An Optimal Method for Tuning a Six-Hole Ocarina

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Abstract

In this paper we propose an optimal method for tuning a six-hole ocarina, this optimization will be subordinated by the minimization of the square norm of square difference between the desired and obtained resonant frequency. The results show that it is not possible to tune a John Taylor style six-hole ocarina with a perfect equal temperament scale, but the minimization rule returns a tuning method with an optimal approximation and heterogeneous errors less than 35.613% of a semitone.

1 Introduction

The ocarina is an extremely ancient musical instrument widely used in worldwide. It is classified as a globular flute and depending on the culture we can find shapes like animals, vegetables or anthropomorphic. Exist vestiges of clay ocarinas in Shang dynasty peoples in Bronze Age China, Bantu peoples of Southern Africa, pre-Columbian cultures in Peru, etc. [Ape69, pp. 589] [Sac12, pp. 166-168] [Lei20, pp. 31] [Ros+20].

The ocarinas around the world present many fingering tune methods and musical scales, one the last fingering style for 4 holes pendant-type ocarina was developed around 1964 by the English mathematician John Taylor to approximate the tuning of an equal temperament scale [Met+85, pp. 79] [Soc01, pp. 10]. In this method, the degree of approximation to an equal temperament scale depends a lot on the ability of the potter or craftsman in to choose the correct size of area and clay thickness in all the tone-holes.

The work presented here will study the math behind the whistles (an ocarina without tone-holes) or more formally a Helmholtz resonant cavity [Cor11] excited by a jet-edge-resonator system [Gib13, pp. 3] [NWS53, pp. 138]. Additionally, will be studied some proposed models to describe in whistles with tone-holes (an ocarina), the resonant frequency in function of the size area and clay thickness in the tone-holes [Mp10] [Cab02, pp. 44] [Hop99, pp. 16]. Finally, using a cost function an optimal tuning method will be presented and the approximation error will be shown.

2 Fingering chart in six-hole ocarina

The Fig. 1 represents the hole distribution of the ocarina used in this work, where S_0 represents the sound hole (voicing) and the holes S_1, S_2, S_3, ... and S_6 represent the tone-holes. The fingering

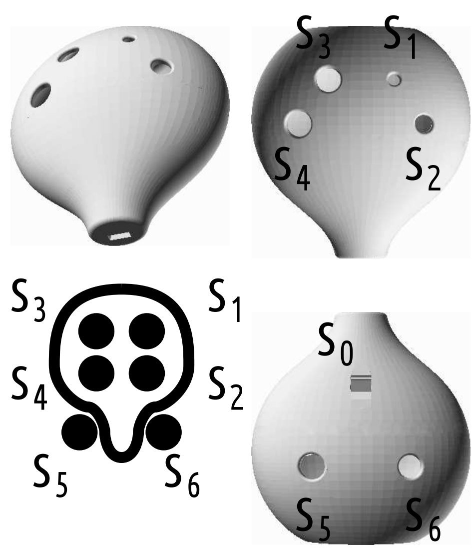

Figure 1: View of six-hole ocarina.

chart used in this ocarina model is shown in the Table 1 and it is based in the 4 holes fingering style proposed by John Taylor [Met+85, pp. 79, 146] [Hop99, pp. 30]. This table indicates the configuration of open or close holes to produce the tones from C until D (a higher octave), where a value $S_i = 1$ represents an open hole and the value $S_i = 0$ a closed hole.

Note	S_1	S_2	S_3	S_4	S_5	S_6
C	0	0	0	0	0	0
$C\#$	0	0	0	0	0	1
D	1	0	0	0	0	0
$D\#$	1	0	0	0	0	1
E	0	1	0	0	0	0
F	1	1	0	0	0	0
$F\#$	0	0	1	0	0	0
G	1	0	1	0	0	0
$G\#$	0	1	1	0	0	0
A	1	1	1	0	0	0
$A\#$	1	0	1	1	0	0
B	0	1	1	1	0	0
C	1	1	1	1	0	0
$C\#$	0	1	1	1	1	0
D	1	1	1	1	1	0

Table 1: Fingering chart

3 Math and music

In this paper we analyze a tuning method with 12 tone in an equal temperament scale; so that, the frequency interval between any pair of adjacent notes has the same ratio $\rho = \sqrt[12]{2}$. The Table 2 shows how are distributed the musical notes from tone C until D (a higher octave), where f_0 represents the frequency of C note and the next notes follow a geometric progression $\hat{f}_i = \rho^i f_0$ with i representing the separation degree with the C note.

