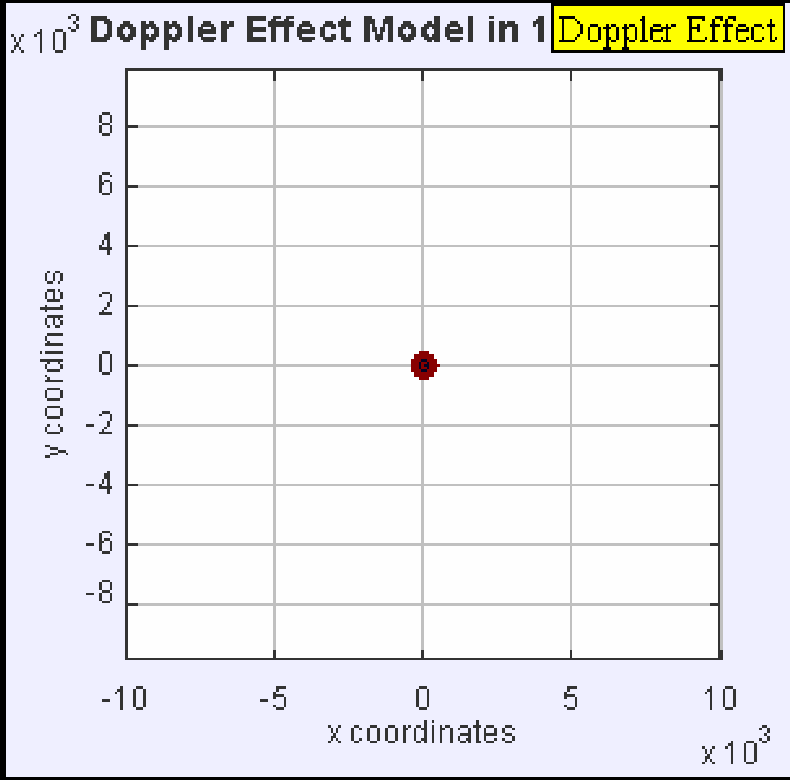
The Radial Velocity Method

The second most successful method for finding exoplanets is the radial velocity or Doppler method.

The radial velocity method complements the transit method, both in its biases and in the additional types of information it can provide.

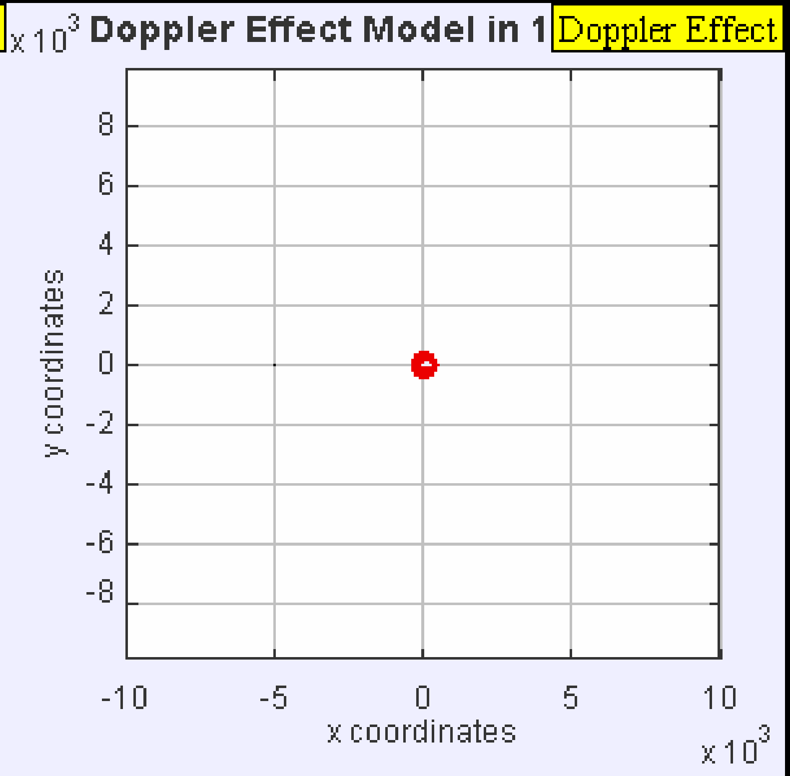
Let's begin by understanding the Doppler Effect.

In the Doppler Effect, relative motion of the source and observer affects the measured wavelength of a wave.



source doesn't move

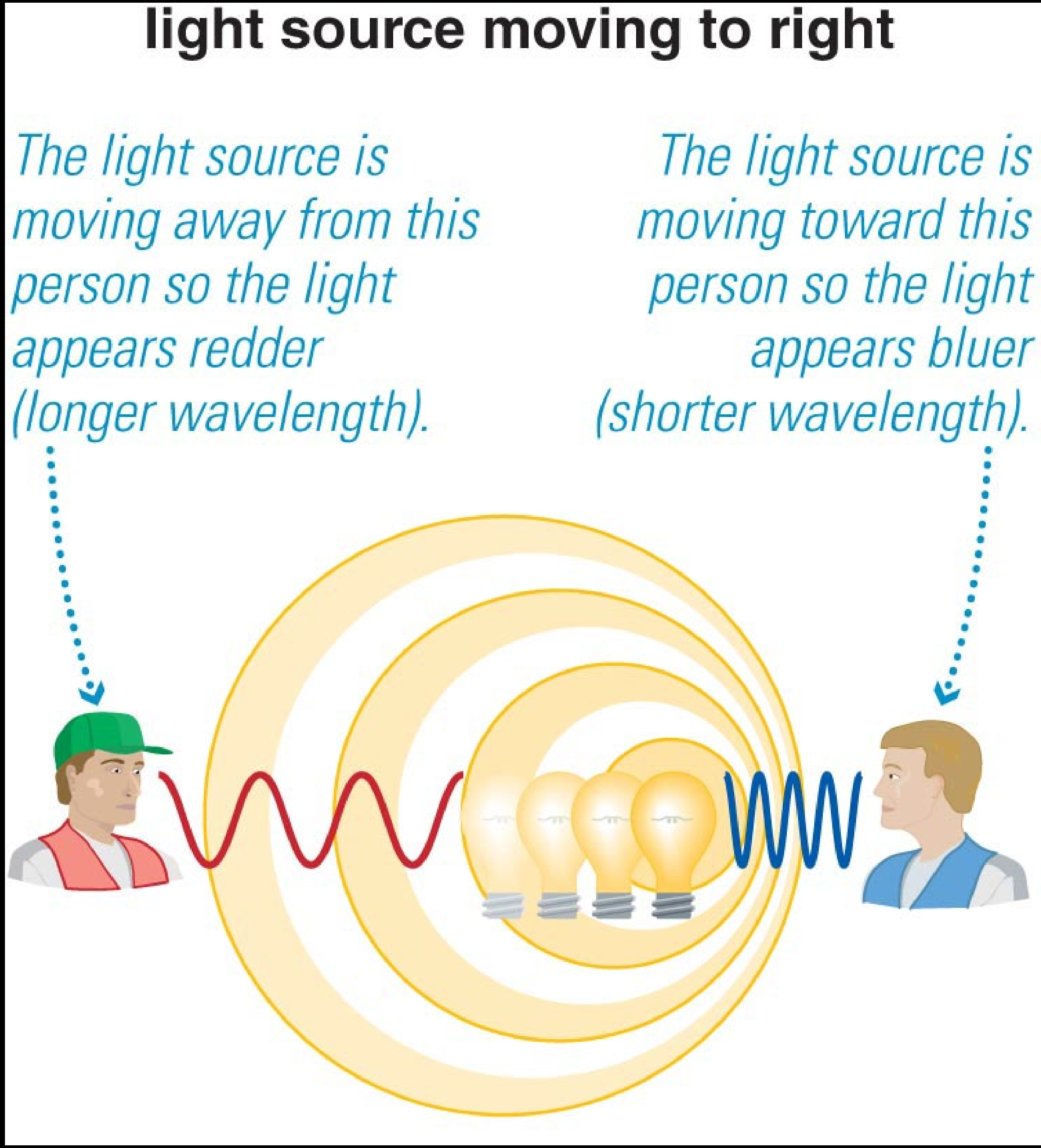
(relative to observer)



source moves

(relative to observer)

In the case of light, we perceive the change in wavelength as a change in colour.

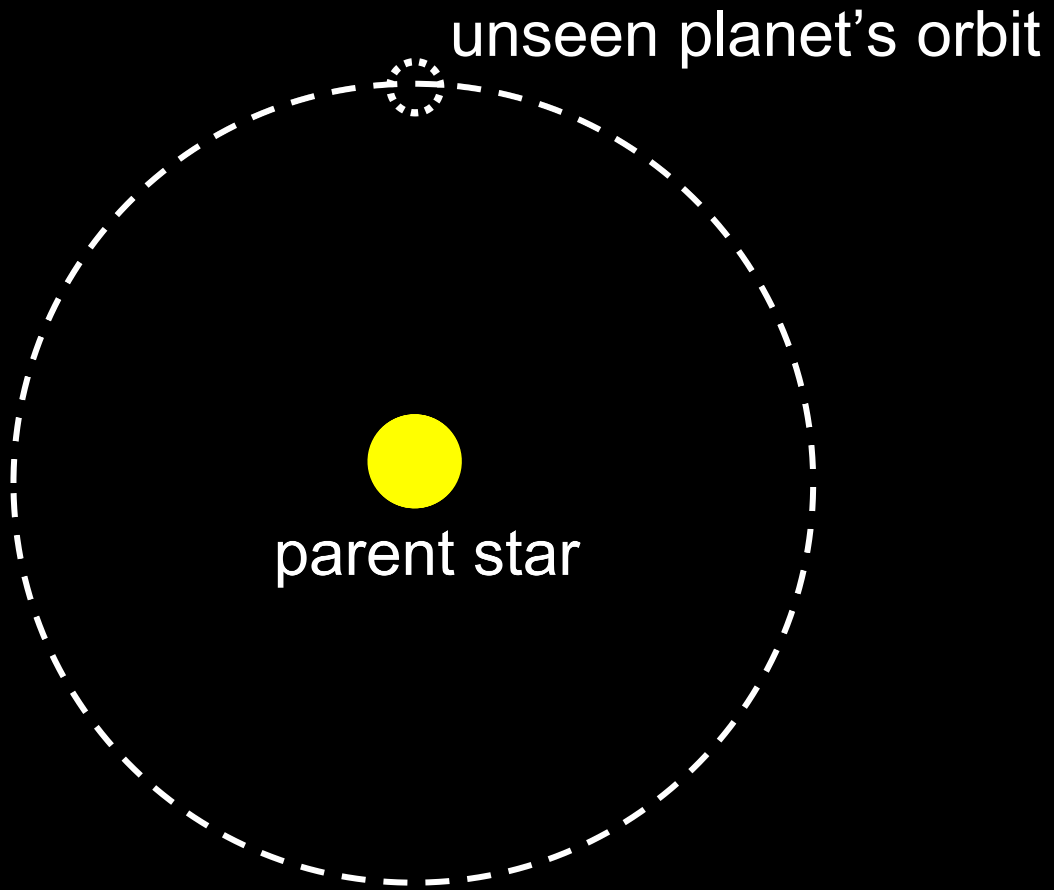


Credit: Pearson Education

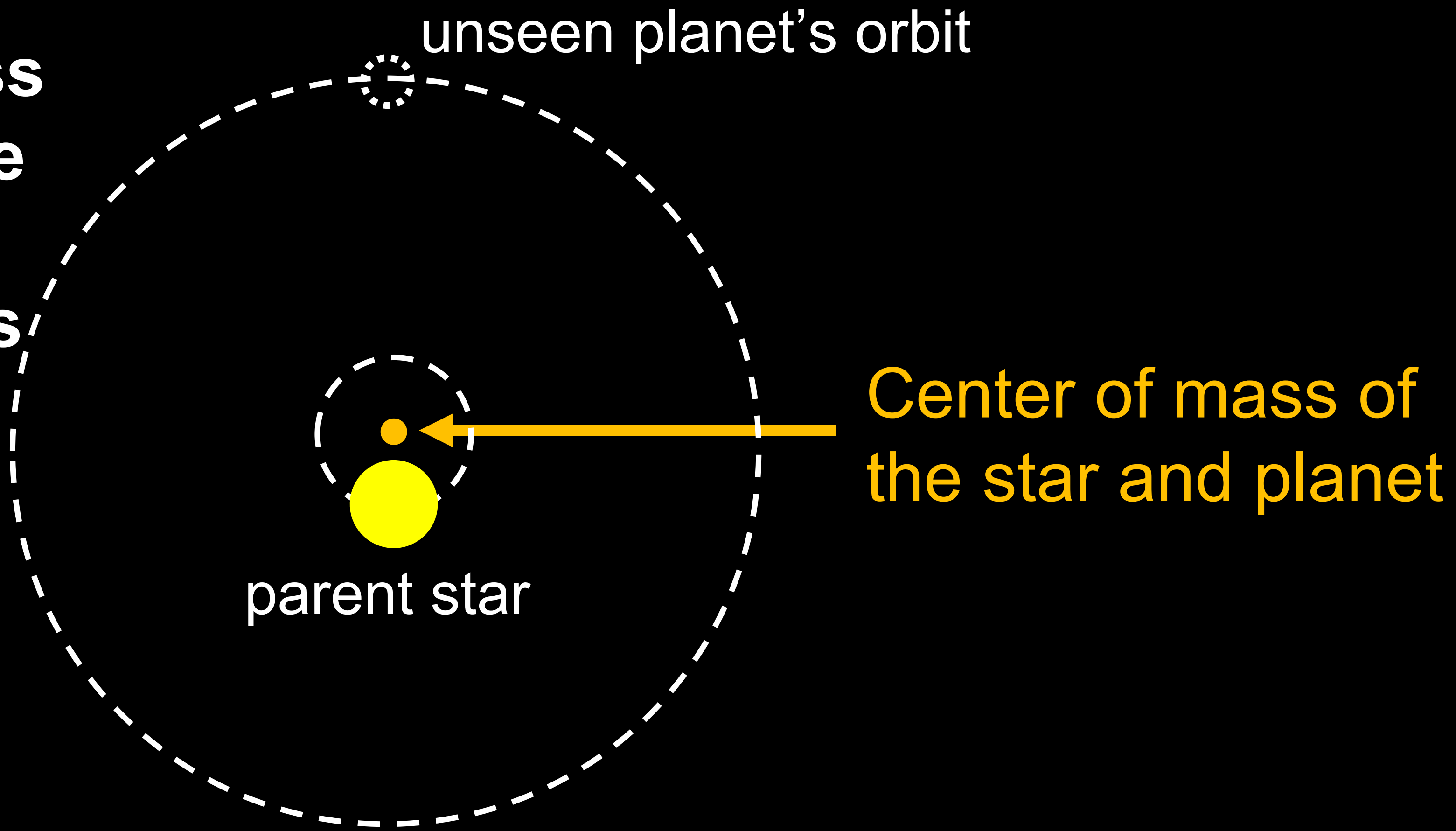
We say “planets orbit stars”, but actually they both orbit their common centre of mass.*

So, remember that with Kepler’s First Law, we should actually say that the star-planet center of mass is at one focus of the planet’s orbit, not the star itself.

