Anschutz

February 19, 2019

Bonferroni is too conservative when tests are correlated

Proof for two tests

Consider the following 2 by 2 table, depicting the joint likelihood that two tests will be accepted or rejected.

		X = Test 2		
		X=1 (Reject)	X=0 (Do not reject)	
Y= Test 1	Y=1 (Reject)	π_{11}	π_{22}	$\pi_{1\cdot}$
	Y=0 (Do not reject)	π_{21}	π_{22}	$\pi_{2\cdot}$
		$\pi_{\cdot 1}$	$\pi_{\cdot 2}$	$\pi_{\cdot\cdot}$

Goal of Bonferroni is to control the family wise error rate (FWER), i.e., FWER$< \alpha$.

FWER $= P(X = 1 \bigcup Y = 1) = P(X = 1) + P(Y = 1) - P(X = 1 \bigcap Y = 1) = \pi_{1\cdot} + \pi_{\cdot 1} - \pi_{11}$

Note that $\sigma^2_{XY} = E[XY] - E[X]E[Y] = \pi_{11} - \pi_{\cdot 1}\pi_{1\cdot}$

Thus FWER $= \pi_{1\cdot} + \pi_{\cdot 1} - \sigma^2_{XY} - \pi_{\cdot 1}\pi_{1\cdot}$

Under the Bonferroni correction, $\pi_{1\cdot} = \pi_{\cdot 1} = \alpha/2$

FWER $= \alpha - \sigma^2_{XY} - \frac{\alpha^2}{4} < \alpha$ (When the two tests are correlated, $\sigma^2_{XY} > 0$).

Thus we see Bonferroni is conservative when tests are correlation.

Question: won't it be conservative even when the tests aren't correlated?

Research goals: big picture ideas

Limitations of methods for analysis of correlated data

	Repeated Covariates	Non-normal	Missing	Mistimed	Nominal Type I error
Multivariate					X
Mixed Model	X		X	X	
Generalized Mixed Model	X	X	X	X	
GEE	X	X	X	X	
Quazi (Ringham)			X	X	X (?)

How we do inference with frequentest statistics

Example: $\dfrac{\overline{x}-\mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1}$

$$\begin{array}{c} \text{Data} \\ \downarrow \text{estimation} \\ \text{Parameter estimates (i.e., } \overline{x},\ S\text{)} \\ \downarrow \\ \text{Test Statistics} \\ \downarrow \\ \text{Specify the distribution of the test statistic under the null } (t_{n-1}) \\ \downarrow \\ \text{P-value} \\ \downarrow \\ \text{Tenure} \end{array}$$

In order to specify the distribution of the test statistic under the null, we must *understand the parameter estimate distribution* (i.e., $\dfrac{\overline{x}-\mu}{\frac{S}{\sqrt{n}}} \sim \frac{N(0,1)}{\sqrt{\chi^2_{n-1}/(n-1)}} = t_{n-1}$). This will be your job this spring.