Graduation Project - Total Variational Image Denoising with U-Net
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1 Introduction

- Rewrite parts written by ChatGPT, or written specifically for mini project but not relevant anymore

- Check consistent notations

- Redraw U-Net diagram

- Arrange figures in correct locations

In this dissertation we will describe the theory and application of the total variational (TV) method to image denoising. We will also describe the implementation of the method and an improvement using a U-Net architecture to find the regularisation parameters more efficiently.

Image processing, and image denoising in particular, has seen significant advancements since the U-Net was first applied in 2015. This project aims to build upon these advancements by implementing and experimenting with a U-Net-based model for image denoising. In particular, it combines U-Net with a traditional image-denoising algorithm called the Primal Dual Hybrid Gradient (PDHG) algorithm [1]. Specifically, a PDHG solver is appended to the end of the U-Net, treating the solver as the final layer of the entire network to create an end-to-end unsupervised model.

One notable aspect of using U-Net in this problem is that, instead of acting as a black box and outputting a denoised version of each input image, it outputs a parameter map used in another algorithm, demonstrating the flexibility of neural networks.

Here we combine the traditional TV method with the U-Net to produce a more efficient method for image denoising.

1.1 Total Variational Denoising

The total variational (TV) method is a method for image denoising that is based on the principle of minimising the total variation of the image. The total variation of an image is a measure of the amount of variation in the intensity of the image. The TV method works by finding the image that has the smallest total variation and is closest to the noisy image in terms of the mean squared error.

The TV method is based on the idea that images that are smooth have a small total variation, while images that are noisy have a large total variation. By minimising the total variation of the image, the TV method is able to remove the noise from the image and produce a smoother image.

Denoising problems can be described as

$$\mathbf{z} = \mathbf{x}_{\text{true}} + \mathbf{e} \tag{1}$$

where $\mathbf{x}_{\text{true}} \in \mathbb{R}^{m \times n}$ is the object to be imaged, $\mathbf{e} \in \mathbb{R}^{m \times n}$ denotes some random noise component, and $\mathbf{z} \in \mathbb{R}^{m \times n}$ represents the measured data. The goal is to recover $\mathbf{x}_{\text{true}}$ or at least a good enough reconstruction given $\mathbf{z}$.

A prominent approach is to formulate the reconstruction as a minimisation problem:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} d(\mathbf{x}, \mathbf{z}) + \mathcal{R}(\mathbf{x}) \tag{2}$$

where $d(\cdot, \cdot)$ is a data fidelity term and $\mathcal{R}(\cdot)$ is a regularisation term.

Here we will look at Gaussian noise. The data fidelity term appropriate for Gaussian noise is the square of the ℓ_2 norm:

$$d(\mathbf{x}, \mathbf{z}) = \frac{1}{2}\|\mathbf{x} - \mathbf{z}\|_2^2 \tag{3}$$

The regularisation term $\mathcal{R}(\cdot)$ is the total variation of the image, which is a measure of the amount of variation in the intensity of the image. The total variation of an image can be calculated using the following formula:

$$\mathcal{R}(\mathbf{x}) = \lambda\|\nabla\mathbf{x}\|_1 \tag{4}$$

where $\nabla\mathbf{x}$ is the gradient of the image $\mathbf{x}$ and $\lambda \in \mathbb{R}^+$ is a regularisation parameter that controls the trade-off between the total variation of the image and the mean squared error between the denoised image $\mathbf{x}$ and the noisy image $\mathbf{z}$.

∇ denotes a finite-differences operator. Finite difference operators include, forward difference operator, backward difference operator, shift operator, central difference operator and mean operator. Let us focus on the forward difference operator. In particular, in the discrete case, given a matrix $\mathbf{x}$, $\nabla\mathbf{x}$ is two matrices, one for the vertical gradient $\nabla_x\mathbf{x}$ and one for the horizontal gradient $\nabla_y\mathbf{x}$.

$$\nabla\mathbf{x} = \begin{bmatrix} \nabla_x\mathbf{x} \\ \nabla_y\mathbf{x} \end{bmatrix} \tag{5}$$

For the forward gradient, the vertical gradient is calculated as:

$$\begin{aligned} \nabla_x\mathbf{x} &= \begin{bmatrix} \mathbf{x}_{2,1} - \mathbf{x}_{1,1} & \mathbf{x}_{2,2} - \mathbf{x}_{1,2} & \dots & \mathbf{x}_{2,n-1} - \mathbf{x}_{1,n-1} & \mathbf{x}_{2,n} - \mathbf{x}_{1,n} \\ \mathbf{x}_{3,1} - \mathbf{x}_{2,1} & \mathbf{x}_{3,2} - \mathbf{x}_{2,2} & \dots & \mathbf{x}_{3,n-1} - \mathbf{x}_{2,n-1} & \mathbf{x}_{3,n} - \mathbf{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{x}_{m,1} - \mathbf{x}_{m-1,1} & \mathbf{x}_{m,2} - \mathbf{x}_{m-1,2} & \dots & \mathbf{x}_{m,n-1} - \mathbf{x}_{m-1,n-1} & \mathbf{x}_{m,n} - \mathbf{x}_{m-1,n} \\ \mathbf{x}_{1,1} - \mathbf{x}_{m,1} & \mathbf{x}_{1,2} - \mathbf{x}_{m,2} & \dots & \mathbf{x}_{1,n-1} - \mathbf{x}_{m,n-1} & \mathbf{x}_{1,n} - \mathbf{x}_{m,n} \end{bmatrix} \\ &= \begin{bmatrix} -1 & 1 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & 1 \\ 1 & 0 & \dots & 0 & -1 \end{bmatrix} \mathbf{x} \end{aligned} \tag{6}$$

