$$
\begin{aligned}
\delta_{ii} &= 3\,, \\
\delta_{ij}\delta_{jk} &= \delta_{ik}\,, \\
\epsilon_{ijk}\epsilon_{imn} &= \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}\,, \\
\epsilon_{ijk}\epsilon_{ijm} &= 2\delta_{km}\,, \\
\epsilon_{ijk}\epsilon_{ijk} &= 6\,.
\end{aligned}
$$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \epsilon_{ijk} a_i b_j \mathbf{e}_k$$

Gradient of a Scalar Field

$$\nabla\phi = \mathbf{e}_i \partial_i \phi$$

Divergence of a Vector Field

$$\operatorname{div} \mathbf{v} = \partial_i v_i$$

Curl of a Vector Field

$$\operatorname{curl} \mathbf{v} = \epsilon_{ijk} \partial_i v_j \mathbf{e}_k$$

Gradient of a Vector Field

$$\operatorname{grad} \mathbf{v} = \partial_j v_i \mathbf{e}_i \otimes \mathbf{e}_j$$

Divergence of a Tensor Field

$$\operatorname{div} \mathbf{A} = \partial_j A_{ij} \mathbf{e}_i$$

$$\boldsymbol{u} = \boldsymbol{\varphi}(\mathbf{X}) - \mathbf{X}$$

$$\mathbf{F}(\mathbf{X}) = \nabla\boldsymbol{\varphi}(\mathbf{X}) = \mathbf{I} + \nabla\boldsymbol{u}(\mathbf{X})$$

$$\operatorname{Cof} \mathbf{F} = \det \mathbf{F} \mathbf{F}^{-T}$$

$$\mathbf{C} = \mathbf{F}^T \mathbf{F}$$

$$
\begin{aligned}
\mathbf{E} &= \frac{1}{2}(\mathbf{C} - \mathbf{I}) \\
&= \frac{1}{2}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T + \nabla\boldsymbol{u}^T \nabla\boldsymbol{u})
\end{aligned}
$$

$$e_1 = \sqrt{1 + 2E_{11}} - 1$$

$$\sin\gamma_{12} = \frac{2E_{12}}{\sqrt{1+2E_{11}}\sqrt{1+2E_{22}}}$$

$$\frac{\mathrm{d}\psi(\boldsymbol{x},t)}{\mathrm{d}t} = \frac{\partial\psi(\boldsymbol{x},t)}{\partial t} + \boldsymbol{v}(\boldsymbol{x},t)\cdot\frac{\partial\psi(\boldsymbol{x},t)}{\partial\boldsymbol{x}}$$

$$\mathbf{L}(\boldsymbol{x},t) = \operatorname{grad} \boldsymbol{v}(\boldsymbol{x},t)$$

$$\dot{\mathbf{F}} = \mathbf{L}_m \mathbf{F}$$

$$\mathbf{L} = \mathbf{D} + \mathbf{W}$$

$$\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T)$$

$$\mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T)$$

$$\mathbf{W}\boldsymbol{v} = \frac{1}{2}\boldsymbol{\omega} \times \boldsymbol{v}$$

$$\dot{\overline{\det \mathbf{F}}} = \det \mathbf{F} \operatorname{div} \boldsymbol{v}$$

Piola Transform

$$\mathbf{T}_0(\mathbf{X}) = \mathbf{T}(\boldsymbol{x}) \operatorname{Cof} \mathbf{F}(\mathbf{X})$$

$$\operatorname{Div} \mathbf{T}_0 = \det \mathbf{F} \operatorname{div} \mathbf{T}$$

$$\mathbf{T}_0 \boldsymbol{n}_0 \, dA_0 = \mathbf{T} \boldsymbol{n} \, dA$$

$$dA = \det \mathbf{F} \|\mathbf{F}^{-T} \boldsymbol{n}_0\| dA_0$$

$$\boldsymbol{n} = \frac{\operatorname{Cof} \mathbf{F} \boldsymbol{n}_0}{\|\operatorname{Cof} \mathbf{F} \boldsymbol{n}_0\|}$$

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$$

$$\mathbf{C} = \mathbf{U}^2$$

$$\mathbf{B} = \mathbf{V}^2$$

$$(\mathbf{C} - \lambda\mathbf{I})\boldsymbol{m} = \mathbf{0}$$

$$
\begin{aligned}
\det(\mathbf{C} - \lambda\mathbf{I}) = -\lambda^3 + I(\mathbf{C})\lambda^2 \\
- II(\mathbf{C})\lambda + III(\mathbf{C})
\end{aligned}
$$

$$
\begin{aligned}
I(\mathbf{C}) \quad &= \operatorname{tr} \mathbf{C} \\
&= \lambda_1 + \lambda_2 + \lambda_3 \\
II(\mathbf{C}) \quad &= \operatorname{tr} \operatorname{Cof} \mathbf{C} \\
&= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 \\
III(\mathbf{C}) \quad &= \det \mathbf{C} \\
&= \lambda_1\lambda_2\lambda_3
\end{aligned}
$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\omega \Psi \, dx = \int_\omega \frac{\partial\Psi}{\partial t} \, dx + \int_{\partial\omega} \Psi \boldsymbol{v}\cdot\boldsymbol{n} \, dA$$

$$I(\mathcal{B},t) = \int_{\Omega_t} \rho\boldsymbol{v} \, dx$$

$$H(\mathcal{B},t) = \int_{\Omega_t} \boldsymbol{x} \times \rho\boldsymbol{v} \, dx$$

$$\frac{\mathrm{d}I(\mathcal{B},t)}{\mathrm{d}t} = \mathcal{F}(\mathcal{B},t)$$

Cauchy Stress:

$$\mathbf{T} = \frac{\mathbf{P}\mathbf{F}^{\mathbf{T}}}{\det \mathbf{F}} = \frac{\mathbf{F}\mathbf{S}\mathbf{F}^T}{\det \mathbf{F}}$$

