



Lecture 02

Understanding Interest Rates

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- The relationship between the coupon rate, interest rate and price relative to par value
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- Real and nominal interest rates
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Readings

Mishkin (2015), *The Economics of Money, Banking, and Financial Markets*, 11th edition, Pearson, Chapter 4.

Cecchetti and Schoenholtz (2012), *Money, Banking, and Financial Markets*, 4th edition, McGraw-Hill, Chapter 4.

Introduction

[The tricky business of raising interest rates](#) | Jon Talton

[The Seattle Times](#) - 14 hours ago
The **Federal Reserve** is ready to **raise interest rates**. But these are no ordinary and the central bank faces myriad risks in its next moves.

[Fed rates moves: lower for longer or is higher now here](#)
[The Australian Financial Review](#) - 3 hours ago

He rattled out plenty of evidence for a **rate rise** – such as the f. investors in Europe and the US are accepting negative or zero returns ...

[Federal Reserve needs a few good rules: Bloomberg View Opinion](#) - [OregonLive.com](#) - Aug 17, 2015

[Explore in depth](#) (295 more articles)

[Gundlach: Rate hike a 'bad idea' due to junk bond prices](#)
[CNBC](#) - 13 hours ago
DoubleLine Capital's co-founder Jeffrey Gundlach warned on Tu that it might be difficult for the U.S. **Federal Reserve** to **raise interest** ...

[GUNDLACH: It's a 'bad idea' for Fed to hike rates right now](#)
[Business Insider](#) - 12 hours ago

[Explore in depth](#) (15 more articles)

[Why a rise in US interest rates is still risky](#)
[BBC News](#) - Aug 17, 2015

There was never going to be a risk-free, stress-free moment for Fed, the US central bank, to **increase the interest rate** it con after almost ...

[Lãi suất VND chịu sức ép tỷ giá](#)
[tinhhanhchungkhoan](#) - 2 hours ago
Tuy nhiên, theo ông Quang, nếu lấy thông tin này để nhận định khoản của thị trường kém, gây áp lực có thể làm **tăng lãi suất** ...

[Phụ thuộc tin dụng, ngân hàng "sợ" giảm lãi suất cho vay?](#)
[XãLuân.com tin tức việt nam 24h cập nhật](#) - 7 hours ago

[Explore in depth](#) (11 more articles)

[Tin dụng tăng, lãi suất tăng theo](#)

[Tiền Phong](#) - Aug 17, 2015

TP - Theo thống kê, hiện, mặt bằng **lãi suất huy động** bằng VND phổ biến ở ... đến ngày 20/7, tin dụng toàn hệ thống ngân hàng **tăng** khá ...

[Lãi suất tăng "ăn theo" tin dụng?](#)

[Cafef.vn](#) - Aug 11, 2015

Có ngân hàng **huy động** vốn **tăng** mạnh tới 45%, trong khi, **ti** **tăng** tin ... Chẳng hạn, **lãi suất huy động** vốn kỳ hạn dưới 6 các ngân ...

[Sắp có lần sống giảm lãi suất huy động?](#)

[Lao động](#) - Aug 12, 2015

[Explore in depth](#) (30 more articles)

[Lãi huy động trái phiếu chính phủ tăng](#)

[Thời báo Tài chính Việt Nam](#) - Aug 13, 2015

(TBTCO) - Ngày 13/8/2015, Kho bạc Nhà nước đã **huy động** : công ... **Lãi suất** tăng **thấu** **tăng** 10 điểm so với phiên **trung** t trước.

[Sắp diễn ra một lần sống cắt giảm lãi suất huy động?](#)

[Cafef.vn](#) - Aug 10, 2015

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Introduction

- Interest rates are among the most closely watched variables in the economy.
- An interest rate is the price paid by a borrower to a lender for the use of resources that will be used during some time period then returned
- Interest rates
 - Link the present to the future
 - Tell the future reward for lending today
 - Tell the cost of borrowing now and repaying later
- In this lecture, we will explore what an interest rate is, and the relationship between interest rates, bond prices, and returns

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Interest rate defined

Dual" Definition:

- **Borrowing:** the cost of borrowing or the price (%) paid for the "rental" of funds.
 - A financial liability for "deficit" (borrowing) entities.
- **Saving:** the return from investing funds or the price (%) paid to delay consumption.
 - A financial asset for "surplus" (lending, investing) entities.

