CS 763: Problem Set: Due: 10:00 PM, 18-Feb

- Please write (only if true) the honor code. If you used any source (person or thing) explicitly state it.
- Important: This is an INDIVIDUAL assignment.
- Always provide a brief explanation. (The length of the explanation required has been forecasted with the amount of space provided.)
- Submit following files in folder name lab03_roll_XX :
  1. readme.txt (case sensitive name). This text file contains identifying information, honor code, links to references used
  2. ReflectionEssay.pdf is optional but a brief one would be nice.
  3. lab03_roll_XX.pdf (includes all solutions).
  4. All relevant tex source (and images only if necessary). No other junk files, please.

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1. State whether or not the following points are the same and explain why.

   (a) $A[2,-1,3]$, $B[4,-2,6]$ (b) $A[\sqrt{2}/2,-1,0]$, $B[1,-\sqrt{2},0]$

   In $\mathbb{P}^{\nvDash}$ space all points of the form $k\mathbf{v}$ are equivalent, where k is a scalar.
   For (a) we have $2A = B$, hence they are equal
   For (b) $\sqrt{2}A = B$ hence they are equal. (Though they are ideal points)

2. In projective three-space, what are the standard homogeneous coordinates of (a) the origin and (b) ideal points determined by the intersections of the extensions of the coordinate axes and the ideal plane?
   (a) Origin is (0,0,0,1) (b) The Ideal plane is given by (0,0,0,1) and the x axis is given by (a,0,0,b), y by (0,a,0,b) and z by (0,0,a,b).
   So the intersection of x-axis and the plane is (1,0,0,0),y-axis and the plane is (0,1,0,0) and z-axis and the plane is (0,0,1,0)

3. Write standard homogeneous coordinates for the points specified in uppercase characters. (Use left and right to distinguish.)
   Left : $A = (-1.5, 1), B = (3, 1), C = (5, 1), D = (5.5, 1), E = (1, 0)$
   Right : $A = (0, 0, 1), B = (2, 0, 1), C = (3, 1, 1), D = (1, 1, 0), E = (-1, 4.5, 1), F = (-1, 1, 0), G = (-3, 4, 1), H = (-4, 3, 1), I = (-1, 1, 1), J = (-4, -2, 1), K = (1, 4, 1), L = (1.5, -0.5, 1), M = (0, -1, 0)$

4. Which of the following points lie on the line $3p_1 - 2p_2 + 5p_3 = 0$? Why?

(a) $A[1, 1, 2]$

(b) $B[4, 1, -2]$

For a point to lie on a line it should satisfy the equation. So substituting $A[p_1, p_2, p_3]$ into line equation we get
For a we have

$$3 * 1 - 2 * 1 + 5 * 2 \neq 0$$

Doesn't satisfy the line equation
For b we have

$$3 * 4 - 2 * 1 + 5 * (-2) = 0$$

Hence satisfies the line equation and lies on the line

5. Write the coordinates of the lines that are the extensions to the projective plane of the following Euclidean lines.

(a) $3x + 2y = 6$

(b) $4x + 5y + 7 = 0$

Replacing x by X/Z and y by Y/Z we get $3X + 2Y - 6Z = 0$ $4X + 5Y + 7Z = 0$

6. Sketch each line in the projective plane whose equation is given.

(a) $2p_1 + 3p_2 + 5p_3 = 0$

(b) $3p_1 - 2p_2 - p_3 = 0$

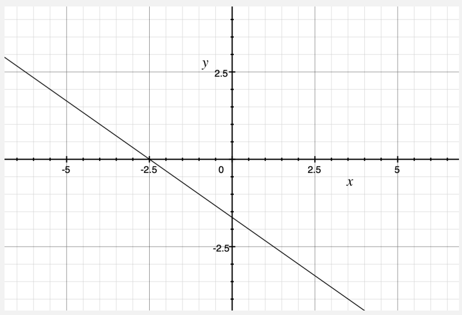

Figure 1: (a)

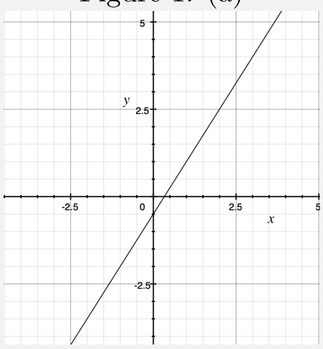

Figure 2: (b)

7. In each of the following cases, sketch the line determined by the two given points; then find the equation of the line.

(a) $A[3,1,2]$, $B[1,2,-1]$ (b) $A[2,1,3]$, $B[1,2,0]$

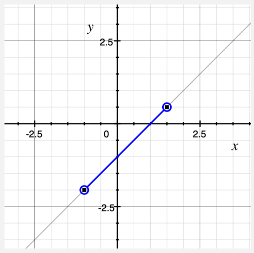

Figure 3: (a)

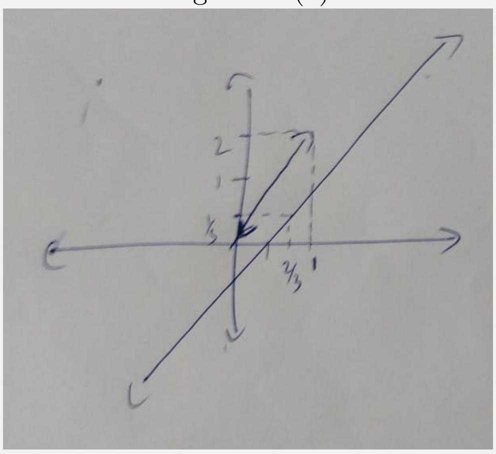

Figure 4: (b)

For (a) the equation of line using points (1.5,0.5) and (-1,-2) is

$$y = x - 1$$

for (b) using point (2/3,1/3) and slope of 2/1 we get

$$y = 2x - 1$$

8. Find the standard homogeneous coordinates of the point of intersection for each pair of lines.

(a) $p_1 + p_2 - 2p_3 = 0, 3p_1 + p_2 + 4p_3 = 0$ (b) $p_1 + p_2 = 0, 4p_1 - 2p_2 + p_3 = 0$

The point of intersection of lines in the cross product of the triplets defining the lines. Hence, for (a) we have

$$(1,1,-2) \times (3,1,4) = (6,-10,-2) = (-3,5,1)$$

For (b) we have

$$(1,1,0) \times (4,-2,1) = (1,-1,-6) = (-1/6,1/6,1)$$

9. Determine which of the following sets of three points are collinear.

(a) $A[1,2,1]$, $B[0,1,3]$, $[2,1,1]$ (b) $A[1,2,3]$, $B[2,4,3]$, $[1,2,-2]$

For 3 points to be collinear their determinent has to be 0. The theorem can be found on Slide 9 of "FriJan29.pdf"
For (a) we have

$$|[A,B,C]| = 8 \neq 0$$

hence they aren't collinear.
For (b) we have

$$7A - 5B + 3C = 0$$

which implies collinearity of A, B, C

10. Determine which of the following sets of three lines meet in a point.

(a) $l[1,0,1], m[1,1,0], n[0,1,-1]$ (b) $l[1,0,-1], m[1,-2,1], n[3,-2,-1]$

For 3 lines in $\mathbb{P}^2$ to intersect in a point, their determinant has to be 0. This can also be thought of as a result of duality between points and lines in $\mathbb{P}^{\nvDash}$. For (a) the lines can be written as

$$l + n - m = 0$$

which implies the determinant is 0. Hence they meet in a point. For (b), they can be written as

$$2l + m - n = 0$$

which again gives 0 determinant implying they meet in a point.