

Scenario 5 Validation Results: Time by Treatment Interaction using Orthogonal Polynomial Contrast for Time

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1 Study Design

Scenario 5 emulates a balanced, two-sample design with five repeated measurements collected over time. We devise a general linear hypothesis test of the time trend by treatment interaction. This scenario aims to demonstrate the validity of the missing data assumptions using an orthogonal polynomial contrast for the effect of time. Several conditional linear missing data processes and Markov missing data processes are considered.

1.1 Linear Mixed Model Inputs

1.1.1 Type I Error Rates (α)

0.0500

1.1.2 Beta Scale Values (δ_β)

1.0000

1.1.3 Sigma Scale Values (δ_σ)

1.0000

1.1.4 Planned Sample Sizes (N)

20, 40, 80

1.1.5 Matrix Inputs

$$Es(\mathbf{X}_M) = \begin{bmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 1.0000 \\ 1.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} -0.632 & -0.316 & 0.000 & 0.316 & 0.632 & 0.632 & 0.316 & 0.000 & -0.316 & -0.632 \\ 0.534 & -0.267 & -0.534 & -0.267 & 0.534 & -0.534 & 0.267 & 0.534 & 0.267 & -0.534 \\ -0.316 & 0.632 & 0.000 & -0.632 & 0.316 & 0.316 & -0.632 & 0.000 & 0.632 & -0.316 \\ 0.119 & -0.478 & 0.717 & -0.478 & 0.119 & -0.119 & 0.478 & -0.717 & 0.478 & -0.119 \end{bmatrix}$$

$$\boldsymbol{\theta}_0 = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_M = \begin{bmatrix} 1.5000 & 0.3750 & 0.3750 & 0.3750 & 0.3750 \\ 0.3750 & 1.5000 & 0.3750 & 0.3750 & 0.3750 \\ 0.3750 & 0.3750 & 1.5000 & 0.3750 & 0.3750 \\ 0.3750 & 0.3750 & 0.3750 & 1.5000 & 0.3750 \\ 0.3750 & 0.3750 & 0.3750 & 0.3750 & 1.5000 \end{bmatrix}$$

2 Conditional Linear Missing Data Models

Pattern Index	π	$\gamma_{jj'}$	Correlation Pattern
1	(1.0000, 1.0000, 1.0000, 1.0000, 1.0000)	0.25	AR(1)
2	(0.9000, 0.9000, 0.9000, 0.9000, 0.9000)	0.25	AR(1)
3	(0.8000, 0.9000, 0.9000, 0.9000, 0.9000)	0.25	AR(1)
4	(0.9000, 0.8000, 0.8000, 0.8000, 0.8000)	0.25	AR(1)
5	(0.8000, 0.8000, 0.8000, 0.8000, 0.8000)	0.25	AR(1)

3 Validation Results

3.1 Summary Statistics

Maximum Deviation from the Complete Case Scenarios	0.0454
Maximum Deviation from the Observed Case Scenarios	0.0348

3.2 Full Validation Results

3.2.1 Complete Case Analysis

Missing Process	Pattern Index	δ_σ	δ_β	N	$\mathcal{E}(N_c)$	Analytical Power	Empirical Power	Absolute Deviation	Iterations	Converged
CLP	1	1	1	20	20	0.5098	0.5072	0.0026	10000	10000
CLP	2	1	1	20	13.1777	0.2648	0.2764	0.0117	9999	9988
CLP	3	1	1	20	11.8718	0.2190	0.2295	0.0105	10000	9919
CLP	4	1	1	20	9.2119	0.1345	0.1629	0.0284	9954	8884
CLP	5	1	1	20	8.3521	0.1109	0.1564	0.0454	9891	7974
CLP	1	1	1	40	40	0.9025	0.9008	0.0017	10000	10000
CLP	2	1	1	40	26.3554	0.6724	0.6728	4e-04	10000	10000
CLP	3	1	1	40	23.7436	0.6016	0.5933	0.0083	10000	10000
CLP	4	1	1	40	18.4238	0.4329	0.4509	0.018	10000	10000
CLP	5	1	1	40	16.7042	0.3737	0.3855	0.0117	9999	9998
CLP	1	1	1	80	80	0.9989	0.9988	1e-04	10000	10000
CLP	2	1	1	80	52.7108	0.971	0.9687	0.0023	10000	10000
CLP	3	1	1	80	47.4872	0.9496	0.9465	0.0031	10000	10000

CLP	4	1	1	80	36.8475	0.8579	0.8504	0.0075	10000	10000
CLP	5	1	1	80	33.4084	0.8072	0.8128	0.0056	10000	10000

3.2.2 Observed Case Analysis

Missing Process	Pattern Index	δ_σ	δ_β	N	$\mathcal{E}(N_m)$	Analytical Power	Empirical Power	Absolute Deviation	Iterations	Converged
CLP	1	1	1	20	20	0.5098	0.5072	0.0026	10000	10000
CLP	2	1	1	20	18	0.44	0.4479	0.0079	10000	9991
CLP	3	1	1	20	17.6	0.4256	0.4220	0.0036	1000	9937
CLP	4	1	1	20	16.4	0.3821	0.4068	0.0247	10000	9082
CLP	5	1	1	20	16	0.3675	0.3672	2e-04	10000	8466
CLP	1	1	1	40	40	0.9025	0.9008	0.0017	10000	10000
CLP	2	1	1	40	36	0.8568	0.8435	0.0133	10000	10000
CLP	3	1	1	40	35.2	0.8458	0.811	0.0348	10000	10000
CLP	4	1	1	40	32.8	0.8084	0.7967	0.0117	10000	10000
CLP	5	1	1	40	32	0.7945	0.7618	0.0327	10000	9999
CLP	1	1	1	80	80	0.9989	0.9988	1e-04	10000	10000
CLP	2	1	1	80	72	0.9969	0.9959	0.001	10000	10000
CLP	3	1	1	80	70.4	0.9962	0.9926	0.0036	10000	10000
CLP	4	1	1	80	65.6	0.9933	0.9933	0	10000	10000
CLP	5	1	1	80	64	0.9919	0.9883	0.0036	10000	10000