# An Optimal Method for Tuning a Six Holes Ocarina

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## Abstract

In this paper we propose an optimal method for tuning a six holes ocarina, this optimization will be subordinated by the minimization of the square norm of square difference between the desired and obtained resonant frequency. The results show that it is not possible tuning a John Taylor style six holes ocarina with a perfect equal temperament scale, but the minimization rule returns a tuning method with an optimal approximation and heterogeneous errors less than 43.697% of a semitone.


## 1 Introduction

The ocarina is an extremely ancient musical instrument widely used in worldwide. It is classified as a globular flute, depending on the culture we can find shapes like animals, vegetables or anthropomorphic. Exist vestiges of clay ocarinas in Shang dynasty peoples in Bronze Age China, Bantu peoples of Southern Africa, pre-Columbian cultures in Peru, etc. [Ape69, pp. 589] [Sac40] [Lei20, pp. 31] [Ros+20].

The ocarinas around the world present many fingering tune methods and musical scales, one the last fingering style for 4 holes pendant-type ocarina was developed around 1964 by the English mathematician John Taylor to approximate the tuning of a equal temperament scale [Met+85, pp. 79] [Soc01, pp. 10]. In this method, the degree of approximation to an equal temperament scale depends a lot on the ability of the potter or craftsman in to choose the correct size of area and clay thickness in all the tone holes.

The work presented here, will study the math behind the whistles (an ocarina without tone holes) or more formally a Helmholtz resonant cavity [Cor11] excited by a jet-edge-resonator system [Gib13, pp. 3] [NWS53, pp. 138]. Additionally will be studied some proposed models to describe in a whistles with tone holes (an ocarina) the resonant frequency in function of the size area and clay thickness in the tone holes [MP10] [Cab02, pp. 44] [Hop99]. Finally, using a cost function an optimal tuning method will be presented and the approximation error will be shown.

## 2 Fingering chart in six holes ocarina

The Fig. 1 represents the hole distribution of the ocarina used in this work, where $S_0$ represent the sound hole (voicing) and the holes $S_1$, $S_2$, $S_3$, ... and $S_6$ represent the tone-holes. The fingering

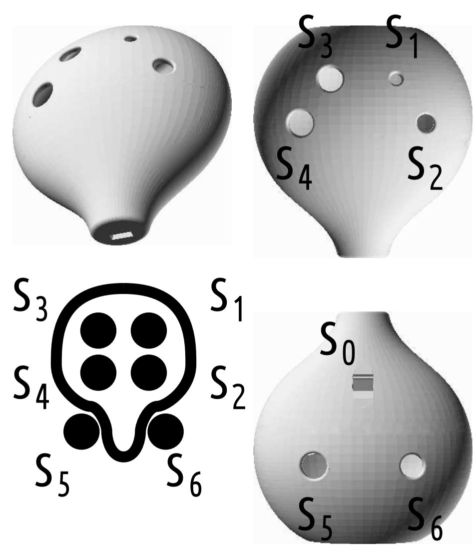


Figure 1: View of six holes ocarina.

chart used in this ocarina model is based in the 4 holes fingering style proposed by John Taylor [Met+85, pp. 79, 146], the Table 1 shows the fingering chart, it indicates the configuration of open or close holes to produce the tones from $C$ until $D$ (a higher octave ), where a value $S_i = 1$ represents an open hole and the value $S_i = 0$ a closed hole.