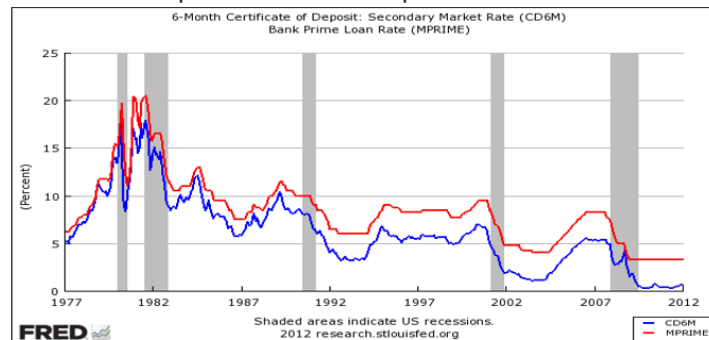
Both concepts are expressed as a percentage per year (Percent per annum; "p.a."). This is true regardless of maturity of instrument of the financial liability or financial asset. Thus, all observed interest rate data is annualized.

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Interest rate defined

Savings and Borrowing Rates: They Move Together, 1977– 2011

- Regression analysis: 1964 – 2010 (monthly data, 564 observations); CD rate as dependent variable. R-squared = 88.55%



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Interest rate defined

Basis Point: A unit that is equal to 1/100th of 1%, and is used to denote the changes in interest rates or differences in interest rates between various debt instruments.

The relationship between interest rate changes (or differentials) and basis points can be summarized as follows: 1% change (or difference) = 100 basis points.

- ✓ Example 1: If Bond A's yield increases from 5% to 6.5%, then Bond A's yield increased 150 basis points.
- ✓ Example 2: If Bond B's yield falls from 7.00% to 6.93%, then Bond B's yield decreased 7 basis points.
- ✓ Example 2: If Bond C has a yield of 6% and Bond D a yield of 2%, then Bond C is 400 basis points more than Bond D.

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Commonly used interest rate measures

- There are four important ways of measuring (and reporting) interest rates on financial instruments.
- When people talk about bonds they use the terms yield and interest rate interchangeably, so we will too.

These are:

- **Coupon Yield:** The “promised” annual percent return on a coupon instrument.
- **Current Yield:** Bond’s annual coupon payment divided by its current market price.

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Commonly used interest rate measures

These are:

- **Yield to Maturity:** The interest rate that equates the future payments to be received from a financial instrument (coupons plus maturity value) with its market price today (i.e., to its present value).
- **Discount Yield and Investment Yield:** These are yields on short term (one year or less) debt instruments that have no coupon payments and are selling at a discount of their par values. These interest rates are the “implied” returns from buying a debt instrument at a price below its par value.

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Coupon yield

- Coupon yield is the annual interest rate which was promised by the issuer when a bond was first sold.
- Coupon information is found in the bond’s indenture (legal contract). Indenture will state the coupon payment (as a percent of the bond’s par value) and the schedule of payments (semi-annual or annual).
- The coupon yield on a bond will not change during the lifespan of the bond.
- Go to Bloomberg to view coupon yields:
<http://www.bloomberg.com/>
- Note: U.S. Treasuries, 12 months and less have no coupons. Same is true for short term government bonds in other countries.

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Coupon yield

	CPN (%)	PRC CHG	YLD (%)	YLD CHG
30-Year Bond	2.375	0/32	2.246	0.000
10-Year Note	1.75	0/32	1.789	0.000
7-Year Note	1.75	0/32	1.708	0.000
5-Year Note	1.75	0/32	1.593	0.000
3-Year Note	1.625	0/32	1.546	0.000
2-Year Note	1.625	0/32	1.533	0.000
1-Year Bill	0	0/32	1.548	0.000
6-Month Bill	0	0/32	1.549	0.000
3-Month Bill	0	0/32	1.520	0.000
1-Month Bill	0	0/32	1.514	0.000

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Current Yield



- The **current yield** (or **flat yield**, or **interest yield**) is the coupon expressed as a percentage of the current price; this is the simplest of all return measures
- This provides us with a measure of the "current" interest yield obtained at the bond's current market price (i.e., cost associated with investing in a particular bond).
- Current yield = annual coupon payment/market price
- A 6 year, 1.50% bond selling at \$1,003.75 (thus it is a premium bond). Thus the current yield = $\$15.00/\$1,003.75 = 1.4944\%$

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Current Yield



- Premium bonds:** The current yield on these bonds will always be below the coupon yield.
- Current Yield = Annual coupon payment/ $>\$1,000$
 - Using the 1.5% coupon bond:
 - Current yield = $\$15.00/\$1,003.75 = 1.4944\%$
- Discount bonds:** The current yield on these bonds will always be above the coupon yield (assume a market price of \$985).
- Current Yield = Annual coupon payment/ $<\$1,000$
 - Current yield = $\$15.00/\$985 = 1.5228\%$

