

# Combinacional

## Algebra booliana

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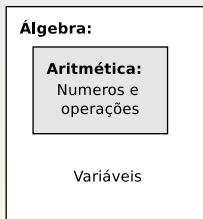
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# Álgebra booliana [1]



George Boole



Abstrata

Elementar

Booliana

Álgebra:

**Aritmética:**

**Estado1:** 0,F

**Estado2:** 1,V

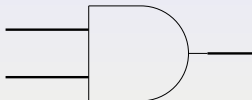
**e lógico:** and,\*,.,,^

**ou lógico:** or,+,v

**negação:** not,~, -

# AND

interseção



<b>A</b>	<b>B</b>	<b>A and B</b>
F	F	F
F	V	F
V	F	F
V	V	V

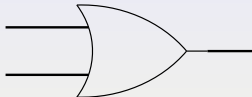
<b>A</b>	<b>B</b>	<b>A and B</b>
0	0	0
0	1	0
1	0	0
1	1	1

SE: **é redonodo** **A** **e** **rebota** **B** então **é uma bola**

**and** **→**

# OR

União



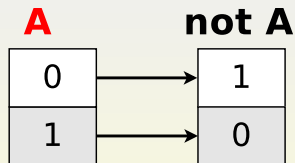
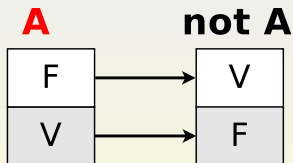
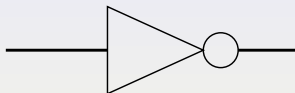
<b>A</b>	<b>B</b>	<b>A or B</b>
F	F	F
F	V	V
V	F	V
V	V	V

<b>A</b>	<b>B</b>	<b>A or B</b>
0	0	0
0	1	1
1	0	1
1	1	1

SE: **está triste** **A** ou **está calado** **B** então ele perdeu

**or** **B**  $\longrightarrow$

# NOT



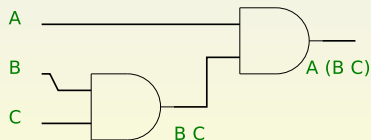
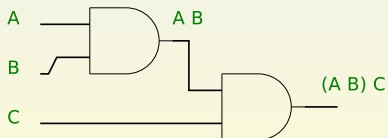
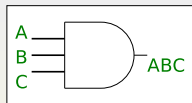
SE: **não** está salgado                      então                      Posso comer  
**not**                      **A**                      →

# Propriedades

## Propriedade associativa:

$(A \text{ and } B) \text{ and } C \equiv A \text{ and } (B \text{ and } C)$

$(A B) C \equiv A (B C)$

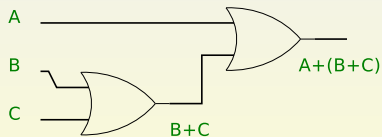
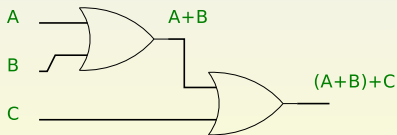
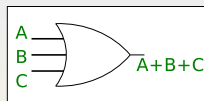


# Propriedades

## Propriedade Associativa:

$$(A \text{ or } B) \text{ or } C \equiv A \text{ or } (B \text{ or } C)$$

$$(A+B)+C \equiv A+(B+C)$$

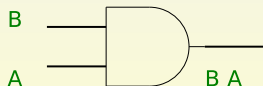
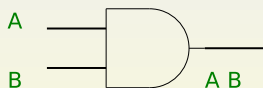


# Propriedades

## Propriedades comutativas

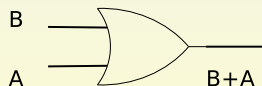
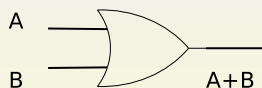
$$A \text{ and } B \equiv B \text{ and } A$$

$$A B \equiv B A$$



$$A \text{ or } B \equiv B \text{ or } A$$

$$A+B \equiv B+A$$



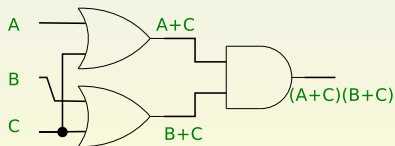
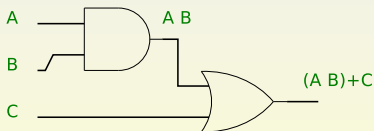


# Propriedades

## Propriedade distributiva:

$$(A \text{ and } B) \text{ or } C \equiv (A \text{ or } C) \text{ and } (B \text{ or } C)$$

$$(A B) + C \equiv (A+C)(B+C)$$

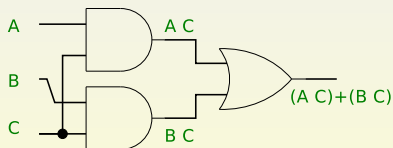
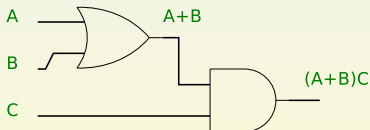