$\hat{f}_0$	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$	$\hat{f}_4$
C	$C\#$	D	$D\#$	E
f_0	$\rho^1 f_0$	$\rho^2 f_0$	$\rho^3 f_0$	$\rho^4 f_0$
$\hat{f}_5$	$\hat{f}_6$	$\hat{f}_7$	$\hat{f}_8$	$\hat{f}_9$
F	$F\#$	G	$G\#$	A
$\rho^5 f_0$	$\rho^6 f_0$	$\rho^7 f_0$	$\rho^8 f_0$	$\rho^9 f_0$
$\hat{f}_{10}$	$\hat{f}_{11}$	$\hat{f}_{12}$	$\hat{f}_{13}$	$\hat{f}_{14}$
$A\#$	B	C	$C\#$	D
$\rho^{10} f_0$	$\rho^{11} f_0$	$\rho^{12} f_0$	$\rho^{13} f_0$	$\rho^{14} f_0$

Table 2: Musical notes

4 Math and ocarina

4.1 Analyzing a whistle

We can think of an ocarina as a whistle with tone-holes in its resonant cavity, so that when we open and close some of these holes, we can modify the pitch of generated sound. Thus, to understand the math of an ocarina first we should to know the science behind the whistles or more formally a Helmholtz resonant cavity [Cor11] (see Fig. 2) excited by a jet-edge-resonator system (see Fig. 3) [Gib13, pp. 3] [NWS53, pp. 138]. In this sense the Eq. 1

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{A_0}{l_0 V}} \tag{1}$$

shows the way of to calculate the frequency f_0 of a theoretic[1] whistle [Gib13, pp. 3] [Kob+09, pp. 5] [Oka+19, pp. 265] in relation with the constants π and c (the velocity of sound), and the variables: V the volume of its resonant cavity, A_0 its cross-section neck area and l_0 the neck effective length in the resonator.

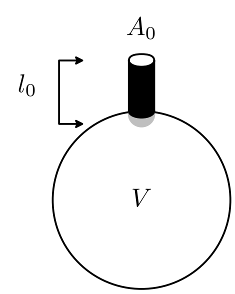

Figure 2: whistle.

[1] In a sphere-like resonant cavity, the open area of the neck should be much less than the total surface area of the resonant cavity.

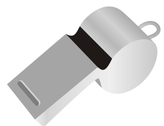

Figure 3: whistle.

4.2 Analyzing an ocarina

The fundamental resonant frequency f of an ocarina, that have at least one circular tone-hole in addition to the sound hole (see Fig. 4), can be represented like a function of the total tone-holes area A.

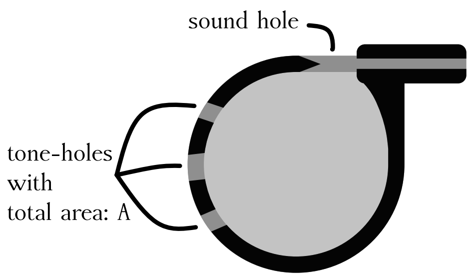

Figure 4: Theoretic ocarina

In the article "La Ocarina de Zanahoria a Carrot Ocarina" [Mp10] were analyzed data about of the increment of resonant frequency f in relation to the augment of area A, showing that in that case study the relationships between the variables can be approximated with a line

$$f = c_1 + c_2\ A. \tag{2}$$

where $c_1 = 595.859069$, $c_2 = 4.324273$ and a mean square error $MSE = 8051.1$. The Fig. 5 shows this linear curve in relation to the collected data.

By other side, if we follow the equation proposed by Bart Hopkin in its book "Air Columns and Toneholes" [Cab02, pp. 44] [Hop99, pp. 16] and we assume that all the neck effective lengths in the tone-holes are equals, we can fit the data of Fig. 5 with a square root function

$$f = d_1\sqrt{d_2 + A}, \tag{3}$$

where $d_1 = 91.215856$, $d_2 = 33.262474$ and a mean square error $MSE = 5993.3$. We must highlight that in this model d_2 represents the area of the sound hole.

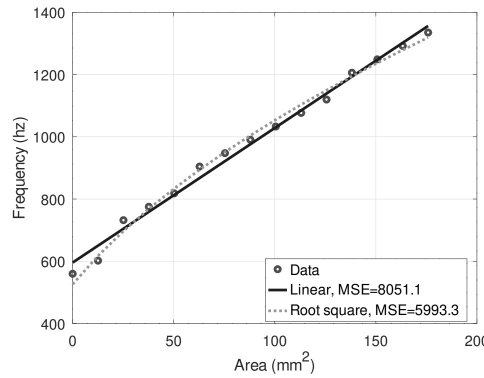

Figure 5: Comparing models.