First Piola-Kirchhoff Stress:

$$\mathbf{P} = (\det \mathbf{F})\mathbf{T}\mathbf{F}^{-T} = \mathbf{T} \operatorname{Cof} \mathbf{F} = \mathbf{F}\mathbf{S}$$

Second Piola-Kirchhoff Stress:

$$\mathbf{S} = (\det \mathbf{F})\mathbf{F}^{-1}\mathbf{T}\mathbf{F}^{-T} = \mathbf{F}^{-1}\mathbf{P}$$

$$
\begin{aligned}
\mathcal{P} &= \int_{\Omega_t} \mathbf{f}\cdot\boldsymbol{v} \, dx + \int_{\partial\Omega_t} \boldsymbol{\sigma}(\boldsymbol{n})\cdot\boldsymbol{v} \, dA \\
&= \frac{\mathrm{d}\kappa}{\mathrm{d}t} + \int_{\Omega_t} \mathbf{T} : \mathbf{D} \, dx
\end{aligned}
$$

$$\mathbf{D} = \frac{1}{2}(\operatorname{grad} \boldsymbol{v} + \operatorname{grad} \boldsymbol{v}^T)$$

$$
\begin{aligned}
&\int_{\Omega_0} \mathbf{f}_0 \cdot \dot{\boldsymbol{u}} \, dX + \int_{\partial\Omega_0} \mathbf{P}\boldsymbol{n}_0 \cdot \dot{\boldsymbol{u}} \, dA_0 \\
&= \int_0 \mathbf{P} : \dot{\mathbf{F}} \, dX + \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}\int_{\Omega_0} \rho_0 \dot{\boldsymbol{u}}\cdot\dot{\boldsymbol{u}} \, dX
\end{aligned}
$$

$$
\begin{aligned}
&\int_{\Omega_0} \mathbf{f}_0 \cdot \dot{\boldsymbol{u}} \, dX + \int_{\partial\Omega_0} \mathbf{F}\mathbf{S}\boldsymbol{n}_0 \cdot \dot{\boldsymbol{u}} \, dA_0 \\
&= \int_0 \mathbf{S} : \dot{\mathbf{E}} \, dX + \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{2}\int_{\Omega_0} \rho_0 \dot{\boldsymbol{u}}\cdot\dot{\boldsymbol{u}} \, dX
\end{aligned}
$$

$$
\begin{aligned}
Q &= \int_{\partial\Omega_t} -\mathbf{q}\cdot\boldsymbol{n} \, dA + \int_{\Omega_t} r \, dx \\
&= \int_{\partial\Omega_0} -\mathbf{q}_0\cdot\boldsymbol{n}_0 \, dA_0 + \int_{\Omega_0} r_0 \, dX
\end{aligned}
$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\kappa + U) = \mathcal{P} + Q$$

Principle of Determinism
Principle of Material Frame Indifference
Principle of Physical Consistency
Principle of Material Symmetry
Principle of Local Action
Dimensional consistency
Existence, well-posedness
Equipresence

$$\psi = e - \theta\eta$$

$$\rho\frac{\mathrm{d}\psi}{\mathrm{d}t} - \rho\eta\frac{\mathrm{d}\theta}{\mathrm{d}t} + \mathbf{T} : \mathbf{D} - \frac{\mathbf{q}}{\theta}\cdot\operatorname{grad}\theta \geq 0$$

$$\rho_0\dot{\psi}_0 - \rho_0\eta_0\dot{\theta} + \mathbf{S} : \dot{\mathbf{E}} - \frac{\mathbf{q}_0}{\theta}\cdot\nabla\theta \geq 0$$

$$\mathbf{S} = \rho_0\frac{\partial\Psi}{\partial\mathbf{E}}$$

$$\eta_0 = -\frac{\partial\Psi}{\partial\theta}$$

Lagrangian	Eulerian
Conservation of Mass	
$\rho_0 = \rho \det \mathbf{F}$	$\dfrac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{v}) = 0$
Conservation of Linear Momentum	
$\rho_0 \ddot{\boldsymbol{u}} = \operatorname{Div} \mathbf{F}\mathbf{S} + \mathbf{f}_0$	$\rho \dfrac{\partial \boldsymbol{v}}{\partial t} + \rho \boldsymbol{v} \cdot \operatorname{grad} \boldsymbol{v} = \operatorname{div} \mathbf{T} + \mathbf{f}$
Conservation of Angular Momentum	
$\mathbf{S} = \mathbf{S}^T$	$\mathbf{T} = \mathbf{T}^T$
Conservation of Energy	
$\rho_0 \dot{e}_0 = \mathbf{S} : \dot{\mathbf{E}} - \operatorname{Div} \mathbf{q}_0 + r_0$	$\rho \dfrac{\mathrm{d}e}{\mathrm{d}t} = \mathbf{T} : \mathbf{D} - \operatorname{div} \mathbf{q} + r$
Second Law of Thermodynamics	
$\rho_0 \dot{\eta}_0 + \operatorname{Div} \dfrac{\mathbf{q}_0}{\theta} - \dfrac{r_0}{\theta} \geq 0$	$\rho \dfrac{\partial \eta}{\partial t} + \rho \boldsymbol{v} \cdot \operatorname{grad} \eta + \operatorname{div} \dfrac{\mathbf{q}}{\theta} - \dfrac{r}{\theta} \geq 0$