DATA.ML.300 Computer Vision Exercise 5

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1a. Denote $cos(\theta) = Co$ and $sin(\theta) = Si$. We have:

$$X' = s \begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} * X + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

$$V = X_2 - X_1 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$=> ||V|| = \sqrt{\Delta x^2 + \Delta y^2}$$

$$V' = X_2' - X_1' = \begin{pmatrix} x_2' - x_1' \\ y_2' - y_1' \end{pmatrix} = \begin{pmatrix} \Delta x' \\ \Delta y' \end{pmatrix}$$

$$\begin{aligned} => V' &= s \begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} * (X_2 - X_1) \\ &= s \begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \\ &= \begin{pmatrix} sCo\Delta x - sSi\Delta y \\ sSi\Delta x + sCo\Delta y \end{pmatrix} \end{aligned}$$

$$\begin{aligned} => ||V'|| &= \sqrt{s^2Co^2\Delta x^2 + s^2Si^2\Delta y^2 + s^2Si^2\Delta x^2 + s^2Co^2\Delta y^2 - 2s^2CoSi\Delta x\Delta y + 2s^2CoSi\Delta x\Delta y} \\ &= s\sqrt{\Delta x^2(Co^2 + Si^2) + \Delta y^2(Co^2 + Si^2)} \\ &= s\sqrt{\Delta x^2 + \Delta y^2} \\ &= s||V|| \end{aligned}$$

$$=> s = \frac{||V'||}{||V||} = \frac{\sqrt{\Delta x'^2 + \Delta y'^2}}{\sqrt{\Delta x^2 + \Delta y^2}}$$

1b. Using the V' equation

$$
\begin{aligned}
V' = s\begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} * (X_2 - X_1) &= s\begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} V \\
<=> \frac{V'}{||V'||} &= \frac{s}{||V'||}\begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} V \\
<=> \frac{V'}{||V'||} &= \begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} \frac{V}{||V||} \\
<=> V'_{unit} &= \begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} V_{unit}
\end{aligned}
$$

Because V'_{unit} and V_{unit} are unit vectors. So $V'_{unit}V_{unit} = Co = cos(\theta)$

$$
\begin{aligned}
=> \theta = acos(V'_{unit}V_{unit}) &= acos(\frac{V'V}{||V'||||V||}) \\
&= acos(\frac{\Delta x'\Delta x + \Delta y'\Delta y}{\sqrt{\Delta x'^2 + \Delta y'^2}\sqrt{\Delta x^2 + \Delta y^2}})
\end{aligned}
$$

1c. Substitute s and θ, using point X_1 and X'_1 we can get t_1 and t_2 as follow:

$$
\begin{aligned}
X'_1 = s\begin{pmatrix} Co & -Si \\ Si & Co \end{pmatrix} * X + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \\
<=> \begin{pmatrix} x'_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} sCox_1 - sSiy_1 \\ sSix_1 + sCoy_1 \end{pmatrix} + \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \\
=> t_1 = x'_1 - sCox_1 + sSiy_1 \\
t_2 = y'_1 - sSix_1 - sCoy_1
\end{aligned}
$$

1d. Given $\{(\frac{1}{2}, 0) \to (0, 0)\}$ and $\{(0, \frac{1}{2}) \to (-1, -1)\}$. We have:

$$
\begin{aligned}
V = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} => ||V|| &= \frac{1}{\sqrt{2}} \\
V' = \begin{pmatrix} -1 \\ -1 \end{pmatrix} => ||V'|| &= \sqrt{2} \\
=> s = \frac{||V'||}{||V||} &= 2 \\
\theta = acos(\frac{\Delta x'\Delta x + \Delta y'\Delta y}{\sqrt{\Delta x'^2 + \Delta y'^2}\sqrt{\Delta x^2 + \Delta y^2}}) \\
= acos(-1 * (-0.5) + -1 * 0.5) &= acos(0) = \pm\pi/2
\end{aligned}
$$

With $\theta = \pi/2$

$$
\begin{aligned}
t_1 &= 0 - 2 * 1 * 0 = 0 \\
t_2 &= 0 - 2 * 1 * 0.5 = -1
\end{aligned}
$$

