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Economics (İbn Haldun Üniversitesi)

Chapter 3:

Elements of Chance: Probability Methods

3.1 $\overline{A}$ is the complement of event *A* and contains all of the sample points that are not in event *A*. Therefore, $\overline{A} = (E_2, E_4, E_5, E_7, E_8, E_{10})$

3.2 a. *A* intersection *B* contains the sample points that are in both *A* and *B*. The intersection $= (E_3, E_9)$
b. *A* union *B* contains the sample points in *A* or *B* or both. The union $= (E_1, E_2, E_3, E_7, E_8, E_9)$
c. *A* union *B* is not collectively exhaustive – it does not contain all of the possible sample points.

3.3 Given $\overline{A}$ and $\overline{B}$, $A = (E_2, E_4, E_5, E_6, E_8, E_{10})$ and $B = (E_1, E_4, E_5, E_6, E_7, E_{10})$
a. *A* intersection $B = (E_4, E_5, E_6, E_{10})$
b. *A* union $B = (E_1, E_2, E_4, E_5, E_6, E_7, E_8, E_{10})$
c. The union of *A* and *B* is not collectively exhaustive – it does not contain all of the sample points in the sample space.

3.4 a. *A* intersection $B = (E_3, E_6)$
b. *A* union $B = (E_3, E_4, E_5, E_6, E_9, E_{10})$
c. *A* union *B* is not collectively exhaustive – it does not contain all of the possible sample points.

3.5 a. The complement of event *A* is that it will take 4 days or less before the machinery becomes operational.
b. The intersection of *A* and *B* will be the event that it takes 5 days before the machinery becomes operational.
c. The union of *A* and *B* is the event of 1 day, 2 days, 3 days, 4 days, 5 days, 6 days, or 7 days.
d. *A* and *B* are not mutually exclusive because $P(A\cap B) \neq 0$.
e. Yes, *A* and *B* are collectively exhaustive because they include all of the possible sample points.
f. $(A\cap B)$ is the event that it takes 5 days. $(\overline{A}\cap B)$ is the event that it takes 4 days, 3 days, 2 days, 1 day. The union between these two events is that it takes less than 6 days (1 through 5) before the machinery is operational. This is the definition of event *B*, therefore $(A\cap B) \cup (\overline{A}\cap B)$ = event *B*.
g. $(\overline{A}\cap B)$ is the event that it takes 4 days, 3 days, 2 days, 1 day. Since *A* is the event of 5 days, 6 days, 7 days, then $A\cup(\overline{A}\cap B)$ will be the event of 1 through 7 days. This is the event of $A\cup B$. Therefore, $A\cup(\overline{A}\cap B)$ must equal $A\cup B$.

3.6 a. $(A\cap B)$ is the event that the Dow-Jones average rises on both days which is O_1. $(\overline{A}\cap B)$ is the event the Dow-Jones average does not rise on the first day but it rises on the second day which is O_3. The union between these two will be O_1 or O_3 either of which by definition is event *B*: the Dow-Jones average rises on the second day.

b. Since ($\overline{A} \cap B$) is the event the Dow-Jones average does not rise on the first day but rises on the second day which is O_3 and because *A* is the event that the Dow-Jones average rises on the first day, then the union will be O_1, either the Dow-Jones average does not rise on the first day but rises on the second day or the Dow-Jones average rises on the first day or both. This is the definition of $A \cup B$.

3.7 a. Sample points in the sample space include the following 20 simple events:

1. M1,M2	5. M2,M1	9. M3,M1	13. T1,M1	17. T2,M1
2. M1,M3	6. M2,M3	10.M3,M2	14. T1,M2	18. T2,M2
3. M1,T1	7. M2,T1	11. M3,T1	15. T1,M3	19. T2,M3
4. M1,T2	8. M2,T2	12. M3,T2	16. T1,T2	20. T2,T1

b. Event *A* is that at least one of the two cars selected is a Toyota, outcomes 3, 4, 7, 8, 11-20.

c. Event *B* is that the two cars selected are of the same model, outcomes 1, 2, 5, 6, 9, 10, 16, 20.

d. The complement of *A* is the event that the customers do not select at least one Toyota, outcomes 1, 2, 5, 6, 9, 10.

e. ($A \cap B$) is the event that at least one Toyota is selected and two cars of the same model are selected, outcomes 16, 20. Since ($\overline{A} \cap B$) is the event that the customers do not select at least one Toyota and two cars of the same model are selected, outcomes 1, 2, 5, 6, 9, 10, then the union between the two events will be the event that two cars of the same model are selected, 1, 2, 5, 6, 9, 10, 16, or 20. This is the definition of event *B*.

f. ($\overline{A} \cap B$) is the event that the customers do not select at least one Toyota and two cars of the same model are selected. Since *A* is the event that the customers select at least one Toyota, then the union between the two will be the event that at least one Toyota is selected or the two cars of the same model are selected or both. This is the definition of event $A \cup B$.

3.8 The total number of outcomes in the sample space,

$$N = C_2^{12} = \frac{12!}{2!(12-2)!} = \frac{12!}{2!10!} = 66 .$$

The number of ways to select 1 A from the 5 available, $C_1^5 = \frac{5!}{1!(5-1)!} = 5$.

The number of ways to select 1 B from the 7 available, $C_1^7 = \frac{7!}{1!(7-1)!} = 7$.

The number of outcomes that satisfy the condition of 1 A and 1 B is,
$N_A = 5 \times 7 = 35$.
Therefore, the probability that a randomly selected set of 2 will include 1A and 1 B is

$$P_A = \frac{N_A}{N} = \frac{C_1^5 C_1^7}{C_2^{12}} = \frac{5 \times 7}{66} = .53 .$$

3.9 The total number of outcomes in the sample space,

$N = C_3^{10} = \dfrac{10!}{3!(10-3)!} = 120$.