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Yield to Maturity



The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today

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Yield to maturity



- Four Types of Credit Market Instruments
- Simple Loan
 - Fixed Payment Loan
 - Coupon Bond
 - Discount Bond

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Simple Loan

PV = amount borrowed = \$100
CF = cash flow in one year = \$110

n = number of years = 1

$$100 = \frac{110}{(1+i)^1}$$

$$(1+i)100 = 110$$

$$(1+i) = \frac{110}{100}$$

$$i = 0.10 = 10\%$$

For simple loans, the simple interest rate equals the yield to maturity

Test question

What is the YTM of a simple loan whose today's value is \$1000 and next year's value is \$1100?

Fixed Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \dots + \frac{FP}{(1+i)^n}$$

Test question

What is the YTM of a fixed payment loan whose today's value is \$1000 and the yearly payment is \$126 for the next 25 years?

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Coupon Bond

Using the same strategy used for the fixed-payment loan:

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Test question

What is the YTM of a \$1000 face value 10% annual coupon bond with 10 years to maturity that now sells for \$900?

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Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate

The price of a coupon bond and the yield to maturity are negatively related

The yield to maturity is greater than the coupon rate when the bond price is below its face value

Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

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Consol or Perpetuity



A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$P = C / i_c$$

P_c = price of the consol

C = yearly interest payment

i_c = yield to maturity of the consol

can rewrite above equation as this : $i_c = C / P_c$

For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity

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Yield to Maturity



Government Bonds

4:58 PM Coordinated Universal Time 1/03/20

10-year bonds.

COUPON(%)	COUNTRY	YIELD(%)	YIELD CHG	LATEST SPREAD OVER TREASURY*
1.750	U.S.	1.789	0.000	---
0.000	Germany	-0.280	0.000	-207.0
1.625	U.K.	0.739	0.000	-105.0
0.100	Japan	-0.016	0.000	-180.5
2.750	Australia	1.266	0.000	-52.3
3.290	China	3.189	0.000	140.0
3.000	New Zealand	1.597	0.000	-19.2
0.000	France	0.021	0.000	-176.8
1.350	Italy	1.345	0.000	-44.4
0.600	Spain	0.388	0.000	-140.1

*in basis points

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Discount yields and investment yields



- Discount yields and investment yields are calculated for U.S. T-bills and other short term money market instruments (e.g., commercial paper and bankers' acceptances) where there are no stated coupons (and thus the assets are quoted at a discount of their maturity value).
- The discount yield relates the return to the instrument's par value (or face or maturity). The discount yield is sometimes called the bank discount rate or the discount rate.

$$r_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}}$$

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Discount yields and investment yields



The investment yield relates the return to the instrument's current market price.

- The investment yield is sometimes called the coupon equivalent yield, the bond equivalent rate, the effective yield or the interest yield.
- The investment yield is generally calculated so that we can compare the return on T-bills to "coupon" investment options

$$r = \frac{F - P}{P} \times \frac{365(366)}{\text{days to maturity}}$$

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Discount yields and investment yields

- What is the discount yield for a 182-day T-bill, with a market price of \$965.93 (per \$1,000 par, or face, value)?
- What is the investment yield for a 182-day T-bill, with a market price of \$965.93 (per \$1,000 par, or face, value)?

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The Distinction Between Interest Rates and Returns

- Rate of Return:

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

RET = return from holding the bond from time t to time $t + 1$

P_t = price of bond at time t

P_{t+1} = price of the bond at time $t + 1$

C = coupon payment

$\frac{C}{P_t}$ = current yield = i_c

$\frac{P_{t+1} - P_t}{P_t}$ = rate of capital gain = g

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The Distinction Between Interest Rates and Returns (cont'd)

- Note that **the return on a bond will not necessarily equal the yield to maturity on that bond**
- The return equals the yield to maturity only if the holding period equals the time to maturity
- To calculate the return over the life of the bond, we must also include the interest generated by reinvesting the coupons that are paid, which is known as **reinvestment income**

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The Distinction Between Interest Rates and Returns (cont'd)

- It is typically assumed that the future coupons will be reinvested at the same rate of return that the bond currently offers in terms of current yield and capital gain. Then we can think of the total return on the bond as being defined as

Total return = current yield + capital gain

- In equilibrium, this return must be equal to the interest rate, otherwise there is an arbitrage opportunity

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The Distinction Between Interest Rates and Returns (cont'd)



The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change

The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate

Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

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Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

