1 - p_B) + p_A(1 - p_B) \\ & = & p_A + p_B - 2p_A p_B, \end{array} \tag{6}$$

■

Lemma 2 *Known two BSC channels in cascade with error probabilities p_A and p_B respectively. These are statistically equivalent to only one BSC channel with error probability $p_A || p_B$*

Proof: Considering that the BSC channels are replace for the or exclusive (XOR) sum in cascade of two independent binary random variables A and B, with error probabilities $Pr(A = 1) = p_A$ and $Pr(B = 1) = p_B$ respectively. Thus, these two sources have in joint an error probability $Pr((A \oplus B) = 1)$. Using the Lemma 1 we obtain that $Pr((A \oplus B) = 1) = p_A || p_B$. ■

Lemma 3 *Considering a m-th transmit channel in the Fig. 1 as in the Fig. 2a, where we have a source U_0 that passes through of BSC channel with error probability p_m and the output is codified with a systematic LDGM matrix G, with rate r and Y ones per column in the parity matrix P. This*

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is statistically identical to have a source U_0 that is codified with a matrix G and pass through a BSC channel with error probability $\hat{p}_m$, as is drawn in the Fig. 2b. Where

$$\hat{p}_m = rp_m + (1-r)(||^Y p_m), \tag{7}$$

Fig. 2. Equivalent Model of LDGM Matrix Later of BSC Channel.

Proof: In the Fig 1 each vector $X_m^N = [\ x_{m(1)}, ..., x_{m(K)}, x_{m(K+1)}, ..., x_{m(N)}]$ can be seen as a noise codified version of vector $U_0^K = [\ u_{0(1)}, ..., u_{0(K)}]$ of source U_0. The first K bits of X_m^N have a error probability p_m and the next $N-K$ bits have a probability error of $\mathring{p}_m$.

For to calculate $\mathring{p}_m$ first is necessary to define a vector $U_m^K = [\ u_{m(1)}, ..., u_{m(K)}]$ of source U_m as

$$u_{m(k)} = u_{0(k)} \oplus e_{m(k)}, \tag{8}$$

$\forall k \in \{1, 2, ..., K\}$, where $e_{m(k)}$ is a element of vector $E_m^N = [\ e_{m(1)}, ..., e_{m(K)}]$ with $Pr(E_m = 1) = p_m$ and $\oplus$ is the operator XOR. Thus, each coded bit in $x_{m(n)}$, $\forall n \in \{K+1, K+2, ..., N\}$, is equal to

$$x_{m(n)} = \sum_{l \in \varphi_n}^{\oplus} u_{m(l)} \tag{9}$$

$$= \sum_{l \in \varphi_n}^{\oplus} u_{0(l)} \oplus \sum_{l \in \varphi_n}^{\oplus} e_{m(l)}, \tag{10}$$

where $\sum^{\oplus}$ is a XOR summatory and φ_n is a subset with the index bits for calculate to n-th parity check bit $x_{m(n)}$ using the parity matrix P. φ_n has Y elements. Thus, the error probability $\mathring{p}_m$ is equal to

$$\mathring{p}_m = Pr(\sum_{l \in \varphi_n}^{\oplus} e_{m(l)} = 1). \tag{11}$$

Using the Lemma 1 for Y sources independents and identically distributed

$$\mathring{p}_m = ||^Y p_m. \tag{12}$$

Know it can be say that in the Fig. 2a each bit of coded vector X_m^N of source U_0 has in average a probability error of

$$\hat{p}_m = rp_m + (1-r)\mathring{p}_m, \tag{13}$$

with the first K bits with probability p_m and the last $N-K$ with probability $\mathring{p}_m$. This is statistically identical to have a source U_0 that is codified with a matrix G and pass through a BSC channel with error probability $\hat{p}_m$, as is drawn in the Fig. 2b. ■

III. Performance Limit of Systematic LDGM Codes

In [4] can be seen a lower bound limit in the prediction of the BER for the sources U_m, $\forall m \in \{1, 2, ..., M\}$, in the case of systematic LDGM codes over BSC channels. This BER is giving for $Pr(U_m \neq \hat{U}_m)$ and it is in function of the quantity of ones for line X in the parity matrix P and the channel error probability p_c, where

$$f(X,p_c) \approx \begin{cases} \sum_{i=(X+1)/2}^{X} \binom{X}{i} p_c^i (1-p_c)^{X-i} & \text{if } X\ odd \\ \binom{X}{\frac{X}{2}} p_c^{\frac{X}{2}} (1-p_c)^{\frac{X}{2}} + \sum_{i=\frac{X}{2}+1}^{X} \binom{X}{i} p_c^i (1-p_c)^{X-i} & \text{if } X\ even \end{cases} \tag{14}$$

,

$$Pr(U_m \neq \hat{U}_m) = f(X, p_c). \tag{15}$$

A. Joint Performance Limit of Systematic LDGM Codes

Fig. 3

Fig. 3. Equivalent Model of m-th transmit Channel.

IV. Final Remarks and Conclusions

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