and the horizontal gradient is calculated as:

$$\nabla_y \mathbf{x} = \begin{bmatrix} \mathbf{x}_{1,2} - \mathbf{x}_{1,1} & \mathbf{x}_{1,3} - \mathbf{x}_{1,2} & \dots & \mathbf{x}_{1,n} - \mathbf{x}_{1,n-1} & \mathbf{x}_{1,1} - \mathbf{x}_{1,n} \\ \mathbf{x}_{2,2} - \mathbf{x}_{2,1} & \mathbf{x}_{2,3} - \mathbf{x}_{2,2} & \dots & \mathbf{x}_{2,n} - \mathbf{x}_{2,n-1} & \mathbf{x}_{2,1} - \mathbf{x}_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{x}_{m-1,2} - \mathbf{x}_{m-1,1} & \mathbf{x}_{m-1,3} - \mathbf{x}_{m-1,2} & \dots & \mathbf{x}_{m-1,n} - \mathbf{x}_{m-1,n-1} & \mathbf{x}_{m-1,1} - \mathbf{x}_{m-1,n} \\ \mathbf{x}_{m,2} - \mathbf{x}_{m,1} & \mathbf{x}_{m,3} - \mathbf{x}_{m,2} & \dots & \mathbf{x}_{m,n} - \mathbf{x}_{m,n-1} & \mathbf{x}_{m,1} - \mathbf{x}_{m,n} \end{bmatrix} \\ = \mathbf{x} \begin{bmatrix} -1 & 0 & \dots & 0 & 1 \\ 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & 0 \\ 0 & 0 & \dots & 1 & -1 \end{bmatrix} \tag{7}$$

We calculate the gradient in the horizontal and vertical directions separately, and then combine them to get the total variation of the image.

$$\|\nabla \mathbf{x}\|_1 = \|\nabla_x \mathbf{x}\|_1 + \|\nabla_y \mathbf{x}\|_1 \tag{8}$$

For the forward gradient, the norm-1 of the vertical gradient is calculated as:

$$\|\nabla_x \mathbf{x}\|_1 = \left(\sum_{i=1}^{m-1} \sum_{j=1}^{n} |\mathbf{x}_{i+1,j} - \mathbf{x}_{i,j}| \right) + \sum_{j=1}^{n} |\mathbf{x}_{1,j} - \mathbf{x}_{m,j}| \tag{9}$$

and norm-1 of the horizontal gradient is calculated as:

$$\|\nabla_y \mathbf{x}\|_1 = \left(\sum_{i=1}^{m} \sum_{j=1}^{n-1} |\mathbf{x}_{i,j+1} - \mathbf{x}_{i,j}| \right) + \sum_{i=1}^{m} |\mathbf{x}_{i,1} - \mathbf{x}_{i,n}| \tag{10}$$

where $\mathbf{x}_{i,j}$ is the intensity value of pixel (i, j) in the image $\mathbf{x}$. Note that $\mathbf{x}_{i,j}$ can be a scalar for a black-and-white image or a vector of 3 values for a colour image.

Note that, we can move λ inside the norm, so the regularisation term can be written as:

$$\mathcal{R}(\mathbf{x}) = \lambda \|\nabla \mathbf{x}\|_1 = \|\lambda \nabla \mathbf{x}\|_1 = \|\lambda \nabla_x \mathbf{x}\|_1 + \|\lambda \nabla_y \mathbf{x}\|_1 \tag{11}$$

The regularisation parameter does not need to be a scalar. Since regularisation is multiplied by the gradient, higher λ values will penalize areas with many changes (typically areas with more details), while lower λ values prioritize detail preservation over noise reduction. Using a single λ value penalises all regions of the image equally, which is suboptimal. Instead, it would be better to assign higher penalties to smooth regions and lower penalties to detailed regions to preserve image details. We can swap the scalar λ with a regularisation parameter map $\mathbf{\Lambda}$,

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}^{(x)} \\ \mathbf{\Lambda}^{(y)} \end{bmatrix} \tag{12}$$

where $\mathbf{\Lambda}^{(x)}, \mathbf{\Lambda}^{(y)} \in \mathbb{R}^{m \times n}$ are matrices that contain the regularisation parameters for the vertical and horizontal gradients respectively. Here we will assume $\mathbf{\Lambda}^{(x)} = \mathbf{\Lambda}^{(y)}$ and denote them as $\mathbf{\Lambda}^{(xy)}$, i.e.

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}^{(xy)} \\ \mathbf{\Lambda}^{(xy)} \end{bmatrix} \tag{13}$$

This gives us a new regularisation term that can be written as

$$\mathcal{R}(\mathbf{x}) = \|\mathbf{\Lambda} \circ \nabla \mathbf{x}\|_1 = \|\mathbf{\Lambda}^{(xy)} \circ \nabla_x \mathbf{x}\|_1 + \|\mathbf{\Lambda}^{(xy)} \circ \nabla_y \mathbf{x}\|_1 \tag{14}$$

where $\circ$ denotes the Hadamard or element-wise product.

Putting it all together, the solution to the total variational denoising problem can be defined using the following formula:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{x} - \mathbf{z}\|_2^2 + \lambda \|\nabla \mathbf{x}\|_1 \tag{15}$$

if we use a scalar λ regularisation parameter, or

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{x} - \mathbf{z}\|_2^2 + \|\mathbf{\Lambda} \circ \nabla \mathbf{x}\|_1 \tag{16}$$

if we use a regularisation parameter map $\mathbf{\Lambda}$ of regularisation parameters.

In this dissertation, we will use U-Net to help us find the regularisation parameter map $\mathbf{\Lambda}$ given any (noisy) image. We will compare the results to the best-case scenario where we use a scalar regularisation parameter λ.