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Price-yield curves



The price-yield curve is the plot of the price of a bond against its yield to maturity

Consider three bonds as follows

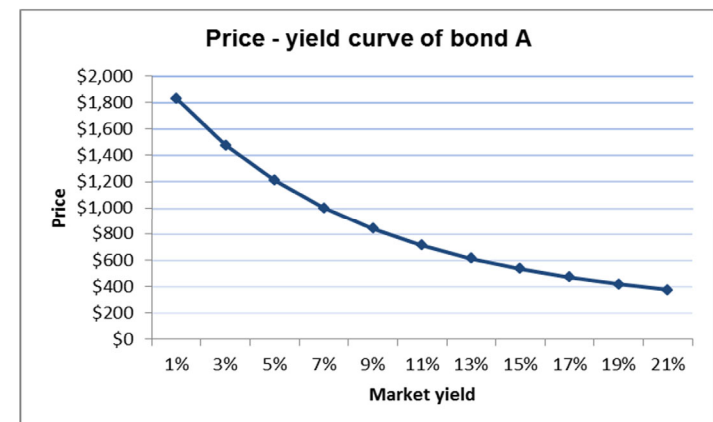
	A	B	C	D	E	F	G	H
52	Bond maturity and price							
53								
54	YTM for all 3 bonds			10%		Initial price, Bond A:		\$771.82
55	Par (or face) value			\$1,000		Initial price, Bond B:		\$1,000.00
56	Initial years to maturity			15		Initial price, Bond C:		\$1,228.18
57	Coupon rate, Bond A:			7%				
58	Coupon rate, Bond B:			10%				
59	Coupon rate, Bond C:			13%				
60								

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Price-yield curves

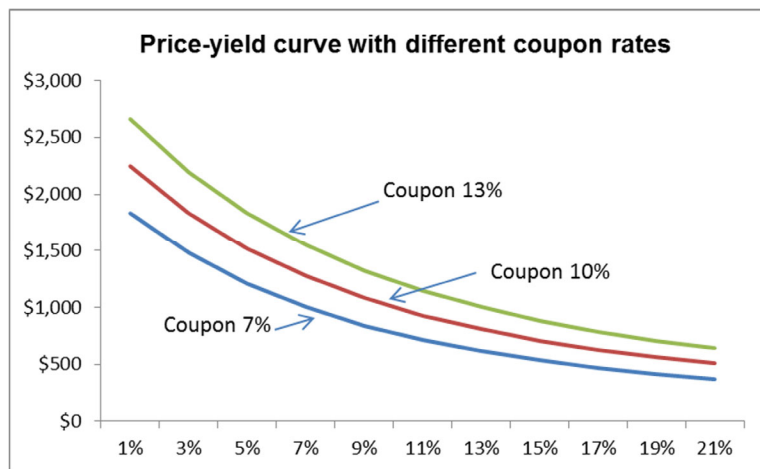


For our example, the price-yield curve for bond A is as follows



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Price-yield curves



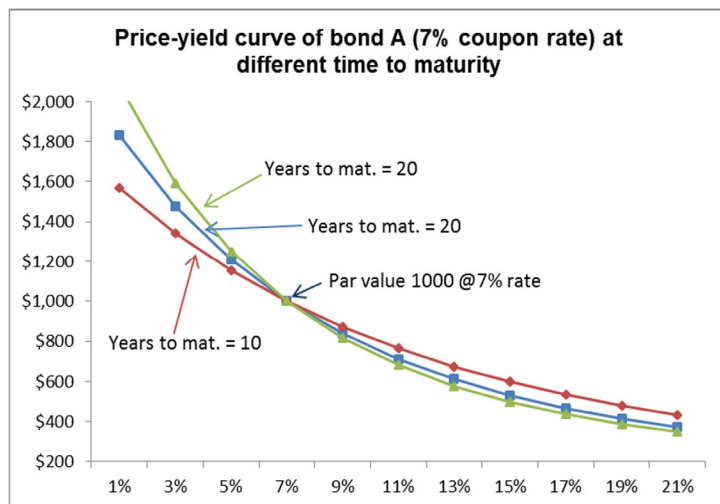
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Price-yield curves

- There is a negative relationship between the price of a bond and its yield
- The price yield-curve is convex: the change in the price of a bond brought about by a change in the yield is greater the higher the yield
- The intercept of the price-yield curve is the total undiscounted value of all of the bond's cash payments
- The price of a bond approaches zero as the yield to maturity goes to infinity
- For a given maturity, the higher the coupon, the higher the curve, because higher coupon bonds are more expensive for a given yield to maturity