| Note | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ |
|---|---|---|---|---|---|---|
| $C$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $C\#$ | 0 | 0 | 0 | 0 | 0 | 1 |
| $D$ | 1 | 0 | 0 | 0 | 0 | 0 |
| $D\#$ | 1 | 0 | 0 | 0 | 0 | 1 |
| $E$ | 0 | 1 | 0 | 0 | 0 | 0 |
| $F$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $F\#$ | 0 | 0 | 1 | 0 | 0 | 0 |
| $G$ | 1 | 0 | 1 | 0 | 0 | 0 |
| $G\#$ | 0 | 1 | 1 | 0 | 0 | 0 |
| $A$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $A\#$ | 1 | 0 | 1 | 1 | 0 | 0 |
| $B$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $C$ | 1 | 1 | 1 | 1 | 0 | 0 |
| $C\#$ | 0 | 1 | 1 | 1 | 1 | 0 |
| $D$ | 1 | 1 | 1 | 1 | 1 | 0 |

Table 1: Fingering chart

## 3 Math and music

In this paper we analyze a tuning method with 12 tone equal temperament; so that, the frequency interval between any pair of adjacent notes has the same ratio $\rho = \sqrt[12]{2}$. The Table 2 shows how are distributed the musical notes from tone $C$ until $D$ (a higher octave ), where $f_0$ represent the frequency of $C$ note and the next notes follow the geometric progression $\hat{f}_i = \rho^i f_0$ with $i$ representing the separation degree with $C$.

| | | | | |
|---|---|---|---|---|
| $\hat{f}_0$ | $\hat{f}_1$ | $\hat{f}_2$ | $\hat{f}_3$ | $\hat{f}_4$ |
| $C$ | $C\#$ | $D$ | $D\#$ | $E$ |
| $f_0$ | $\rho^1 f_0$ | $\rho^2 f_0$ | $\rho^3 f_0$ | $\rho^4 f_0$ |
| $\hat{f}_5$ | $\hat{f}_6$ | $\hat{f}_7$ | $\hat{f}_8$ | $\hat{f}_9$ |
| $F$ | $F\#$ | $G$ | $G\#$ | $A$ |
| $\rho^5 f_0$ | $\rho^6 f_0$ | $\rho^7 f_0$ | $\rho^8 f_0$ | $\rho^9 f_0$ |
| $\hat{f}_{10}$ | $\hat{f}_{11}$ | $\hat{f}_{12}$ | $\hat{f}_{13}$ | $\hat{f}_{14}$ |
| $A\#$ | $B$ | $C$ | $C\#$ | $D$ |
| $\rho^{10} f_0$ | $\rho^{11} f_0$ | $\rho^{12} f_0$ | $\rho^{13} f_0$ | $\rho^{14} f_0$ |

Table 2: Musical notes

## 4 Math and ocarina

### 4.1 Analyzing a whistle

We can think of an ocarina as a whistle with tone-holes in its resonant cavity, so that when we open and close some of these holes we can modify the tone of generated sound. Thus to understand the math of an ocarina first we should to know the science behind the whistles or more formally a Helmholtz resonant cavity [Cor11] (see Fig. 2) excited by a jet-edge-resonator system (see Fig. 3) [Gib13, pp. 3] [NWS53, pp. 138]. In this sense the Eq. 1

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{A_0}{l_0 V}} \tag{1}$$

shows the way of to calculate the frequency $f_0$ of a theoretic[1] whistle [Gib13, pp. 3] [Kob+09, pp. 5] [Oka+19, pp. 265] in relation with the constants $\pi$ and $c$ (the velocity of sound), and the variables: $V$ the volume of its resonant cavity, $A_0$ is its cross-section area and $l_0$ the effective length of the neck in the resonator.

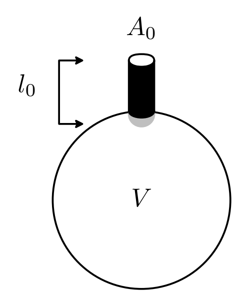


Figure 2: whistle.

### 4.2 Analyzing an ocarina

The fundamental resonant frequency $f$ of an ocarina, that have at least one circular tone-hole in addition to the sound hole (see Fig. 4), can be

[1] A sphere-like resonant cavity and the open area of the neck is much less than the total surface area of the resonant cavity. .

