Filtering system

Impulse response

$$\begin{array}{ccccccccccc} h[n] & = & h_0\,\delta[n] & + & h_1\,\delta[n-1] & + & h_2\,\delta[n-2] & \dots & + & h_M\,\delta[n-M] \\ H(Z) & = & h_0 & + & h_1\,Z^{-1} & + & h_2\,Z^{-2} & \dots & + & h_M\,Z^{-M} \end{array} \tag{1}$$

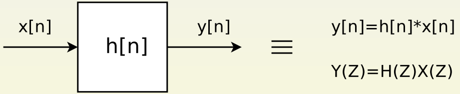


Figure: Filtering system

Filtering system

Impulse response

$$\begin{array}{ccccccccccc} h[n] & = & h_0\delta[n] & + & h_1\delta[n-1] & + & h_2\delta[n-2] & \dots & + & h_M\delta[n-M] \\ H(Z) & = & h_0 & + & h_1 Z^{-1} & + & h_2 Z^{-2} & \dots & + & h_M Z^{-M} \end{array} \quad (2)$$

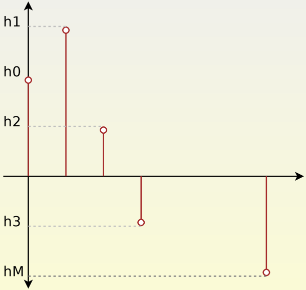


Figure: h[n] Impulse response

Knowing the Fourier transform of $h_a[n]$

Impulse response

$$\begin{array}{ccccccccccc} h_a[n] & = & h_0\delta[n] & + & h_1\delta[n-1] & + & h_2\delta[n-2] & \dots & + & h_M\delta[n-M] \\ H_a(Z) & = & h_0 & + & h_1 Z^{-1} & + & h_2 Z^{-2} & \dots & + & h_M Z^{-M} \end{array} \quad (3)$$

Z transform $\xrightarrow{Z=e^{jw}}$ Fourier transform

$$H_a(Z) \xrightarrow{Z=e^{jw}} H_a(e^{jw}) \quad (4)$$

$$h_a[n] = \begin{cases} \frac{sin(\pi \ a \ n)}{\pi \ n} & \text{if } n \neq 0 \\ a & \text{if } n = 0 \end{cases} \quad (5)$$

$$H_a(e^{jw}) = \begin{cases} 1 & \text{if } W \leq a\pi \\ 0 & \text{if } W > a\pi \end{cases} \quad (6)$$

Knowing the Fourier transform of $h_a[n]$

Function $h_a[n]$

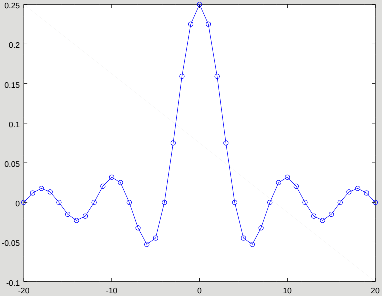


Figure: $h_a[n]$ with $a = 0.25$ and 41 points

Knowing the Fourier transform of $h_a[n]$

Fourier transform of $h_a[n]$

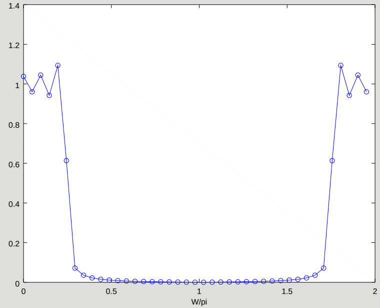


Figure: $H_a(W)$ with $a = 0.25$ and 41 points

Knowing the Fourier transform of $h_a[n]$

Fourier transform of $h_a[n]$

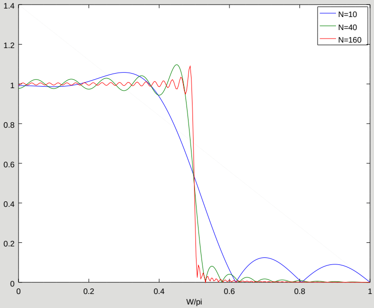


Figure: $H_a(W)$ with $a = 0.5$

Janelamento da função sinc

Janela de Hanning de $N = 61$ amostras

$$hanning[n] = 0.5(1 - cos(\frac{2\pi n}{N-1})) \tag{7}$$

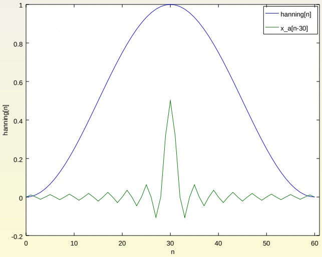


Figure: Hanning window and $h_a[n-30]$ for $a = 0.5$

Janelamento da função sinc

Fourier transform de uma janela de Hanning de $N = 61$ amostras

$$HANNING(W) = FFT\{hanning[n]\} \quad (8)$$

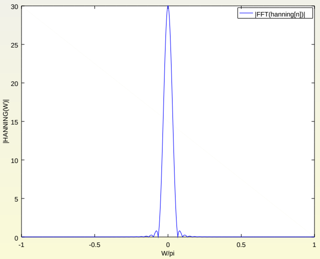
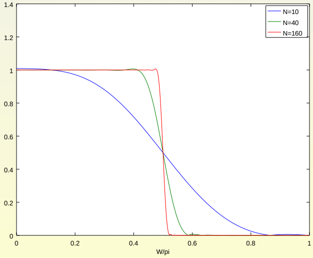


Figure: Fourier transform of Hanning window

Conhecendo que a transformada de fourier de $h_a[n]\ hanning[n+D]$

Fourier transform of $h_a[n]\ hanning[n+D]$

$$H_a(W) * hanning(W) \equiv convolution\{H_a(W), hanning(W)\} \quad (9)$$



Tipos de janelas

Hanning

$$hanning[n] = 0.5(1 - cos(\frac{2\pi n}{N-1})) \tag{10}$$

Hamming

$$hamming[n] = \alpha - \beta \ \cos\left(\frac{2\pi n}{N-1}\right), \alpha + \beta = 1 \tag{11}$$

Blackman

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) \tag{12}$$

$$a_0 = \frac{1-\alpha}{2}; \quad a_1 = \frac{1}{2}; \quad a_2 = \frac{\alpha}{2} \tag{13}$$

FIR Low-pass filter with cut-off at $a\pi$ (normalized to 2π)

$$\begin{array}{cccccccccccc} h_a[n] & = & h_0\delta[n] & + & h_1\delta[n-1] & + & h_2\delta[n-2] & \dots & + & h_M\delta[n-M] \\ H_a(Z) & = & h_0 & + & h_1 Z^{-1} & + & h_2 Z^{-2} & \dots & + & h_M Z^{-M} \end{array} \tag{14}$$

If M is even then: High pass filter with cut-off at $a\pi$

$$Z^{-M/2} - H_a(Z) \tag{15}$$

High pass filter with cut-off at $(1-a)\pi$

$$H_a(-Z) \tag{16}$$

Band pass filter with cut-off at $(1-a)\pi$ and $b\pi$

$$H_b(Z) * H_a(-Z) \tag{17}$$