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Price-yield curves



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Interest-Rate Risk

- The sensitivity of the price of a bond to the interest rate is known as **interest rate risk**, and is captured by the slope of the price-yield curve
- We have seen that, for a given coupon rate, bonds with long maturities have greater interest rate risk than bonds with short maturities; similarly, for a given maturity, bonds with low coupons have greater interest rate risk than bonds with high coupons
- There is no interest-rate risk for any bond whose time to maturity matches the holding period

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Interest-Rate Risk



- Essentially, the interest rate risk of a bond is determined by how long you have to wait 'on average' to receive the bond's cash flows;
- This concept is encapsulated in the bond's **duration**, which is defined as (the negative of) the weighted average time to maturity of the bond's cash flows, with weights proportional to the present value of the bond's cash flows; It is straightforward to show that duration is (the negative of) the first derivative of the bond's price yield curve
- The second derivative of the price-yield curve is closely related to a measure known as **convexity**

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Reinvestment risk



- Reinvestment risk occurs because of the need to "roll over" securities at maturity, i.e., reinvesting the par value into a new security.
- Problem for bond holder: The interest rate you can obtain at roll over is unknown while you are holding these outstanding securities.
- Issue: What if market interest rates fall?
- You will then re-invest at a lower interest rate than the rate you had on the maturing bond.
- Potential reinvestment risk is greater when holding shorter term fixed income securities.
- With longer term bonds, you have locked in a known return over the long term.

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Risk free and risk-adjusted interest rates



- When cash flows are riskless (lenders can receive for sure), the appropriate interest rate (or discount rate) is the **risk-free rate**, often denoted R_f
- When cash flows are not certain, i.e. when they are **risky**, investors generally demand a higher interest rate that reflects the risk
- An interest rate that reflects the risk of the cash flows is known as a **risk-adjusted interest rate**, or a **risk-adjusted discount rate**

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Risk free and risk-adjusted interest rates



- We can think of the risk-adjusted interest rate as being equal to the risk-free rate plus a **risk premium**

$$R_{Risk-Adjusted} = R_f + RP$$

- For example, suppose that the risk free rate is 5.5%, but the appropriate risk premium for an investment is 2.1%, the risk-adjusted discount rate to be used to discount the cash flows from this investment would be $5.5\% + 2.1\% = 7.6\%$

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The Distinction Between Real and Nominal Interest Rates



Nominal interest rate makes no allowance for inflation

Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing

Ex ante real interest rate is adjusted for expected changes in the price level

Ex post real interest rate is adjusted for actual changes in the price level

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Fisher Equation



$$i = i_r + \pi^e$$

i = nominal interest rate

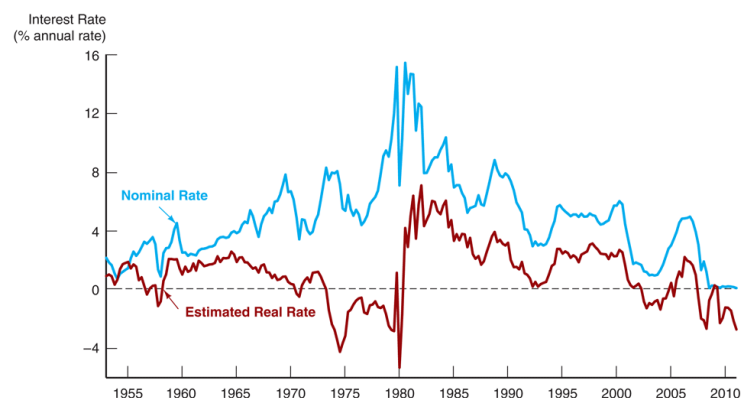
i_r = real interest rate

π^e = expected inflation rate

When the real interest rate is low,
there are greater incentives to borrow and fewer incentives to lend.
The real interest rate is a better indicator of the incentives to
borrow and lend.

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Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2011



Sources: Nominal rates from www.federalreserve.gov/releases/H15 and inflation from <http://ftp.bis.gov/special.requests/cpi/cpia.txt>. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.

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Other readings



Readings:

Interest rates:

<http://www.econlib.org/library/Enc/InterestRates.html>

Bonds: <http://www.econlib.org/library/Enc/Bonds.html>

Understanding the Subprime Mortgage Crisis:

<http://useconomy.about.com/od/economicindicators/tp/Subprime-Mortgage-Primer.htm>

Why a U.S. Subprime Mortgage Crisis Is Felt Around the World:

<http://www.nytimes.com/2007/08/31/business/worldbusiness/31derivatives.html?pagewanted=all>

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