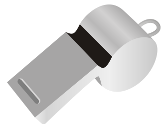

Figure 3: whistle.

represented like a function of the total tone-holes area $A$.

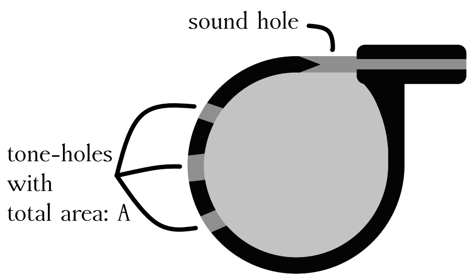


Figure 4: Theoretic ocarina

In the article "La Ocarina de Zanahoria a Carrot Ocarina" [MP10] were analyzed data about of the increment of resonant frequency $f$ in relation to the augment of area $A$, showing that in that study case the relationships between the variables can be approximated with a line

$$f = c_1 + c_2\ A. \tag{2}$$

where $c_1 = 595.859069$, $c_2 = 4.324273$ and a mean square error $MSE = 8051.1$. The Fig. 5 show the linear curve in relation to the collected data.

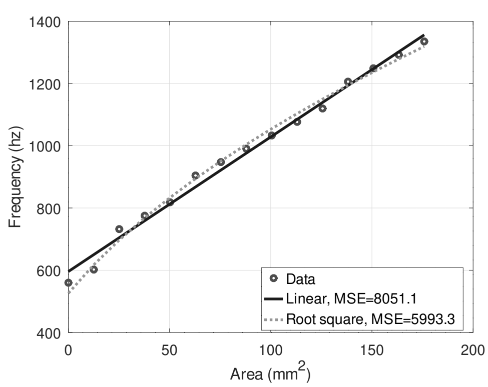


Figure 5: Comparing models.

By other side, if we follow the equation proposed by Bart Hopkin in its book "Air Columns and Toneholes" [Cab02, pp. 44] [Hop99], we can fit the data of Fig. 5 with a root square function

$$f = d_1 \sqrt{d_2 + A}, \tag{3}$$

where $d_1 = 91.215856$, $d_2 = 33.262474$ and a mean square error $MSE = 5993.3$. We must highlight that $d_2$ represents the area of sound hole.

This last model fit better the data, also the equation proposed by Hopkin keep coherence with the Helmholtz resonator equation (see Eq. 1). In the next section we use the model proposed by Hopkin to obtain a optimal tuning in John Taylor style six holes ocarina.

### 4.3 Resonance frequency in ocarinas

If we assume the equation 4 proposed by Bart Hopkin [Cab02, pp. 44] [Hop99],

$$f = \frac{c}{2\pi} \sqrt{\frac{\frac{A_0 S_0}{l_0} + \frac{A_1 S_1}{l_1} + \frac{A_2 S_2}{l_2} + \ldots}{V}} \tag{4}$$

as a good approximation to estimate the fundamental frequency $f$ of globular resonators that have at least one circular pitch hole in addition to the sound hole (a.k.a. an ocarina). in which $A_i$ and $l_i$ are the area an the effective length of the $S_i$ tone-hole, so that the variable $S_i$ has only have two values $\{0, 1\}$ according the Table 1, additionally $S_0 = 1$ and represent the sound hole, being $A_0$ and $l_0$ their corresponding variables.

#### 4.3.1 Reordering the resonance equation

We can rearrange the Eq. 4 defining the variable $h_i \propto \frac{A_i}{l_i}$, so that

$$h_i \equiv \left( \frac{c^2}{4\pi^2 V} \right) \left( \frac{A_i}{l_i} \right); \tag{5}$$