# Propriedades

## Propriedade distributiva:

$$(A \text{ or } B) \text{ and } C \equiv (A \text{ and } C) \text{ or } (B \text{ and } C)$$

$$(A+B)C \equiv (AC)+(BC)$$

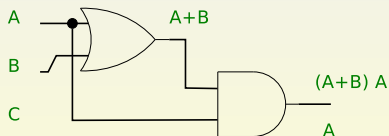
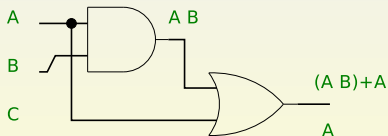


# Propriedades

**Propriedade absorbtiva:** (similar a conjuntos or:união and:interseção)

$$(A \text{ and } B) \text{ or } A \equiv (A \text{ or } B) \text{ and } A \equiv A$$

$$(A B) + A \equiv (A+B) A \equiv A$$

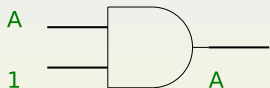


# Propriedades

## Propriedades de elementos neutros:

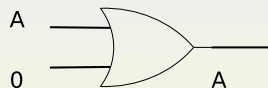
$$A \text{ and } 1 \equiv A$$

$$A 1 \equiv A$$



$$A \text{ or } 0 \equiv A$$

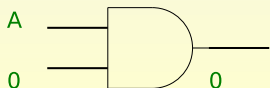
$$A+0 \equiv A$$



## Elementos absorventes:

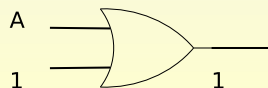
$$A \text{ and } 0 \equiv 0$$

$$A 0 \equiv 0$$



$$A \text{ or } 1 \equiv 1$$

$$A+1 \equiv 1$$

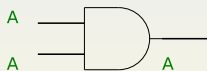


# Propriedades

## Propriedades idempotentes:

$$A \text{ and } A \equiv A$$

$$A A \equiv A$$



$$A \text{ or } A \equiv A$$

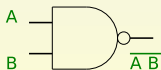
$$A + A \equiv A$$



## Leis de "De Morgan": (Augustus De Morgan)

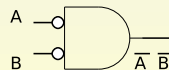
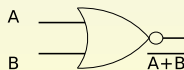
$$\text{not } (A \text{ and } B) \equiv (\text{not } A) \text{ or } (\text{not } B)$$

$$\overline{A B} \equiv \overline{A} + \overline{B}$$

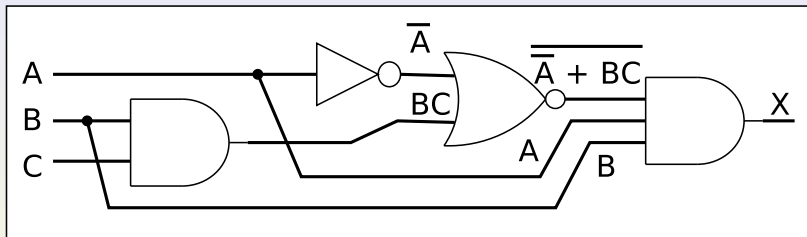


$$\text{not } (A \text{ or } B) \equiv (\text{not } A) \text{ and } (\text{not } B)$$

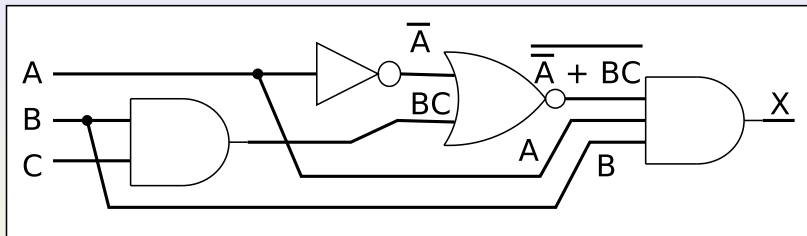
$$\overline{A + B} \equiv \overline{A} \overline{B}$$



