

February 19, 2019

Bonferroni is too conservative when tests are correlated

Proof for two tests

Consider the following 2 by 2 table, depicting the joint likelihood that two tests will be accepted or rejected.

		X = Test 2		
		X=1 (Reject)	X=0 (Do not reject)	
Y= Test 1	Y=1 (Reject)	π_{11}	π_{22}	$\pi_{1\cdot}$
	Y=0 (Do not reject)	π_{21}	π_{22}	$\pi_{2\cdot}$
		$\pi_{\cdot 1}$	$\pi_{\cdot 2}$	$\pi_{\cdot\cdot}$

Goal of Bonferroni is to control the family wise error rate (FWER), i.e., $\text{FWER} < \alpha$.

$$\text{FWER} = P(X = 1 \cup Y = 1) = P(X = 1) + P(Y = 1) - P(X = 1 \cap Y = 1) = \pi_{1\cdot} + \pi_{\cdot 1} - \pi_{11}$$

Note that $\sigma_{XY}^2 = E[XY] - E[X]E[Y] = \pi_{11} - \pi_{\cdot 1}\pi_{1\cdot}$.

$$\text{Thus FWER} = \pi_{1\cdot} + \pi_{\cdot 1} - \sigma_{XY}^2 - \pi_{\cdot 1}\pi_{1\cdot}.$$

Under the Bonferroni correction, $\pi_{1\cdot} = \pi_{\cdot 1} = \alpha/2$

$$\text{FWER} = \alpha - \sigma_{XY}^2 - \frac{\alpha^2}{4} < \alpha \text{ (When the two tests are correlated, } \sigma_{XY}^2 > 0).$$

Thus we see Bonferroni is conservative when tests are correlation.

Question: won't it be conservative even when the tests aren't correlated?

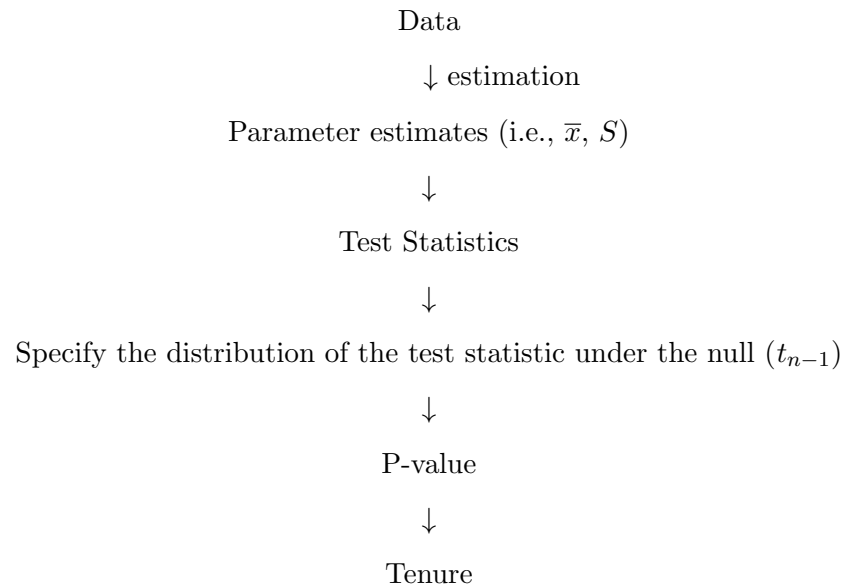
Research goals: big picture ideas

Limitations of methods for analysis of correlated data

	Repeated Covariates	Non-normal	Missing	Mistimed	Nominal Type I error
Multivariate					X
Mixed Model	X		X	X	
Generalized Mixed Model	X	X	X	X	
GEE	X	X	X	X	
Quazi (Ringham)			X	X	X (?)

How we do inference with frequentest statistics

Example: $\frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$



In order to specify the distribution of the test statistic under the null, we must *understand the parameter estimate distribution* (i.e., $\frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \sim \frac{N(0,1)}{\sqrt{\chi_{n-1}^2/(n-1)}} = t_{n-1}$). This will be your job this spring.