We typically imagine that planets orbit stationary stars.

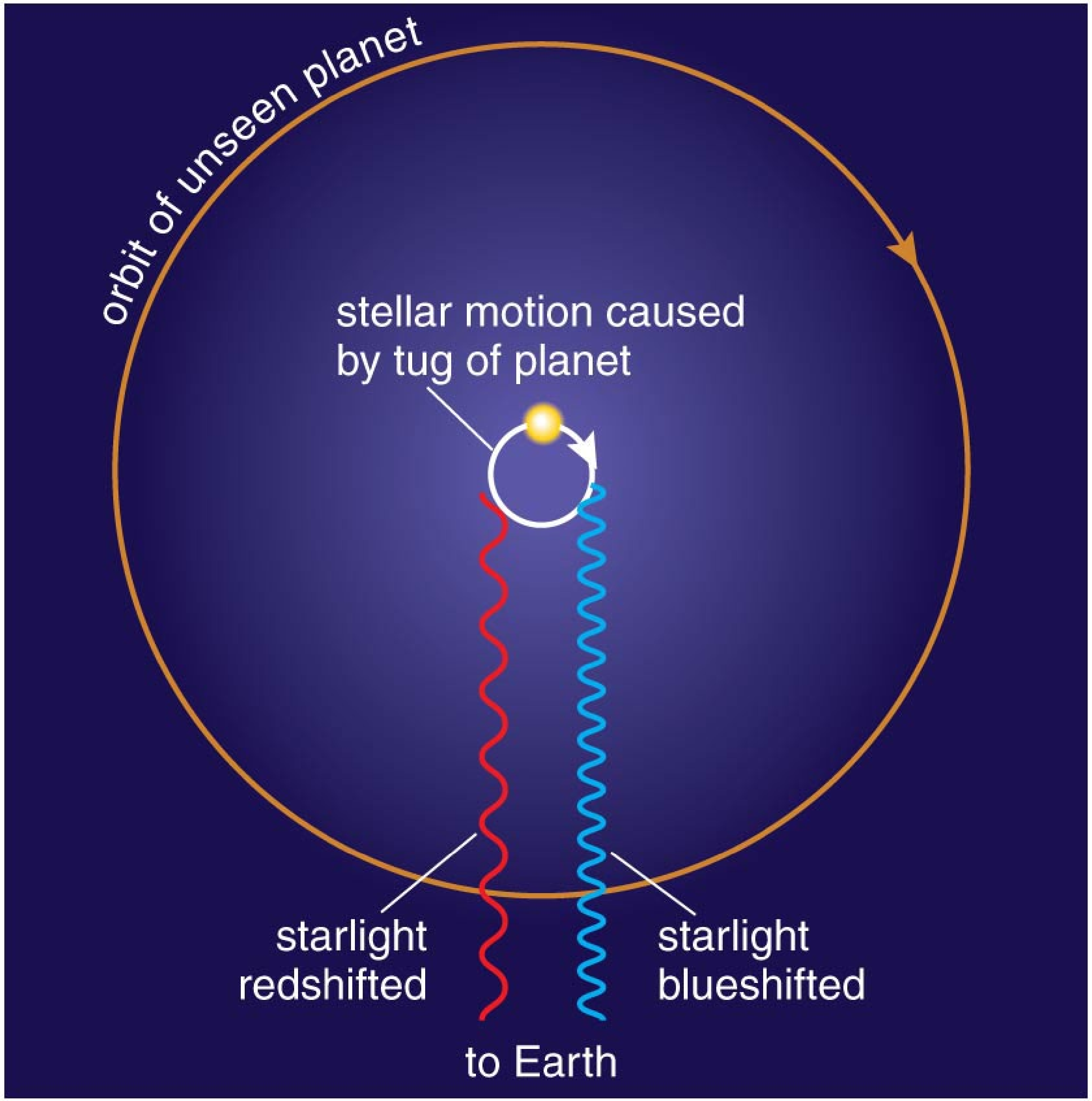


In fact, planets and stars orbit their common centre of mass (which is typically quite close to the centre of the star; the distance is exaggerated here).



If we can detect the motion of the star, we can deduce the existence of the unseen planet!

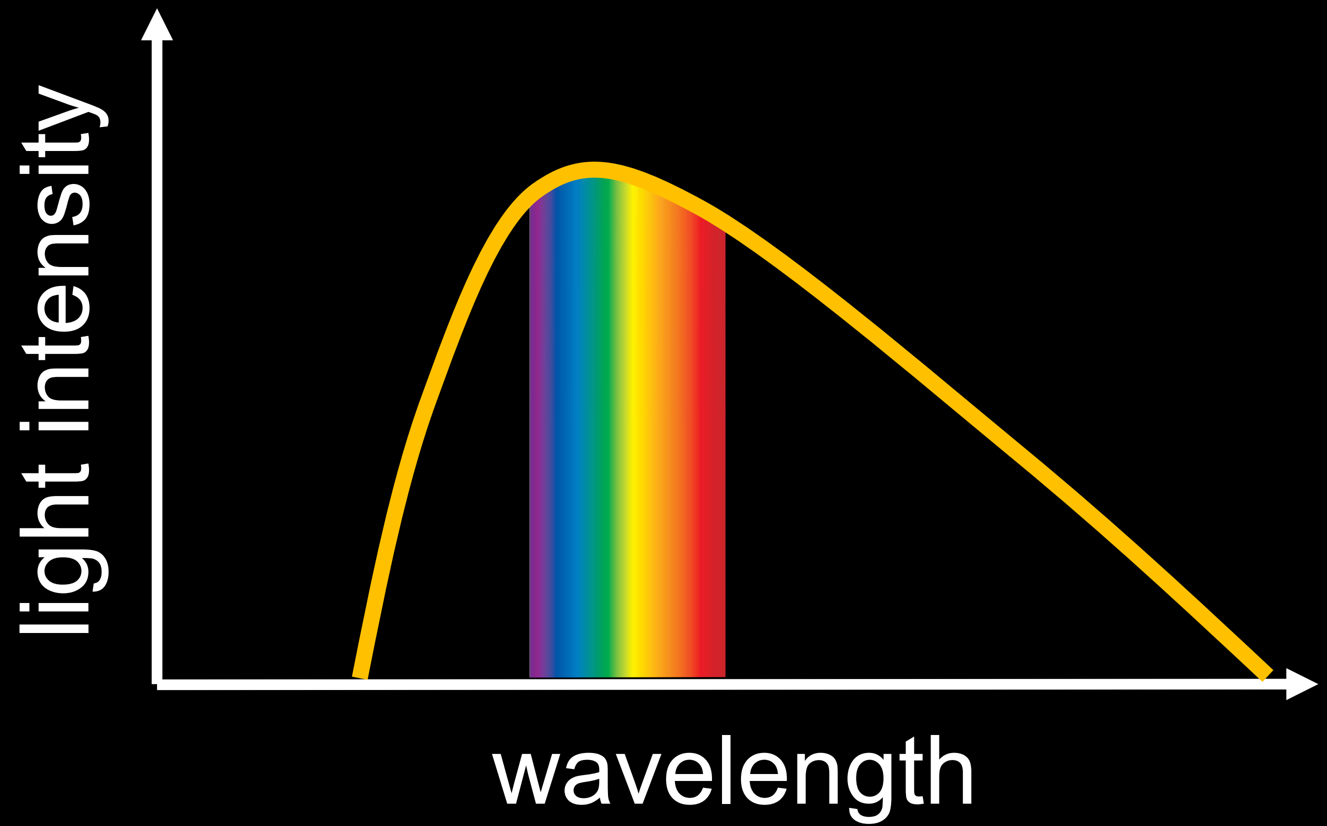
Thanks to the Doppler effect, the light from the star shifts colour toward the red when the star is moving away from us, and toward the blue when it is moving toward us.



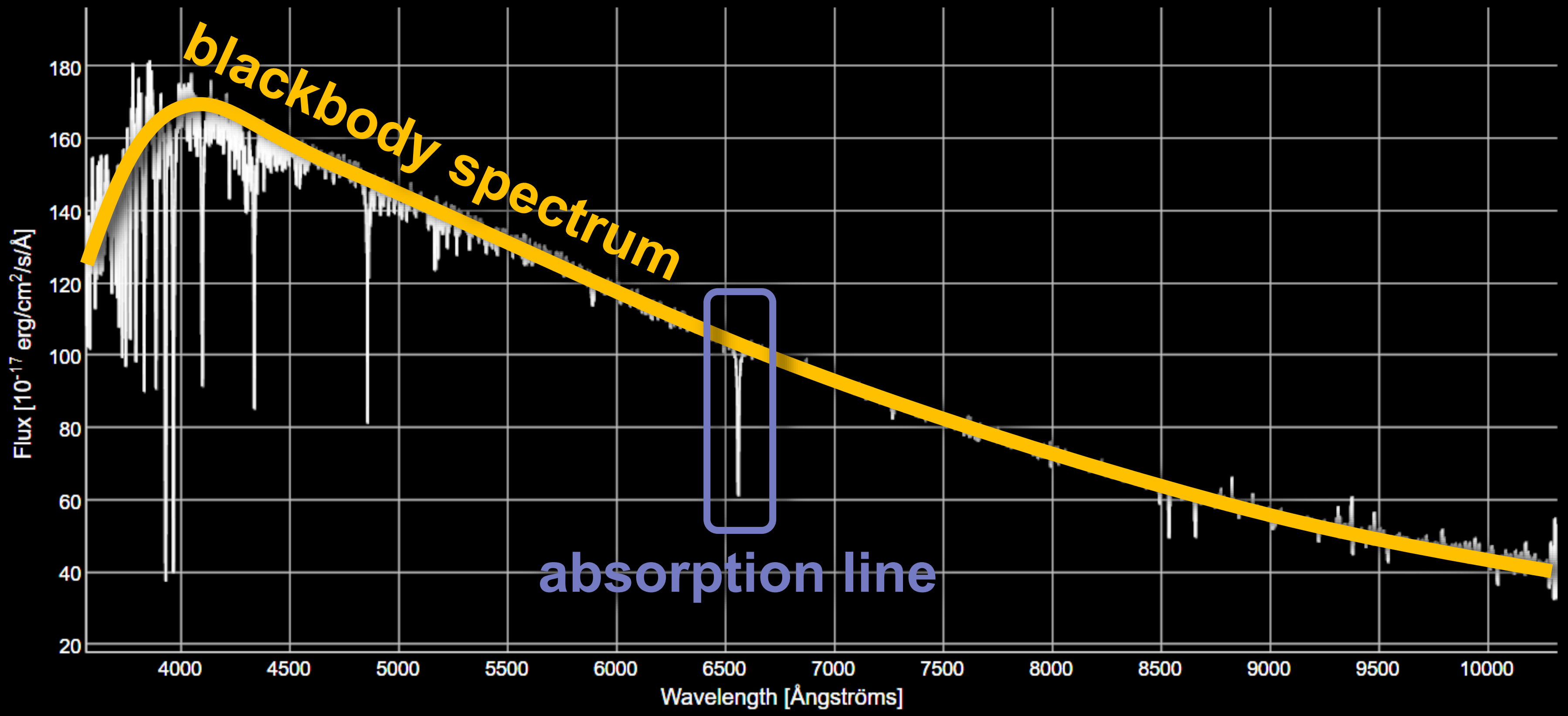
The Doppler shift caused by unseen exoplanets is very small. It doesn’t *visibly* change the colour of a star. Instead, we must measure it spectroscopically.

We can use a spectrometer to spread out the many different wavelengths or colours that make up a beam of light.

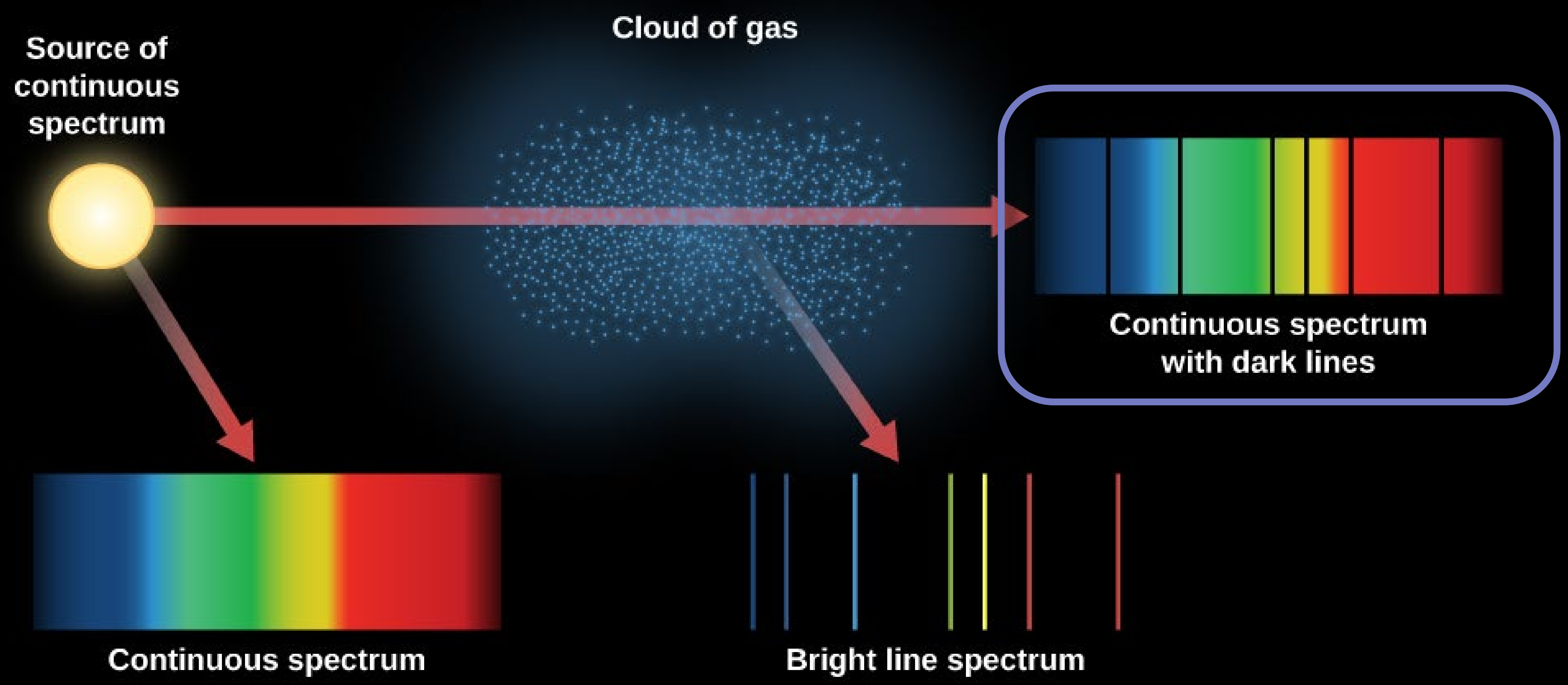
Previously, we said that stars produce a continuous spectrum following the blackbody distribution.