This last model fit better the data, also the equation proposed by Hopkin keep coherence with the Helmholtz resonator equation (seen in Eq. 1). In the next section we use the model proposed by Hopkin to obtain an optimal tuning in John Taylor style six-hole ocarina.

4.3 Resonance frequency in ocarinas

For what is explained in the Sec. 4.2, here we use the equation 4, proposed by Bart Hopkin [Cab02, pp. 44] [Hop99, pp. 16],

$$f = \frac{c}{2\pi}\sqrt{\frac{\frac{A_0 S_0}{l_0} + \frac{A_1 S_1}{l_1} + \frac{A_2 S_2}{l_2} + ...}{V}} \tag{4}$$

as a good approximation to estimate the fundamental frequency f of globular resonators that have at least one circular pitch hole in addition to the sound hole (a.k.a. an ocarina). in which A_i and l_i are the area and the effective length of the S_i tone-hole, so that the variable S_i has only two values $\{0, 1\}$ according to the Table 1; additionally, $S_0 = 1$ and represents the sound hole, being A_0 and l_0 their corresponding variables.

4.3.1 Reordering the resonance equation

We can rearrange the Eq. 4 defining the variable $h_i \propto \frac{A_i}{l_i}$ and the constant k, so that

$$k \equiv \frac{c}{2\pi\sqrt{V}}, \qquad h_i \equiv \frac{A_i}{l_i}. \tag{5}$$

This variable change imply that the Eq. 4 can be rewritten as

$$f = k\sqrt{h_0 + \sum_{i=1}^{6} S_i h_i}, \tag{6}$$

It is important to remember that following the Eqs. 1 and 6, when all the tone-holes are closed we obtain that

$$f_0 = k\sqrt{h_0}. \tag{7}$$

4.3.2 Generated frequencies in a six-hole ocarina

If our intention is to generate by approximation all the 14 frequencies in a six-hole ocarina similar to seen in Table 2, then we use the Eq. 6 to approximate the (note) frequency $f_j(\mathbf{h})$ of the j-th row in the Table 1,

$$f_j(\mathbf{h}) = k\sqrt{h_0 + \sum_{i=1}^{6} S_i^j h_i}, \tag{8}$$

$$\mathbf{f}(\mathbf{h}) = \left[f_1(\mathbf{h}), f_2(\mathbf{h}), ..., f_{14}(\mathbf{h})\right]^T, \tag{9}$$

where the unknown variable $\mathbf{h} = [h_1,\ h_2,\ h_3,\ h_4,\ h_5,\ h_6]^T$ and the constant S_i^j is the element of the row j and column i in the matrix $\mathbf{S}$,

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}. \tag{10}$$

5 Problem Statement

5.1 Minimization rule and optimal value

Known and calculated the desired ($\hat{f}_j$) and generated ($f_j(\mathbf{h})$) frequencies seen in the Table 2 and Eq. 8, respectively. We propose the next minimization rule

$$\begin{aligned} e^2(\mathbf{h}) &= \sum_{j=1}^{14} \left(\frac{f_j(\mathbf{h}) - \hat{f}_j}{\hat{f}_j}\right)^2 \\ &= \sum_{j=1}^{14} \left(\frac{1}{\rho^j}\sqrt{1 + \sum_{i=1}^{6} \frac{S_i^j h_i}{h_0}} - 1\right)^2. \end{aligned} \tag{11}$$

If we define

$$\hat{\mathbf{h}} = \mathbf{h}/h_0, \tag{12}$$

then $e^2(\mathbf{h})$ can be rewritten as

$$e^2(\mathbf{h}) \equiv \hat{e}^2(\hat{\mathbf{h}}) = \left\|\mathbf{g}(\hat{\mathbf{h}}) - \mathbf{q}\right\|^2 \tag{13}$$

where

$$\mathbf{q} = [1,\ 1,\ 1,\ ...,\ 1]^T, \tag{14}$$

$$\mathbf{g} = [g_1(\hat{\mathbf{h}}),\ g_2(\hat{\mathbf{h}}),\ g_3(\hat{\mathbf{h}}),\ ...,\ g_{14}(\hat{\mathbf{h}})]^T, \tag{15}$$

$$g_j(\hat{\mathbf{h}}) = \frac{1}{\rho^j}\sqrt{1 + \sum_{i=1}^{6} S_i^j \hat{h}_i}. \tag{16}$$