This variable change imply that the Eq. 4 can be rewrite as

$$f^2 = \sum_{i=0}^{6} S_i h_i. \tag{6}$$

It is important remember that following the Eqs. 1 and 6, when all the tone-holes are close we obtain that

$$f_0^2 = h_0. \tag{7}$$

## 5 Problem Statement

**Definition 5.1 ( $\hat{\mathbf{z}}$ - Square value of the desired frequencies)** *Known the frequency relation between the notes showed in the Table 2, we define the variable* $\hat{z}_i \equiv \hat{f}_i^2 = \left(\rho^i f_0\right)^2$*, so that the column vector* $\hat{\mathbf{z}} = [\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_{14}]^T$*, that contain the square value of the desired frequencies, can be represented as in the Eq. 8.*

$$\hat{\mathbf{z}} \equiv \mathbf{p} f_0^2 \tag{8}$$

*where* $\mathbf{p} = [\rho^2,\ \rho^4,\ \rho^6,\ \rho^8,\ ...,\ \rho^{28}]^T$.

**Definition 5.2 ( $\mathbf{z}(\mathbf{h})$ - Square value of the generated frequencies)** *If we rearrange the Eqs. 6 and 7 for all notes in Table 1, we obtain the next equation*

$$\mathbf{z}(\mathbf{h}) = \mathbf{q} h_0 + \mathbf{A}\mathbf{h} \tag{9}$$

*where*

$$\mathbf{q} = \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix}0&0&0&0&0&1\\1&0&0&0&0&0\\1&0&0&0&0&1\\0&1&0&0&0&0\\1&1&0&0&0&0\\0&0&1&0&0&0\\1&0&1&0&0&0\\0&1&1&0&0&0\\1&1&1&0&0&0\\1&0&1&1&0&0\\0&1&1&1&0&0\\1&1&1&1&0&0\\0&1&1&1&1&0\\1&1&1&1&1&0\end{bmatrix}, \tag{10}$$

$\mathbf{h} = [h_1,\ h_2,\ h_3,\ h_4,\ h_5,\ h_6]^T$ *and* $\mathbf{z}(\mathbf{h}) = [z_1(\mathbf{h}),\ z_2(\mathbf{h}),\ ...,\ z_{14}(\mathbf{h})]^T$*. Being* $z_i(\mathbf{h})$ *the square value of* $i-th$ *frequency generate by the ocarina given a* $\mathbf{h}$ *relation of areas and effective lengths in its tone-holes.*

### 5.1 Minimization rule and optimal value

Known and calculated the desired and generated frequencies seen in the Eqs. 8 and 9, we propose the next minimization rule

$$e^2(\mathbf{h}) = ||\hat{\mathbf{z}} - \mathbf{z}(\mathbf{h})||^2, \tag{11}$$

where the unknown variable is the vector $\mathbf{h}$. This problem can be fitted as a case of multiple linear regression that can be solved using the Tikhonov regularization [Puj20, pp. 79, 80]. Thus, following this method, we obtain the vector $\mathbf{h}^*$ that minimize the function $e^2(\mathbf{h})$,

$$\mathbf{h}^* = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\left(\mathbf{p} - \mathbf{q}\right) h_0, \tag{12}$$

if we use this results joint the Eqs. 7 and 9, we obtain that

$$\mathbf{z}^* = \left(\mathbf{q} + \mathbf{A}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\left(\mathbf{p} - \mathbf{q}\right)\right) f_0^2, \tag{13}$$

being $\mathbf{z}^*$ the square value of the generated frequencies that minimize the function $e^2(\mathbf{h})$.

## 6 Results

**Definition 6.1 ($E_1$ - relative tuning error )** *The first type of error used in this work is*

$$E_1 = 100\ \frac{f_b - f_a}{f_a}\ \%, \tag{14}$$

*that measures the difference error of* $f_b$ *in relation to the target frequency (*$f_a$*) or relative tuning error.*

