Scenario 4 Validation Results: Difference-in-Difference Hypothesis

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1 Study Design

Scenario 4 emulates a balanced, four-sample design with three repeated measurements collected over time. We approximate the power for a general linear hypothesis test of the group by time interaction. A compound symmetric covariance model is specified for the simulation of the outcome variables. This scenario aims to demonstrate the validity of our methodology when assumptions of sphericity are unlikely to hold. Several conditional linear missing data processes and Markov missing data processes are considered.

1.1 Linear Mixed Model Inputs

1.1.1 Type I Error Rates (α)

0.0100

1.1.2 Beta Scale Values (δ_β)

0.0000, 0.5000, 1.0000, 1.5000, 2.0000

1.1.3 Sigma Scale Values (δ_σ)

1.0000, 2.0000

1.1.4 Planned Sample Sizes (N)

$10, 20$

1.1.5 Matrix Inputs

$$Es(\mathbf{X}_M) = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$\boldsymbol{\beta} = \begin{bmatrix} 1.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\theta}_0 = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$\boldsymbol{\Sigma}_M = \begin{bmatrix} 1.0000 & 0.4000 & 0.4000 \\ 0.4000 & 1.0000 & 0.4000 \\ 0.4000 & 0.4000 & 1.0000 \end{bmatrix}$$

2 Conditional Linear Missing Data Models

Pattern Index	$\boldsymbol{\pi}$	$\gamma_{jj'}$	Correlation Pattern
1	(1.0000, 1.0000, 1.0000)	0.25	CS
2	(0.9000, 0.9000, 0.9000)	0.25	CS
3	(0.8000, 0.9000, 0.9000)	0.25	CS
4	(0.9000, 0.8000, 0.8000)	0.25	CS
5	(0.8000, 0.8000, 0.8000)	0.25	CS

3 Validation Results

3.1 Summary Statistics

Maximum Deviation from the Complete Case Scenarios	0.0340
Maximum Deviation from the Observed Case Scenarios	0.0556

3.2 Full Validation Results

3.2.1 Complete Case Analysis

Missing Process	Pattern Index	δ_σ	δ_β	N	$\mathcal{E}(N_c)$	Analytical Power	Empirical Power	Absolute Deviation	Iterations	Converged
CLP	1	1	0	20	20	0.01	0.0108	8e-04	10000	10000
CLP	2	1	0	20	15.4012	0.01	0.0096	4e-04	9968	9968
CLP	3	1	0	20	13.875	0.01	0.0109	9e-04	9902	9899
CLP	4	1	0	20	12.75	0.01	0.0109	9e-04	9751	9738
CLP	5	1	0	20	11.56	0.01	0.0097	3e-04	9495	9447
CLP	1	1	0.5	20	20	0.0193	0.0206	0.0013	10000	10000