1.2 U-Net

The U-Net is a type of neural network that is commonly used for image segmentation [4]. The U-Net is an encoder-decoder network that is designed to take an image as input and produce a segmentation mask as output. The U-Net is made up of a series of convolutional layers that downsample the image and a series of transposed convolutional layers that upsample the image. The U-Net is able to learn to segment images by training on a large dataset of images and their corresponding segmentation masks.

We refer to the set of trainable parameters in the U-Net as Θ, and the output of the U-Net as Λ_Θ.

In our case, we will optimise a U-Net model to find the regularisation parameter map Λ_Θ that produces the best denoised image for any particular noisy image.

1.3 Related Work

1.4 Contribution

- Adapt the existing code of the dynamic image denoising project in [1] to static image denoising. Train and evaluate the model using a new dataset - Chest X-ray dataset.

- Present the theory of the total variational denoising method and the experimental set-up in detail so that a fellow student can understand the method and implement it themselves.

2 Method and Implementation

2.1 Primal Dual Hybrid Gradient (PDHG)

To solve equation (14) we can use an iterative method called the Primal Dual Hybrid Gradient (PDHG).

The algorithm is

Algorithm 1 PDHG algorithm for image denoising with fixed regularization parameter-map $\mathbf{\Lambda}$ (adapted from ...)

Input: $L = \|[\mathbf{I}, \nabla]^{\mathrm{T}}\|$, $\tau = 1/L$, $\sigma = 1/L$, $\theta = 1$, noisy image $\mathbf{x}_0$
Output: reconstructed image $\hat{\mathbf{x}}$
$\bar{\mathbf{x}}_0 = \mathbf{x}_0$
$\mathbf{p}_0 = \mathbf{0}$
$\mathbf{q}_0 = \mathbf{0}$
for $k < T$ **do**
    $\mathbf{p}_{k+1} = (\mathbf{p}_k + \sigma(\bar{\mathbf{x}}_k - \mathbf{x}_0)) / (1 + \sigma)$
    $\mathbf{q}_{k+1} = \mathrm{clip}_{\mathbf{\Lambda}} (\mathbf{q}_k + \sigma \nabla \bar{\mathbf{x}}_k)$
    $\mathbf{x}_{k+1} = \mathbf{x}_k - \tau \mathbf{p}_{k+1} - \tau \nabla^{\mathrm{T}} \mathbf{q}_{k+1}$
    $\bar{\mathbf{x}}_{k+1} = \mathbf{x}_{k+1} + \theta(\mathbf{x}_{k+1} - \mathbf{x}_k)$
end for
$\hat{\mathbf{x}} = \mathbf{x}_T$

where $\mathbf{I}$ is the identity matrix, $\mathrm{clip}_{\mathbf{\Lambda}}$ is a function that clips the values of $\mathbf{q}_{k+1}$ to the range $[\mathbf{0}, \mathbf{\Lambda}]$.

The choice of L is taken from [5] https://iopscience.iop.org/article/10.1088/0031-9155/57/10/3065/pdf. The number of iterations T is a hyperparameter that we can choose for the training process.

(TODO: Understand what L is. Do we need to include $\theta = 1$? Can we write $\bar{\mathbf{x}}_{k+1} = 2\mathbf{x}_{k+1} - \mathbf{x}_k$? In the code, initially $\mathbf{p}_0 = \mathbf{x}_0$?)

The challenge lies in finding the regularisation parameter map $\mathbf{\Lambda}$. Narrowing down the search space is challenging, as we potentially have d^{mn} different maps to consider, where d is the number of λ values (typically around 100, ranging from 0.01 to 1) and n is the image dimension. This is computationally infeasible. The U-Net can be used to find the regularisation parameter map $\mathbf{\Lambda}$ more efficiently.

3 Architecture

For this project, our model comprises two primary components:

1. A U-Net, denoted as NET_{Θ}, which is responsible for learning the regularisation parameters $\mathbf{\Lambda}_{\Theta}$ from an input image $\mathbf{x}_0$.
2. A Primal Dual Hybrid Gradient (PDHG) algorithm solver which utilizes both $\mathbf{z}$ and $\mathbf{\Lambda}_{\Theta}$ to transform $\mathbf{x}_0$ into $\hat{\mathbf{x}}$.

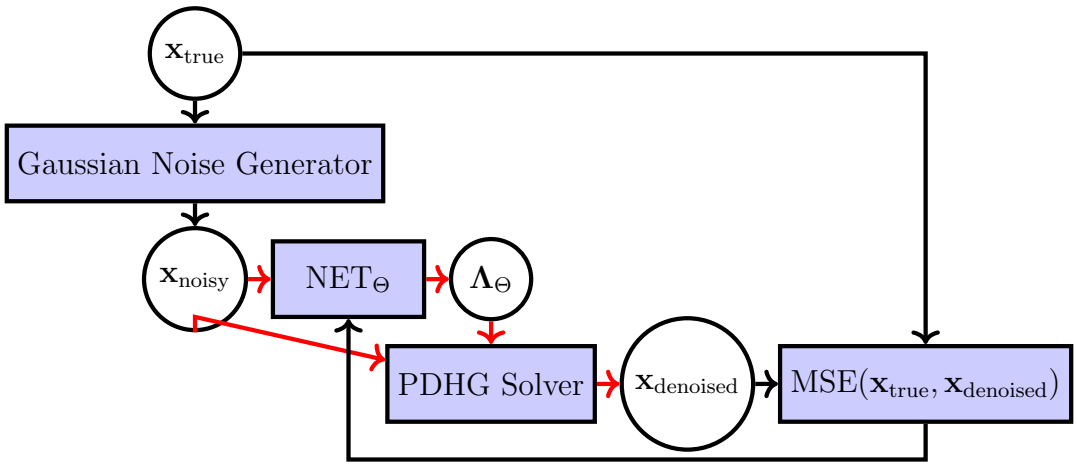


Figure 1: The general architecture

The general architecture is depicted in shown in Figure 1,

where

- $\mathbf{x}_{\text{true}}$ represents the clean image, serving as the ground truth.
- $\mathbf{x}_{\text{noisy}}$ denotes the noisy image inputted to NET_Θ.
- $\mathbf{\Lambda}_\Theta$ is the regularisation parameters map which is the final output from the U-Net.