The number of ways to select 1 A from the 6 available, $C_1^6 = \dfrac{6!}{1!(6-1)!} = 6$.

The number of ways to select 2 Bs from the 4 available, $C_2^4 = \dfrac{4!}{2!(4-2)!} = 6$.

The number of outcomes that satisfy the condition of 1A and 2 Bs is,
$N_A = 6 \times 6 = 36$.
Therefore, the probability that a randomly selected set of 3 will include 1A and 2 Bs is

$$P_A = \frac{N_A}{N} = \frac{C_1^6 C_2^4}{C_3^{10}} = \frac{6 \times 6}{120} = .30 .$$

3.10 The total number of outcomes in the sample space,

$N = C_4^{16} = \dfrac{16!}{4!(16-4)!} = 1{,}820$.

The number of ways to select 2 As from the 10 available, $C_2^{10} = \dfrac{10!}{2!(10-2)!} = 45$.

The number of ways to select 2 Bs from the 6 available, $C_2^6 = \dfrac{6!}{2!(6-2)!} = 15$.

The number of outcomes that satisfy the condition of 2 As and 2 Bs is,
$N_A = 45 \times 15 = 675$.
Therefore, the probability that a randomly selected set of 4 will include 2 As and 2 Bs is

$$P_A = \frac{N_A}{N} = \frac{C_2^{10} C_2^6}{C_4^{16}} = \frac{45 \times 15}{1{,}820} = .3709 .$$

3.11 $P(A) = \dfrac{n_A}{n} = \dfrac{20{,}000}{120{,}000} = .1667$.

3.12 $P(A) = \dfrac{n_A}{n} = \dfrac{20,000}{180,000} = .1111$.

The probability of a random sample of 2 people from the city will contain 2 legal immigrants from Latin America is (.1111)(.1111) = .0123.

An alternative method to obtain the probability is,

$$P(A) = \frac{n_A}{n} = \frac{C_2^{20,000}}{C_2^{180,000}} = .0123.$$

3.13
a. $P(A) = P(5 \text{ days} \cup 6 \text{ days} \cup 7 \text{ days}) = .41 + .20 + .07 = .68$
b. $P(B) = P(3 \text{ days} \cup 4 \text{ days} \cup 5 \text{ days}) = .08 + .24 + .41 = .73$
c. $P(\bar{A}) = P(3 \text{ days} \cup 4 \text{ days}) = .08 + .24 = .32$
d. $P(A \cap B) = P(5 \text{ days}) = .41$
e. $P(A \cup B) = .08 + .24 + .41 + .2 + .07 = 1.0$

3.14
a. Event A= 89/1500=0.059
b. Event D= (1500-450-750-89)/1500=0.140
c. Complement of B= 1-0.5=0.5
e. Complement of C= 1-0.3=.7
f. The probability of the events A or D = 0.059+0.140=0.199

3.15
a. $P(A) = 4/8 = 0.5$
b. $P(B) = 2/8 = 0.25$
c. $P(A \cap B) = P(B) = 0.25$

3.16 Events A and B of Exercise 3.2 are not mutually exclusive.
$P(A) = .4$, $P(B) = .4$, and $P(A \cup B) = .6$. $P(A \cup B) = P(A) + P(B) = .4 + .4 = .8 > .6$.
Therefore, if two events are not mutually exclusive, the probability of their union cannot equal the sum of their individual probabilities.

3.17
a. $P(A) = .39 + .23 + .15 + .06 + .03 = .86$
b. $P(B) = .14 + .39 + .23 + .15 = .91$
c. $P(\bar{A}) = 1 - P(A) = 1 - .86 = .14$
d. $P(A \cup B) = .14 + .39 + .23 + .15 + .06 + .03 = 1.00$
e. $P(A \cap B) = .39 + .23 + .15 = .77$
f. Check if $P(A \cap B) = 0$. Because $P(A \cap B) = .77 \neq 0$, A and B are not mutually exclusive.
g. Yes, because $P(A \cup B) = 1$, A and B are collectively exhaustive.

3.18
a. $P(X < 3) = .29 + .36 + .22 = .87$
b. $P(X > 1) = .22 + .10 + .03 = .35$
c. By the third probability postulate, the probabilities of all outcomes in the sample space must sum to one.

3.19 $P(A) = .60$, $P(B) = .45$, $P(A \cup B) = .80$
By the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Therefore, $.80 = .60 + .45 - P(A \cap B)$
$P(A \cap B) = .60 + .45 - .80 = .25$

3.20 $P(A) = .40$, $P(B) = .45$, $P(A \cup B) = .85$
By the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Therefore, $.85 = .40 + .45 - P(A \cap B)$
$P(A \cap B) = .40 + .45 - .85 = 0$

3.21 $P(A) = .60$, $P(B) = .40$, $P(A \cup B) = .76$
By the Addition Rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Therefore, $.76 = .60 + .40 - P(A \cap B)$
$P(A \cap B) = .60 + .40 - .76 = .24$

3.22 $P(A) = .60$, $P(B) = .45$, $P(A \cap B) = .30$
By the Addition Rule,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Therefore, $P(A \cup B) = .60 + .45 - .30 = .75$

3.23 $P(A) = .60$, $P(B) = .45$, $P(A \cap B) = .30$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.30}{.45} = .6667$$

A and B are not independent since the $P(A|B)$ of .6667 does not equal the $P(A)$ of .60.