CLP	2	1	0.5	20	15.4012	0.0152	0.0137	0.0015	9978	9978
CLP	3	1	0.5	20	13.875	0.0141	0.0129	0.0012	9885	9879
CLP	4	1	0.5	20	12.75	0.0133	0.0128	4e-04	9765	9755
CLP	5	1	0.5	20	11.56	0.0125	0.0129	4e-04	9431	9383
CLP	1	1	1	20	20	0.066	0.0671	0.0011	10000	10000
CLP	2	1	1	20	15.4012	0.0375	0.0405	0.003	9977	9977
CLP	3	1	1	20	13.875	0.0302	0.0368	0.0065	9903	9899
CLP	4	1	1	20	12.75	0.0256	0.0279	0.0023	9759	9751
CLP	5	1	1	20	11.56	0.0213	0.0246	0.0033	9450	9401
CLP	1	1	1.5	20	20	0.2015	0.1978	0.0037	10000	10000
CLP	2	1	1.5	20	15.4012	0.0964	0.1097	0.0133	9972	9972
CLP	3	1	1.5	20	13.875	0.0705	0.0867	0.0162	9878	9877
CLP	4	1	1.5	20	12.75	0.0546	0.066	0.0114	9750	9733
CLP	5	1	1.5	20	11.56	0.0407	0.0525	0.0118	9498	9456
CLP	1	1	2	20	20	0.4506	0.444	0.0066	10000	10000
CLP	2	1	2	20	15.4012	0.2161	0.242	0.0259	9967	9967
CLP	3	1	2	20	13.875	0.1517	0.1801	0.0284	9886	9886
CLP	4	1	2	20	12.75	0.1117	0.1406	0.0289	9744	9736
CLP	5	1	2	20	11.56	0.0771	0.1098	0.0326	9482	9427
CLP	1	2	0	20	20	0.01	0.0078	0.0022	10000	10000
CLP	2	2	0	20	15.4012	0.01	0.0102	2e-04	9982	9982
CLP	3	2	0	20	13.875	0.01	0.0106	6e-04	9892	9891
CLP	4	2	0	20	12.75	0.01	0.012	0.002	9742	9734
CLP	5	2	0	20	11.56	0.01	0.0086	0.0014	9487	9442
CLP	1	2	0.5	20	20	0.0142	0.0156	0.0014	10000	10000
CLP	2	2	0.5	20	15.4012	0.0125	0.0117	8e-04	9973	9973
CLP	3	2	0.5	20	13.875	0.012	0.0107	0.0012	9898	9897
CLP	4	2	0.5	20	12.75	0.0116	0.0127	0.0012	9749	9738
CLP	5	2	0.5	20	11.56	0.0112	0.0109	3e-04	9464	9414
CLP	1	2	1	20	20	0.0316	0.0319	3e-04	10000	10000
CLP	2	2	1	20	15.4012	0.0216	0.0216	0	9972	9972
CLP	3	2	1	20	13.875	0.0188	0.0209	0.0021	9891	9890
CLP	4	2	1	20	12.75	0.0169	0.0193	0.0024	9736	9727
CLP	5	2	1	20	11.56	0.0152	0.0147	4e-04	9479	9431
CLP	1	2	1.5	20	20	0.0765	0.0789	0.0024	10000	10000
CLP	2	2	1.5	20	15.4012	0.0422	0.0481	0.0059	9966	9966
CLP	3	2	1.5	20	13.875	0.0335	0.0379	0.0044	9874	9872
CLP	4	2	1.5	20	12.75	0.028	0.031	0.003	9749	9742
CLP	5	2	1.5	20	11.56	0.023	0.0283	0.0053	9455	9397
CLP	1	2	2	20	20	0.1697	0.1669	0.0028	10000	10000
CLP	2	2	2	20	15.4012	0.0825	0.087	0.0044	9971	9971
CLP	3	2	2	20	13.875	0.0612	0.075	0.0139	9891	9887
CLP	4	2	2	20	12.75	0.048	0.059	0.0111	9765	9757
CLP	5	2	2	20	11.56	0.0363	0.0474	0.011	9488	9437
CLP	1	1	0	40	40	0.01	0.0118	0.0018	10000	10000

