CS 763: Problem Set: Due: 10:00 PM, 19-Feb

- Please write (only if true) the honor code. If you used any source (person or thing) explicitly state it.
- Important: This is an INDIVIDUAL assignment.
- Always provide a brief explanation. (The length of the explanation required has been forecasted with the amount of space provided.)
- Submit following files in folder name lab03_roll_XX :
  1. readme.txt (case sensitive name). This text file contains identifying information, honor code, links to references used
  2. ReflectionEssay.pdf is optional but a brief one would be nice.
  3. lab03_roll_XX.pdf (includes all solutions).
  4. All relevant tex source (and images only if necessary). No other junk files, please.

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1. State whether or not the following points are the same and explain why.

   (a) $A[2, -1, 3]$, $B[4, -2, 6]$

   (b) $A[\sqrt{2}/2, -1, 0]$, $B[1, -\sqrt{2}, 0]$

   Solution:
   Condition: The representation $\mathbf{x}$ of a geometric object is homogeneous if $\mathbf{x}$ and k$\mathbf{x}$ represent the same object ($k \neq 0$)
   Assuming the given points are in homogeneous coordinates, we have:

   (a) $2 \times A = B$ Yes, they are the same point

   (b) $\sqrt{2} \times A = B$ Yes, they are the same point

2. In projective three-space, what are the standard homogeneous coordinates of (a) the origin and (b) ideal points determined by the intersections of the extensions of the coordinate axes and the ideal plane?

Solution:

Standard homogeneous coordinates are as follows:

(a) Origin: $[0, 0, 0, 1]^T$

(b) Ideal points: $[1, 0, 0, 0]^T$, $[0, 1, 0, 0]^T$, $[0, 0, 1, 0]^T$

We illustrate with the example of intersection of the extension of the X-axis and the ideal plane:

Ideal plane contains all points at ∞ and thus represented as $[0, 0, 0, 1]^T$

The X-axis is represented as linear combination of 2 planes(say, XY and XZ)

$$\mathbf{L} = m \times [0, 1, 0, 0]^T + n \times [0, 0, 1, 0]^T$$

Thus, our desired ideal point is at the intersection of 3 planes $[0, 0, 0, 1]^T$, $[0, 1, 0, 0]^T$ and $[0, 0, 1, 0]^T$. Solving the four 3×3 determinants of the 3×4 matrix, we obtain the ideal point: $[1, 0, 0, 0]^T$. Similarly intersection with the Y-axis and Z-axis can be shown.

3. Write standard homogeneous coordinates for the points specified in uppercase characters. (Use left and right to distinguish.)

Solution:

(a) **Left** A: $[-1.5, 1]^T$ B: $[3, 1]^T$ C: $[5, 1]^T$ D: $[5.5, 1]^T$ E: $[1, 0]^T$

(b) **Right** A: $[0, 0, 1]^T$ B: $[2, 0, 1]^T$ C: $[3, 1, 1]^T$ D: $[1, 1, 0]^T$ E: $[-1, 4.5, 1]^T$
F: $[-1, -1, 0]^T$ G: $[-3, 4, 1]^T$ H: $[-4, 3, 1]^T$ I: $[-1, 1, 1]^T$ J: $[-4, -2, 1]^T$
K: $[1, -4, 1]^T$ L: $[1.5, -0.5, 1]^T$ M: $[0, -1, 0]^T$

4. Which of the following points lie on the line $3p_1 - 2p_2 + 5p_3 = 0$? Why?

(a) $A[1, 1, 2]$

(b) $B[4, 1, -2]$

Solution:

The projective line $\mathbf{u}$ is represented by the triplet $[3, -2, 5]^T$. All points $\mathbf{p}$ lying on the line satisfy the equation $\mathbf{u}^T\mathbf{p} = 0$

(a) $\mathbf{u}^T\mathbf{p} = 11$ No, it does not lie on $\mathbf{u}$

(b) $\mathbf{u}^T\mathbf{p} = 0$ Yes, it lies on $\mathbf{u}$

5. Write the coordinates of the lines that are the extensions to the projective plane of the following Euclidean lines.

(a) $3x + 2y = 6$

(b) $4x + 5y + 7 = 0$

Solution:

(a) $[3, 2, -6]^T$

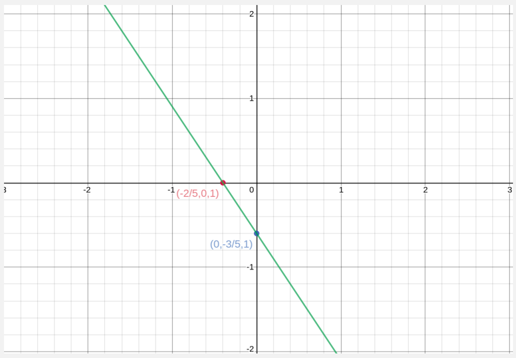
(b) $[4, 5, 7]^T$

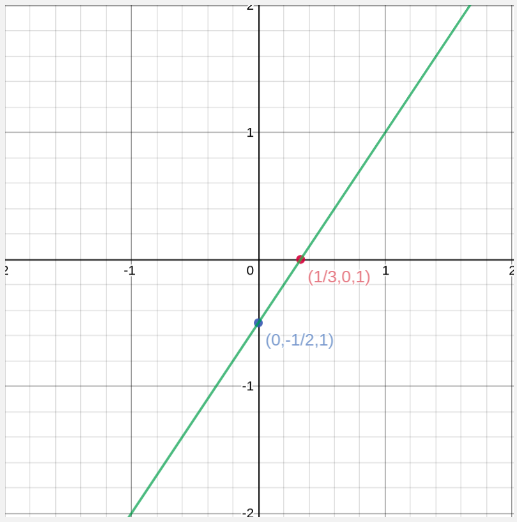
6. Sketch each line in the projective plane whose equation is given.

(a) $2p_1 + 3p_2 + 5p_3 = 0$

(b) $3p_1 - 2p_2 - p_3 = 0$

Solution:

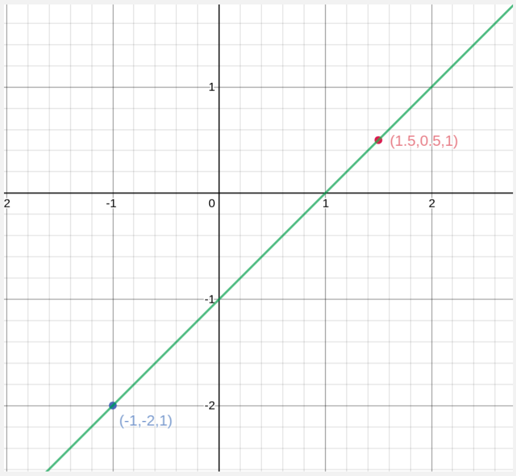




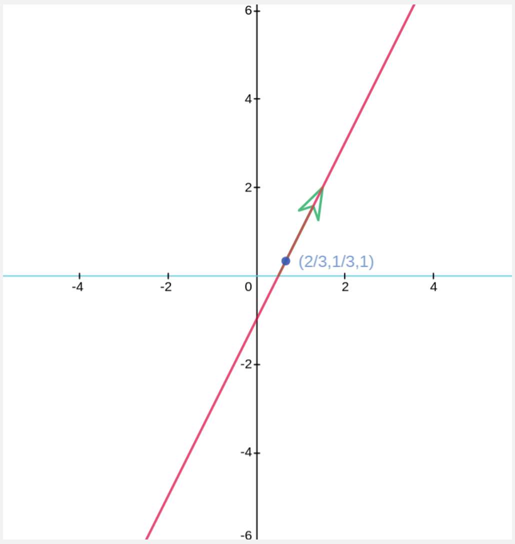
7. In each of the following cases, sketch the line determined by the two given points; then find the equation of the line.

   (a) $A[3, 1, 2]$, $B[1, 2, -1]$ (b) $A[2, 1, 3]$, $B[1, 2, 0]$

Solution:



$$-p_1 + p_2 + p_3 = 0$$



$$-2p_1 + p_2 + p_3 = 0$$

8. Find the standard homogeneous coordinates of the point of intersection for each pair of lines.

   (a) $p_1 + p_2 - 2p_3 = 0, 3p_1 + p_2 + 4p_3 = 0$ (b) $p_1 + p_2 = 0, 4p_1 - 2p_2 + p_3 = 0$

Solution:
The point of intersection for pair of lines is obtained by taking cross product of the line coordinates

(a) $[1, 1, -2]^T \times [3, 1, 4]^T = [6, -10, -2]^T$

(b) $[1, 1, 0]^T \times [4, -2, 1]^T = [1, -1, -6]^T$

9. Determine which of the following sets of three points are collinear.

(a) $A[1,2,1]$, $B[0,1,3]$, $[2,1,1]$

(b) $A[1,2,3]$, $B[2,4,3]$, $[1,2,-2]$

Solution:
Points **p**, **q** and **r** are collinear if their corresponding determinant is 0

(a) Not collinear

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 8$$

(b) Collinear

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 3 \\ 1 & 2 & -2 \end{vmatrix} = 0$$

10. Determine which of the following sets of three lines meet in a point.

(a) $l[1,0,1], m[1,1,0], n[0,1,-1]$

(b) $l[1,0,-1], m[1,-2,1], n[3,-2,-1]$

Solution:
Lines **l**, **m** and **n** intersect at a point if their corresponding determinant is 0
Reason: The cross product of first two line coordinates when scalar multiplied with third line coordinate should be 0. This is the scalar triple product that is easily obtained as the corresponding determinant

(a) Yes, intersect

$$\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 10 \\ 0 & 1 & -1 \end{vmatrix} = 0$$

(b) Yes, intersect

$$\Delta = \begin{vmatrix} 1 & 0 & -1 \\ 1 & -2 & 1 \\ 3 & -2 & -1 \end{vmatrix} = 0$$