We can treat the PDHG solver as the final hidden layer in our network. The result of the loss function $\text{MSE}(\mathbf{x}_{\text{true}}, \mathbf{x}_{\text{denoised}})$ is then used for backpropagation to train the parameters Θ.

As previously mentioned, the specific architecture for NET_Θ is the U-Net, a special type of convolutional neural network architecture.

3.1 Convolutional Neural Network (CNN)

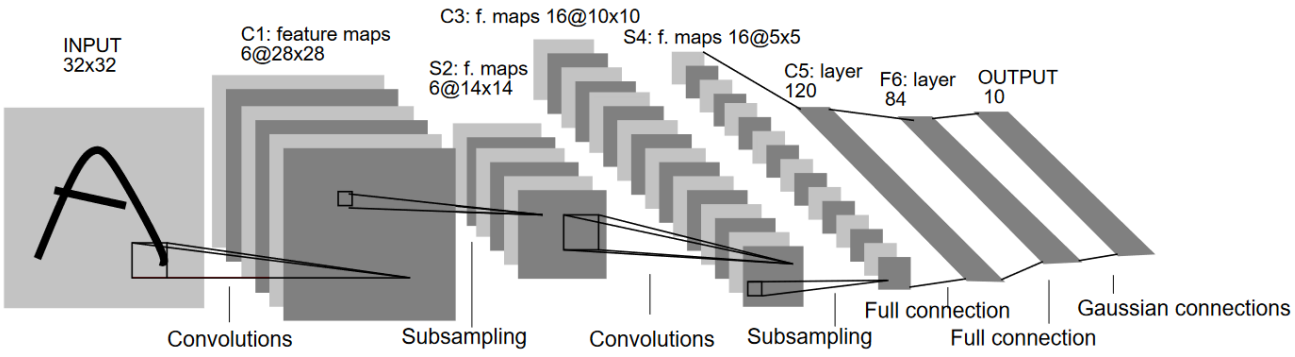


Figure 2: Architecture of LeNet-5, one of the earliest Convolutional Neural Networks [2]

Convolutional Neural Networks (CNNs) are specialised types of artificial neural networks optimised for analyzing image data and other grid-like structures. We base our understanding on a two-dimensional (2D) CNN architecture, where each layer is comprised of one or more 2D matrices, in contrast to the vectors used in standard "vanilla" neural networks.

Inspiration and Function

The design of CNNs draws inspiration from the human visual cortex, which processes visual stimuli by sequentially extracting features, ranging from simple edges and shapes to complex objects. CNNs replicate this hierarchical feature extraction using convolutional layers and pooling layers.

Convolutional Layers

In a convolutional layer, each output element results from a linear combination of inputs, processed through an activation function—akin to "vanilla" neural networks. However, the critical difference lies in the utilization of filters (or kernels). Each filter traverses the input matrix, computes the dot product with the corresponding input elements, and channels the result through an activation function. A single filter generates one output matrix, termed a feature map, acting as a feature extractor that emphasizes specific characteristics of the input. When multiple filters are employed in a layer, they produce multiple feature maps, with each map corresponding to a different channel. This design significantly reduces the proportion of trainable parameters relative to the number of output values, enhancing efficiency for image-related tasks.

Pooling Layers

An essential component of CNNs is the pooling layer, typically implemented as a max-pooling layer. This layer simplifies the output by retaining only the maximum values from the defined regions in the feature map. The objective is to preserve the most significant features while reducing data dimensionality. In this project, max-pooling is a crucial technique used to enhance model performance.

The convolutional and pooling layers constitute the foundational blocks of the U-Net architecture.

3.2 U-Net

A U-Net, as introduced by Ronneberger et al. [4], extends the conventional CNN architecture by incorporating skip connections, akin to those in a residual network. These skip connections concatenate the outputs of distinct convolutional layers to generate a larger composite output.

Encoder

The U-Net architecture is divided into two principal components: an Encoder and a Decoder. The Encoder functions similarly to a standard CNN, utilizing a combination of convolutional layers and max-pooling layers organized into distinct "blocks." In this project, each Encoder block comprises a pair of convolutional layers followed by a max-pooling layer, designed to successively double the number of channels (or feature maps) while concurrently reducing the feature map size due to the pooling.

Decoder

Conversely, the Decoder is structured around a series of blocks that progressively halve the number of channels while enlarging the feature map size, effectively "unrolling" the data. Unlike the Encoder blocks, each Decoder block concludes with an up-convolutional layer followed by a

skip connection. This up-convolutional layer, essentially a regular convolutional layer, outputs data that is then concatenated with the output from a corresponding Encoder block, facilitating the feature integration across different layers of the network.