CLP	2	1	0	40	30.8025	0.01	0.0101	1e-04	10000	10000
CLP	3	1	0	40	27.75	0.01	0.0105	5e-04	10000	10000
CLP	4	1	0	40	25.5	0.01	0.0089	0.0011	9997	9997
CLP	5	1	0	40	23.12	0.01	0.0099	1e-04	9996	9996
CLP	1	1	0.5	40	40	0.0392	0.0395	3e-04	10000	10000
CLP	2	1	0.5	40	30.8025	0.0283	0.0293	0.001	10000	10000
CLP	3	1	0.5	40	27.75	0.0251	0.0258	7e-04	10000	10000
CLP	4	1	0.5	40	25.5	0.0229	0.0245	0.0016	9999	9999
CLP	5	1	0.5	40	23.12	0.0207	0.0243	0.0036	9994	9994
CLP	1	1	1	40	40	0.2415	0.2395	0.002	10000	10000
CLP	2	1	1	40	30.8025	0.1425	0.1452	0.0027	10000	10000
CLP	3	1	1	40	27.75	0.1145	0.1164	0.002	9999	9999
CLP	4	1	1	40	25.5	0.0957	0.1022	0.0065	9999	9999
CLP	5	1	1	40	23.12	0.0777	0.0811	0.0033	9993	9993
CLP	1	1	1.5	40	40	0.6833	0.6755	0.0078	10000	10000
CLP	2	1	1.5	40	30.8025	0.4543	0.4648	0.0105	10000	10000
CLP	3	1	1.5	40	27.75	0.3697	0.3884	0.0187	10000	10000
CLP	4	1	1.5	40	25.5	0.3078	0.3278	0.02	9998	9998
CLP	5	1	1.5	40	23.12	0.2448	0.2655	0.0207	9993	9993
CLP	1	1	2	40	40	0.9554	0.9525	0.0029	10000	10000
CLP	2	1	2	40	30.8025	0.8117	0.8016	0.0101	10000	10000
CLP	3	1	2	40	27.75	0.7197	0.7212	0.0015	10000	10000
CLP	4	1	2	40	25.5	0.6353	0.6435	0.0081	9999	9999
CLP	5	1	2	40	23.12	0.5329	0.5455	0.0126	9993	9993
CLP	1	2	0	40	40	0.01	0.0108	8e-04	10000	10000
CLP	2	2	0	40	30.8025	0.01	0.0106	6e-04	10000	10000
CLP	3	2	0	40	27.75	0.01	0.0113	0.0013	9999	9999
CLP	4	2	0	40	25.5	0.01	0.0087	0.0013	9998	9998
CLP	5	2	0	40	23.12	0.01	0.0083	0.0017	9991	9991
CLP	1	2	0.5	40	40	0.0221	0.0242	0.0021	10000	10000
CLP	2	2	0.5	40	30.8025	0.0179	0.019	0.0011	10000	10000
CLP	3	2	0.5	40	27.75	0.0167	0.016	7e-04	10000	10000
CLP	4	2	0.5	40	25.5	0.0158	0.0164	6e-04	9999	9999
CLP	5	2	0.5	40	23.12	0.0149	0.0149	0	9991	9991
CLP	1	2	1	40	40	0.0891	0.0869	0.0022	10000	10000
CLP	2	2	1	40	30.8025	0.0566	0.059	0.0024	10000	10000
CLP	3	2	1	40	27.75	0.0476	0.0533	0.0057	10000	10000
CLP	4	2	1	40	25.5	0.0415	0.0405	0.001	9997	9997
CLP	5	2	1	40	23.12	0.0356	0.0433	0.0078	9993	9993
CLP	1	2	1.5	40	40	0.2869	0.2912	0.0043	10000	10000
CLP	2	2	1.5	40	30.8025	0.1692	0.1789	0.0097	10000	10000
CLP	3	2	1.5	40	27.75	0.1354	0.1482	0.0128	10000	10000
CLP	4	2	1.5	40	25.5	0.1126	0.1295	0.0169	10000	10000
CLP	5	2	1.5	40	23.12	0.0908	0.1034	0.0126	9993	9993
CLP	1	2	2	40	40	0.6066	0.6122	0.0056	10000	10000

CLP	2	2	2	40	30.8025	0.3887	0.3984	0.0097	10000	10000
CLP	3	2	2	40	27.75	0.3134	0.3335	0.0201	10000	10000
CLP	4	2	2	40	25.5	0.2597	0.2832	0.0236	9999	9999
CLP	5	2	2	40	23.12	0.2061	0.2326	0.0266	9994	9994