*To obtain the biggest accepted relative tuning error, It is necessary to know the fact that between any two consecutive notes, with frequencies* $f_a$ *and* $\rho f_a$*, exist a proportion of* $\rho \approx 1.059463$ *(a semitone), this mean that the next consecutive note is ever a* $+5.9463\%$ *that the last note. Thus, if we set as maximum error the midway position in geometric proportion between notes, them the biggest accepted positive relative tuning error to avoid a tone recognition mistake, it is equal to* $+2.9302\% \equiv 100(\sqrt{\rho} f_a - f_a)/f_a\ \%$*. By other side the biggest accepted negative relative tuning error to avoid a tone recognition mistake, it is equal to* $-2.8468\% \equiv 100(f_a/\sqrt{\rho} - f_a)/f_a\ \%$*.*

**Definition 6.2 ($E_2$ - semitone error)** *The logarithmic difference error between two frequencies* $f_a$ *and* $f_b$ *in relation to a semitone (*$\rho$*) is defined as*

$$E_2 = 100\ \frac{ln(f_b) - ln(f_a)}{ln(\rho)}\ \%, \tag{15}$$

*called here as semitone error.*

*To obtain the biggest accepted semitone error, we use a similar criterion as seen in the Def. 6.1, so that if we set as maximum error the midway position in geometric proportion between notes (ex: C and C#), them the biggest accepted positive tuning error to avoid a tone recognition mistake, it is equal to* $+50\% \equiv 100\ \frac{ln(\sqrt{\rho} f_a)-ln(f_a)}{ln(\rho)}\ \%$*. By other side the biggest accepted negative semitone error to avoid a tone recognition mistake, it is equal to* $-50\% \equiv 100\ \frac{ln(f_a/\sqrt{\rho})-ln(f_a)}{ln(\rho)}\ \%$*.*

### 6.1 Analyzing the optimal $\mathbf{h}^*$

In the Eq. 12 was calculated $\mathbf{h}^* = [h_1^*,\ h_2^*,\ ...,\ h_6^*]^T$, the optimal vector that minimize the function $e(\mathbf{h})$. The Table 3 shows each element of this vector in relation to $h_0$, being that $h_i^*/h_0 \equiv \left(\frac{l_0}{A_0}\right)\left(\frac{A_i}{l_i}\right)$.

| $h_1^*/h_0$ | $h_2^*/h_0$ | $h_3^*/h_0$ | $h_4^*/h_0$ | $h_5^*/h_0$ | $h_6^*/h_0$ |
|---|---|---|---|---|---|
| 29.3% | 58.1% | 96.1% | 103.5% | 104.1% | 12.2% |

Table 3: Percentage ratio between $h_i^*$ and $h_0$.

In this table we observe that if we consider $l_i$ a constant in all ocarina then $h_i^*/h_0 \propto A_i$ and the percentages presented in the Table 3 describes the ratio of proportions between areas $A_i$. So that below this conditions the optimal area $A_2$ is about twice $A_1$, the optimal area $A_3$ is about $3.27 A_1$ and so on.

### 6.2 Analyzing the optimal frequencies

The Tabla 4 shows a comparison between the desired and optimal generated frequencies for each note in a six holes ocarina. The first column shows all notes used in the optimization calculation. The second column has the desired frequencies (equal temperament scale), this column is equivalent to the root square value of $\hat{\mathbf{z}}$. The third column has the optimal generated frequencies, this column is equivalent to the root square value of $\mathbf{z}^*$. The fourth and fifth columns have the errors described in the Definitions 6.1 and 6.1, respectively.