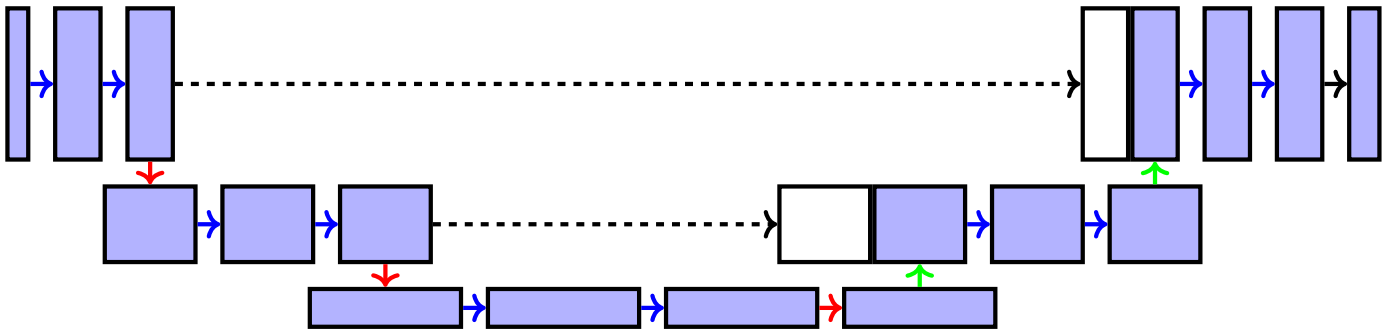


Figure 3: Our NET_Θ Architecture

In our model, the Encoder consists of three encoding blocks. First we increase the number of channels to 32. The first block the double the number of channels to 64 feature maps. The second and third blocks increase the number of channels further to 128 and 256, respectively.

Similarly we have three decoding blocks. Each decoding block in our model concludes with an up-convolutional layer, which serves the same function as a typical convolutional layer, followed by a skip connection. The first skip connection takes the 32-channel output from the second encoding block and merges it with the 32-channel output from the first up-convolutional layer. This combination produces a 64-channel output, which is then processed by the first decoding block. The second skip connection similarly merges the 16-channel output from the first encoding block with the 16-channel output from the second up-convolutional layer, resulting in a 32-channel input for the second decoding block.

The final stage in our architecture employs a convolutional layer to convert the 16-channel output from the last decoding block into a 2-channel output, thus producing two matrices as the ultimate output of our model.

4 Experimental Set-Up

We used the Chest X-Ray Images (Pneumonia) dataset which was downloaded from https://www.kaggle.com/datasets/paultimothymooney/chest-xray-pneumonia. This is a dataset for binary classification of normal and pneumonia chest X-ray images. The dataset consists of 5,863 X-Ray images (JPEG) and 2 categories (Pneumonia/Normal). All images are black-and-white and are in the JPEG format. The X-rays images have various resolutions.

At the time of download, the original dataset had already been split into a training set, a validation set, and a test set. We picked 800 images for the training dataset for training and 100 images from the test set for testing. The validation set consists of only 8 images which were all used for validation. We used only images which were classified as normal.

For consitent input, we cropped the images to squares and then resized them to 256×256. For each image we generated a random value of σ uniformly in the range from 0.1 to 0.5, and noised the image with Gaussian noise with mean 0 and the corresponding standard deviation σ.

For learning the regularisation parameter map $\mathbf{\Lambda}_\Theta$, we used a U-Net structure with 1 initial double convolutional block, followed by 3 encoding blocks, 3 decoding blocks, and 1 final fully

connected layer. The number of initial filters, or output channels of the first convolution layer, is set to 32. As commonly done, each encoding/decoding block contains a(n) downsampling/upsampling step, followed by 2 (fully) convolutional layers, in which the first convolutional layer doubles/halves the number channels which the second maintains the number of channels. In other words, the number of output channels of each subsequent encoding block is doubled, while the number of output channels of each subsequent decoding block is halved. The U-Net has 1 input channel and 2 final output channels. This leads to a 1-32-64-128-256-128-64-32-2 structure.

All convolutional layers have a kernel size of 3×3 and a stride of 1. Each side of a feature map has zero-padding of size 1 to maintain the size of the feature map after the convolution. Keeping the number of size of the feature map constant after each convolution has the advantage of making the implementation of the skip connections simpler by not having to crop the output of the encoding blocks to match the size of the input of the decoding blocks.

Each encoding block begins with a max pooling layer with 2×2 kernels and a step size of 2. Prior to the max pooling, a zeo-padding of size 1 is added in order to exactly halve the length of each side of the output. On the other hand, all upsampling steps are done with linear interpolation with a scale factor of 2 which doubles the length of each side of the feature map. In the end, the total number of trainable parameters of the set Θ is 1,796,034.

For the activation function, we used the Leaky ReLU with a negative slope of 0.01. For the primal dual solver, we set the up-bound parameter to 0 and T_{train} to 256. During testing we also set T_{test} to 256.

For reproducibility, we manually set a seed value to 42 for the random number generator at the beginning of the training process. We use the Adam optimiser with an initial learning rate of 1e-4, a batch size of 1, and the Mean Square Error (MSE) loss function. We trained for a total of 500 epochs.

It is worth noting that, for the implementation of the U-Net, we used the 3D implementations of the convolutional layers as well as the downsampling and upsmapling steps from the PyTorch library [3]. The main reasons we went with 3D instead of the 2D implementations are convenience and generality. The 3D U-Net implementation is adapted from the dynamic image denoising application in [1], and can therefore be extended to 3D dynamic imaging applications in the future.

5 Results

As previously mentioned, we used 100 images with label "normal" from the test set provided in the Chest X-ray dataset for testing. Each test image was noised with a Gaussian noise of mean 0 and standard deviation $\sigma = 0.5$. For each test image, we also did a grid search with 81 equally spaced values between 0 and 0.8 in order to find the best scalar λ for the total variational denoising method, which we used as a baseline to show the effectiveness of our method.