With $\theta = -\pi/2$

$$t_1 = 0 + 2 * 1 * 0 = 0$$
$$t_2 = 0 + 2 * 1 * 0.5 = 1$$

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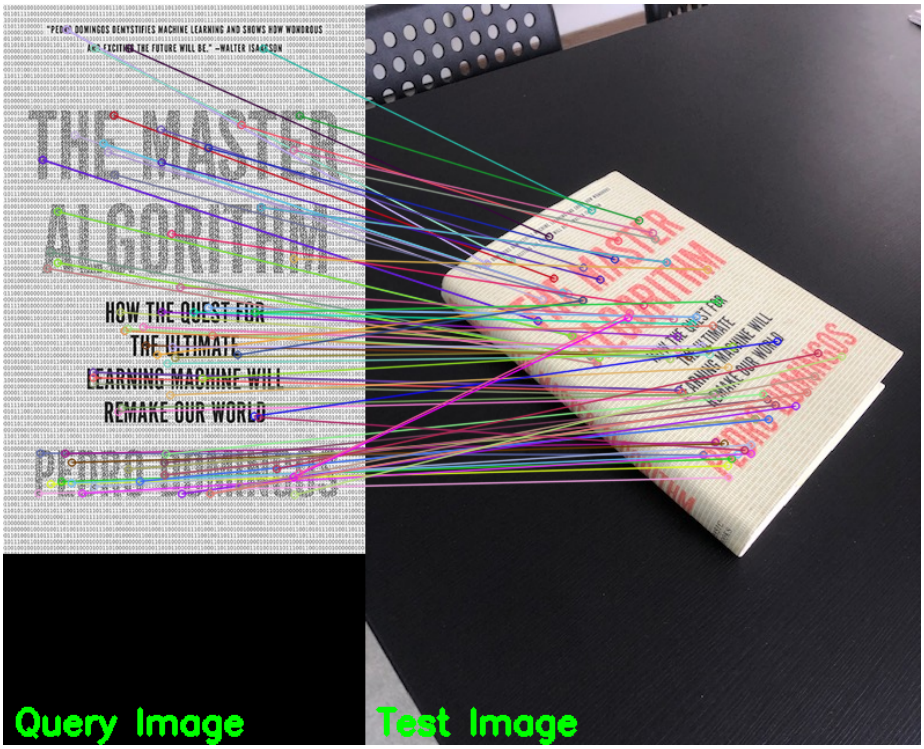