# Projetando um circuito lógico

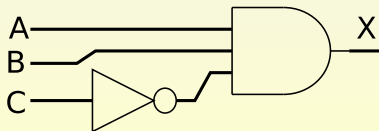


# Projetando um circuito lógico

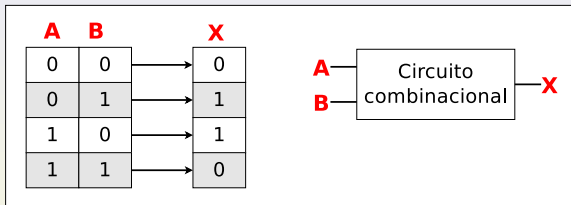


$$\begin{aligned}
 X &= (\overline{A} + BC) A B \\
 &= (A \overline{BC}) A B \\
 &= \overline{BC} A B \\
 &= (\overline{B} + \overline{C}) A B \\
 &= \overline{B} A B + \overline{C} A B
 \end{aligned}$$

$$X = A B \overline{C}$$

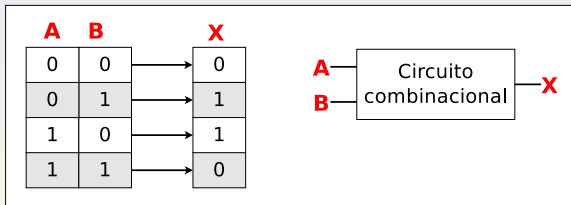


# Projetando um circuito lógico

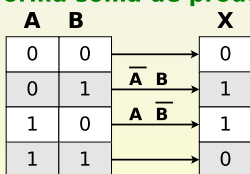




# Projetando um circuito lógico

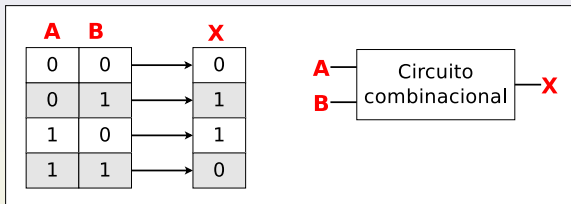


Forma soma de produtos

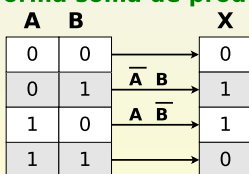


$$X = \bar{A}B + A\bar{B}$$

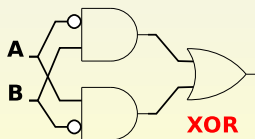
# Projetando um circuito lógico



## Forma soma de produtos



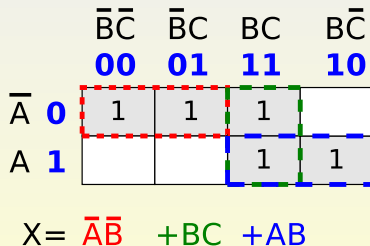
$$X = \bar{A}B + A\bar{B}$$



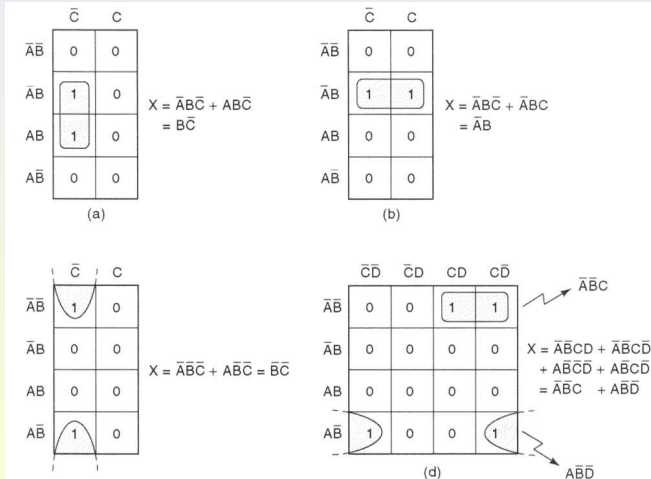
# Projetando: Mapa de Karnaugh [1]

A	B	C		X
0	0	0	$\bar{A} \bar{B} \bar{C}$	1
0	0	1	$\bar{A} \bar{B} C$	1
0	1	0		0
0	1	1	$\bar{A} B C$	1
1	0	0		0
1	0	1		0
1	1	0	$A B \bar{C}$	1
1	1	1	$A B C$	1

- \* Divide em dois grupos
- \* Ordena seguindo minima distância de Hamming
- \* Agrupa e potenciais de 2



# Projetando: Mapa de Karnaugh



# Projetando: Mapa de Karnaugh

	$\bar{C}$	$C$
$\bar{A}\bar{B}$	0	1
$\bar{A}B$	0	1
$AB$	0	1
$A\bar{B}$	0	1

(a)  $X = C$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
$AB$	1	1	1	1
$A\bar{B}$	0	0	0	0

(b)  $X = AB$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
$AB$	0	1	1	0
$A\bar{B}$	0	0	0	0

(c)  $X = BD$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
$AB$	1	0	0	1
$A\bar{B}$	1	0	0	1

(d)  $X = A\bar{D}$

	$\bar{C}\bar{D}$	$\bar{C}D$	$CD$	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	0	0	0	0
$AB$	0	0	0	0
$A\bar{B}$	1	0	0	1

(e)

$$X = \bar{B}\bar{D}$$

# References I

- [1] Ronald J Tocci, Neal S Widmer, and Gregory L Moss. *Sistemas digitais: princípios e aplicações*, volume 8. Prentice Hall, 2003.