3.24 $P(A) = .80$, $P(B) = .10$, $P(A \cap B) = .08$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.08}{.10} = .80$$

A and B are independent since the $P(A|B)$ of .80 equals the $P(A)$ of .80

3.25 $P(A) = .30$, $P(B) = .40$, $P(A \cap B) = .30$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.30}{.40} = .75$$

A and B are not independent since the $P(A|B)$ of .75 does not equal the $P(A)$ of .30

3.26 $P(A) = .70$, $P(B) = .80$, $P(A \cap B) = .50$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.50}{.80} = .625$$

A and B are not independent since the $P(A|B)$ of .625 does not equal the $P(A)$ of .70

3.27 The probability of the intersection between two independent events is equal to the product of the individual probabilities.
Therefore, the probability of both Superior Packaging and Intense Media campaign = (1/3)(1/3) = 1/9

3.28 a. $P_7^7 = 7! = 5{,}040$

b. The probability that the guess will turn out to be correct is 1 / 5,040 = 0.0001984

3.29 $P_2^{50} = 50!/48! = 2{,}450$

3.30 $P_3^6 = 6!/3! = 120$. Therefore, the probability of selecting in the correct order, the three best performing stocks by chance is 1/120 = .00833

3.31 If A is the event 'no graduate student is selected', then the number of combinations of 3 chosen from 6 is $C_3^6 = 6!/3!3! = 20$ and the number of combinations of 0 objects chosen from 2 is $C_0^2 = 2!/0!2! = 1$. $P(A) = 1/20 = .05$

3.32 Ignoring the possibility of ties, the number of different predictions which could be done is $P_3^5 = 5!/2! = 60$. Therefore, the probability of making the correct prediction by chance is 1/60 = .0167

3.33 $C_2^8 = 8!/2!6! = 28$

3.34 a. $P_2^7 = 7!/5! = 42$

b. $P_1^6 = 6!/5! = 6$

c. $P_1^6 = 6!/5! = 6$

d. Probability of being chosen as the heroine = 6 chances out of 42 = 6/42 = 1/7 = .1429. More direct way: Since there are seven candidates for 1 part – a randomly chosen candidate would have a 1 in 7 chance of getting any specific part.

e. Since being chosen as the heroine or as the best friend are mutually exclusive, the $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/7 + 1/7 - 0 = 2/7 = .2857$.
More direct way: Since there are seven candidates for 2 parts – a randomly chosen candidate would have a 2 in 7 chance of getting apart.

3.35 a. $C_2^5 = 5!/2!3! = 10$, $C_4^6 = 6!/4!2! = 15$. Since the selections are independent, there are (10)(15) = 150 possible combinations.

b. P(select a brother who is a craftsman) = $C_1^4/10 = [4!/1!3!]/10 = 4/10$. Because there are only 5 craftsmen, once a brother has been selected as a craftsman there are only four ways to fill the second craftsman spot on the work crew.
P(select a brother who is a laborer) = $C_3^5/15 = [5!/2!3!]/10 = 10/15$. Multiply the two probabilities together to find their intersection: P(selecting both brothers) = (4/10)(10/15) = .2667

c. The probability of the complements is 1 minus the probability of the event. Therefore, P(not selecting a brother who is a craftsman) = 1 – 4/10 = 6/10. P(not selecting a brother who is a laborer) = 1-10/15 = 5/15. Multiply the two probabilities together to find their intersection: P(neither brother will be selected) = (6/10)(5/15) = .20

3.36 a. $C_2^6 = 6!/2!4! = 15$, $C_2^4 = 4!/2!2! = 6$. Because the selections are independent, there are (15)(6) = 90 different sets of funds from which to choose.

b. P(no U.S. fund under performs) = $C_2^5/15 = 5!/2!3!/15 = 10/15$. P(no foreign fund under performs) = $C_2^3/6 = 3!/1!2!/6 = 3/6$, P(at least one fund under performs) = 1 – P(no fund under performs) = 1-(10/15)(3/6) = 2/3 = 0.6667.

3.37 Let A – employment concern, B – grade concerns, $A \cap B$ – both. Then, P(concerned about at least one of these two) = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .25 - .20 = .35$

3.38 Let A – customer asks for assistance, B – customer makes a purchase, $A \cap B$ – both. Then, P(a customer does at least one of these two) = $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .20 - .15 = .35$

3.39 a. Let A – Immediate donation, B-Information request, C-No interest, D-donate later. Then $P(A) = .05$, $P(B) = .25$, $P(C)=.7$. Then $P(4 \text{ misses}) = (.95)^4 = .8145$.
b. $P(\text{donation}) = P(A) + P(B) = .05 + .25(.2) = .10$
Then $P(4 \text{ misses}) = (.9)^4 = .6561$

3.40 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) = .02 + .01 + .04 - .0002 - .0008 - .000 = .069$

3.41 Let A – graduating with adequate grades, B – passing the standardized test. $P(A) = 1-.02 = .98$, $P(B) = 1-.15 = .85$. Probability that the player will be eligible to play is $(.98)(.85) = .833$

3.42 Let A – watch a TV program oriented to business and financial issues, B – read a publication, then the $P(A) = .18$, $P(B) = .12$ and $P(A \cap B) = .10$
a. Find $P(B|A) = P(A \cap B) / P(A) = .10/.18 = .5556$
b. Find $P(A|B) = P(A \cap B) / P(B) = .10/.12 = .8333$

3.43
a. 0.8091
b. 0.0091

3.44 The number of ways of randomly choosing 2 stocks in order out of 4 is: $P_2^4 = 4!/2! = 12$, the number of ways of randomly choosing 2 bonds in order from 5: $P_2^5 = 5!/3! = 20$. Then the probability of choosing either the stocks in order or the bonds in order is the union between the two events which is equal to the sum of the individual probabilities minus the probability of the intersection. $= 1/12 + 1/20 - 1/240 = .1292$

3.45 Let event A – high risk borrowers, B – loan in default, then $P(A) = .15$, $P(B) = .05$ and $P(A|B) = .40$. Then, $P(B|A) = [P(A|B)P(B)] / P(A) = [(.4)(.05)]/.15 = .1333$

3.46 Let event A—portfolio management was attended, B – Chartism attended, C – random walk attended, then $P(A) = .4$, $P(B) = .5$, $P(C) = .80$.
a. Find $P(A \cup B) = .4 + .5 - 0 = .9$
b. Find $P(A \cup C)$ if A and C are independent events $= .4 + .8 - .32 = .88$
c. If the $P(C|B) = .75$, then $P(B \cap C) = P(C|B)P(B) = (.75)(.5) = .375$. $P(C \cup B) = P(C) + P(B) - P(C \cap B) = .8 + .5 - .375 = .925$

3.47 Let A – picking 3 high tech stocks in order, B – picking 3 airline stocks in order out of five. The number of ways of choosing three high tech stocks out of 5 or three airline stocks out of 5 in order is $P_3^5 = 5!/2! = 60$. Then $P(A \cup B) = 1/60 + 1/60 - 1/3600 = .0331$. Therefore, the probability of getting either the high tech stocks or the airline stocks picked correctly is relatively small, and would be an accomplishment.

