Footstep Planning on 2D Environments using Mixed-integer Quadratic Program for Biped Robots*

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***Abstract*— In this paper, we implement a footstep planner for a biped robot on discontinuous 2D environments. We use a simplified version of the method proposed in R. Tedrake's paper [1]. This implementation represents the footstep planning problem as a mixed-integer quadratic program and handles kinematic reachability. The mixed-integer formulation is able to view the non-convex constraints as a collection of convex regions.**

I. INTRODUCTION

The advantage of legged locomotion is realized via footstep planning. The case of narrow, uneven terrains is the most interesting because that is where wheeled robots fail. The goal is to find a list of locations that a walking robot can follow safely to reach some destination. Footstep planning lies at the highest level of the legged controller stack (Fig. 1) [2]. In this problem, we significantly simplify motion planning through contact by approximating or ignoring whole-body kinematics and dynamics. This enables us to produce a tractable problem and warm start our lower-level motion planners.

In the past, there have been two approaches to footstep planning: discrete searches and continuous optimizations. In turn, there have been two approaches to discrete search methods: representing the action set as a set of allowed movements from one footstep to the next, or representing a set of possible footholds in the environment. These actions can be chained together to form a tree and we can apply graph-search algorithms to find the best set of actions. Action set approaches using a pre-computed set of allowed movements have been used by Hornung [3], Michel [4], Baudoin [5], Chestnutt [6], [7], Kuffner [8], [9], Shkolnik [10]. The fixed set of footholds representation has been used by Bretl [11] and Neuhaus [12].

The alternate approach involves optimizing directly on the poses of the footsteps as continuous decision variables. Team MIT applied non-convex optimization during the DARPA Robotics Challenge 2013 Trials [13], but this optimization could not guarantee its solutions' optimality or find paths around obstacles. Tedrake [1] implemented a mixed-integer quadratically constrained quadratic program (MIQCQP) to provide a more capable continuous footstep planner. Their approach performs continuous optimization of the footstep locations while the integer variables absorb the non-convex constraints. In this paper, we shall implement a simplified version of their method using a mixed-integer quadratic program (MIQP). We will only deal with non-convex constraints from obstacle avoidance leading to discontinuous safe regions in the 2D environment. We also assume that we already have the obstacle-free convex terrain regions (shown in Fig. 1). The code is available on GitHub[1].

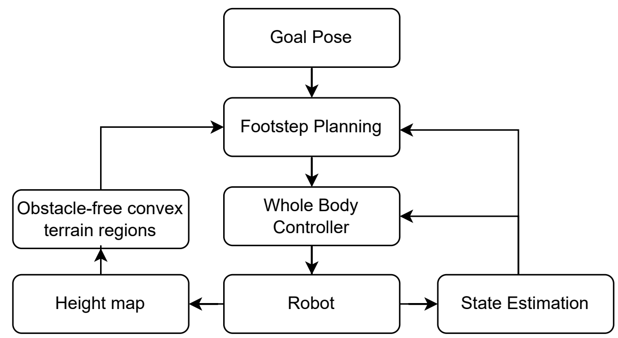

Fig. 1. Stack for legged locomotion control

II. TECHNICAL APPROACH

A. Complete Formulation

The below equations describe the exact implementation and reflect the variable choice in our GitHub repository.

$$\begin{aligned}\underset{f_{L,t}, f_{R,t}, \delta^L_{t,i}, \delta^R_{t,i}}{\text{minimize}} \quad & \left(\left(f_{\{L,R\},N} - g_{\{L,R\}}\right)^\top Q_g \right. \\ & \left.\left(f_{\{L,R\},N} - g_{\{L,R\}}\right)\right) \\ & + \sum_{t=0}^{N-1} \left(\left(f_{\{L,R\},t+1} - f_{\{L,R\},t}\right)^\top \right. \\ & \left. Q_r \left(f_{\{L,R\},t+1} - f_{\{L,R\},t}\right)\right)\end{aligned}$$

where

$f_{L,t}$: Left foot position at time step t
$t = 0, \ldots, N$ (Number of time steps for each foot)
$i = 1, \ldots, S$ (Number of safe convex stones)
$\{L, R\}$: Sum over both left foot and right foot
Q_g : Objective weights on distance to the goal
Q_r : Objective weights on distance between steps
$\delta^L_{t,i}$: True if left foot is on stone i at time step t

subject to

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[1]https://github.com/trunc8/2D-footstep-planning

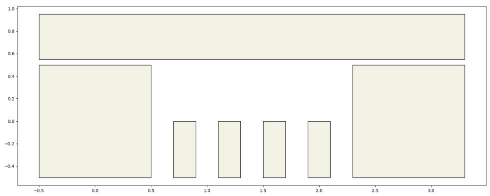

Fig. 2. Terrain A with a complete bridge

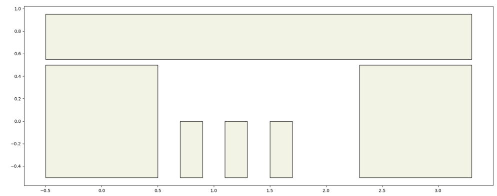

Fig. 3. Terrain B with a missing block in the bridge

Approximate reachability as square centered on the other foot:

$$|f_{R,t} - f_{L,t}| \leq \frac{\text{step\_span}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \forall t \in [0, N]$$