Real stellar spectra have essentially blackbody shapes, but with lots of dips in brightness called "absorption lines".

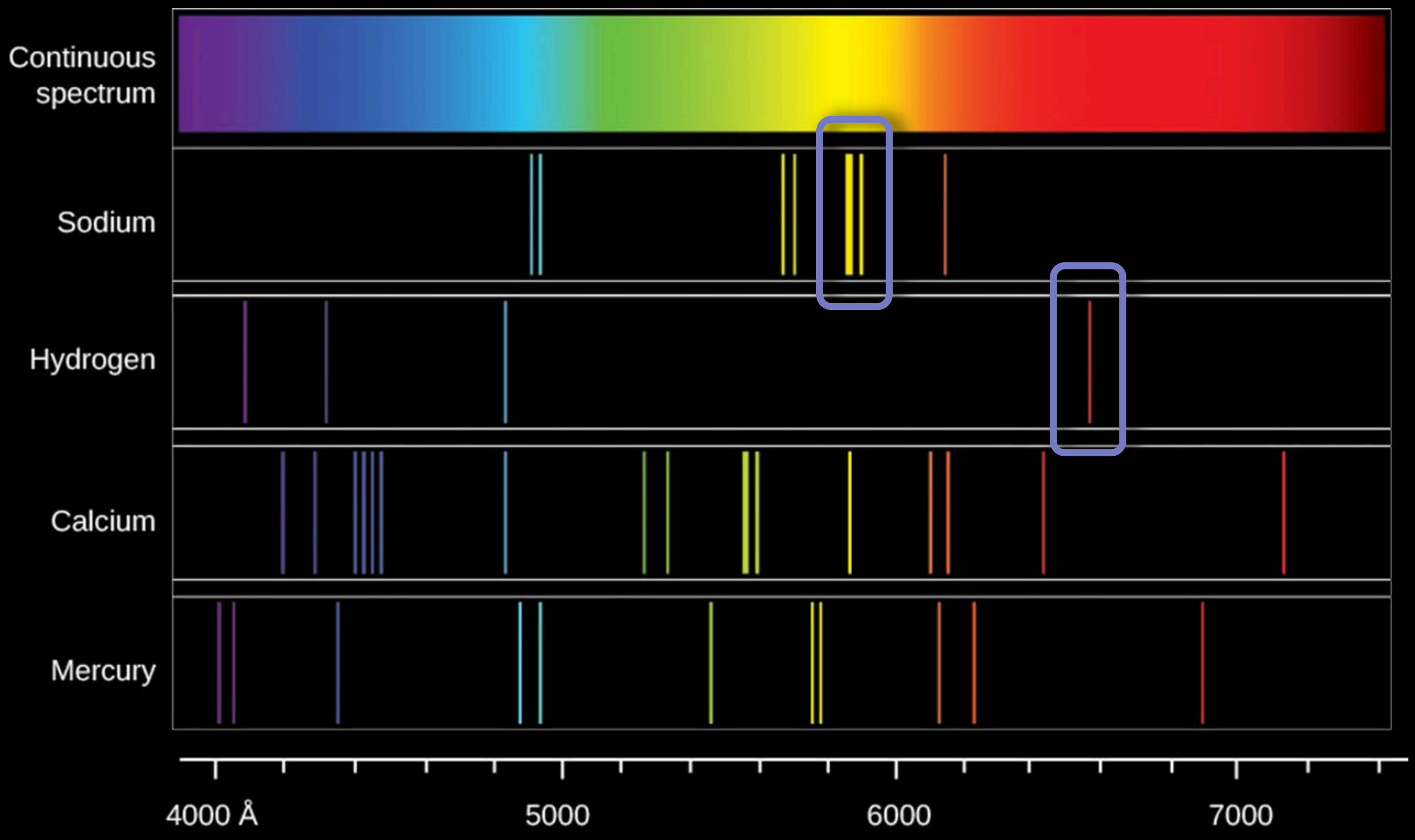


Absorption lines are formed when a thin gas absorbs some of the light produced by a star.



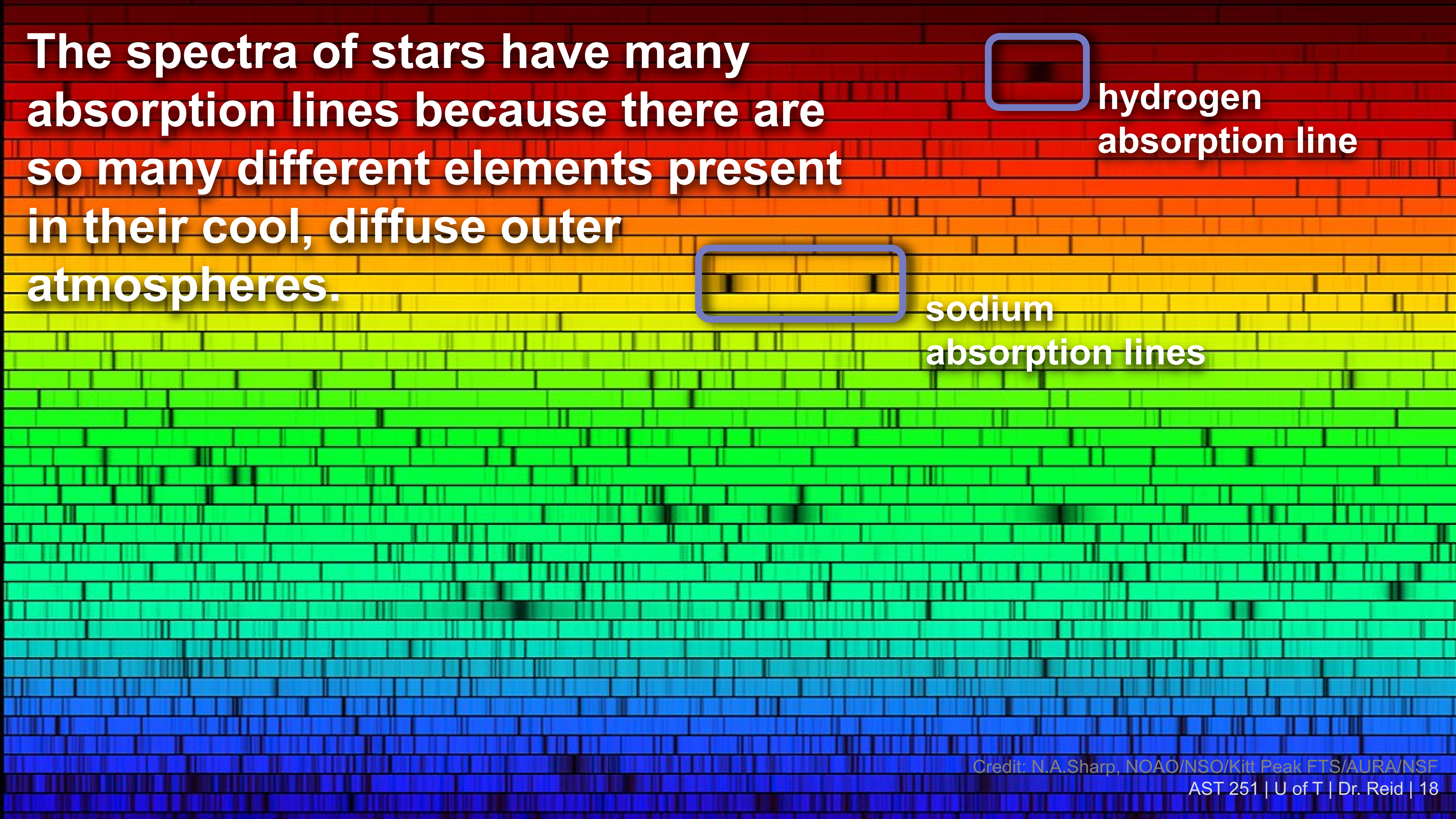
Credit: OpenStax Astronomy

Each chemical element or molecule is capable of absorbing or emitting only very specific wavelengths of light.



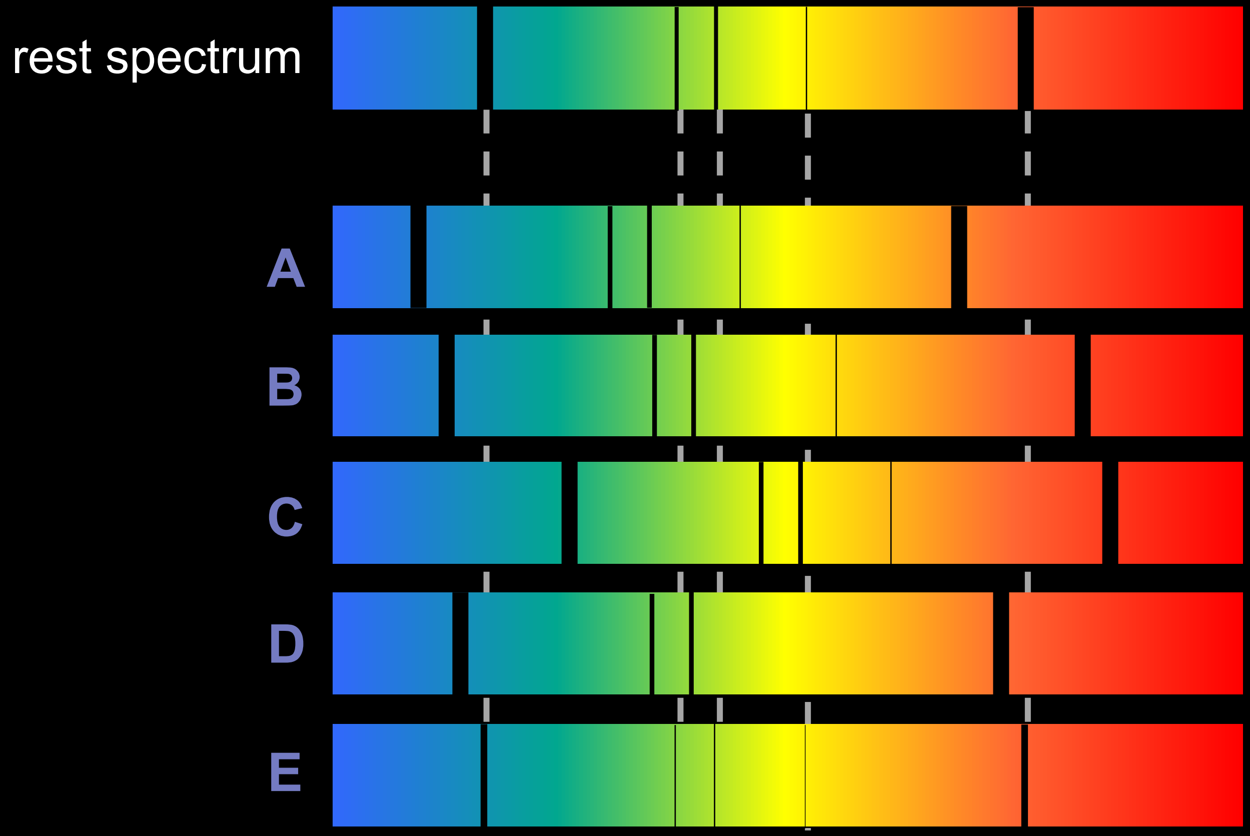
Credit: OpenStax Astronomy

The spectra of stars have many absorption lines because there are so many different elements present in their cool, diffuse outer atmospheres.



Concept Check

Which of these spectra corresponds to a star that is moving toward us?



