Detection of Coffee Seeds in Images by the Circularity of the Binary Shapes

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Here is presented a list of methods and techniques necessary to detection of coffee seeds in images by the calculus of the circularity in the binary shapes.

1 Relation γ between the perimeter and the root square value of area

In this section we analyze the relation γ, between the perimeter P and the root square value of area A, so that

$$\gamma = \frac{P}{\sqrt{A}}. \tag{1}$$

They will be calculated the γ value of a triangle, a rectangle, and a circle.

1.1 γ value of an ellipse or γ_e

The perimeter of an ellipse, with width $2a$ and height $2b$; it is equal to $P_e = 4\int_0^{\frac{\pi}{2}} \sqrt{a^2 - (a^2 - b^2) sin^2(\theta)} d\theta$; with an area $A_e = \pi ab$ [1, pp. 702]. Thus, the value

$$\gamma_e(a,b) = \frac{P_e}{\sqrt{A_e}} = \frac{4\int\limits_0^{\frac{\pi}{2}} \sqrt{a^2 - (a^2 - b^2) sin^2(\theta)} d\theta}{\sqrt{\pi ab}}. \tag{2}$$

The $\gamma_e(a,b)$ function generates a surface as showed in the Fig. 1. To obtain the minimum value of this

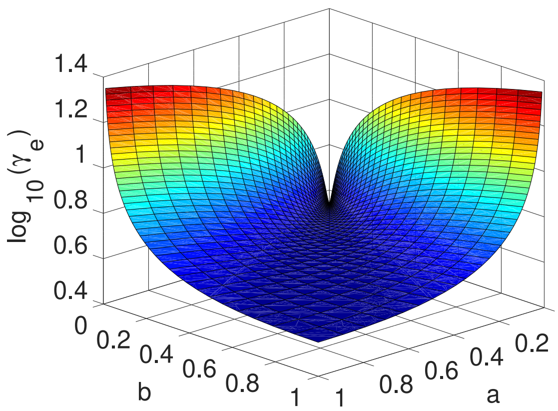

Figure 1: *Surface generated by some possible values of $log_{10}(\gamma_e(a,b))$.*

surface we calculate $\frac{\partial \gamma_e(a,b)}{\partial a} = 0$ and $\frac{\partial \gamma_e(a,b)}{\partial b} = 0$, so that

$$\frac{\partial \gamma_e(a,b)}{\partial a} = \frac{1}{\sqrt{\pi ab}}\left(\frac{\partial P_e(a,b)}{\partial a} - \frac{P_e(a,b)}{2a}\right) = 0, \quad (3)$$

$$\frac{\partial \gamma_e(a,b)}{\partial b} = \frac{1}{\sqrt{\pi ab}}\left(\frac{\partial P_e(a,b)}{\partial b} - \frac{P_e(a,b)}{2b}\right) = 0. \quad (4)$$

Of this equation we deduce that the minimum value is $\gamma_e(a,b)$ is reach when

$$P_e(a,b) = k_a\sqrt{b} = k_b\sqrt{a} = k\sqrt{ab}. \quad (5)$$

This can be obtained if $a = b$, where $P_e(a,a) = 2\pi\sqrt{a^2}$ and $\gamma_e(a,a) = 2\sqrt{\pi} \approx 3.5449$; so that

$$2\sqrt{\pi} \leq \gamma_e. \quad (6)$$

1.2 γ value of a rectangle or γ_r

The perimeter of a rectangle, of sides with lengths a and b; it is equal to $P_r = 2(a+b)$; with an area $A_r = ab$. Thus, the value

$$\gamma_r(a,b) = \frac{P_r}{\sqrt{A_r}} = \frac{2(a+b)}{\sqrt{ab}}. \quad (7)$$

The $\gamma_r(a,b)$ function generates a surface as showed in the Fig. 2. To obtain the minimum value of this

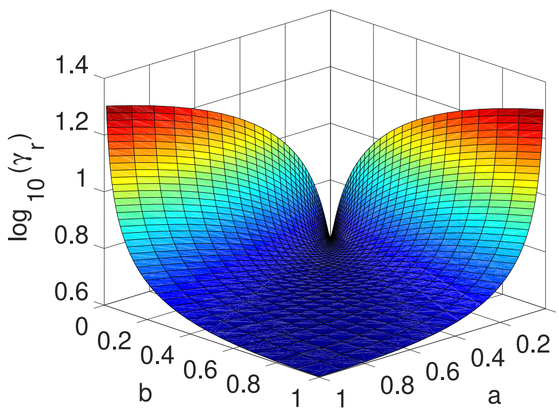

Figure 2: *Surface generated by some possible values of $log_{10}(\gamma_r(a,b))$.*

surface we calculate $\frac{\partial \gamma_r(a,b)}{\partial a} = 0$ and $\frac{\partial \gamma_r(a,b)}{\partial b} = 0$, so that

$$\frac{\partial \gamma_r(a,b)}{\partial a} = \frac{2}{\sqrt{ab}} - \frac{a+b}{a\sqrt{ab}} = 0, \quad (8)$$

$$\frac{\partial \gamma_r(a,b)}{\partial b} = \frac{2}{\sqrt{ab}} - \frac{a+b}{b\sqrt{ab}} = 0. \quad (9)$$

Of this equation we deduce that the minimum value is $\gamma_r(a,b)$ is reach when $a = b$; this mean when $\gamma_r(a,a) = 4$; so that

$$4 \leq \gamma_r. \quad (10)$$

1.3 γ value of a triangle or γ_t

The perimeter of a triangle, of sides with lengths a, b and c; it is equal to $P_t = a + b + c$; with an area[1] $A_t = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = (a+b+c)/2$ [3, pp. 253]. Thus, we can calculate the value

$$\gamma_t(a,b,c) = \frac{P_t}{\sqrt{A_t}} = \frac{a+b+c}{\sqrt[4]{s(s-a)(s-b)(s-c)}}. \quad (11)$$

The $\gamma_t(a,b,c)$ function generates a solid with different values γ_t to each point $\{a,b,c\}$ that can describe a tringale, with this purpose we use the triangle inequality[2] [2, pp. 17]; the Fig. 3 show this result.