The minimization problem of Eq. 11 can be fitted as a case of multiple nonlinear regression that can be solved using regularization [Puj20, pp. 129]. Thus, following this method, we obtain that the vector $\hat{\mathbf{h}}^*$,

$$\hat{\mathbf{h}}^* = \underset{\hat{\mathbf{h}}}{\arg\min}\ \hat{e}^2(\hat{\mathbf{h}}) \tag{17}$$

can be calculated following iteratively the Eq. 18

$$\begin{aligned} &\hat{\mathbf{h}}_l = \hat{\mathbf{h}}_{l-1} + \\ &\left\{\mathbf{A}(\hat{\mathbf{h}}_{l-1})^T \mathbf{A}(\hat{\mathbf{h}}_{l-1}) + \alpha\mathbf{I}\right\}^{-1} \mathbf{A}(\hat{\mathbf{h}}_{l-1})^T \left\{\mathbf{q} - \mathbf{g}(\hat{\mathbf{h}}_{\mathbf{l-1}})\right\}, \end{aligned} \tag{18}$$

until $\hat{\mathbf{h}}_l \approx \hat{\mathbf{h}}_{l-1}$, where we declare that $\hat{\mathbf{h}}^* \equiv \hat{\mathbf{h}}_l$.

The variables in the Eq. 18 are the regularization factor $0 \leq \alpha \leq 1.0$ (chosen heuristically by the user), the identity matrix $\mathbf{I}$ and

$$\mathbf{A}(\hat{\mathbf{h}}) = \begin{bmatrix} \frac{\partial g_1(\hat{\mathbf{h}})}{\partial \hat{h}_1} & \frac{\partial g_1(\hat{\mathbf{h}})}{\partial \hat{h}_2} & \cdots & \frac{\partial g_1(\hat{\mathbf{h}})}{\partial \hat{h}_6} \\ \frac{\partial g_2(\hat{\mathbf{h}})}{\partial \hat{h}_1} & \frac{\partial g_2(\hat{\mathbf{h}})}{\partial \hat{h}_2} & \cdots & \frac{\partial g_2(\hat{\mathbf{h}})}{\partial \hat{h}_6} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_{14}(\hat{\mathbf{h}})}{\partial \hat{h}_1} & \frac{\partial g_{14}(\hat{\mathbf{h}})}{\partial \hat{h}_2} & \cdots & \frac{\partial g_{14}(\hat{\mathbf{h}})}{\partial \hat{h}_6} \end{bmatrix}, \quad (19)$$

$$\frac{\partial g_j(\hat{\mathbf{h}})}{\partial \hat{h}_l} = \frac{1}{2\rho^j} \frac{S_l^j}{\sqrt{1 + \sum_{i=1}^{6} S_i^j \hat{h}_i}}. \quad (20)$$

Thus, following the Eqs. 12, 8 and 7; we obtain the optimum value,

$$\mathbf{h}^* = \hat{\mathbf{h}}^* \; h_0, \quad (21)$$

so that

$$f_j(\hat{\mathbf{h}}^* \; h_0) = f_0 \sqrt{1 + \sum_{i=1}^{6} S_i^j \hat{h}_i^*}, \quad (22)$$

and consequently (using the Eq. 9)

$$\mathbf{f}(\hat{\mathbf{h}}^* \; h_0) = f_0 \sqrt{1 + \mathbf{S}\hat{\mathbf{h}}^*}. \quad (23)$$

5.2 Iterative calculation

Using iteratively the Eq. 18 from $\hat{\mathbf{h}}_0$,

$$\hat{\mathbf{h}}_0 = [0.25,\; 0.25,\; 0.75,\; 1.0,\; 1.0,\; 0.125]^T \quad (24)$$

until $||\hat{\mathbf{h}}_l - \hat{\mathbf{h}}_{l-1}|| \leq 0.0001$, we obtain

$$\hat{\mathbf{h}}_7 = \begin{bmatrix} 0.26344 \\ 0.56628 \\ 0.98485 \\ 1.02421 \\ 1.04440 \\ 0.13352 \end{bmatrix}, \quad (25)$$

with an error $||\hat{\mathbf{h}}_7 - \hat{\mathbf{h}}_6|| \equiv 0.000087817$ and $e^2(\hat{\mathbf{h}}_7 h_0) \equiv \hat{e}^2(\hat{\mathbf{h}}_7) \equiv 0.0014474$, so that $\hat{\mathbf{h}}^* \equiv \hat{\mathbf{h}}_7$.