3.48 Let A – work related problem occurs on Monday and B – work related problem occurs in the last hour of the day's shift, then $P(A) = .3$, $P(B) = .2$ and $P(A \cap B) = .04$, $P(A \cap \bar{B}) = P(A) - P(A \cap B) = .3 - .04 = .26$

a. $P(\bar{B}|A) = P(A \cap \bar{B})/P(A) = .26/.3 = .867$

b. Check if $P(A \cap B) = P(A)P(B)$. Since $.04 \neq .06$, the two events are not independent events.

3.49 Let A – Reading class, B—Math class, then $P(A) = .4$, $P(B) = .5$ and $P(B|A) = .3$

a. $P(A \cap B) = P(B|A)P(A) = (.3)(.4) = .12$

b. $P(A|B) = P(A \cap B)/P(B) = .12 / .5 = .24$

c. $P(A \cup B) = .4 + .5 - .12 = .78$

d. Check if $P(A \cap B) = P(A)P(B)$. Since $.12 \neq .2$ the two events are not independent events.

3.50 Let A – new customer, B – call to a rival service customer, then $P(A) = .15$, $P(B) = .6$ and $P(B|A) = .8$. $P(A|B) = P(A \cap B)/ P(B)$ where $P(A \cap B) = P(B|A)P(A)$. $[(.8)(.15)]/.6 = .2$

3.51 Given that $P(A \cap B \cap C) = .02$, $P(B) = .2$, and $P(A|B) = .8$, then find the $P(C|(A \cap B) = P(A \cap B \cap C)/[PA|B)P(B)] = .02/.16 = .125$

3.52 $P(\mathit{High\ Income} \cap \mathit{Never}) = .05$

3.53 $P(\mathit{Low\ Income} \cap \mathit{Regular}) = .05$

3.54 $P(\mathit{Middle\ Income} \cap \mathit{Never}) = .05$

3.55 $P(\mathit{Middle\ Income} \cap \mathit{Occasional}) = .20$

3.56 $P(\mathit{High\ Income}|\mathit{Never}) = \dfrac{P(\mathit{High\ Income} \cap \mathit{Never})}{P(\mathit{Never})} = \dfrac{.05}{.30} = .1667$

3.57 $P(\mathit{Low\ Income}|\mathit{Occasional}) = \dfrac{P(\mathit{Low\ Income} \cap \mathit{Occasional})}{P(\mathit{Occasional})} = \dfrac{.10}{.40} = .25$

3.58 $P(\mathit{Regular}|\mathit{High\ Income}) = \dfrac{P(\mathit{Regular} \cap \mathit{High})}{P(\mathit{High})} = \dfrac{.10}{.25} = .40$

3.59 Odds $= \dfrac{.8}{1 - .8}$ = 4 to 1 odds

3.60 Odds = $\frac{.5}{1-.5}$ = 1 to 1 odds

3.61 P(High scores|>25 hours) = .80, P(Low scores|>25) = .40, $\frac{.8}{.4} = 2.00$. Studying increases the probability of achieving high scores.

3.62 P(High scores|>25 hours) = .40, P(Low scores|>25) = .20, $\frac{.4}{.2} = 2.00$. Studying increases the probability of achieving high scores.

3.63 P(High scores|>25 hours) = .20, P(Low scores|>25) = .40, $\frac{.2}{.4} = .50$. Studying decreases the probability of achieving high scores.

3.64 Let F – frequent, I – Infrequent, O – Often, S – Sometimes and N- Never.

a. $P(F \cap O) = .12$
b. $P(F|N) = P(F \cap N)/P(N) = .19 / .27 = .7037$
c. Check if $P(F \cap N) = P(F)P(N)$. Since $.19 \neq .2133$, the two events are not independent.
d. $P(O|I) = P(I \cap O) / P(I) = .07/.21 = .3333$
e. Check if $P(I \cap O) = P(I)P(O)$. Since $.07 \neq .0399$, the two events are not independent.
f. $P(F) = .79$
g. $P(N) = .27$
h. $P(F \cup N) = P(F) + P(N) - P(F \cap N) = .79 + .27 - .19 = .87$

3.65 Let PH – Predicted High, PN – Predicted Normal, PL – Predicted Low, H – High Outcome, N – Normal Outcome, L – Low Outcome

a. P(PH) = .3
b. P(H) = .38
c. P(PH|H) = P(PH∩H)/P(H) - .23/.38 = .6053
d. P(H|PH) = P(PH∩H)/P(PH) = .23/.3 = .7667
e. P(L|PH) = P(PH∩L) / P(PH) = .01/.3 = .0333

3.66 Let A – Regularly read business section, B – Occasionally, C – Never, TS – Traded stock

a. $P(C) = .25$
b. P(TS) = .32
c. $P(TS|C) = P(TS \cap C)/P(C) = .04/.25 = .16$
d. $P(C|TS) = P(TS \cap C)/ P(TS) = .04/.32 = .125$
e. $P(TS|\bar{A}) = P(TS \cap (B \cup C))/P(B \cup C) = (.10 + .04)/(.41+.25) = .2121$

3.67 Let G – good part, D – defective part

a. $P(\text{D}) = .1$
b. $P(B) = .35$
c. $P(\text{D}|B) = P(B \cap \text{D})/P(B) = .05/.35 = .1429$
d. $P(B|\text{D}) = P(B \cap \text{D})/P(\text{D}) = .05/.1 = .5$
e. No, since $P(\text{G}\cap B)$ which is $.3 \neq P(\text{G})P(B)$ which is .315
f. $P(\text{G}|A) = .27/.29 = .931$, $P(\text{G}|B) = .3/.35 =.857$ $P(\text{G}|C) = .33/.36 = .917$. Therefore, Subcontractor A is the most reliable.

