Decodification cost in CEO problem: test_ber_vs_hbu0omega.m

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Abstract

test_ber_vs_hbu0omega compare the relation between $P(\hat{U}_0 \neq U_0)$ and $H(U_0|\Omega_M)$ in the model system.

1 Introduction

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test_ber_vs_hbu0omega
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In the Figure 1 the source U_0, $P(U_0 = 1) = 0.5$, trough across M BSC channels, with error probability P_s, generating the sources U_m, $\forall m \in S = \{1, 2, ..., M\}$. In this context we said $\Omega_M = \{U_m \in S\}$. In the Figure 2 also can be seen that $\Omega_M = g(U_0)$. Known Ω_M, this values are decoded for to get $\hat{U}_0$, a approximate

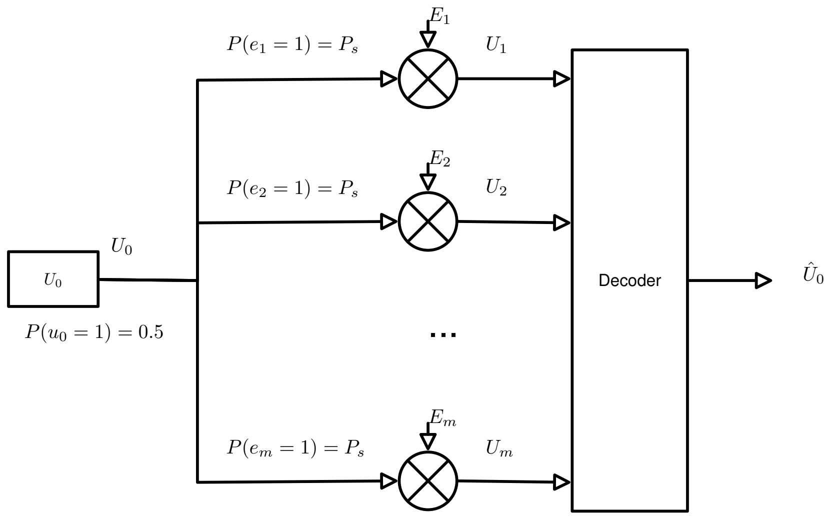

Figure 1: System Model (In extend).

version of U_0, so that $\hat{U}_0 = f(\Omega_M)$.

$$H(U_0|f(\Omega_m)) = H(U_0|\hat{U}_0) = h_b(P(\hat{U}_0 \neq U_0)) \tag{1}$$

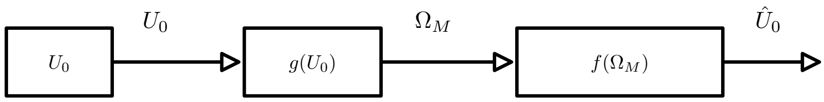

Figure 2: System Model (short).

thereby

$$h_b(P(\hat{U}_0 \neq U_0)) \geq H(U_0|\Omega_M) \tag{2}$$

or if we consider $P(\hat{U}_0 \neq U_0) < 0.5$

$$P(\hat{U}_0 \neq U_0) \geq h_b^{-1}(H(U_0|\Omega_M)) \tag{3}$$

1.1 Working with the probability $P(\hat{U}_0 \neq U_0)$

In [1, 2] is considered a maximum a posteriori (MAP) fusion rule $f(\Omega_M)$, where the output value $\hat{u}_0$ of $\hat{U}_0$ is obtained as

$$\begin{aligned} \hat{u}_0 &= \arg_{u_0} \max P(U_0|\Omega_M) \\ &\equiv \arg_{u_0} \max P(\Omega_M|U_0) \end{aligned} \tag{4}$$

Thus, considering that m_0 is the number of zeros in Ω_M, the decision is simplify to

$$m_0 \underset{\hat{u}_0 = 0}{\overset{\hat{u}_0 = 1}{\substack{\geq \\ <}}} \lfloor \frac{M}{2} \rfloor \tag{5}$$

In this expression is considered that if M is even and $m_0 = M/2$, the decision is arbitrarily assume that $\hat{u}_0 = 1$, so that $P(\hat{U}_0 \neq U_0)$ is $P_e = 0.5\ [P(\hat{u}_0 = 0|u_0 = 1) + P(\hat{u}_0 = 1|u_0 = 0)]$,

$$P_e = 0.5 \sum_{k=0}^{\lfloor \frac{M}{2} \rfloor - 1} \binom{M}{k} (1 - P_s)^k {P_s}^{M-k} + 0.5 \sum_{k=\lfloor \frac{M}{2} \rfloor}^{M} \binom{M}{k} (1 - P_s)^{M-k} {P_s}^k \tag{6}$$

where, $\lfloor . \rfloor$ is the floor function and the value P_e only is valid for values of $P_s \leq 1/2$ [1]. The Equation (6) can be sort as

$$P_e = \begin{cases} \sum\limits_{k=\lfloor \frac{M}{2} \rfloor + 1}^{M} \binom{M}{k} (1 - P_s)^{M-k} P_s^k & \text{if } M \textit{ odd} \\ \\ \sum\limits_{k=\lfloor \frac{M}{2} \rfloor + 1}^{M} \binom{M}{k} (1 - P_s)^{M-k} P_s^k & \text{if } M \textit{ even} \\ \quad + 0.5 \binom{M}{\frac{M}{2}} (1 - P_s)^{\frac{M}{2}} P_s^{\frac{M}{2}} & \end{cases} \tag{7}$$

This form is the form showed in [**?**].

[1] Here is important note that in [1, 2] your value ρ is equal to $1 - P_s$ here, and your result is for $\rho > 0.5$

1.2 Working with the probability $H(U_0|\Omega_M)$

$$H(U_0|\Omega_M) = \sum_{k=0}^{M} \binom{M}{k} p_s^k (1-p_s)^{M-k} log_2 \left(1 + \frac{p_s}{(1-p_s)}^{M-2k}\right) \tag{8}$$

References

[1] Abrardo, A.; Ferrari, G.; Martalò, M.; Perna, F. Feedback Power Control Strategies in Wireless Sensor Networks with Joint Channel Decoding. Sensors 2009, 9, 8776-8809. doi:10.3390/s91108776

[2] Ferrari, G.; Martalo, M.; Abrardo, A.; Raheli, R., "Orthogonal multiple access and information fusion: How many observations are needed?," Information Theory and Applications Workshop (ITA), 2012 , vol., no., pp.311,320, 5-10 Feb. 2012. doi: 10.1109/ITA.2012.6181783