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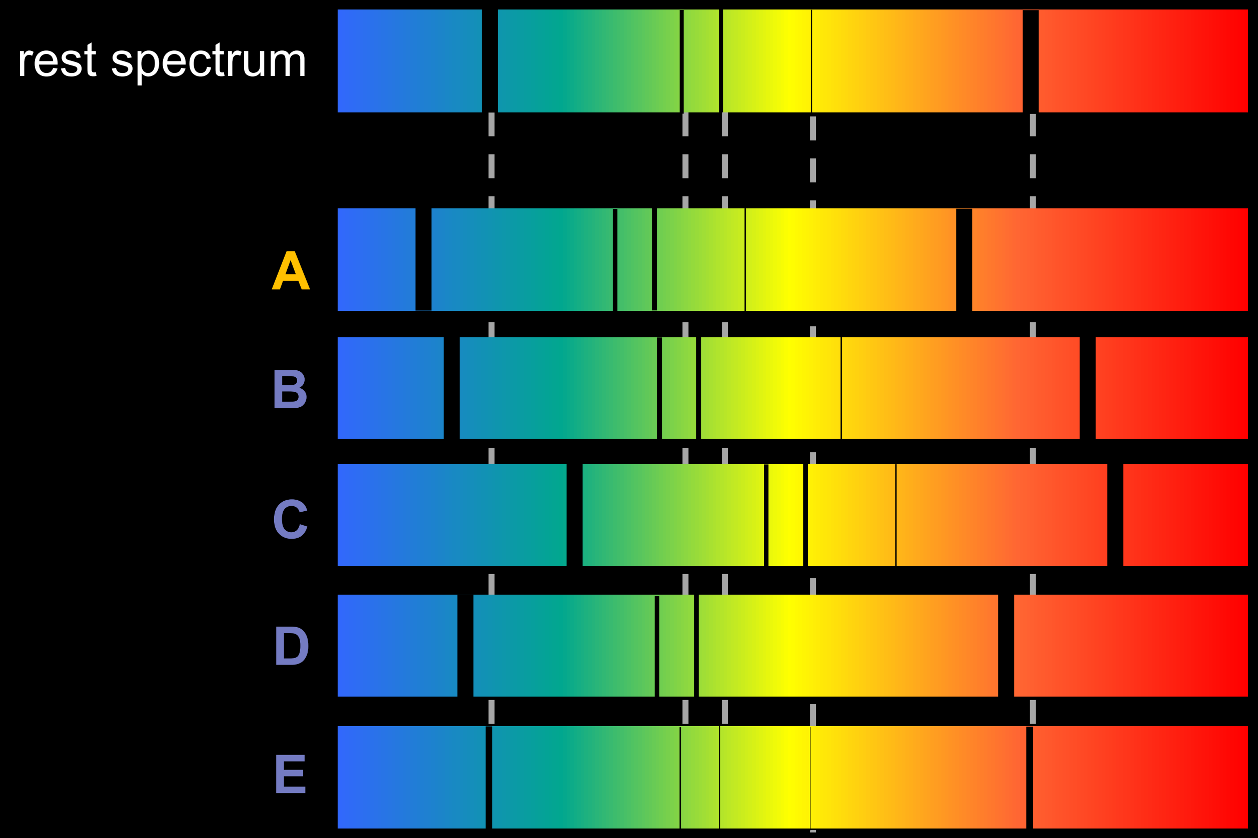


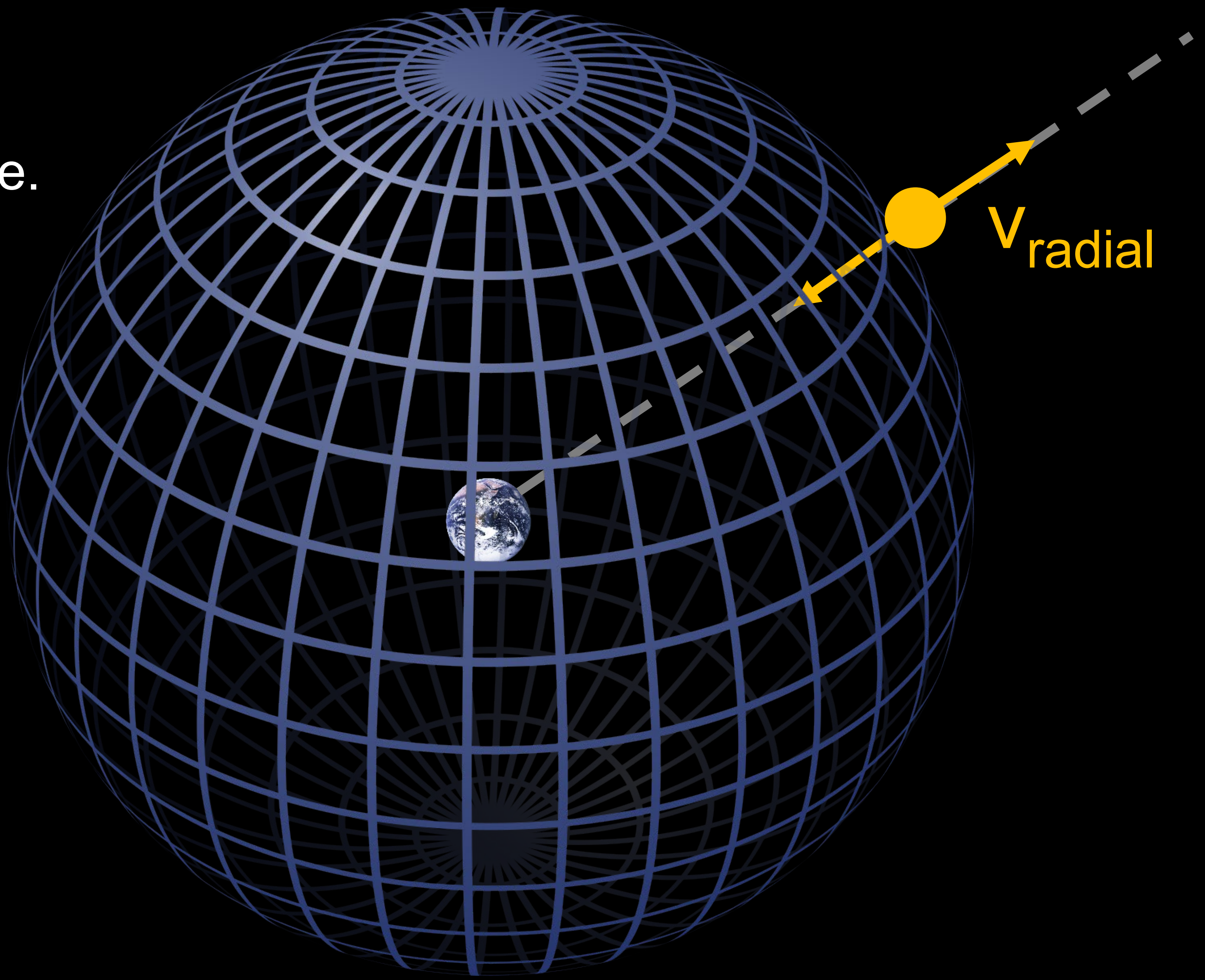
Summary

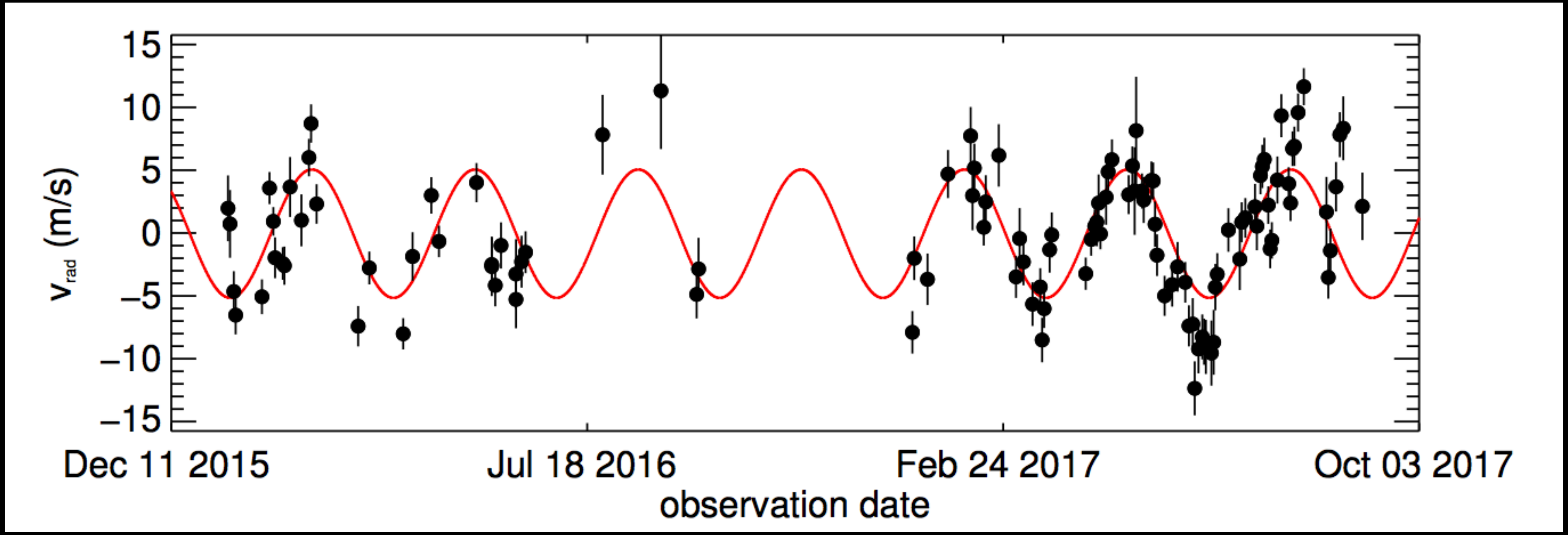
By measuring small Doppler shifts in the wavelengths of a star's spectral lines, we can deduce the existence of unseen planets.

Interpreting Radial Velocity Curves

We can convert measurements of the Doppler shift of a star’s spectrum into measurements of its radial velocity.

We call it the “radial velocity” method because we are measuring motion of the parent star toward and away from us (i.e. along the radius of an imaginary sphere centred on Earth)





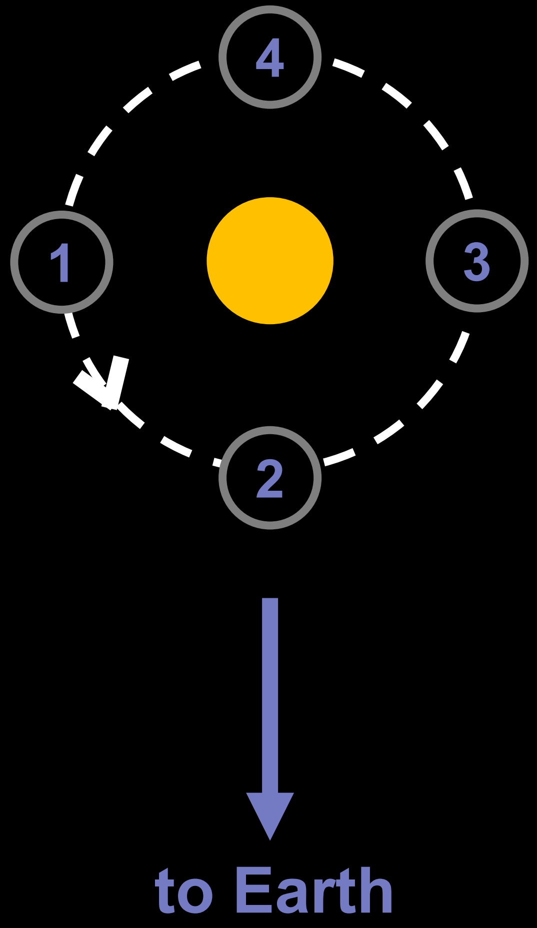
A **radial velocity curve** consists of many measurements of a star’s radial velocity over time.