3.68 Let Y – Problems were worked, N – Problems not worked

a. $P(\text{Y}) = .32$
b. $P(A) = .25$
c. $P(A|\text{Y}) = P(A\cap\text{Y})/P(\text{Y}) = .12/.32 = .375$
d. $P(\text{Y}|A) = P(A\cap\text{Y})/P(A) = .12/.25 = .48$
e. $P(\text{C} \cup \text{below } C|\text{Y}) = P(C \cup \text{below } C \cap \text{Y})/P(\text{Y}) = (.12 + .02)/.32 = .4375$
f. No, since $P(A\cap\text{Y})$ which is $.12 \neq P(A)P(\text{Y})$ which is .08.

3.69 Let event ST– Stayed, L – Left, M – Married and SN – Single

a. $P(\text{M}) = .77$
b. $P(\text{L}) = .19$
c. $P(\text{L}|\text{SN}) = P(\text{L}\cap\text{SN})/P(\text{SN}) = .06/.23 = .2609$
d. $P(\text{M}|\text{ST}) = P(\text{M}\cap\text{ST}) / P(\text{ST}) = .64/.81 = .7901$

3.70 Let R – Readers, V – Voted in the last election

a. $P(\text{V}) = .76$
b. $P(\text{R}) = .77$
c. $P(\bar{V} \cap \bar{R}) = .1$

3.71 Let J – Joined, M – Men, W – Women

a. $P(\text{J}) = P(\text{J} \cap \text{M}) + P(\text{J} \cap \text{W}) = P(\text{J}|\text{M})P(\text{M}) + P(\text{J}|\text{W})P(\text{W}) = (.07)(.4) + (.09)(.6) = .082$
b. $P(\text{W}|\text{J}) = P(\text{J} \cap \text{W})/P(\text{J}) = (.09)(.6)/.082 = .6585$

3.72 Let G – Growth, H – High, L – Low, S – Within. $P(\text{G}|\text{H}) = .1$, $P(\text{G}|\text{L}) = .8$, $P(\text{G}|\text{S}) = .5$, $P(\text{H}) = .25$, $P(\text{L}) = .15$, $P(\text{S}) = .6$

a. $P(\text{G} \cap \text{H}) = P(\text{G}|\text{H})P(\text{H}) = (.1)(.25) = .025$
b. $P(\text{G}) = P(\text{G} \cap \text{H}) + P(\text{G} \cap \text{L}) + P(\text{G} \cap \text{S}) = (.1)(.25) + (.8)(.15) + (.5)(.6) = .445$
c. $P(\text{L}|\text{G}) = P(\text{G} \cap \text{L}) / P(\text{G}) = (.8)(.15)/.445 = .2697$

3.73 Let HC – Health care, WS – Work schedule. $P(\text{HC}) = .42$, $P(\text{WS}) = .22$, $P(\text{WS}|\text{HC}) = .34$

a. $P(\text{HC} \cap \text{WS}) = P(\text{WS}|\text{HC})P(\text{HC}) = (.42)(.34) = .1428$
b. $P(\text{HC} \cup \text{WS}) = P(\text{HC}) + P(\text{WS}) - P(\text{HC} \cap \text{WS}) = .42 + .22 - .1428 = .4972$
c. $P(\text{HC}|\text{WS}) = P(\text{HC} \cap \text{WS})/P(\text{WS}) = .1428 / .22 = .6491$

3.74 Let T– Top quarter, M – Middle half, B – Bottom quarter. $P(10\%|T) = .7$, $P(10\%|M) = .5$, $P(10\%|B) = .2$

a. $P(10\%) = P(10\% \cap T) + P(10\% \cap M) + P(10\% \cap B) = (.7)(.25) + (.5)(.5) + (.2)(.25) = .475$

b. $P(T|10\%) = P(10\% \cap T)/P(10\%) = (.7)(.25)/.475 = .3684$

c. $P(\overline{T}\,|\,\overline{10\%}) = P(\overline{10\%} \cap \overline{T})/P(\overline{10\%}) = P(\overline{10\% \cup T})\ /\ P(\overline{10\%}) =$

$[1 - P(10\% \cup T)]/P(\overline{10\%}) = [1 - (.475 + .25 - (.7)(.25))] / .525 = .8571$

3.75 a. $P(H|F) = P(F \cap H)/P(F) = .173 / .303 = .571$

b. $P(L|U) = P(U \cap L)/P(U) = .141/.272 = .5184$

c. $P(L|(N \cup F)) = P((N \cup F) \cap L) / P(N \cup F) = .155/.728 = .2129$

d. $P((N \cup F)|L) = P((N \cup F) \cap L) / P(L) = .155/.296 = .5236$

3.76 Let M – Faulty machine, I – Impurity

$P(M) = .4$, $P(I|M) = .1$, $P(I) = P(I \cap M) + P(I \cap \overline{M}) = (.4)(.1) + 0 = .04$ $P(M|\overline{I}) = P(\overline{I} \cap M) / P(\overline{I}) = [P(M) - P(I \cap M)] / P(\overline{I}) = (.4 - .04)/.96 = .375$