Alternating step sequence (assuming that we are first stepping with the left foot at $t = 0$):

$$|f_{L,t+1} - f_{L,t}| \leq \text{step\_limit} * is\_even(t)$$

$$|f_{R,t+1} - f_{R,t}| \leq \text{step\_limit} * is\_odd(t)$$

Foot should be on exactly one of the stones at any time t:

$$\sum_{i=1}^{S} \delta^{L}_{t,i} = 1$$

$$\sum_{i=1}^{S} \delta^{R}_{t,i} = 1$$

Constrain each foot to be inside exactly one convex stone region at any time t using the big-M method:

$$A_i f_{\{L,R\},t} \leq b_i + (1 - \delta^{\{L,R\}}_{t,i})M \quad \forall t \in [0, N], i \in [1, S]$$

where

$$A_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

b_i : (4×1 vector) Distance from center to corners of stone
M : (4×1 vector) Large enough values to relax inequality constraint when binary variable is not active

III. RESULTS

We implement the above formulation using the Drake [14] software on Python. We perform mixed-integer branch-and-bound using Drake's OSQP solver to find numerical solutions to the QP sub-problems. We verify our algorithm experimentally on two setups (Fig. 2 and 3) and present the time taken for different parameter values in Tables I and II respectively. The animated gif results can be viewed on the GitHub repository[2].

[2]https://github.com/trunc8/2D-footstep-planning/tree/main/results

In the experimental terrains, the width of the gaps = width of the bridge stones $= 0.2m$. In the tabulated results, step span refers to the side of the reachable square centered around one foot. The number of steps in the current implementation is fixed. The implementation of trimming the extra steps is currently buggy and has not been presented. The objective of trimming is to attach a cost to taking extra steps while allowing flexibility to take more number of steps if needed.

TABLE I
TIME TAKEN IN SECONDS FOR DIFFERENT PARAMETER VALUES IN TERRAIN A

No. of steps (right) Step span in m (down)	8	14	18
0.8	0.83	2.36	3.53
0.42	2.28	3.61	24.67

TABLE II
TIME TAKEN IN SECONDS FOR DIFFERENT PARAMETER VALUES IN TERRAIN B

No. of steps (right) Step span in m (down)	8	14	18
0.8	0.54	56.77	X
0.42	0.96	3.18	30.91

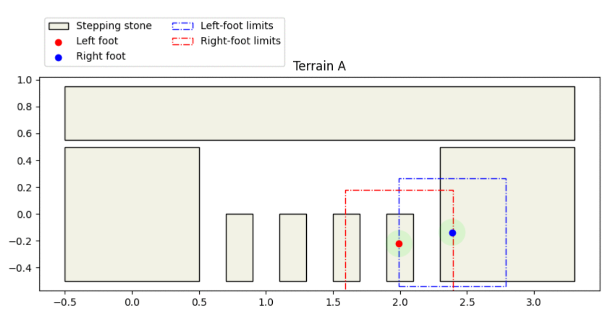

Fig. 4. Snapshot mid-execution of footstep plan in Terrain A with n_steps $= 8$ and step_span $= 0.8m$

We prove from Fig. 4 and 5 that the robot is forced to climb the lateral stone when there is a missing block

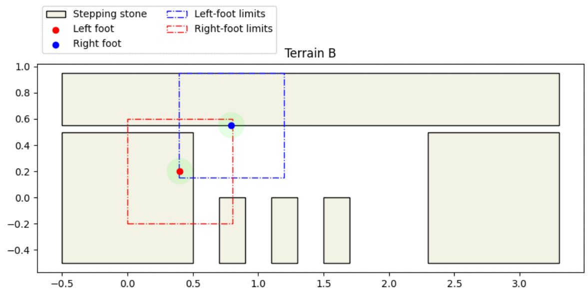

Fig. 5. Snapshot mid-execution of footstep plan in Terrain B with n_steps $= 8$ and step_span $= 0.8m$

in the bridge. Both of these plans reach the goal location successfully. In my original formulation, I implemented the goal position as an equality constraint. A major improvement in the formulation was achieved by writing it instead as a component of the objective function. It cut down the time taken by more than half in some tested cases and also allowed the generation of incomplete plans (as seen in Fig. 6). As we currently have to fix the number of steps, the robot tries its best to reach near the goal position.

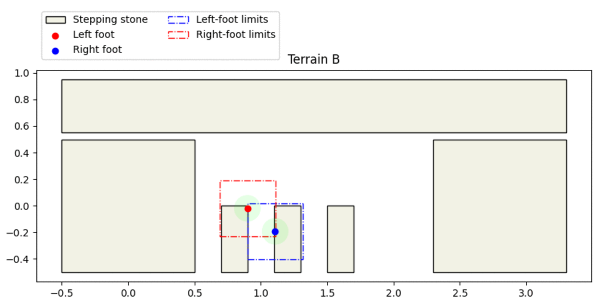

Fig. 6. Snapshot mid-execution of incomplete footstep plan with n_steps $= 8$ and step_span $= 0.42m$

IV. CONCLUSION AND FUTURE WORK

We have demonstrated foostep planning in 2D discontinuous terrains using a mixed-integer quadratic program. As future work, I want to extend this formulation to safe planar regions in 3D and benchmark performance improvements by rewriting the problem using Gurobi's optimizer. The time step trimming functionality failed in my big-M method formulation, but Gurobi provides a special *implies* constraint that can handle it directly. The trimming function will boost the power of this mixed-integer formulation. It would also be very interesting to expand the stack to include extracting obstacle-free convex terrain regions from an input height map (Fig. 1).

V. ACKNOWLEDGMENT

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