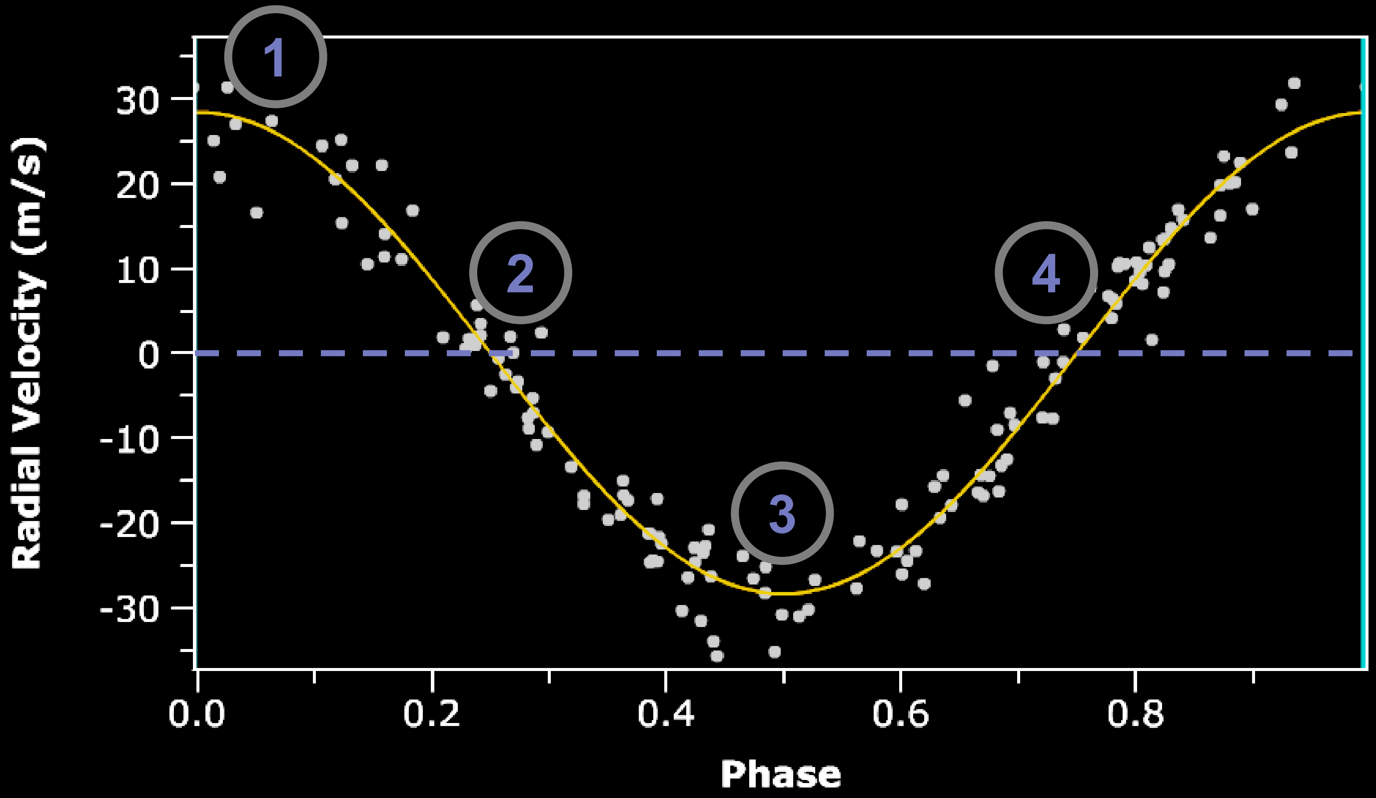
Credit: Reiners et al., A&A, 2017

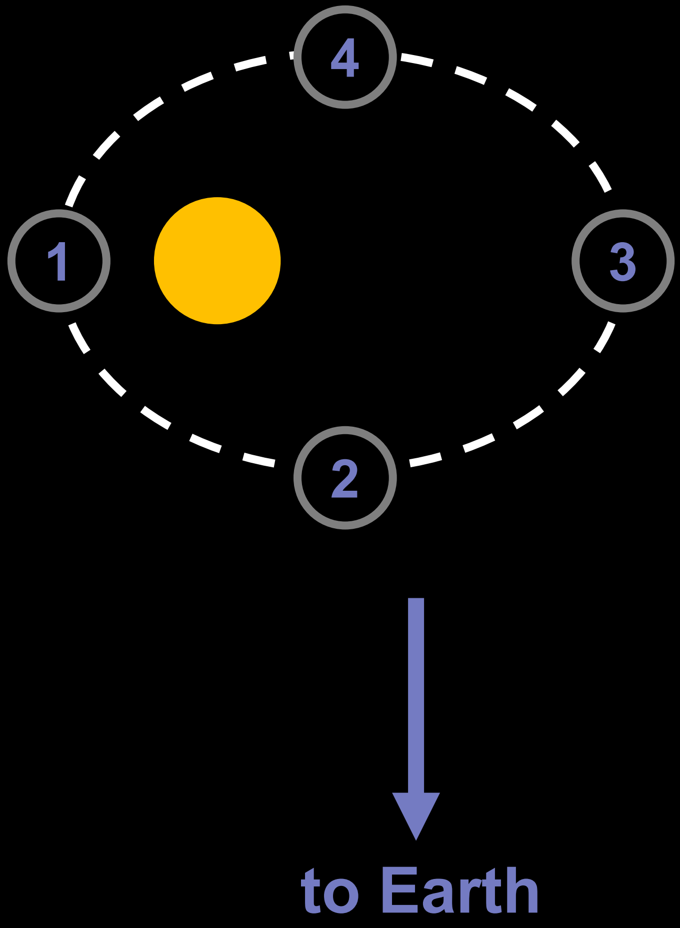
Let’s explore radial velocity curves in real time using a simulator:

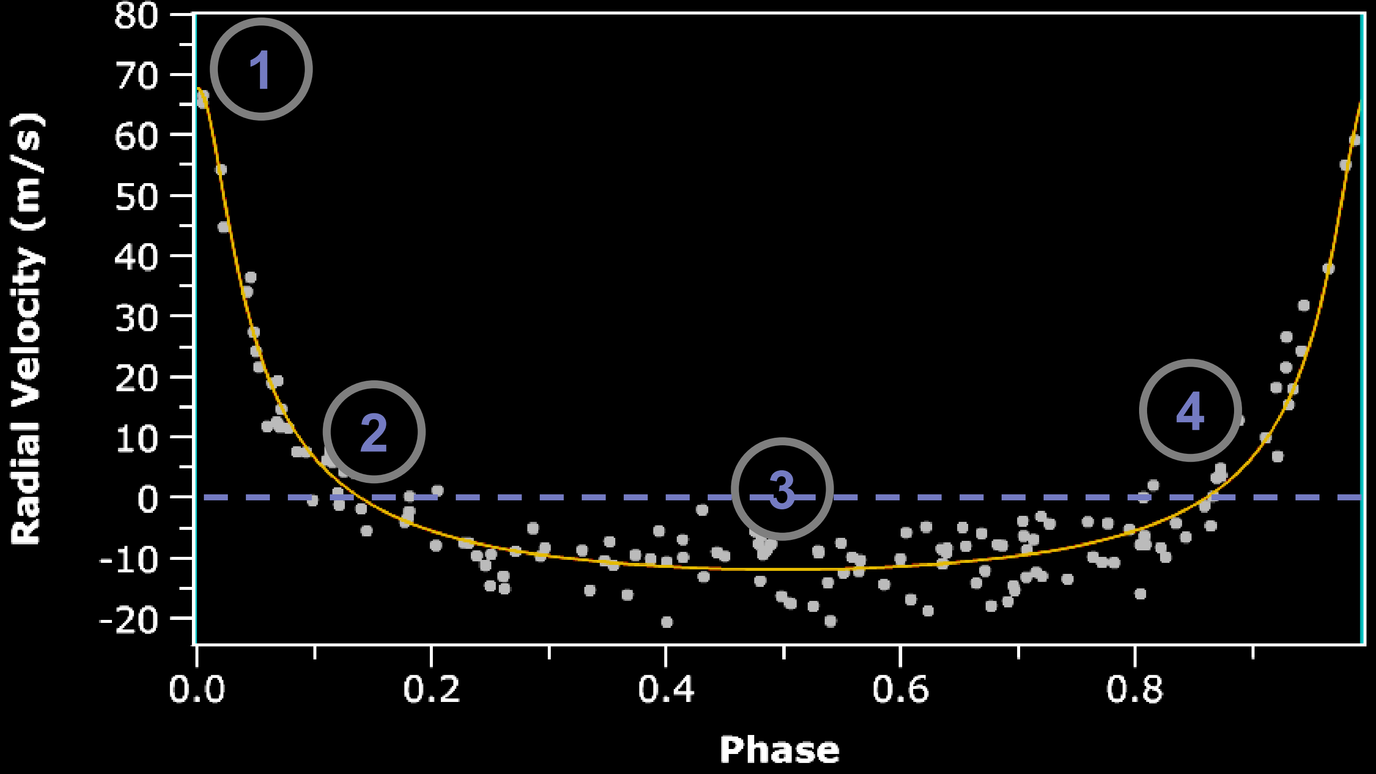
https://astro.unl.edu/nativeapps/

Kepler's Second Law gives us a straightforward way to measure the eccentricity of a planet's orbit using the radial velocity method.


4
1
3
2
to Earth


Radial Velocity (m/s)
30
20
10
0
-10
-20
-30
0.0
0.2
0.4
0.6
0.8
Phase
1
2
3
4


4
1
3
2
to Earth


80
70
60
50
40
30
20
10
0
-10
-20
Radial Velocity (m/s)
1
2
3
4
0.0
0.2
0.4
0.6
0.8
Phase

Using Kepler's Laws, we can derive an equation relating the radial velocity of a star to the properties of its planet:

$$\mathbf{v_{radial}} = \sqrt{\frac{\mathbf{G}}{\mathbf{a(1 - e^2)(M_{star} + M_{planet})}}}\,(\mathbf{M_{planet}\sin i})$$

radial velocity of the star

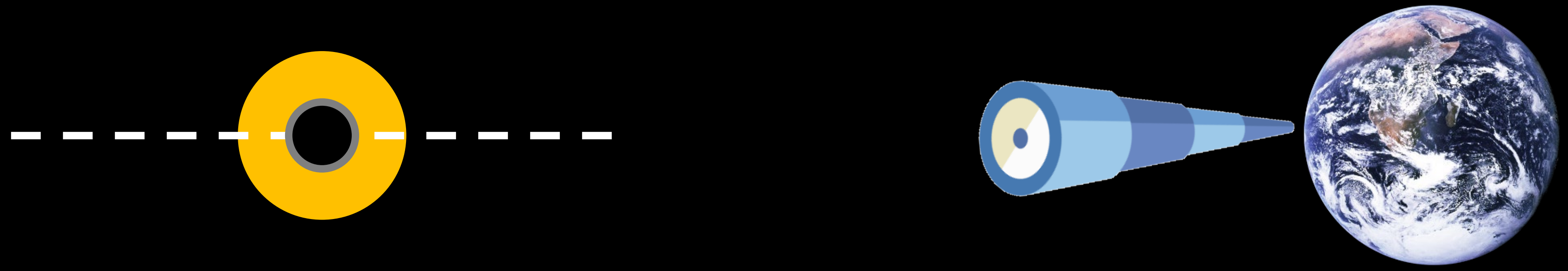
eccentricity of the planet's orbit

inclination of the planet's orbit to our line of sight (0^o for face-on, 90^o for edge-on)

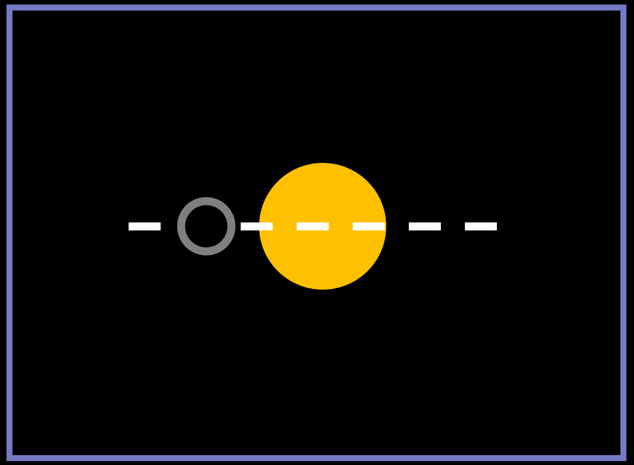
$$v_{radial} = \sqrt{\frac{G}{a(1-e^2)(M_{star} + M_{planet})}}(M_{planet} \sin i)$$

orbital semimajor axis of the planet

masses of the star and planet



view from Earth

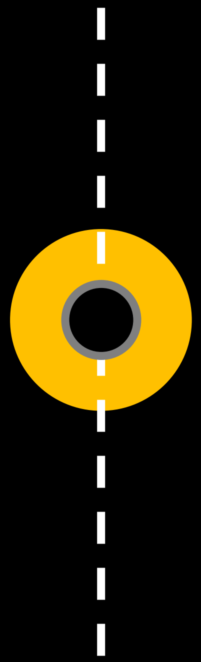


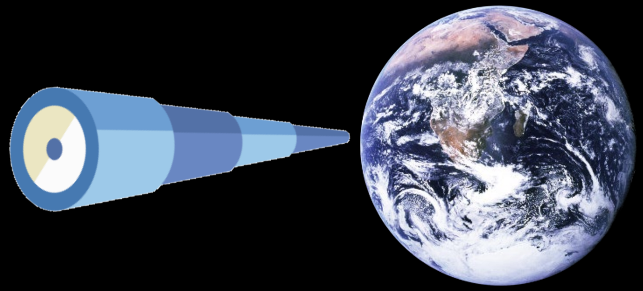
Edge-on orbit:

inclination = 90°

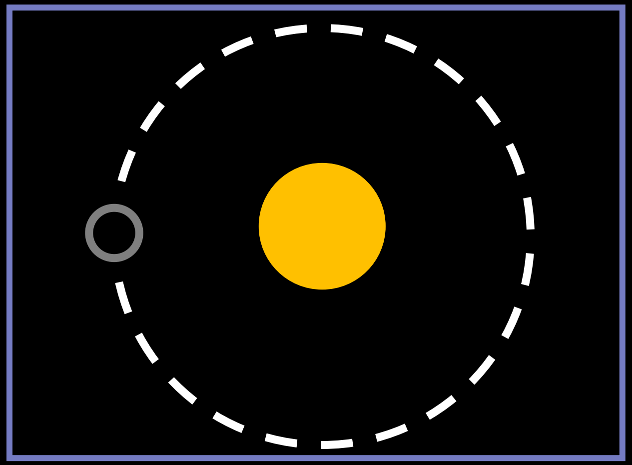
Transit is visible from Earth

Largest possible radial velocity





view from Earth

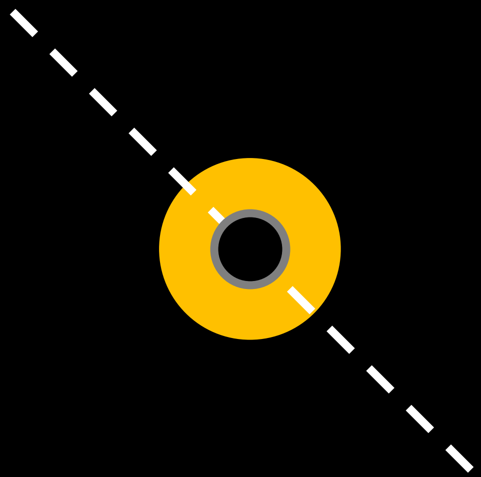


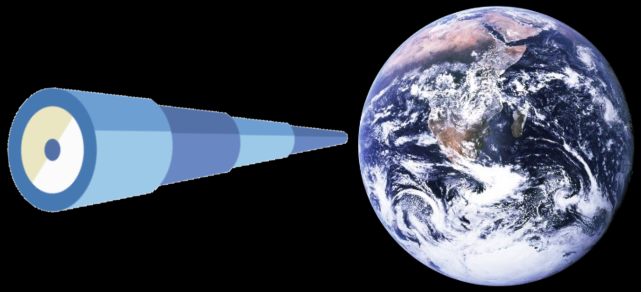
Face-on orbit:

inclination = 0°

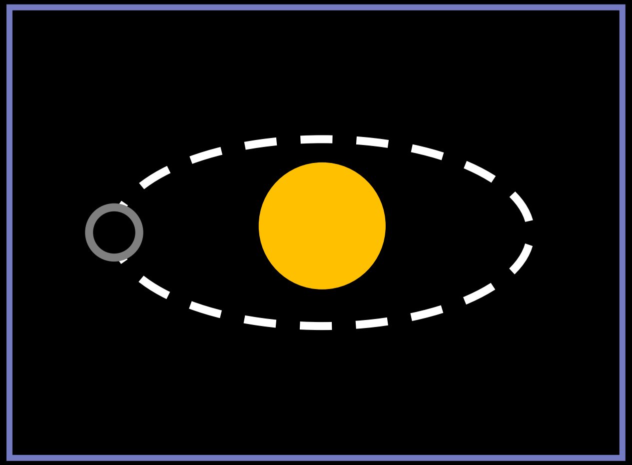
Transit is NOT visible from Earth

NO radial velocity





view from Earth

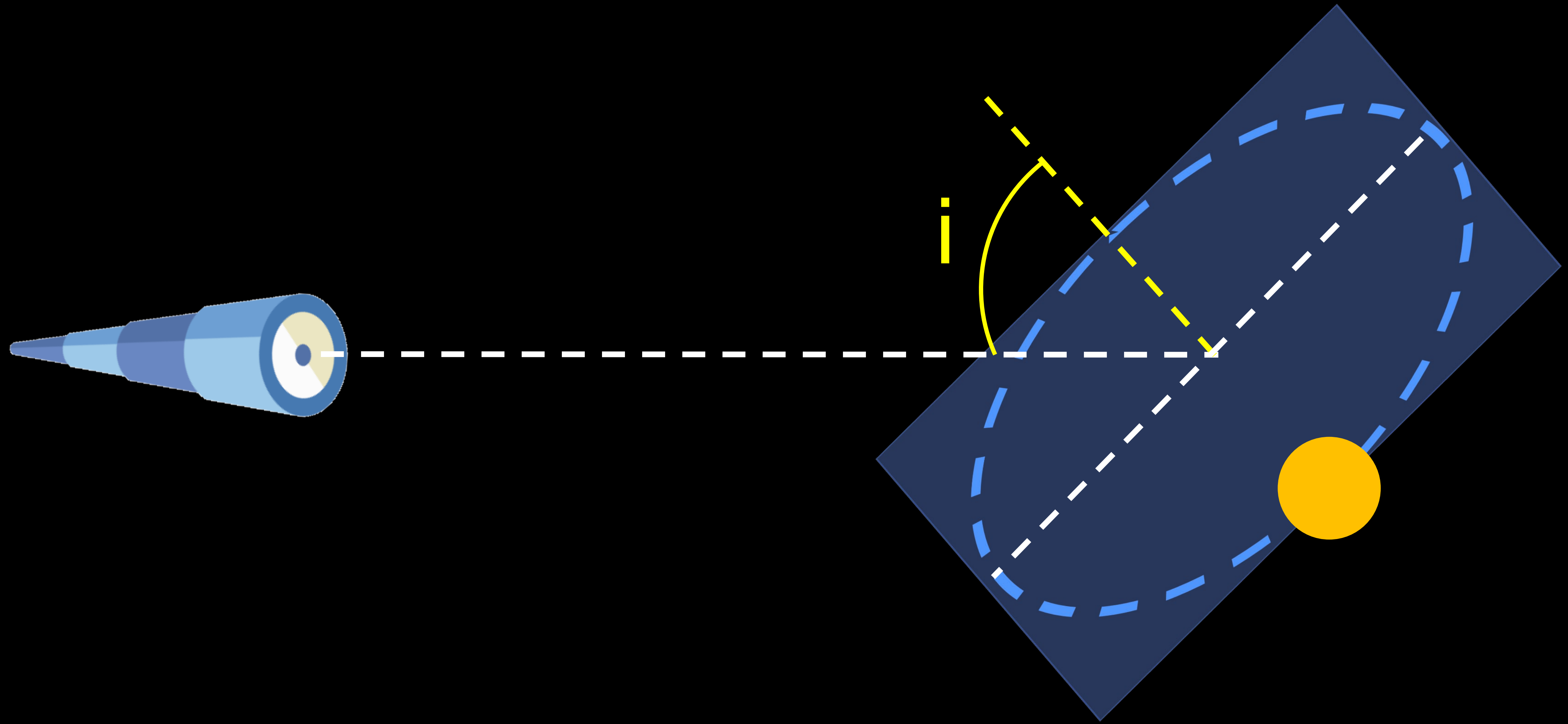


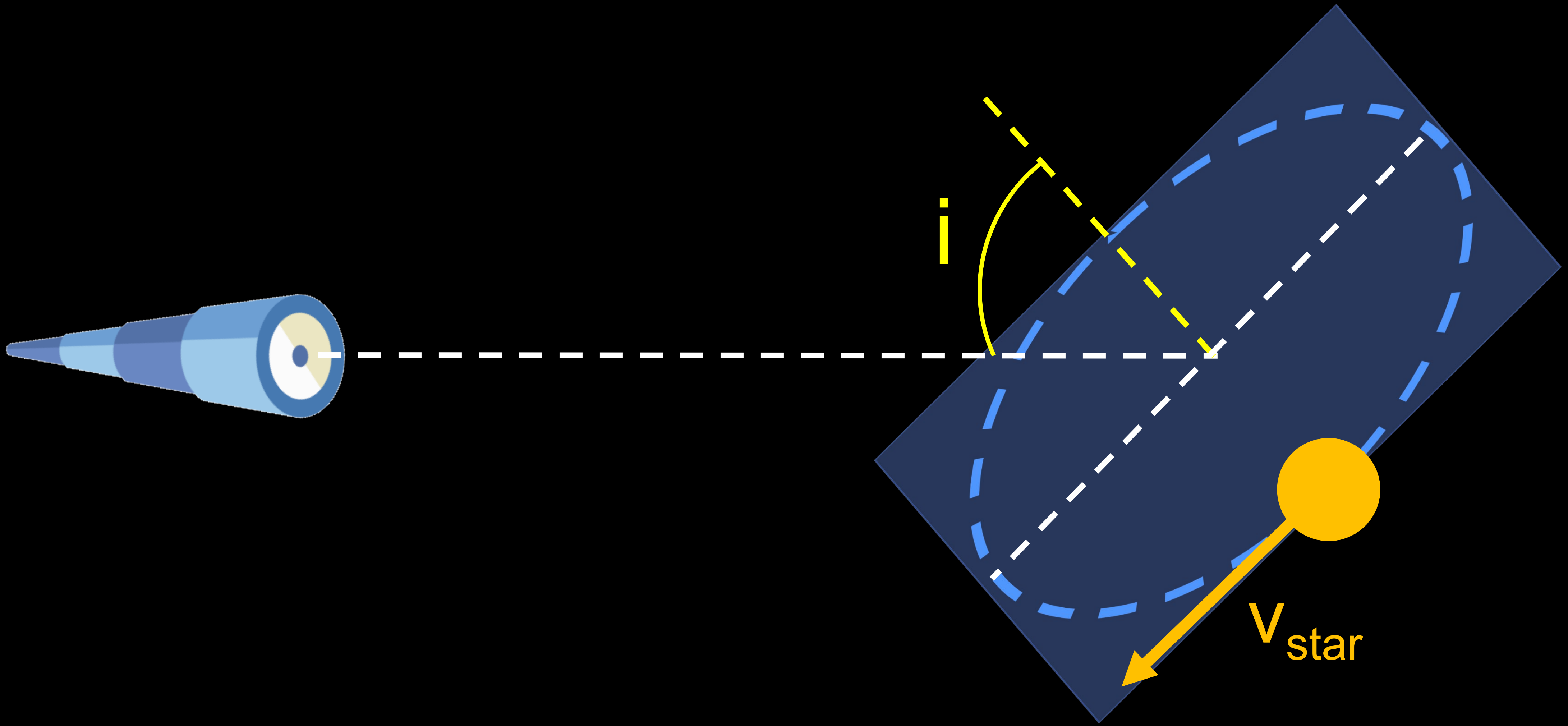
Most other inclinations

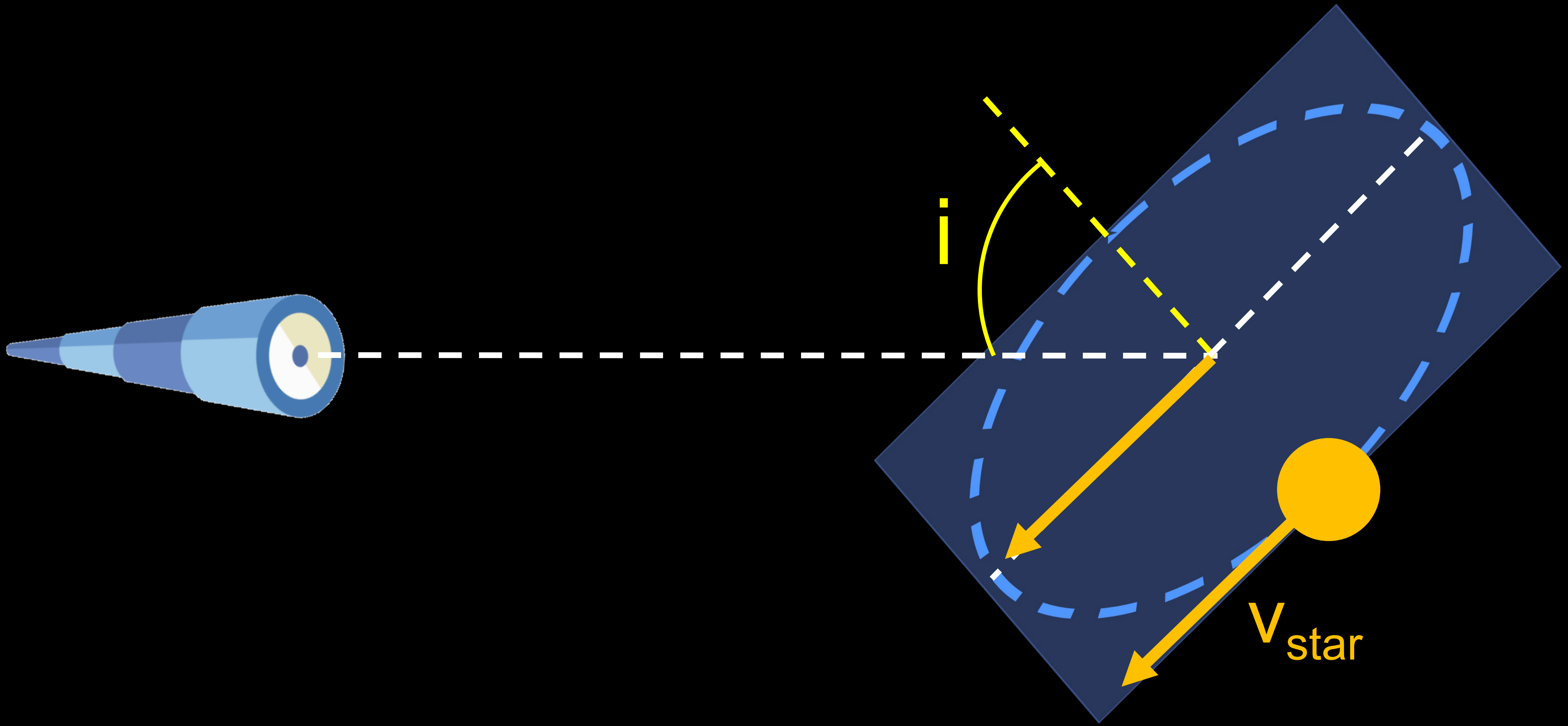
a few degrees < inclination < 90°

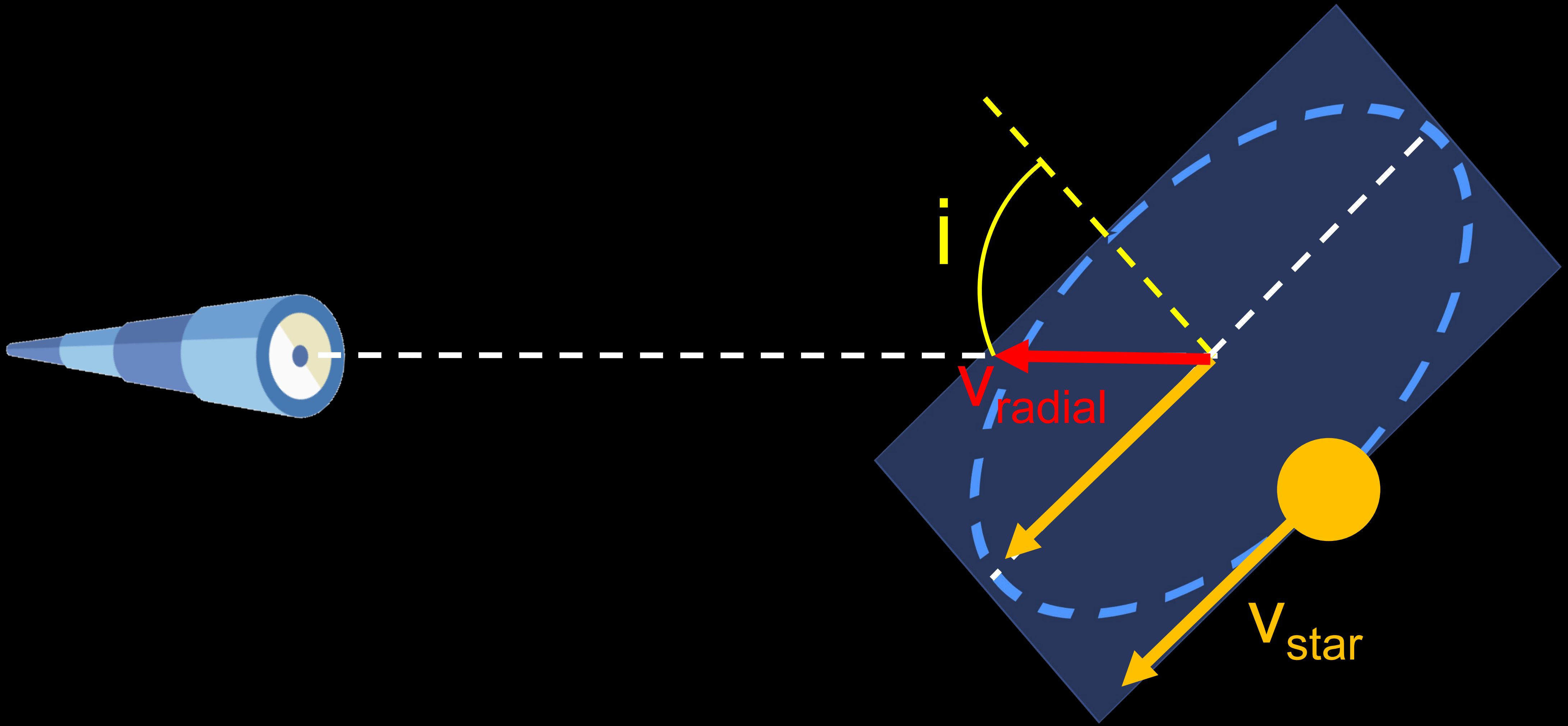
Transit is NOT visible from Earth

Medium radial velocity


i

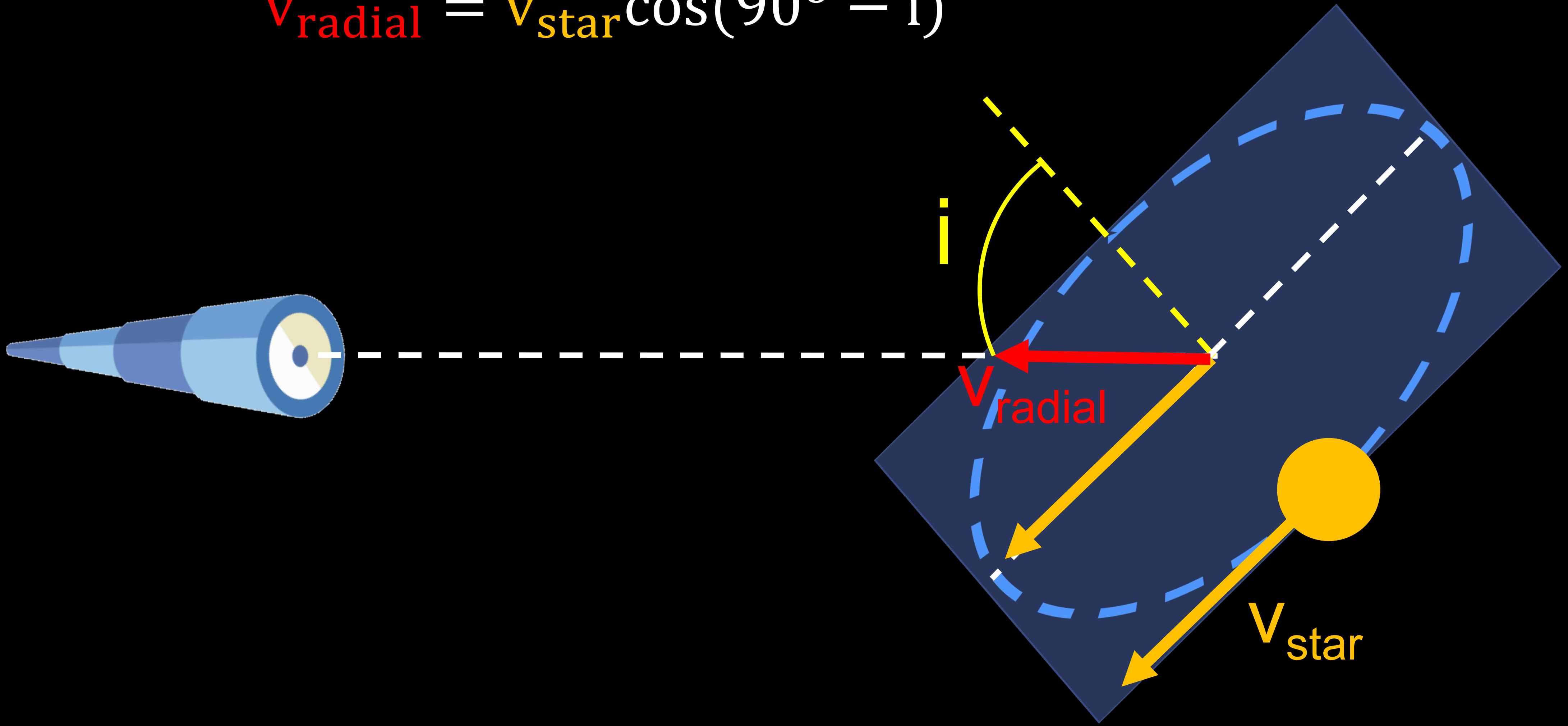

i
V_{star}


i
V_{star}



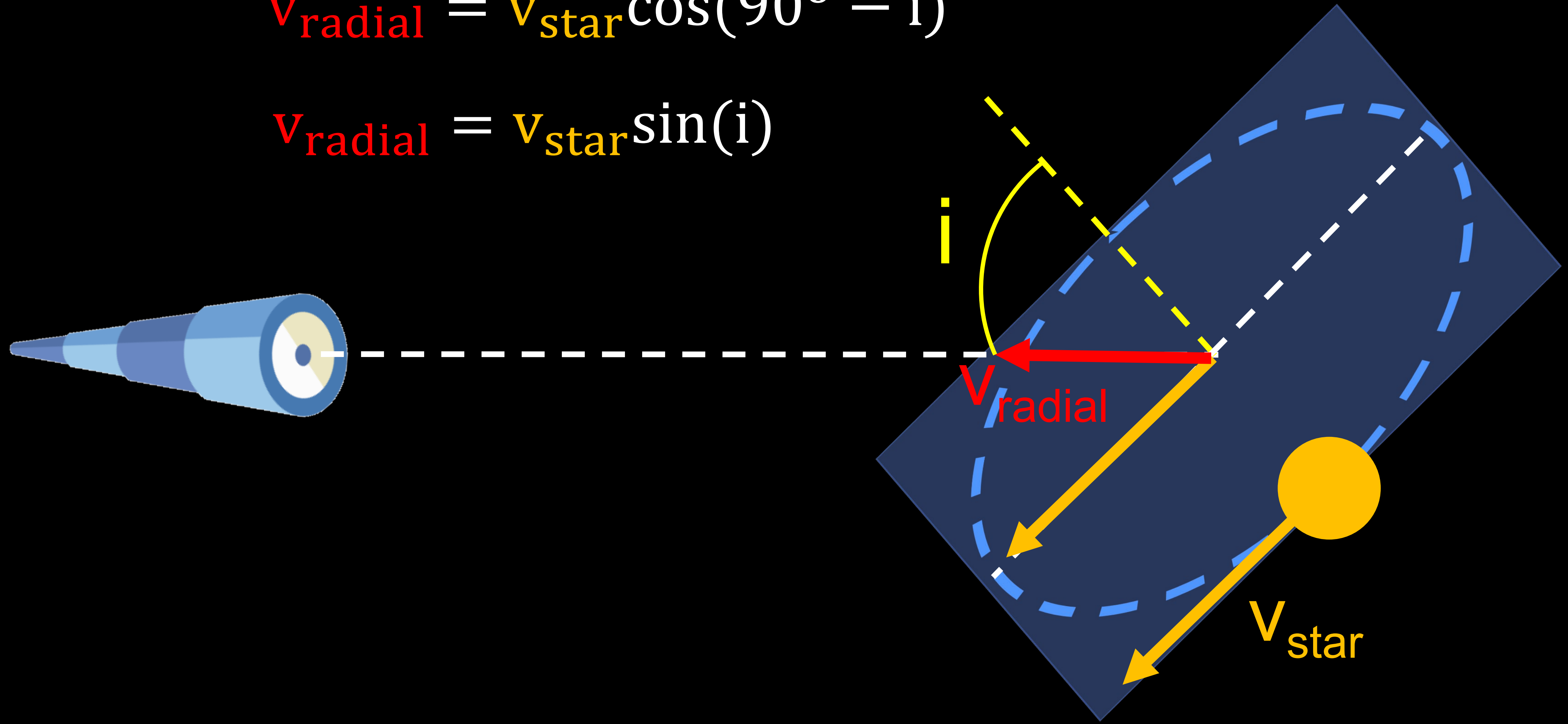
i
V_{radial}
V_{star}

$$v_{radial} = v_{star}\cos(90^o - i)$$



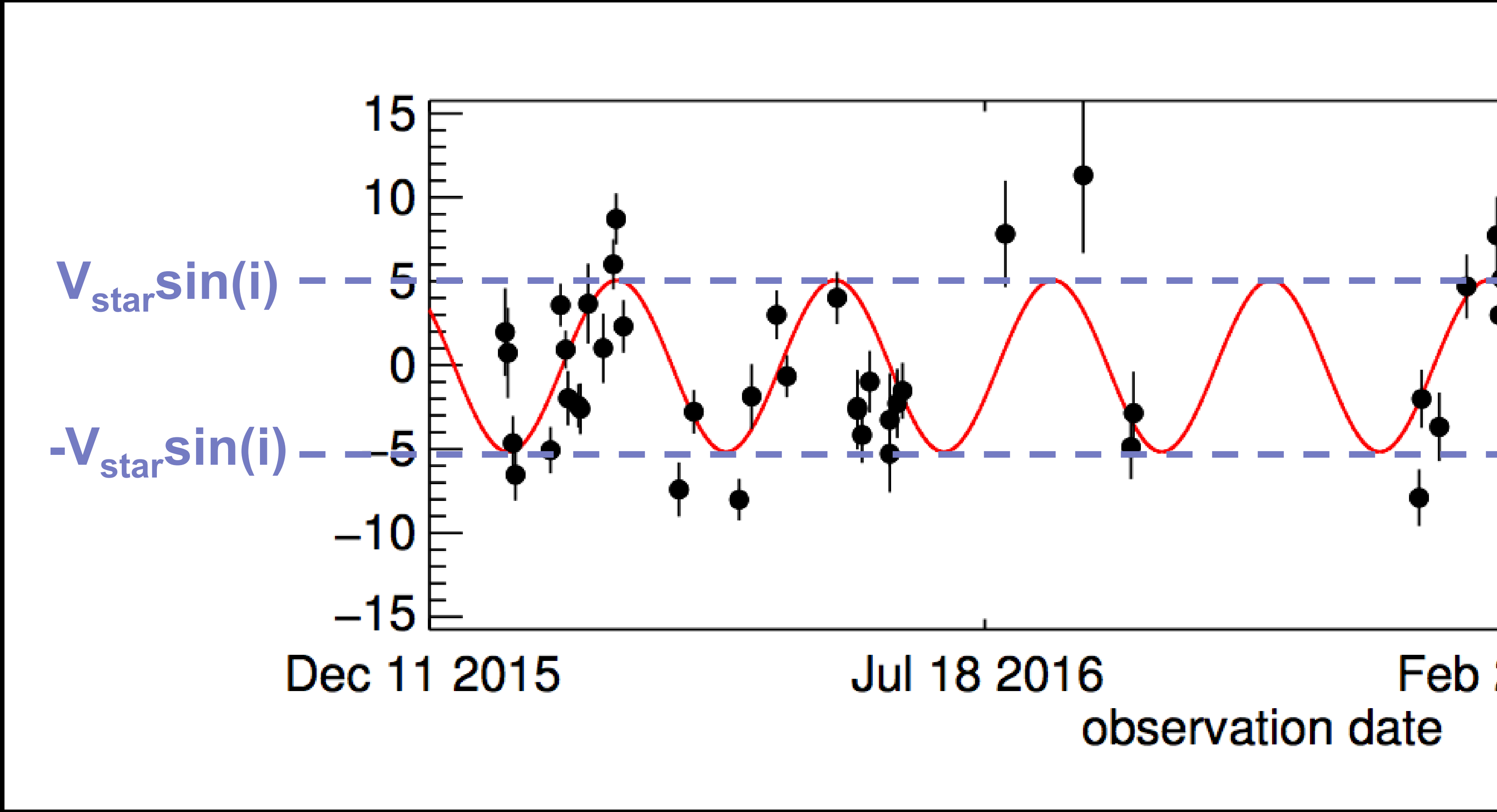
$$v_{radial} = v_{star}\cos(90^o - i)$$

$$v_{radial} = v_{star}\sin(i)$$



Don’t worry if you didn’t follow all of the math.

The important point is that we measure $v_{star}\sin(i)$, but we usually don’t know i.



Credit: Reiners et al., A&A, 2017

So, when we try to solve this equation for the mass of the planet, we are stuck with this factor "sin(i)" that we can't get rid of.