The worst error $E_1$ is equal to $+2.556\%$ and correspond to the note $F$, similarly this note

| Note | Desired (hz) | Optimal (hz) | $E_1$ | $E_2$ |
|---|---|---|---|---|
| $C$ | 1.0000 $f_0$ | 1.0000 $f_0$ | 0.000 % | 0.000 % |
| $C\#$ | 1.0595 $f_0$ | 1.0591 $f_0$ | -0.030 % | -0.524 % |
| $D$ | 1.1225 $f_0$ | 1.1371 $f_0$ | 1.308 % | 22.506 % |
| $D\#$ | 1.1892 $f_0$ | 1.1895 $f_0$ | 0.024 % | 0.415 % |
| $E$ | 1.2599 $f_0$ | 1.2574 $f_0$ | -0.204 % | -3.528 % |
| $F$ | 1.3348 $f_0$ | 1.3690 $f_0$ | 2.556 % | 43.697 % |
| $F\#$ | 1.4142 $f_0$ | 1.4005 $f_0$ | -0.973 % | -16.926 % |
| $G$ | 1.4983 $f_0$ | 1.5015 $f_0$ | 0.210 % | 3.639 % |
| $G\#$ | 1.5874 $f_0$ | 1.5944 $f_0$ | 0.443 % | 7.652 % |
| $A$ | 1.6818 $f_0$ | 1.6838 $f_0$ | 0.122 % | 2.108 % |
| $A\#$ | 1.7818 $f_0$ | 1.8138 $f_0$ | 1.796 % | 30.819 % |
| $B$ | 1.8877 $f_0$ | 1.8915 $f_0$ | 0.198 % | 3.421 % |
| $C$ | 2.0000 $f_0$ | 1.9674 $f_0$ | -1.628 % | -28.417 % |
| $C\#$ | 2.1189 $f_0$ | 2.1490 $f_0$ | 1.419 % | 24.401 % |
| $D$ | 2.2449 $f_0$ | 2.2161 $f_0$ | -1.282 % | -22.333 % |

Table 4: Errors in optimal tuning

present the worst error $E_2$ with a detour of 43.697% of a semitone. The next worst error $E_1$ is equal to $+1.796\%$ and correspond to the note $A\#$, having an error $E_2$ with a detour of 30.819% of a semitone.

By the results show in the Table 4, we observe that the optimal vector $\mathbf{h}^*$ causes that $e^2(\mathbf{h}^*) > 0$ and consequently a perfect equal temperament scale it is not possible to be applied in the tuning of a John Taylor style six holes ocarina only an optimal aproximation.

### 6.3 Proposed tuning method

From the data of the Table 4, we can infer an ocarina construction method that fulfill an optimal tuning following the minimization rule of the Eq. 11. In this sense if we choose of this table a set of notes, in which each current note use a new tone hole in relation to the last tuned note, we can obtain a tuning method or sequence to reach an optimal tuning.

With these considerations, the Table 5 shows one tuning method in which is considered a $C$ note equal to $523.2511hz$. The method consist in follow the table 5 line a line the from top at bottom. So that we set in our ocarina, with each new created tone hole, the semitone error ($E_2$) shown in the fourth column of the table.

| Note | Desired (hz) | Optimal (hz) | $E_2$ |
|---|---|---|---|
| $C$ | 523.2511 | 523.2511 | 0.000 % |
| $C\#$ | 554.3653 | 554.1976 | -0.524 % |
| $D$ | 587.3295 | 595.0147 | 22.506 % |
| $E$ | 659.2551 | 657.9130 | -3.528 % |
| $G$ | 783.9909 | 785.6404 | 3.639 % |
| $B$ | 987.7666 | 989.7206 | 3.421 % |
| $D$ | 1174.6591 | 1159.6030 | -22.333 % |

Table 5: Proposed tuning method.

# 7 Conclusion

The results of Table 4 shows that it is not possible tuning a John Taylor style six holes ocarina with a perfect equal temperament scale, but the minimization rule presented in this work returns a tuning method with an optimal approximation and heterogeneous errors less than 43.697% of a semitone. Additionally, the Table 5 shows one, of several possibles, construction methods, paths or sequences to fulfill an optimal tuning according the minimization rule defined in the Eq. 11 and with the resonance frequency in ocarinas proposed by Bart Hopkin. This method can be complemented with the information of the relations between areas $A_i$ and effective lengths $l_i$ in an optimal tuning shown in the Table 3.

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