

1 Shared memory

2 Message Passing

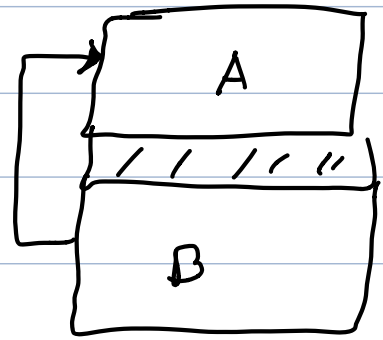
Direct Comm

send (  $P_i$  msg )

receive (  $q_i$  msg )

Indirect comm

↓  
mail box



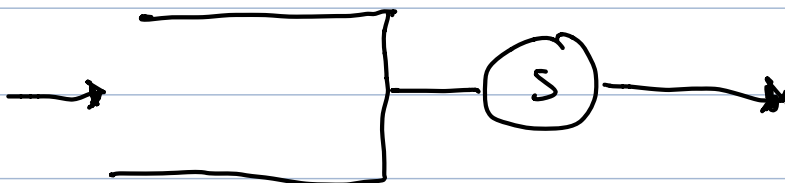
	Shared memory	Message Passing
Small Data to be shared		✓
Faster	✓	
Easier to implement		✓

## Queing Analysis

-> Customers : individuals who want services (process)

servers : satisfy request from customer

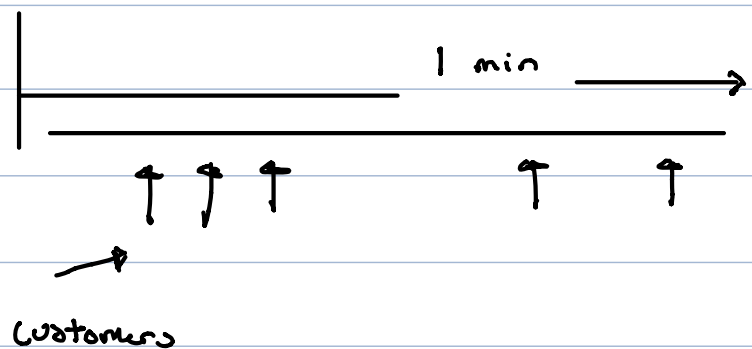
Queues : Waiting areas.



Arrival Process : How customers arrive

$\lambda$  : average arrival rate

$\lambda$  : 5 customers per 1 min.



Poisson Distribution

es// Gives the probability of a given number of events (i.e. arrivals) over a fixed period of time, if the average rate ( $\lambda$ ) is known

$$\text{PDF : } f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

$$\text{Pr [ No arrivals happen ]} = \text{Pr}(x=0) = \frac{5^0}{0!} e^{-5} = e^{-5} = .00674$$

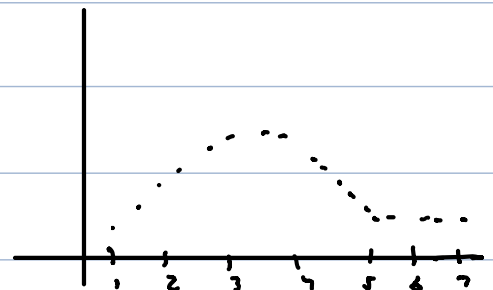
$$\text{Pr [ 1 arrivals happen ]} = \text{Pr}(x=1) = \frac{5^1}{1!} e^{-5} = 5e^{-5} = .033$$

$$\text{[ 2 ]} = .084$$

$$\text{[ 3 ]} = .14$$

$$\text{[ 4 ]} = .175$$

$$\text{[ 5 ]} = .175$$



Memoryless Distribution

- ↓ Does not depend on other factors or what you've seen before
- $\infty$  population

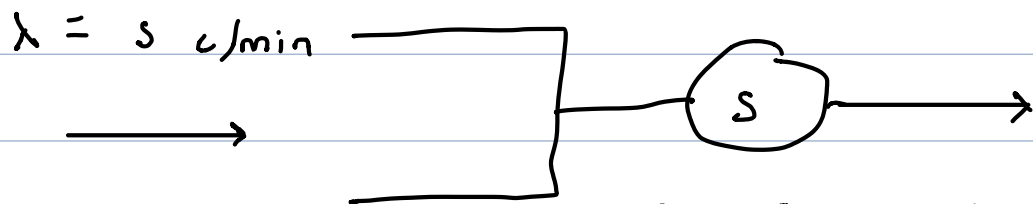
## Departure Process

$T_s$  : Average time to get service  
Exponential Distribution

$\therefore$  therefore  
c : customers

Average service rate ( $\mu$ ) =  $\frac{1}{T_s}$

ex //

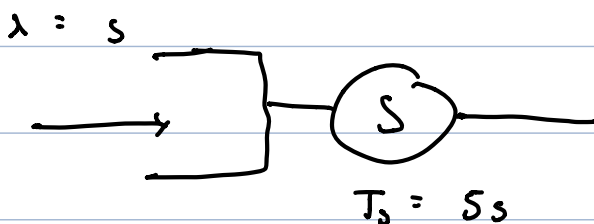


$T_s = 30 \text{ seconds}$

$\mu = 2 \text{ c/min}$

## Problem

Unstable System ( $\lambda > \mu$ )