$$v_{radial} = \sqrt{\frac{G}{a(1-e^2)\left(M_{star}+M_{planet}\right)}}\,(M_{planet}\sin i)$$

BUT…

What about the special case where a planet detected using the radial velocity method is also detected using the transit method?

For a planet to transit, i must be very close to 90°, so sin(i) must be approximately 1.

$$v_{radial} = \sqrt{\frac{G}{a(1-e^2)\left(M_{star} + M_{planet}\right)}}\left(M_{planet}\sin i\right)$$

$$v_{radial} \cong \sqrt{\frac{G}{a(1-e^2)\left(M_{star} + M_{planet}\right)}}M_{planet}$$

If we also assume that the mass of the planet is much less than the mass of the star then $M_{star} + M_{planet} \cong M_{planet}$ and:

$$v_{radial} \cong \sqrt{\frac{G}{a(1-e^2)\left(M_{star} + M_{planet}\right)}} M_{planet}$$

$$v_{radial} \cong \sqrt{\frac{G}{a(1-e^2)M_{star}}} M_{planet}$$

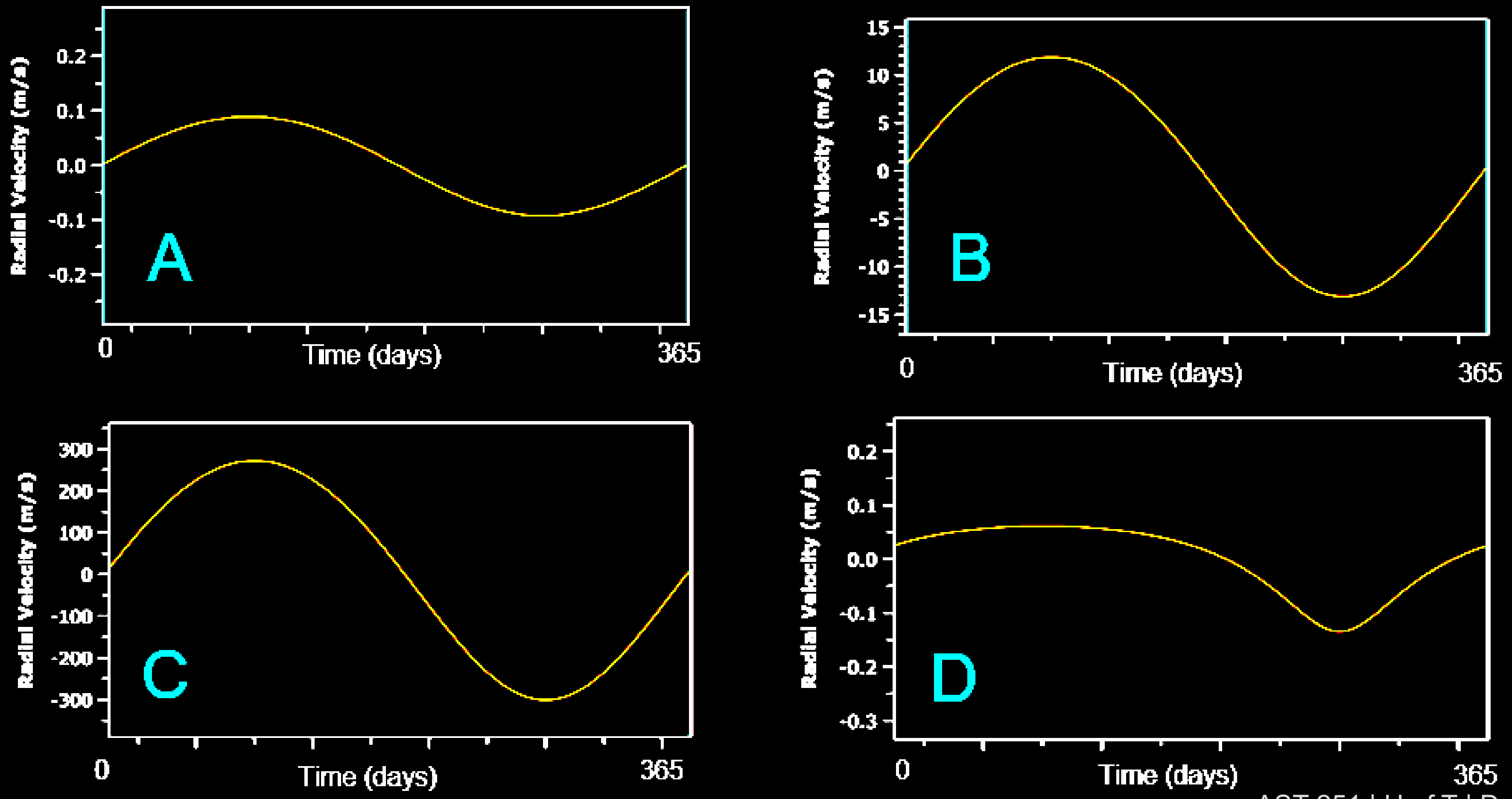
$$M_{planet} \cong \sqrt{\frac{a(1-e^2)M_{star}}{G}} v_{radial}$$

