Algorithm 1 Solve $\min_{u \in U} F_h(u) + \mathrm{TGV}_\alpha^2(u)$

1: Choose $\sigma > 0$, $\tau > 0$ such that $\sigma\tau\frac{1}{2}(17 + \sqrt{33}) \le 1$.
2: Choose $(u^{[0]}, p^{[0]}) \in U \times V$, $(v^{[0]}, w^{[0]}) \in V \times W$ and set $\bar{u}^{[0]} = u^{[0]}$, $\bar{p}^{[0]} = p^{[0]}$.
3: **for** $n = 0, 1, 2, \ldots$ **do**
4:

$$\begin{cases} v^{[n+1]} = \mathcal{P}_{\alpha_1}\left(v^{[n]} + \sigma(\nabla_h \bar{u}^{[n]} - \bar{p}^{[n]})\right), \\ w^{[n+1]} = \mathcal{P}_{\alpha_0}\left(w^{[n]} + \sigma\mathcal{E}_h(\bar{p}^{[n]})\right), \\ u^{[n+1]} = \left(u^{[n]} + \tau(\mathrm{div}_h v^{[n+1]} + f)\right)/(1+\tau), \\ p^{[n+1]} = p^{[n]} + \tau(v^{[n+1]} + \mathrm{div}_h w^{[n+1]}), \\ \bar{u}^{[n+1]} = 2u^{[n+1]} - u^{[n]}, \\ \bar{p}^{[n+1]} = 2p^{[n+1]} - p^{[n]}. \end{cases}$$

5: **end for**
6: Return $u^{[N]}$ for some large N.

where

$$\begin{aligned} \mathcal{P}_{\alpha_1}(v) &= \frac{v}{\max\left(1, \frac{|v|}{\alpha_1}\right)}, \quad v \in V \\ \mathcal{P}_{\alpha_0}(w) &= \frac{w}{\max\left(1, \frac{|w|}{\alpha_0}\right)}, \quad w \in W, \end{aligned} \tag{1}$$

$$\begin{aligned} v \in V: \quad |v|_{i,j} &= \left(\left((v)_{i,j}^{(1)}\right)^2 + \left((v)_{i,j}^{(2)}\right)^2\right)^{1/2}, \\ w \in W: \quad |w|_{i,j} &= \left(\left((w)_{i,j}^{(11)}\right)^2 + \left((w)_{i,j}^{(22)}\right)^2 + 2\left((w)_{i,j}^{(12)}\right)^2\right)^{1/2}. \end{aligned} \tag{2}$$

The x and y forward finite difference operators are (page 13)

$$(\partial_x^+ u)_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & \text{for } 1 \le i < N_1, \\ 0 & \text{for } i = N_1, \end{cases} \tag{3}$$

$$(\partial_y^+ u)_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & \text{for } 1 \le j < N_2, \\ 0 & \text{for } j = N_2, \end{cases} \tag{4}$$

and the backward finite difference operators are (page 13)

$$(\partial_x^- u)_{i,j} = \begin{cases} u_{1,j} & \text{if } i = 1, \\ u_{i,j} - u_{i-1,j} & \text{for } 1 < i < N_1, \\ -u_{N_1-1,j} & \text{for } i = N_1, \end{cases} \tag{5}$$

$$(\partial_y^- u)_{i,j} = \begin{cases} u_{i,1} & \text{if } j = 1, \\ u_{i,j} - u_{i,j-1} & \text{for } 1 < j < N_2, \\ -u_{i,N_2-1} & \text{for } j = N_2, \end{cases} \tag{6}$$

The gradient operator is defined as (page 13)

$$\nabla_h : U \to V, \quad \nabla_h u = \begin{pmatrix} \partial_x^+ u \\ \partial_y^+ u \end{pmatrix}, \tag{7}$$

and the symmetrised gradient operator is defined as (page 14)

$$\mathcal{E}_h : V \to W, \quad \mathcal{E}_h(v) = \begin{pmatrix} \partial_x^-(v)^{(1)} & \frac{1}{2}\left(\partial_y^-(v)^{(1)} + \partial_x^-(v)^{(2)}\right) \\ \frac{1}{2}\left(\partial_y^-(v)^{(1)} + \partial_x^-(v)^{(2)}\right) & \partial_y^-(v)^{(2)} \end{pmatrix}, \tag{8}$$

and

$$\mathrm{div}_h : V \to U, \quad \mathrm{div}_h v = \partial_x^-(v)^{(1)} + \partial_y^-(v)^{(2)}, \tag{9}$$

$$\mathrm{div}_h : W \to V, \quad \mathrm{div}_h w = \begin{pmatrix} \partial_x^+(w)^{(11)} + \partial_y^+(w)^{(12)} \\ \partial_x^+(w)^{(12)} + \partial_y^+(w)^{(22)} \end{pmatrix}. \tag{10}$$