3.77 $P(E) = .7$, $P(B) = .3$, $P(SE|E) = .6$, $P(SE|B) = .25$, $P(SE \cap E) = P(SE|E)P(E) = .42$, $P(SE \cap B) = P(SE|B)P(B) = .075$, $P(SE) = P(SE \cap E) + P(SE \cap B) = .495$

a. $P(E|SE) = P(SE \cap E)/P(SE) = .42/.495 = .8485$, $P(3E|SE) = [P(E|SE)]^3 = .8485^3 = .6109$

b. $P(\text{at least 1E}|SE) = 1 - P(\text{no E}|SE) = 1 - [P(B|SE)]^3 = 1 - [.075/.495]^3 = .9965$

3.78 $P(A_1) = .4$, $P(B_1|A_1)=.6$, $P(B_1|A_2) = .7$

Complements: $P(A_2) = .6$, $P(B_2|A_1)=.4$, $P(B_2|A_2) = .3$

$$P(A_1 | B_1) = \frac{P(B_1 | A_1)P(A_1)}{P(B_1 | A_1)P(A_1) + P(B_1 | A_2)P(A_2)} = \frac{.6(.4)}{.6(.4) + .7(.6)} = .3636$$

3.79 $P(A_1) = .8$, $P(B_1|A_1)=.6$, $P(B_1|A_2) = .2$

Complements: $P(A_2) = .2$, $P(B_2|A_1)=.4$, $P(B_2|A_2) = .8$

$$P(A_1 | B_1) = \frac{P(B_1 | A_1)P(A_1)}{P(B_1 | A_1)P(A_1) + P(B_1 | A_2)P(A_2)} = \frac{.6(.8)}{.6(.8) + .2(.2)} = .9231$$

3.80 $P(A_1) = .5$, $P(B_1|A_1)=.4$, $P(B_1|A_2) = .7$

Complements: $P(A_2) = .5$, $P(B_2|A_1)=.6$, $P(B_2|A_2) = .3$

$$P(A_1 | B_2) = \frac{P(B_2 | A_1)P(A_1)}{P(B_2 | A_1)P(A_1) + P(B_2 | A_2)P(A_2)} = \frac{.6(.5)}{.6(.5) + .3(.5)} = .6667$$

3.81 $P(A_1) = .4$, $P(B_1|A_1)=.6$, $P(B_1|A_2) = .7$

Complements: $P(A_2) = .6$, $P(B_2|A_1)=.4$, $P(B_2|A_2) = .3$

$$P(A_2 | B_2) = \frac{P(B_2 | A_2)P(A_2)}{P(B_2 | A_2)P(A_2) + P(B_2 | A_1)P(A_1)} = \frac{.3(.6)}{.3(.6) + .4(.4)} = .5294$$

3.82 $P(A_1) = .6$, $P(B_1|A_1)=.6$, $P(B_1|A_2) = .4$
Complements: $P(A_2) = .4$, $P(B_2|A_1)=.4$, $P(B_2|A_2) = .6$

$$P(A_1 \mid B_1) = \frac{P(B_1 \mid A_1)P(A_1)}{P(B_1 \mid A_1)P(A_1) + P(B_1 \mid A_2)P(A_2)} = \frac{.6(.6)}{.6(.6) + .4(.4)} = .6923$$

3.83 Let A_1: Professor receives advertising material
A_2: Professor does not receive advertising material
B_1: Adopts the book
B_2: Does not adopt the book
Then, $P(A_1) = .8$, $P(A_2) = 1 - .8 = .2$, $P(B_1| A_1) = .3$, $P(B_1 \mid A_2) = .1$

$$P(A_1 \mid B_1) = \frac{P(B_1 \mid A_1)P(A_1)}{P(B_1 \mid A_1)P(A_1) + P(B_1 \mid A_2)P(A_2)} = \frac{(.3)(.8)}{(.3)(.8) + (.1)(.2)} = .923$$

3.84 E_1: Stock performs much better than the market average
E_2: Stock performs same as the market average
E_3: Stock performs worse than the market average
A: Stock is rated a 'Good Buy'
Given that $P(E_1) = .25$, $P(E_2) = .5$, $P(E_3) = .25$, $P(A| E_1) = .4$, $P(A| E_2) = .2$, $P(A| E_3) = .1$
Then, $P(E_1 \cap A) = P(A| E_1)P(E_1) = (.4)(.25) = .10$
$P(E_2 \cap A) = P(A| E_2)P(E_2) = (.2)(.5) = .10$
$P(E_3 \cap A) = P(A| E_3)P(E_3) = (.1)(.25) = .025$

$$P(E_1 \mid A) = \frac{P(A \mid E_1)P(E_1)}{P(A \mid E_1)P(E_1) + P(A \mid E_2)P(E_2) + P(A \mid E_3)P(E_3)} =$$

$$= \frac{(.40)(.25)}{(.4)(.25) + (.2)(.5) + (.1)(.25)} = .444$$

3.85 E_1: Process is functioning correctly
E_2: Process is malfunctioning
A_1 : Lightbulb is defective
A_2 : Lightbulb is not defective

The following probabilites are given:

$P(E_1) = 0.9$, $P(A_1 \mid E_1) = 0.1$, $P(A_1 \mid E_2) = 0.5$

Next, compute the complements of the given probabilities.