When a planet is detected by BOTH the transit and RV methods, we can get a from either one, and e and v_{radial} from the RV curve, and M_{star} by other means, so we can calculate M_{planet}:

$$\mathbf{M_{planet}} \cong \sqrt{\frac{\mathbf{a(1-e^2)M_{star}}}{\mathbf{G}}}\,\mathbf{v_{radial}}$$

Concept Check

If of these radial velocity curves correspond to planets orbiting stars of the same mass with the same orbital semimajor axis, which planet has the highest mass?



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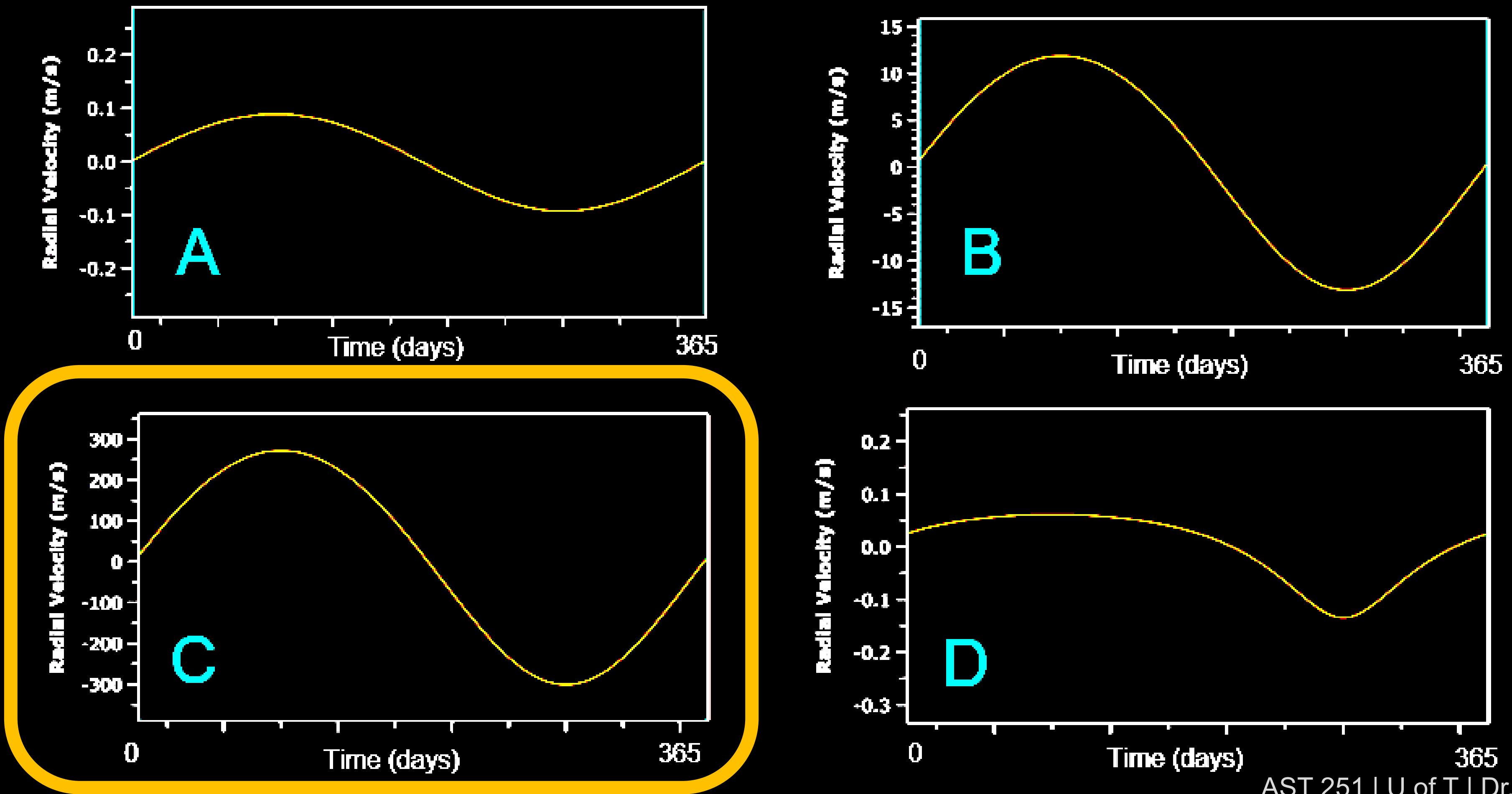
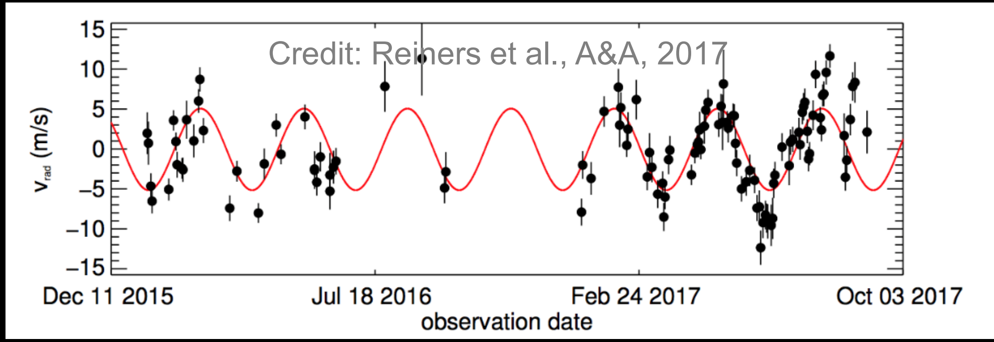
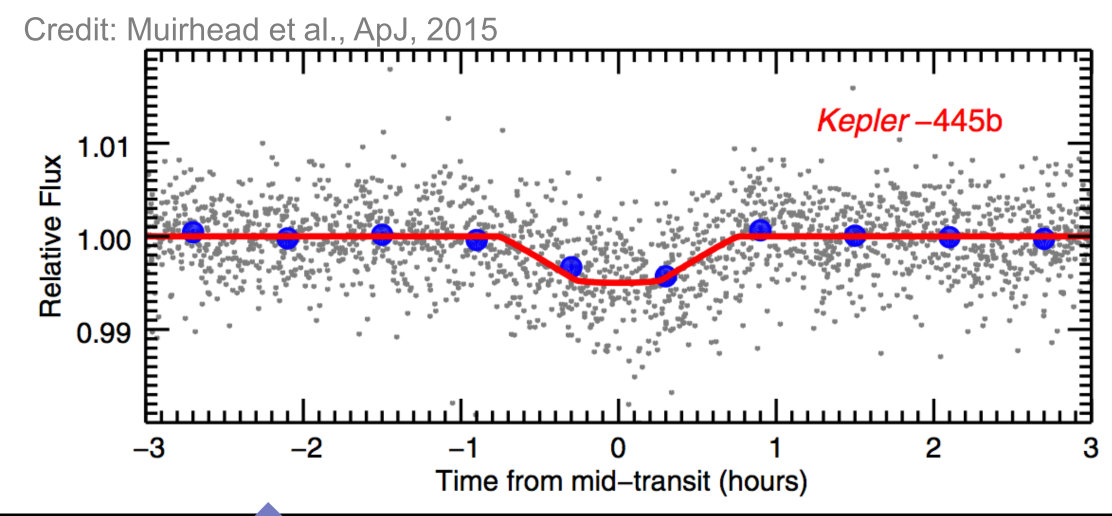


$$v_{radial} \cong \sqrt{\frac{G}{a(1-e^2)M_{star}}} M_{planet}$$

$$v_{radial} \propto M_{planet}$$

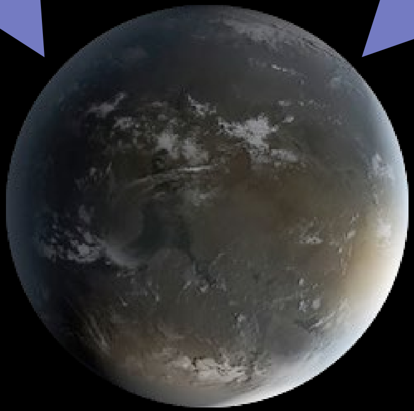
Higher planet mass produces higher radial velocity, all other factors being equal.





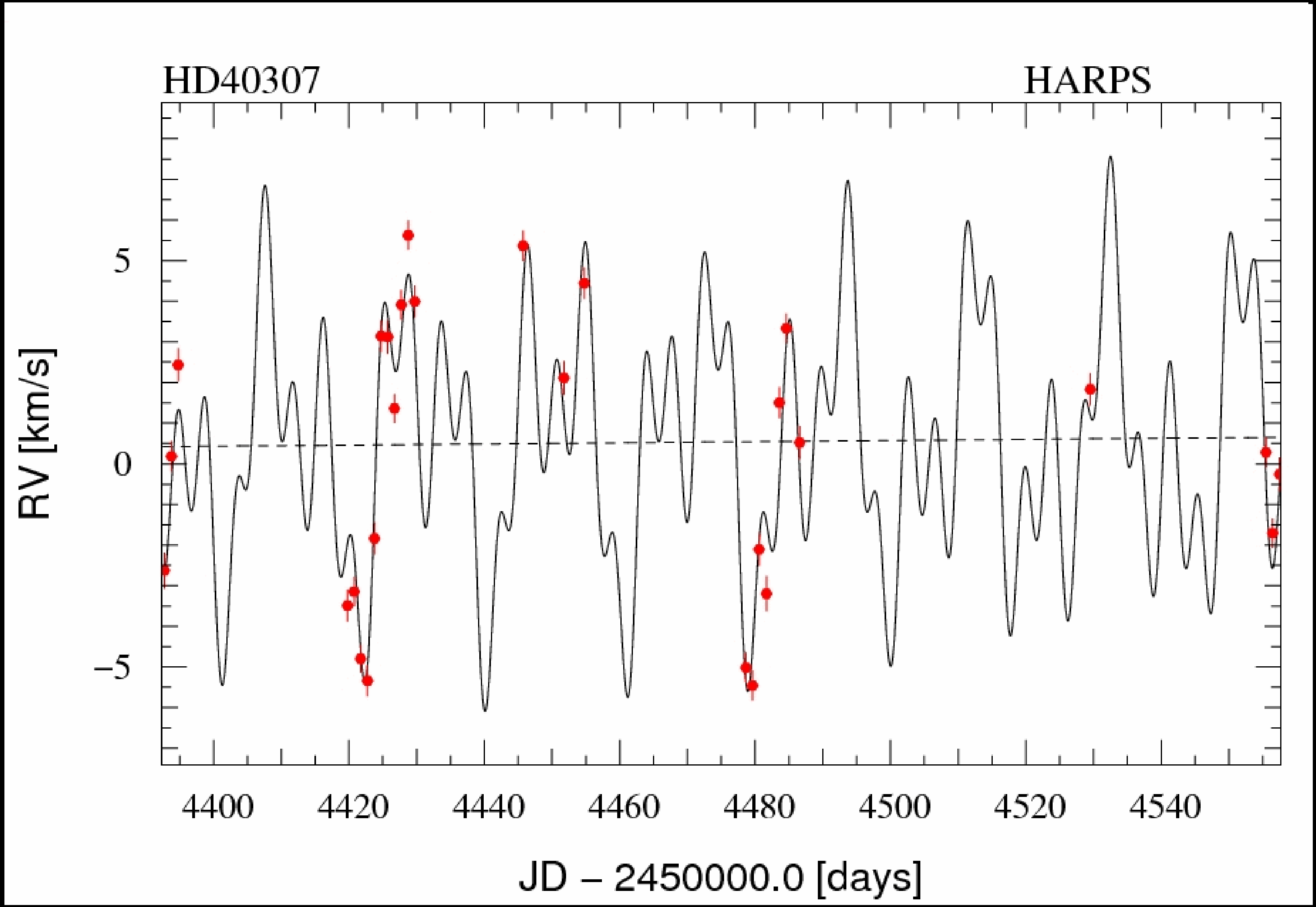
M_{planet}

R_{planet}



Credit: NASA/Kepler

As with the transit method, it is possible to detect multi-planet systems using the radial velocity method.



Credit: Mayor et al., A&A, 2009

Summary

The radial velocity method is especially powerful because it allows us to place limits on the mass of an exoplanet.

In the case where the inclination of a planet's orbit can be determined, we can measure the mass precisely.