As an example, we present the results for one of the test images. The results of the grid search is shown in the line plots in figure 4. For this particular test image, the highest value of PSNR of 29.945 dB was achieved by $\lambda = 0.07$. The regularisation parameter map $\mathbf{\Lambda}_\Theta$ found using NET_Θ resulted in a PSNR of 31.248 dB. The clean and orignal versions as well as the resulting denoised images are shown in Figure 5.

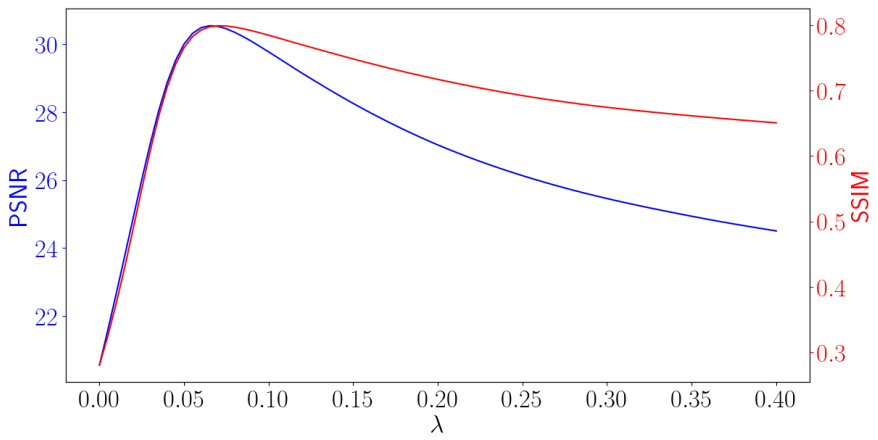


Figure 4: Grid search for the best scalar regularisation parameter λ for the example test image. The highest PSNR of 29.945 dB was achieved by $\lambda = 0.07$.

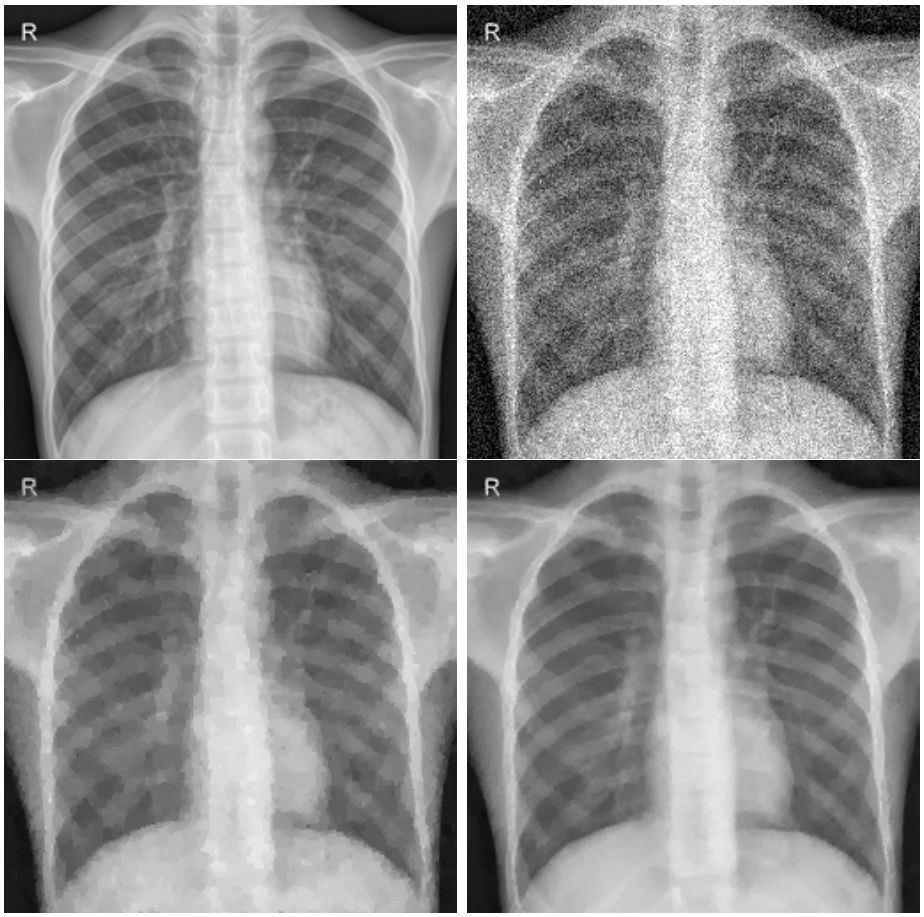
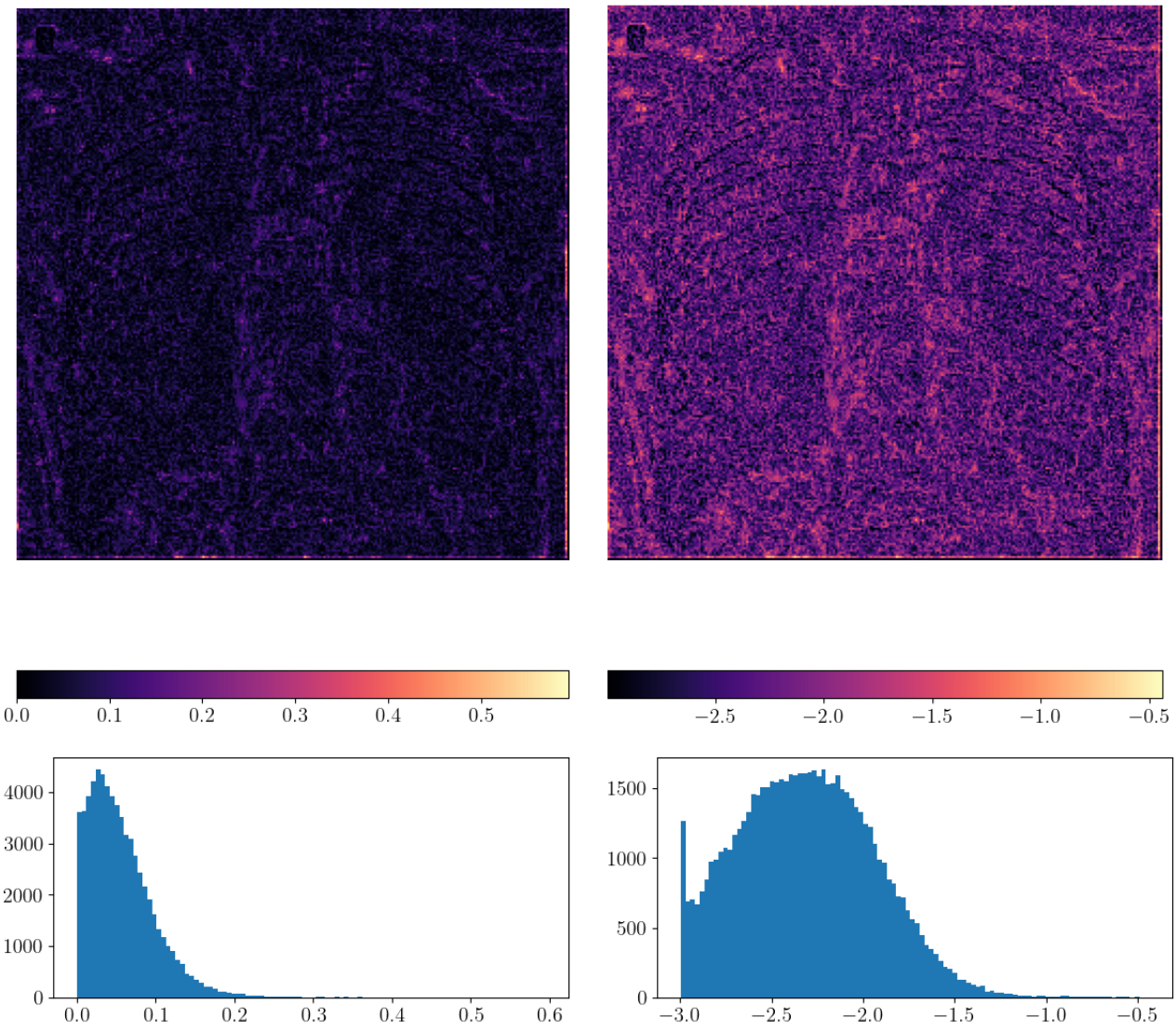