$P(E_2) = 0.1$, $P(A_2 \mid E_1) = 0.9$, $P(A_2 \mid E_2) = 0.5$

The probability that the process is functioning correctly if a defective bulb is found is $P(E_1 \mid A_1)$.

$$P(E_1 \mid A_1) = \frac{P(A_1 \mid E_1)P(E_1)}{P(A_1 \mid E_1)P(E_1) + P(A_1 \mid E_2)P(E_2)} = \frac{(0.1)(0.9)}{(0.1)(0.9) + (0.5)(0.1)} = 0.6429$$

The probability that the process is operating correctly if a nondefective bulb is found is $P(E_1 \mid A_2)$.

$$P(E_1 \mid A_2) = \frac{P(A_2 \mid E_1)P(E_1)}{P(A_2 \mid E_1)P(E_1) + P(A_2 \mid E_2)P(E_2)} = \frac{(0.9)(0.9)}{(0.9)(0.9) + (0.5)(0.1)} = 0.9419$$

3.86 A_1: Chickens are purchased from Free Range Farms
A_2: Chickens are purchased from Big Foods Ltd
B_1: Chickens weigh less than 3 pounds
B_2: Chickens weigh more than 3 pounds

The following probabilites are given:
$P(B_1 \mid A_1) = 0.1$, $P(B_1 \mid A_2) = 0.2$, $P(A_1) = 0.4$,

Next, compute the complements of the given probabilities.
$P(B_2 \mid A_1) = 0.9$, $P(B_2 \mid A_2) = 0.8$, $P(A_2) = 0.6$

The probability the chicken came from Free Range Farms is $P(A_1 \mid B_2)$

$$P(A_1 \mid B_2) = \frac{P(B_2 \mid A_1)P(A_1)}{P(B_2 \mid A_1)P(A_1) + P(B_2 \mid A_2)P(A_2)}$$

$$= \frac{(0.9)(0.4)}{(0.9)(0.4) + (0.8)(0.6)} = 0.4286$$

b. As mentioned earlier,
A_1: Chickens are purchased from Free Range Farms
A_2: Chickens are purchased from Big Foods Ltd
P(A_1) = 0.4 and P(A_2) = 0.6.
P(Out of 5, probability that at least 3 came from Free Range Farms) = $P(A_1 \geq 3)$
= $P(A_1 = 3) + P(A_1 = 4) + P(A_1 = 5)$

P(selecting 3 Chickens are from Free Range Farms)
= 0.4*0.4*0.4*0.6*0.6 = 0.02304.
However, there are ${}^5C_3$ = 10 ways to select 3 chickens from 5.
Therefore $P(A_1 = 3)$ = 10* 0.02304 = 0.2304.

Similarly, P(selecting 4 Chickens are from Free Range Farms)
= 0.4*0.4*0.4*0.4*0.6 = 0.01536.
However, there are ${}^5C_4$ = 5 ways to select 4 chickens from 5
$P(A_1 = 4) = {}^5C_4$ * 0.01536 = 0.0768.

Similarly, $P(A_1 = 5) = {}^5C_5$ * 0.01024 = 0.01024.
Therefore, $P(A_1 \geq 3)$ = 0.2304 + 0.0768 + 0.01024 = 0.31744

If you purchase 5 chickens, then the probability that at least 3 came from Free Range Farms is 0.317

3.87 To pass from odds to probabilities: **P(A) = A / (A+B)**
Therefore P(MU=win) = 2/ (2+8) = 2/10= 0.2

3.88 Mutually exclusive events are events such that if one event occurs, the other event cannot occur. For example, a U.S. Senator voting in favor of a tax cut cannot also vote against it. Independent events are events such that the occurrence of one event has no effect on the probability of the other event. For example, whether or not you ate breakfast this morning is unlikely to have any effect on the probability of a U.S. Senator voting in favor of a tax cut.

3.89 a. True by definition

b. False, only the sum of the probabilities of mutually exclusive events which are collectively exhaustive sum to 1.

c. True, $C_x^n = \frac{n!}{x!(n-x)!} = C_{n-x}^n = \frac{n!}{(n-x)!(n-(n-x))!} = \frac{n!}{(n-x)!x!}$

d. True, $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)} = P(B|A)$ for $P(A) = P(B)$

e. True, $P(A) = 1 - P(A) \Rightarrow 2P(A) = 1 \Rightarrow P(A) = .5$

f. True, $P(\bar{A}\cap\bar{B}) = P(\bar{A}) - P(\bar{A}\cap B)$
$= 1 - P(A) - [P(B) - P(A\cap B)]$
$= 1 - P(A) - P(B) + P(A\cap B)$
$= 1 - P(A) - P(B) + P(A)P(B)$
$= [1 - P(A)][1 - P(B)] = P(\bar{A})P(\bar{B})$

g. False, $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$
$= 1 - P(A\cup B)$
$= 1 - P(A) - P(B) \quad [P(A\cap B) = 0]$
$= P(\bar{A}) + P(\bar{B}) - 1$
$= 0$ which holds if and only if $P(\bar{A}) + P(\bar{B}) = 1$

3.90 Conditional probability is determining the probability of one event given that another event has occurred. This is utilizing prior information that a specific event has occurred and then analyzing how that impacts the probability of another event. The importance is incorporating information about a known event. This has the impact of reducing the sample space in an experiment.

3.91 Bayes' theorem is a summary of the relationship between a specific event that has occurred and the effect on a subsequent event. The occurrence of the specific event is the prior information or 'prior probability' that is known. This prior knowledge can be analyzed to determine the effect on the probability of a subsequent event. The subsequent event is the 'posterior probability'.