Fig. 6 shows the error $||\hat{\mathbf{h}}_l - \hat{\mathbf{h}}_{l-1}||$ in consecutive iterations of $\hat{\mathbf{h}}_l$, similarly the Fig. 7 shows the error $\hat{e}^2(\hat{\mathbf{h}}_l)$.

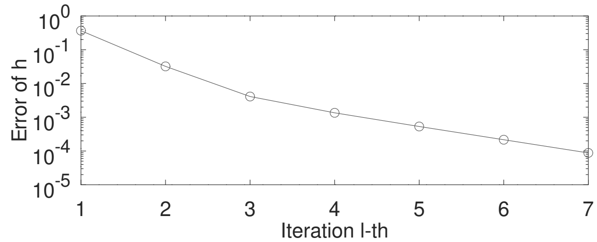

Figure 6: Error $||\hat{\mathbf{h}}_l - \hat{\mathbf{h}}_{l-1}||$ in consecutive iterations of $\hat{\mathbf{h}}_l$.

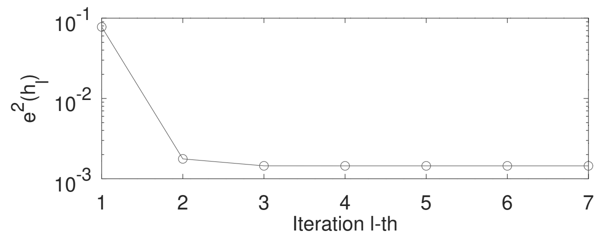

Figure 7: Error $\hat{e}^2(\hat{\mathbf{h}}_l)$ in consecutive iterations of $\hat{\mathbf{h}}_l$.

6 Results

Definition 6.1 (E_1 - relative tuning error) *The first type of error used in this work is*

$$E_1 = 100 \; \frac{f_b - f_a}{f_a} \; \%, \quad (26)$$

that measures the difference error of f_b in relation to the target frequency (f_a), this error also can be called as relative tuning error.

To obtain the biggest accepted relative tuning error, it is necessary to know the fact that between any two consecutive notes, with frequencies f_a and ρf_a, exist a proportion of $\rho \approx 1.059463$ (a semitone), this mean that the next consecutive note is ever a $+5.9463\%$ that the last note. Thus, if we set as maximum error the midway position in geometric proportion between notes, them the biggest accepted positive relative tuning error to avoid a tone recognition mistake, it is equal to $+2.9302\% \equiv 100(\sqrt{\rho} f_a - f_a)/f_a$ %. By other side the biggest accepted negative relative tuning error to avoid a tone recognition mistake, it is equal to $-2.8468\% \equiv 100(f_a/\sqrt{\rho} - f_a)/f_a$ %.

Definition 6.2 (E_2 - semitone error) *The logarithmic difference error between two frequencies f_a (target) and f_b in relation to a semitone*

(ρ) is defined as

$$E_2 = 100\ \frac{ln(f_b) - ln(f_a)}{ln(\rho)}\ \%, \qquad (27)$$

called here as semitone error.

To obtain the biggest accepted semitone error, we use a similar criterion as seen in the Def. 6.1, so that if we set the maximum error as the midway position in geometric proportion between notes (ex: C and C#), them the biggest accepted positive semitone error to avoid a tone recognition mistake, it is equal to $+50\% \equiv 100\ \frac{ln(\sqrt{\rho} f_a) - ln(f_a)}{ln(\rho)}\ \%$*. By other side the biggest accepted negative semitone error to avoid a tone recognition mistake, it is equal to* $-50\% \equiv 100\ \frac{ln(f_a/\sqrt{\rho}) - ln(f_a)}{ln(\rho)}\ \%$*.*

6.1 Analyzing the optimal $\mathbf{h}^*$

In the Eqs. 21 and 25 was calculated $\mathbf{h}^* = [h_1^*, h_2^*, ..., h_6^*]^T$, the optimal vector that minimize the function $e^2(\mathbf{h})$. The Table 3 shows each element of this vector in relation to h_0, being that $h_i^*/h_0 \equiv \left(\frac{l_0}{A_0}\right)\left(\frac{A_i}{l_i}\right)$.

h_1^*/h_0	h_2^*/h_0	h_3^*/h_0	h_4^*/h_0	h_5^*/h_0	h_6^*/h_0
26.3%	56.6%	98.5%	102.4%	104.4%	13.4%

Table 3: Percentage ratio between h_i^* and h_0.