Figure 5: Chest X-ray example. The top row shows the original image on the left and the noisy image, generated with Gaussian noise with $\sigma = 0.5$. The bottom row shows the denoised images using the TV method; the left image with PSNR = 29.945 was produced using the best scalar regularisation parameter $\lambda = 0.07$, while the right image with PSNR = **31.248** was denoised using the regularisation parameter map $\mathbf{\Lambda}_\Theta$ found using NET_Θ.



(a) $\mathbf{\Lambda}_\Theta$ map and histogram of the values (b) $\log(\mathbf{\Lambda}_\Theta + 0.05)$ and histogram of the log values

Figure 6: The produced $\mathbf{\Lambda}_\Theta$ map for the chest x-ray dataset example. Since the values are too close to one another in the lower spectrum, the bottom row shows the log of the map and the histogram of the log values for better separation.

Qualitatively, we can see that there is a good reduction in the stair-case effect in the denoised image using the regularisation parameter map $\mathbf{\Lambda}_\Theta$ compared to the denoised image using the scalar λ.

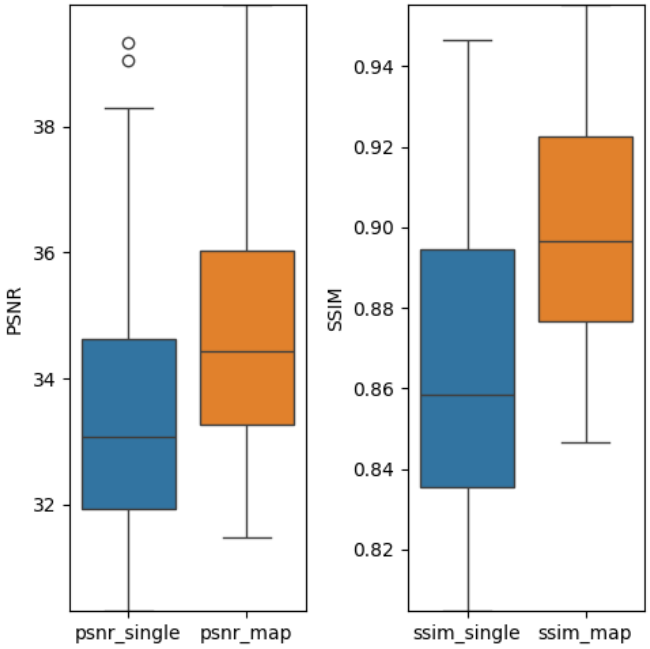


Figure 7: Test results summary

	PDHG - λ	**PDHG - $\mathbf{\Lambda}_\Theta$**
SSIM	0.87 ± 0.04	$\mathbf{0.90} \pm 0.03$
PSNR	33.43 ± 2.16	$\mathbf{34.73} \pm 2.02$

Table 1: Mean and standard deviation of the measures SSIM and PSNR obtained over the test set for the chest x-ray dataset example. The TV-reconstruction using the parameter-map $\mathbf{\Lambda}_\Theta$ improves both measures.