3.2.2 Observed Case Analysis

Missing Process	Pattern Index	δ_σ	δ_β	N	$\mathcal{E}(N_m)$	Analytical Power	Empirical Power	Absolute Deviation	Iterations	Converged
CLP	1	1	0	20	20	0.01	0.0108	8e-04	10000	10000
CLP	2	1	0	20	18	0.01	0.013	0.003	9998	9998
CLP	3	1	0	20	17.3333	0.01	0.0159	0.0059	9983	9981
CLP	4	1	0	20	16.6667	0.01	0.0158	0.0058	9979	9972
CLP	5	1	0	20	16	0.01	0.0205	0.0105	9962	9922
CLP	1	1	0.5	20	20	0.0193	0.0206	0.0013	10000	10000
CLP	2	1	0.5	20	18	0.0174	0.0191	0.0017	9997	9997
CLP	3	1	0.5	20	17.3333	0.0168	0.0219	0.0051	9982	9981
CLP	4	1	0.5	20	16.6667	0.0163	0.0216	0.0053	9981	9972
CLP	5	1	0.5	20	16	0.0157	0.0289	0.0132	9975	9937
CLP	1	1	1	20	20	0.066	0.0671	0.0011	10000	10000
CLP	2	1	1	20	18	0.0524	0.0584	0.006	9996	9996
CLP	3	1	1	20	17.3333	0.0483	0.0588	0.0105	9987	9984
CLP	4	1	1	20	16.6667	0.0444	0.0491	0.0048	9979	9975
CLP	5	1	1	20	16	0.0407	0.0561	0.0155	9968	9921
CLP	1	1	1.5	20	20	0.2015	0.1978	0.0037	10000	10000
CLP	2	1	1.5	20	18	0.1512	0.1539	0.0027	9999	9999
CLP	3	1	1.5	20	17.3333	0.1359	0.1361	2e-04	9988	9987
CLP	4	1	1.5	20	16.6667	0.1215	0.1311	0.0096	9973	9963
CLP	5	1	1.5	20	16	0.1078	0.1176	0.0098	9964	9931
CLP	1	1	2	20	20	0.4506	0.444	0.0066	10000	10000
CLP	2	1	2	20	18	0.3448	0.3311	0.0137	9999	9999
CLP	3	1	2	20	17.3333	0.3103	0.2844	0.0258	9988	9988
CLP	4	1	2	20	16.6667	0.2766	0.2721	0.0044	9975	9969
CLP	5	1	2	20	16	0.244	0.2359	0.0082	9972	9916
CLP	1	2	0	20	20	0.01	0.0078	0.0022	10000	10000
CLP	2	2	0	20	18	0.01	0.0129	0.0029	10000	10000
CLP	3	2	0	20	17.3333	0.01	0.0133	0.0033	9984	9984
CLP	4	2	0	20	16.6667	0.01	0.0179	0.0079	9982	9973
CLP	5	2	0	20	16	0.01	0.0194	0.0094	9962	9919
CLP	1	2	0.5	20	20	0.0142	0.0156	0.0014	10000	10000
CLP	2	2	0.5	20	18	0.0135	0.0161	0.0026	9999	9999
CLP	3	2	0.5	20	17.3333	0.0132	0.0184	0.0052	9992	9991
CLP	4	2	0.5	20	16.6667	0.0129	0.0202	0.0072	9965	9952
CLP	5	2	0.5	20	16	0.0127	0.026	0.0133	9962	9925
CLP	1	2	1	20	20	0.0316	0.0319	3e-04	10000	10000
CLP	2	2	1	20	18	0.0269	0.0303	0.0034	10000	10000