Figure 1: Result of matching

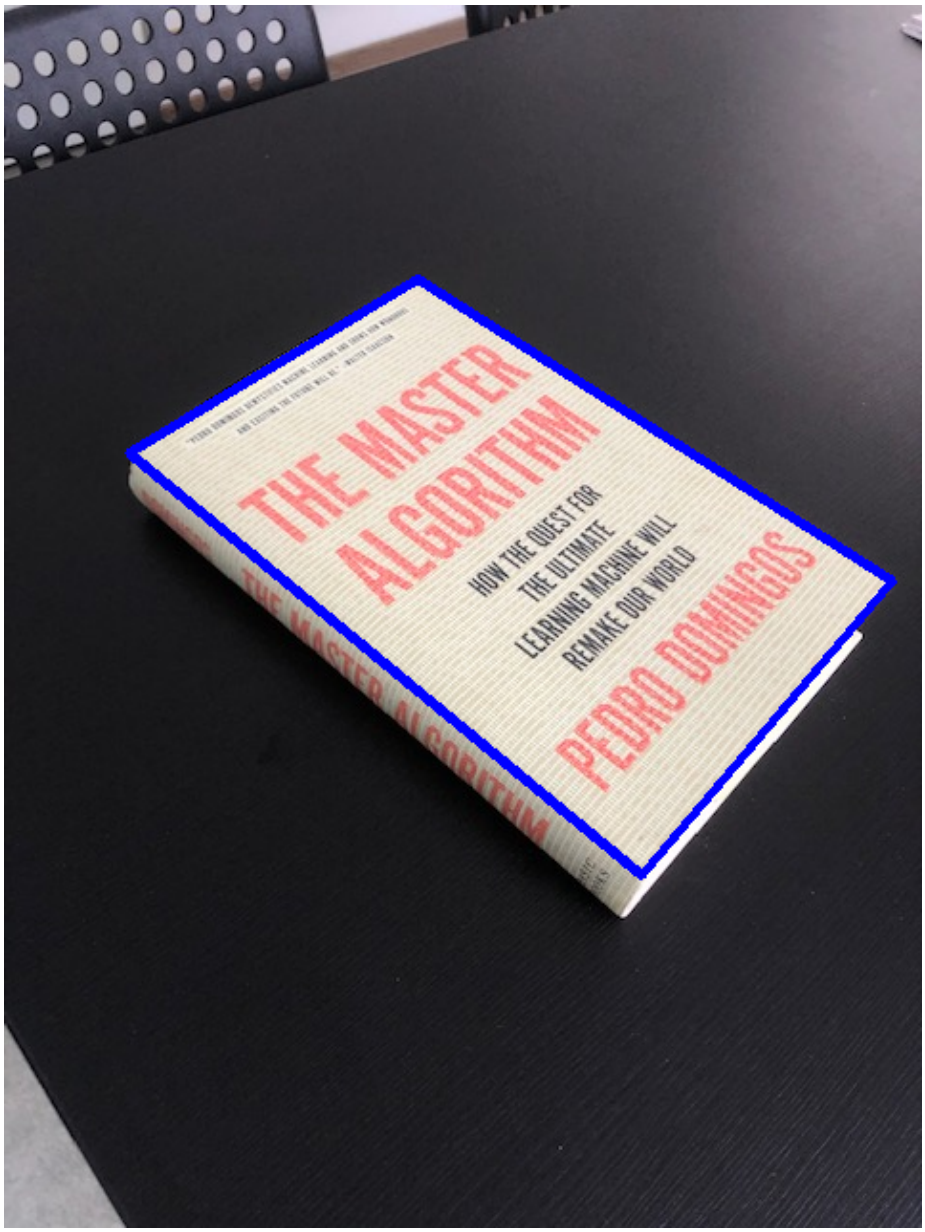

Figure 2: Homography found on test image

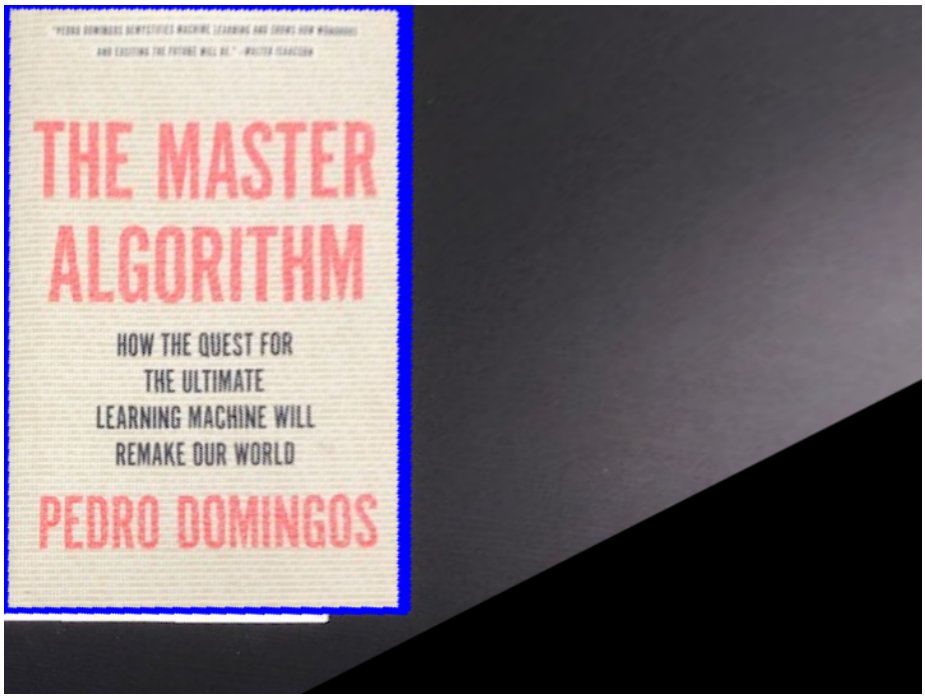

Figure 3: Warped test image result

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3a. 4 parts of the algorithm:
- Object detection (Face detection): Detect face in webcam image
- Cut out the region of interest (the face bounding box) and perform keypoints extraction on the ROI. Convert keypoints back to image coordinate
- If has enough keypoints, put those keypoints along with two consecutive frame into Optical Flow Lucas Kanade to track.
- Keep keypoints if the tracking successfull.

3b.

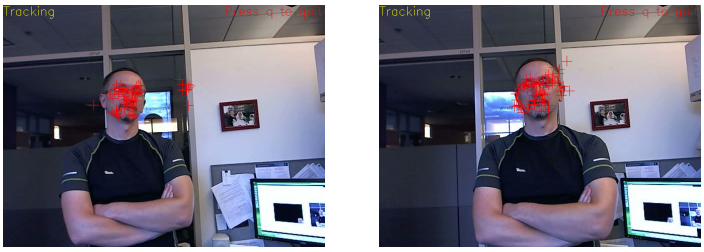

Figure 4: Result of tracking

- Some tracking point are stuck on the door or glass when the person move. This maybe caused by the fast movement of the person which lead to the mismatch of the features. Also, the background seem to have similar color to the key point region on the person. We can avoid this by using coarse-to-fine method for large movement.