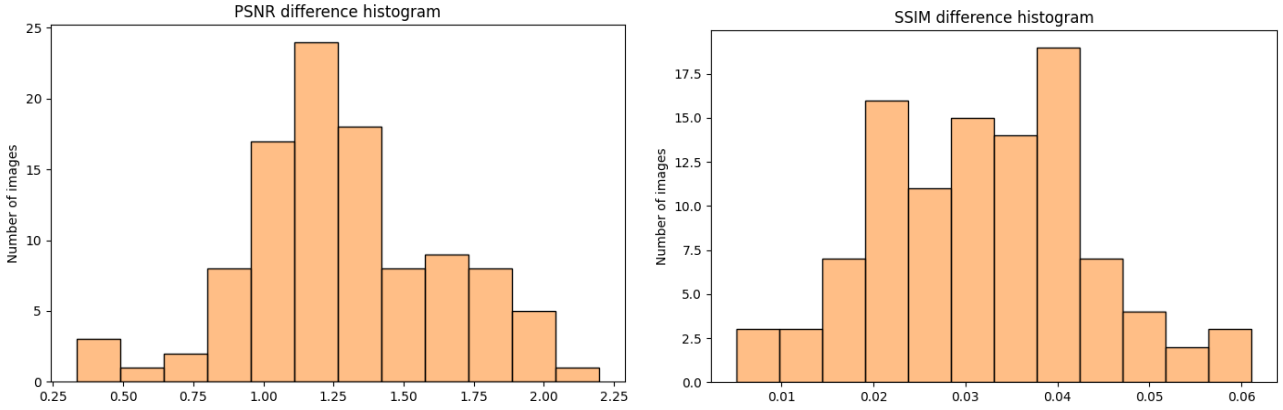


Figure 8: Histograms of the PSNR and SSIM differences between using λ scalar and using $\mathbf{\Lambda}_\Theta$ map for reconstructions of noisy versions of the test images. The $\mathbf{\Lambda}_\Theta$ map method consistently outperforms the scalar λ method.

Plotting the values for all the test images, we can see that the regularisation parameter map consistently outperforms the scalar λ. The box plots in Figure 7 show the distribution of PSNR

and SSIM values for the test images using the two methods. The regularisation parameter map consistently produces higher PSNR and SSIM values than the scalar λ.

- (More experiments) Test with the sigma 0.1 and 0.3 for test images and add: box plots, table rows, and difference histograms.

6 Conclusion

This project showcases the integration of neural networks with traditional algorithms to enhance image denoising capabilities, marking a significant step forward in computational imaging.

6.1 Experimentation and Findings

We experimented with the capabilities of the U-Net architecture to learn and apply regularisation parameters dynamically across different image segments using the PDHG algorithm. Traditional denoising techniques typically rely on a single lambda value for regularisation, which often does not cater to the varying needs across an image. Our approach attempts to transcend this limitation by approximating an optimal lambda map through the U-Net, enhancing the PDHG's performance. This methodology, while computationally intensive, illustrates the potential benefits of segment-specific regularisation in image denoising.

In conclusion, while the experiment demonstrated potential improvements over traditional single-lambda denoising methods, the current computational demands make it impractical for real-world applications.

7 Future Work

References

[1] Andreas Kofler et al. *Learning Regularization Parameter-Maps for Variational Image Reconstruction using Deep Neural Networks and Algorithm Unrolling*. 2023. arXiv: 2301.05888 [math.OC]. URL: https://github.com/koflera/LearningRegularizationParameterMaps.

[2] Y. Lecun et al. "Gradient-based learning applied to document recognition". In: *Proceedings of the IEEE* 86.11 (1998), pp. 2278–2324. DOI: 10.1109/5.726791. URL: https://www.researchgate.net/publication/2985446_Gradient-Based_Learning_Applied_to_Document_Recognition.

[3] Adam Paszke et al. "PyTorch: An Imperative Style, High-Performance Deep Learning Library". In: *Advances in Neural Information Processing Systems 32*. Curran Associates, Inc., 2019, pp. 8024–8035. URL: http://papers.neurips.cc/paper/9015-pytorch-an-imperative-style-high-performance-deep-learning-library.pdf.

[4] Olaf Ronneberger, Philipp Fischer, and Thomas Brox. *U-Net: Convolutional Networks for Biomedical Image Segmentation*. 2015. arXiv: 1505.04597 [cs.CV].

[5] Emil Y Sidky, Jakob H Jørgensen, and Xiaochuan Pan. "Convex optimization problem prototyping for image reconstruction in computed tomography with the Chambolle–Pock algorithm". In: *Physics in Medicine Biology* 57.10 (Apr. 2012), p. 3065. DOI: 10.1088/0031-9155/57/10/3065. URL: https://dx.doi.org/10.1088/0031-9155/57/10/3065.

The following list contain the primary Python libraries used in the implementation of this project.

- `NumPy`: Utilised for numerical representations and operations, including vector and matrix multiplication.
- `PyTorch`: Employed for constructing and training neural network models due to its robust, flexible, and efficient computational dynamics.
- `matplotlib` and `seaborn`: Employed for data visualisation tasks, including the creation of histograms, box plots, and other graphical representations.
- `scikit-learn`: Used for benchmarking our model against traditional machine learning algorithms, providing tools for data preprocessing, cross-validation, and more.
- `random`: Critical for generating random numbers, used extensively in stochastic processes like K-fold cross-validation and bootstrap sampling.
- `wandb`: Integrated for experiment tracking and logging, facilitating the monitoring of model training metrics and performance in real-time.