CLP	3	2	1	20	17.3333	0.0255	0.0321	0.0066	9981	9981
CLP	4	2	1	20	16.6667	0.0241	0.0317	0.0076	9978	9973
CLP	5	2	1	20	16	0.0227	0.0336	0.0108	9962	9924
CLP	1	2	1.5	20	20	0.0765	0.0789	0.0024	10000	10000
CLP	2	2	1.5	20	18	0.0601	0.0644	0.0043	10000	10000
CLP	3	2	1.5	20	17.3333	0.0551	0.06	0.0049	9985	9982
CLP	4	2	1.5	20	16.6667	0.0504	0.0565	0.0061	9970	9964
CLP	5	2	1.5	20	16	0.046	0.0595	0.0135	9963	9919
CLP	1	2	2	20	20	0.1697	0.1669	0.0028	10000	10000
CLP	2	2	2	20	18	0.1279	0.1264	0.0014	9998	9998
CLP	3	2	2	20	17.3333	0.1152	0.1164	0.0012	9986	9982
CLP	4	2	2	20	16.6667	0.1032	0.1136	0.0103	9976	9969
CLP	5	2	2	20	16	0.092	0.1049	0.0129	9959	9916
CLP	1	1	0	40	40	0.01	0.0118	0.0018	10000	10000
CLP	2	1	0	40	36	0.01	0.0107	7e-04	10000	10000
CLP	3	1	0	40	34.6667	0.01	0.011	0.001	10000	10000
CLP	4	1	0	40	33.3333	0.01	0.0115	0.0015	10000	10000
CLP	5	1	0	40	32	0.01	0.0111	0.0011	10000	10000
CLP	1	1	0.5	40	40	0.0392	0.0395	3e-04	10000	10000
CLP	2	1	0.5	40	36	0.0342	0.0345	3e-04	10000	10000
CLP	3	1	0.5	40	34.6667	0.0326	0.0314	0.0012	10000	10000
CLP	4	1	0.5	40	33.3333	0.0311	0.0318	7e-04	10000	10000
CLP	5	1	0.5	40	32	0.0296	0.0318	0.0022	10000	10000
CLP	1	1	1	40	40	0.2415	0.2395	0.002	10000	10000
CLP	2	1	1	40	36	0.1961	0.1877	0.0084	10000	10000
CLP	3	1	1	40	34.6667	0.1817	0.153	0.0287	10000	10000
CLP	4	1	1	40	33.3333	0.1677	0.173	0.0053	10000	10000
CLP	5	1	1	40	32	0.1542	0.1423	0.0119	10000	10000
CLP	1	1	1.5	40	40	0.6833	0.6755	0.0078	10000	10000
CLP	2	1	1.5	40	36	0.591	0.5674	0.0236	10000	10000
CLP	3	1	1.5	40	34.6667	0.5574	0.5049	0.0525	10000	10000
CLP	4	1	1.5	40	33.3333	0.5227	0.5158	0.0069	10000	10000
CLP	5	1	1.5	40	32	0.487	0.4502	0.0368	10000	10000
CLP	1	1	2	40	40	0.9554	0.9525	0.0029	10000	10000
CLP	2	1	2	40	36	0.9131	0.8913	0.0218	10000	10000
CLP	3	1	2	40	34.6667	0.8929	0.8406	0.0523	10000	10000
CLP	4	1	2	40	33.3333	0.8689	0.8511	0.0178	10000	10000
CLP	5	1	2	40	32	0.8408	0.7852	0.0556	10000	10000
CLP	1	2	0	40	40	0.01	0.0108	8e-04	10000	10000
CLP	2	2	0	40	36	0.01	0.0111	0.0011	10000	10000
CLP	3	2	0	40	34.6667	0.01	0.0122	0.0022	10000	10000
CLP	4	2	0	40	33.3333	0.01	0.0108	8e-04	10000	10000
CLP	5	2	0	40	32	0.01	0.0093	7e-04	10000	10000
CLP	1	2	0.5	40	40	0.0221	0.0242	0.0021	10000	10000
CLP	2	2	0.5	40	36	0.0202	0.0192	0.001	10000	10000

CLP	3	2	0.5	40	34.6667	0.0196	0.0192	4e-04	10000	10000
CLP	4	2	0.5	40	33.3333	0.019	0.0205	0.0015	10000	10000
CLP	5	2	0.5	40	32	0.0185	0.0201	0.0016	10000	10000
CLP	1	2	1	40	40	0.0891	0.0869	0.0022	10000	10000
CLP	2	2	1	40	36	0.074	0.074	0	10000	10000
CLP	3	2	1	40	34.6667	0.0693	0.069	3e-04	10000	10000
CLP	4	2	1	40	33.3333	0.0648	0.062	0.0028	10000	10000
CLP	5	2	1	40	32	0.0604	0.0615	0.0011	10000	10000
CLP	1	2	1.5	40	40	0.2869	0.2912	0.0043	10000	10000
CLP	2	2	1.5	40	36	0.2334	0.2285	0.0049	10000	10000
CLP	3	2	1.5	40	34.6667	0.2162	0.1926	0.0236	10000	10000
CLP	4	2	1.5	40	33.3333	0.1995	0.2033	0.0038	10000	10000
CLP	5	2	1.5	40	32	0.1833	0.1754	0.0079	10000	10000
CLP	1	2	2	40	40	0.6066	0.6122	0.0056	10000	10000
CLP	2	2	2	40	36	0.5158	0.4941	0.0217	10000	10000
CLP	3	2	2	40	34.6667	0.4839	0.4339	0.05	10000	10000
CLP	4	2	2	40	33.3333	0.4513	0.4536	0.0023	10000	10000
CLP	5	2	2	40	32	0.4184	0.3946	0.0238	10000	10000