Avg server  
Utilization  
"rho"  
 $\rho = \frac{5}{12}$

$\mu = 12 \text{ c/min} \rightarrow$  This is stable b/c

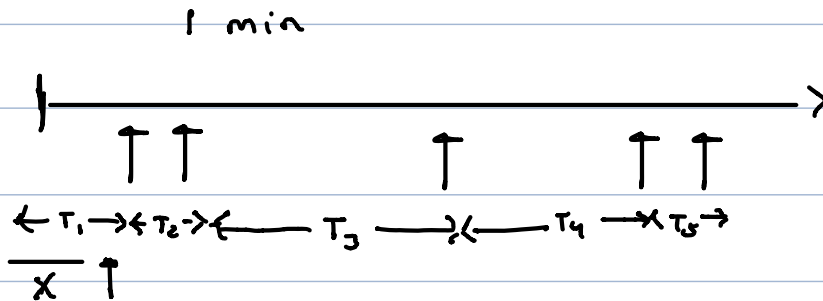
( $\lambda < \mu$ )  $5 < 12$

$\therefore$  Everything checks out

$$\rho = \lambda * T_s = \frac{\lambda}{\mu}$$

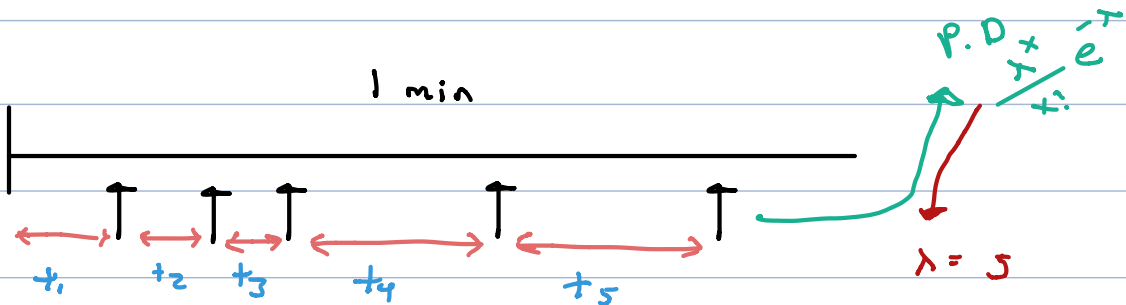
## Exponential Dist

gives prob. distribution of times between events



If the arrivals follow a poisson distribution, the inter-arrival times follow an Exp. Distribution

$$CDF = 1 - e^{-\lambda x}$$



5 customers / min

$T_s = 12$  seconds

Probability [we observe the first arrival after 60 seconds]

Probability [no arrivals happen in the first 60 seconds]

5 customers per min  $\lambda = 5$



$$\frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda} = .00674$$

Prob [we observe the first arrival after 30 seconds]  
 $\equiv$  [no arrivals in the first 30 seconds]

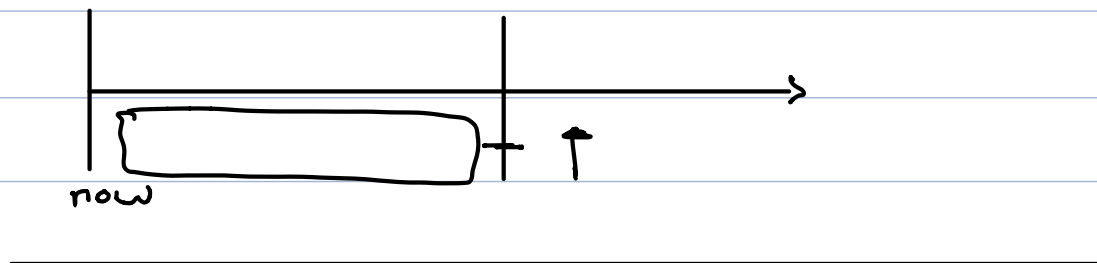
$$\frac{(2.5)^0}{0!} = e^{-2.5} = 0.082$$

Prob [we observe ..... 12 seconds]  
 $\equiv$  [No arrivals in the first 12 seconds]

$$\frac{(1)^0}{0!} e^{-1} = 0.3678$$

Prob we observe ..... 6 seconds  
 $\equiv$  [No arrivals in the first 6 seconds]

$$= e^{-0.5} = 0.605$$



Recap

- Population is infinite (Markov property)
- $\rightarrow$  Queue size is infinite

$M / M / 1$   
Arrival  $\nearrow$   $\uparrow$   $\nwarrow$  # of servers  
Markov  
Departure  
Markov

$M / D / 1$   
 $\uparrow$  Deterministic