EM386M/CAM386M FUNCTIONAL ANALYSIS IN THEORETICAL MECHANICS
List of Theorems

Exam 1:

1. Properties of direct and inverse image

2. Comparability of cardinal numbers

3. Properties of open sets in $\mathbb{R}^n$

4. Properties of closed sets in $\mathbb{R}^n$

5. Relation between open and closed sets

6. Properties of the interior operation

7. Properties of the closure operation

8. Characterization of accumulation (limit) points with sequences

9. Equivalence of continuity and sequential continuity in $\mathbb{R}^n$

10. The Bolzano-Weiestrass Theorem for Sets

11. The Bolzano-Weiestrass Theorem for Sequences

12. The Weierstrass Theorem

Exam 2:

1. Characterization of $\liminf$

2. Characterization of a direct sum of two vector subspaces

3. Characterization of a Hamel basis

4. Existence of a Hamel basis in a vector space (w/o proof)

5. Rank and Nullity Theorem

6. Characterization of a projection

7. Construction of a dual basis in a finite-dimensional space,

8. Properties of orthogonal complements

9. Properties of transpose operators

10. Relation between ranks of a linear map and the rank of its transpose

11. Cauchy-Schwartz Inequality

12. Properties of adjoint operators

Exam 3:

1. Properties of a σ-algebra (Prop. 3.1.1)

2. Properties of an (abstract) measure (Prop. 3.1.6)

3. Properties of Borel sets (Prop. 3.1.4, 3.1.5 combined)

4. Characterization of Lebesgue measurable sets (Prop. 3.2.3, Thm 3.2.1)

5. Properties of measurable (Borel) functions (Prop. 3.4.1)

6. Properties of Lebesgue integral (Prop. 3.5.1)

7. Fatou's Lemma

8. Lebesgue Dominated Convergence Theorem (for non-negative functions, Thm. 3.5.2)

9. Hölder and Minkowski inequalities,

10. Properties of open sets, properties of closed sets, properties of the operations of interior and closure (all in context of general topological spaces),

11. Characterization of open and closed sets in a topological subspace,

12. Characterization of (globally) continuous functions (Prop. 4.3.2),

13. Properties of compact sets,

14. The Heine-Borel Theorem,

15. The Weierstrass Theorem,

16. Properties of sequentially compact sets (Prop. 4.4.5),

Additional material for the final:

1. Hölder and Minkowski inequalities for sequences,

2. Completness of Chebyshev, l^p, and L^p spaces,

3. Bolzano-Weiestrass Theorem,

4. Banach Contractive Map Theorem.