In this table we observe that if we consider l_i a constant throughout the ocarina then $h_i^*/h_0 \propto A_i$ and the percentages presented in the Table 3 describe the proportion ratio between areas A_i. So that below this condition the optimal area A_2 is about twice A_1, the optimal area A_3 is about $3.73 A_1$ and so on.

6.2 Analyzing the optimal frequencies

The Tabla 4 shows a comparison between the desired and optimal generated frequencies for each note in a six-hole ocarina. The first column shows all notes used in the optimization calculation. The second column has the desired frequencies $\hat{f}_j$ (equal temperament scale). The third column has the optimal generated frequencies f_j. The fourth and fifth columns have the errors described in the Definitions 6.1 and 6.2, respectively.

Note	Desired (hz)	Optimal (hz)	E_1	E_2
C	1.0000 f_0	1.0000 f_0	0.000 %	0.000 %
$C\#$	1.0595 f_0	1.0647 f_0	0.491 %	8.483 %
D	1.1225 f_0	1.1240 f_0	0.140 %	2.414 %
$D\#$	1.1892 f_0	1.1819 f_0	-0.612 %	-10.628 %
E	1.2599 f_0	1.2515 f_0	-0.667 %	-11.595 %
F	1.3348 f_0	1.3527 f_0	1.336 %	22.974 %
$F\#$	1.4142 f_0	1.4088 f_0	-0.380 %	-6.584 %
G	1.4983 f_0	1.4994 f_0	0.075 %	1.295 %
$G\#$	1.5874 f_0	1.5972 f_0	0.619 %	10.681 %
A	1.6818 f_0	1.6777 f_0	-0.245 %	-4.252 %
$A\#$	1.7818 f_0	1.8090 f_0	1.527 %	26.234 %
B	1.8877 f_0	1.8909 f_0	0.165 %	2.847 %
C	2.0000 f_0	1.9593 f_0	-2.036 %	-35.613 %
$C\#$	2.1189 f_0	2.1494 f_0	1.436 %	24.685 %
D	2.2449 f_0	2.2098 f_0	-1.565 %	-27.309 %

Table 4: Errors in optimal tuning

The worst error E_1 is equal to -2.036% and correspond to the note C; similarly, this note present the worst error E_2 with a detour of -35.613% of a semitone. The next worst error E_1 is equal to -1.565% and correspond to the note D, having an error E_2 with a detour of -27.309% of a semitone.

By the results show in the Table 4, we observe that the optimal vector $\mathbf{h}^*$ causes that $e^2(\mathbf{h}^*) > 0$ and consequently a perfect equal temperament scale it is not possible to be applied in the tuning of a John Taylor style six-hole ocarina, only it is possible an optimal approximation.

6.3 Proposed tuning method

From the data of the Table 4, we can infer an ocarina construction method that fulfill an optimal tuning following the minimization rule of the Eq. 11. In this sense if we choose of this table a set of notes, in which each current note use a new tone-hole in relation to the last tuned note, we can obtain a tuning method or sequence to reach an optimal tuning.

With these considerations, the Table 5 shows one tuning method, in which is considered a C note equivalent to $523.2511hz$. The method consist in follow the table 5 line a line from top at bottom. So that we set in our ocarina, with each new created tone-hole, the semitone error (E_2)

shown in the fourth column of the Table 5.

Note	Desired (hz)	Optimal (hz)	E_2
C	523.2511	523.2511	0.000 %
$C\#$	554.3653	557.0881	8.483 %
D	587.3295	588.1491	2.414 %
E	659.2551	654.8546	-11.595 %
G	783.9909	784.5777	1.295 %
B	987.7666	989.3923	2.847 %
D	1174.6591	1156.2749	-27.309 %

Table 5: Proposed tuning method.

7 Conclusion

The results of Table 4 shows that it is not possible tuning a John Taylor style six-hole ocarina with a perfect equal temperament scale, but the minimization rule presented in this work returns a tuning method with an optimal approximation and heterogeneous errors less than $|-35.613\%|$ of a semitone. Additionally, the Table 5 shows one, of several possibles, construction methods, paths or sequences to fulfill an optimal tuning according to the minimization rule defined in the Eq. 11 and with the resonance frequency equation in ocarinas proposed by Bart Hopkin. This method can be complemented with the information of the relations between areas A_i and effective lengths l_i in an optimal tuning shown in the Table 3.

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