

## Strong Form

$$\begin{aligned} \frac{1}{\epsilon} \boldsymbol{\sigma} - \nabla u &= 0 & \text{in } \Omega \\ -\nabla \cdot (\boldsymbol{\sigma} - \beta u) &= f & \text{in } \Omega \\ u &= u_0 & \text{in } \partial\Omega \end{aligned}$$

## Weak Form (Residual Form)

CG

$$\begin{aligned} F_u &= \int_{\Omega} (\boldsymbol{\sigma} - \beta u) \cdot \nabla v - f v \, dx - \int_{\partial\Omega} (\sigma_n - \beta_n u) v \, ds &= 0 \quad \forall v \\ F_{\sigma} &= \int_{\Omega} \frac{1}{\epsilon} \boldsymbol{\sigma} \cdot \boldsymbol{\tau} + u \nabla \cdot \boldsymbol{\tau} \, dx - \int_{\partial\Omega} u \boldsymbol{\tau} \cdot \mathbf{n} \, ds &= 0 \quad \forall \boldsymbol{\tau} \end{aligned}$$

Now let

$$u = \sum_i u_i \phi_i, \quad \boldsymbol{\sigma} = \sum_i \boldsymbol{\sigma}_i \psi_i$$

To calculate jacobians, differentiate with respect to  $u_i$  and  $\boldsymbol{\sigma}_i$

$$K_{uu} = \int_{\Omega} -\beta \phi \cdot \nabla v \, dx - \int_{\partial\Omega} -\beta_n \phi v \, ds$$

$$K_{u\sigma} = \int_{\Omega} \boldsymbol{\psi} \cdot \nabla v \, dx - \int_{\partial\Omega} \psi_n v \, ds$$

$$K_{\sigma u} = \int_{\Omega} \phi \nabla \cdot \boldsymbol{\tau} \, dx - \int_{\partial\Omega} \phi \boldsymbol{\tau} \cdot \mathbf{n} \, ds$$

$$K_{\sigma\sigma} = \int_{\Omega} \frac{1}{\epsilon} \boldsymbol{\psi} \cdot \boldsymbol{\tau} \, dx$$

DPG

$$\begin{aligned} F_u &= \int_K (\boldsymbol{\sigma} - \beta u) \cdot \nabla v - f v \, dx - \int_{\partial K} (\widehat{\sigma_n - \beta_n u}) \text{sgn}(n) v \, ds &= 0 \quad \forall v \\ F_{\sigma} &= \int_K \frac{1}{\epsilon} \boldsymbol{\sigma} \cdot \boldsymbol{\tau} + u \nabla \cdot \boldsymbol{\tau} \, dx - \int_{\partial K} \hat{u} \boldsymbol{\tau} \cdot \mathbf{n} \, ds &= 0 \quad \forall \boldsymbol{\tau} \end{aligned}$$