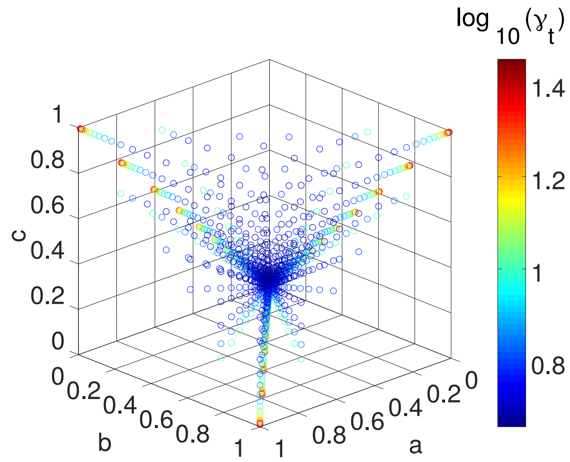

Figure 3: *Solid generated by some possible values of $log_{10}(\gamma_t(a,b,c))$.*

To obtain the minimum value $\gamma_t(a,b,c)$ in this solid we need calculate $\frac{\partial \gamma_t(a,b,c)}{\partial a} = 0$, $\frac{\partial \gamma_t(a,b,c)}{\partial b} = 0$ and $\frac{\partial \gamma_t(a,b,c)}{\partial c} = 0$; but by the symmetry of three equations, and the experience in the calculus made in the Section 1.2, we know that the minimum can be reach when $a = b = c$; this mean when $\gamma_t(a,a,a) = \frac{6}{\sqrt[4]{3}} \approx 4.5590$; so that

$$4.5590 \leq \gamma_t(a,b,c) \quad (12)$$

1.4 Digest of γ values

The γ values obtained in the last sequences can be order as show the Fig. 4. In this figure we can observe that if only we use the value γ, exist a point when an ellipse is indistinguishable of a rectangle, and this happen when $\gamma_e(a,b) = 4$, and consequently

$$4 = \frac{4\int_0^{\frac{\pi}{2}} \sqrt{a^2 - (a^2 - b^2)sin^2(\theta)}d\theta}{\sqrt{\pi ab}}, \quad (13)$$

[1] Calculated using the Heron theorem.
[2] $a < b + c$, $b < a + c$ and $c < a + b$

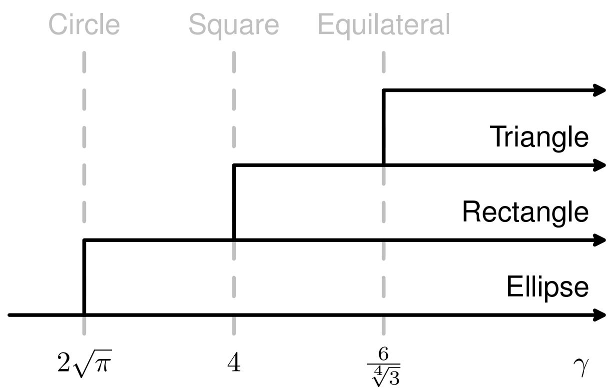

Figure 4: *γ values of some geometric figures.*

$$r = \frac{\left(\int_0^{\frac{\pi}{2}} \sqrt{1-(1-r^2)\, sin^2(\theta)} d\theta\right)^2}{\pi}, \tag{14}$$

being $r = \frac{b}{a}$. Where r can be found using iteratively the equation

$$r_k = \frac{\left(\int_0^{\frac{\pi}{2}} \sqrt{1-(1-{r_{k-1}}^2)\, sin^2(\theta)} d\theta\right)^2}{\pi}, \tag{15}$$

from $r_0 = 1$ until that $r_k \approx r_{k-1}$, where we declare that $r = r_k$. Thus, we obtain with this method that a ellipse with $\gamma_e(a, b) = 4$ have values a and b that fulfill the ratio $\frac{b}{a} \approx 0.437891897721079$. With all these considerations we can show in the Fig. 5 a ellipse and a square with the same area and a $\gamma = 4$.

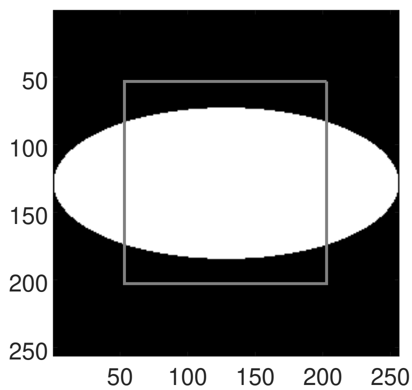

Figure 5: *Ellipse and square with the same area and a $\gamma = 4$.*

References

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