3.92 a. True: By definition, probabilities cannot be negative, hence the union between two events is $P(A) + P(B) - P(A \cap B)$ must be ≥ 0. Adding $P(A \cap B)$ to both sides, it follows that $P(A) + P(B)$ must be $\geq P(A \cap B)$

b. True: this follows from the probability of the union between two events. $P(A) + P(B) - P(A \cap B)$ cannot be larger than $P(A) + P(B)$. Only if the term $P(A \cap B)$ is negative can the statement be false. Since probabilities of events must be between 0 and 1 inclusive, the union between two events cannot be larger than the sum of the individual probabilities.

c. True: events cannot intersect with another event by more than their individual size

d. True: By definition of a complement, if an event occurs, then the event of 'not the event' (its complement) cannot occur.

e. False: Probabilities of any two events can sum to more than 1. Among other conditions, this statement will be true if the two events are mutually exclusive

f. False: there may be more events contained in the sample space. Hence, if two events are mutually exclusive, they may or may not be collectively exhaustive

g. False: the two events could contain common elements and hence their intersection would not be zero

3.93 By definition, *Joint Probability* is the probability that two events will occur together, e.g., *P*(female and Liberal Arts major)

Marginal probability is defined as the probability of an individual event, e.g., *P*(female)

Conditional probability is the probability of occurrence of one event given that another event has occurred, e.g., *P*(female given Liberal Arts major)

3.94 a. False: Given that event B occurs has changed the sample space, hence, the revised probability may be less

b. False: If the probability of its complement is zero, then the event and its complement will be dependent events

c. True: This follows because the probabilities of events must be non-zero. By the Multiplicative Law of Probability: $P(A|B)P(B) = P(A \cap B)$. Since $0 \leq P(B) \leq 1$, then the $P(A|B) \geq P(A \cap B)$.

d. False: this statement is true only when the two events are independent

e. False: the posterior probability of any event could be smaller, larger, or equal to the prior probability

3.95 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - [P(A|B)P(B)]$

$= P(A) + P(B)[1-P(A|B)]$

3.96 Let W – weather condition caused the accident, BI – bodily injuries were incurred. $P(\text{W}) = .3$, $P(\text{BI}) = .2$, $P(\text{W}|\text{BI}) = .4$

a. $P(\text{W} \cap \text{BI}) = P(\text{W}|\text{BI})P(\text{BI}) = (.4)(.2) = .08$

b. No, since $P(\text{W} \cap \text{BI}) = .08 \neq .06 = P(\text{W})P(B)$

c. $P(\text{BI}|\text{W}) = P(\text{W} \cap \text{BI})/P(\text{W}) = .08/.3 = .267$

d. $P(\overline{W} \cap \overline{BI}) = P(\overline{W \cup B}) = 1-P(\text{W} \cup \text{BI}) = 1 - P(\text{W}) - P(\text{BI}) + P(\text{W} \cap \text{BI}) =$
$1-.3-.2+.08 = .58$

3.97 Solve the following for *P*(thick) and *P*(thin):

$.8 = P(\text{thick}) + P(\text{thin})[1-P(\text{thick/thin})] = P(\text{thick}) + .6\ P(\text{thin})$
$.8 = P(\text{thin}) + P(\text{thick})[1-P(\text{thin/thick})] = P(\text{thin}) + .4\ P(\text{thick})$
Solving, $P(\text{thin}) = .6316$

a. $P(\text{thick}) = .8 - .6\ P(\text{thin}) = .8 - (.6)(.6316) = .4211$
b. $P(\text{thin}) = .6316$
c. $P(\text{thick} \cap \text{thin}) = P(\text{thick}|\text{thin})\ P(\text{thin}) = (.4)(.6316) = .2526$

3.98

Let M – analysts have an MBA, A – analysts are over age 35. $P(M) = .35$, $P(A) = .40$, $P(A|M) = .3$

a. $P(M\cap A) = P(A|M)P(M) = (.3)(.35) = .105$
b. $P(M|A) = P(M\cap A)P(A) = (.105)(.4) = .2625$
c. $P(M\cup A) = P(M) + P(A) - P(M\cap A) = .35 + .4 - .105 = .645$
d. $P(\overline{M} \mid \overline{A}) = P(\overline{M} \cap \overline{A})/P(\overline{A}) = [1 - P(M\cup A)]/P(\overline{A}) = .355/.4 = .5917$
e. No, because $P(M \cap A)$ which is $.105 \neq .14$ which is $P(M)P(A)$
f. No, their intersection is not zero, hence the two events cannot be mutually exclusive
g. No, the sum of their individual probabilities is .645 which is less than 1

3.99 Let VM – customer orders a vegetarian meal, S – customer is a student.
$P(\text{VM}) = .35$, $P(\text{S}) = .5$, $P(\text{VM}|\text{S}) = .25$

a. $P(\text{VM}\cap\text{S}) = P(\text{VM}|\text{S})P(\text{S}) = (.25)(.5) = .125$
b. $P(\text{S}|\text{VM}) = P(\text{VM}\cap\text{S})/P(\text{VM}) = .125/.35 = .3571$
c. $P(\overline{VM} \cup \overline{S}) = 1\text{-}P(\text{VM}\cap\text{S}) = 1\text{-}.125 = .875$
d. No, since $P(\text{VM}\cap\text{S})$ which is $.125 \neq .175$ which is $P(\text{VM})P(\text{S})$
e. No, their intersection is not zero, hence the two events cannot be mutually exclusive. $P(\text{VM}\cap\text{S}) = .125 \neq 0$
f. No, the probability of their union does not equal 1. $P(\text{VM}\cup\text{S}) = P(\text{VM}) + P(\text{S}) - P(\text{VM} \cap \text{S}) = .35 + .5 - .125 = .725$ which is less than 1.

3.100 Let 160 – farm size exceeds 160 acres, 50 – farm owner is over 50 years old. $P(160) = .2$, $P(50) = .6$, $P(50|160) = .55$

a. $P(160\cap 50) = P(50|160)P(160) = (.55)(.2) = .11$
b. $P(160 \cup 50) = P(160) + P(50) - P(160\cap 50) = .2 + .6 - .11 = .69$
c. $P(160|50) = P(160\cap 50)/P(50) = .11/.6 = .1833$
d. No, since $P(160\cap 50)$ which is $.11 \neq .12$ which is $P(160)P(50)$

3.101 Let M – men, F – women, G – graduate training, UG – undergraduate training, HS – High School training. $P(\text{M}) = .8$, $P(\text{F}) = .2$, $P(\text{HS}|\text{M}) = .6$, $P(\text{UG}|\text{M}) = .3$, $P(\text{G}|\text{M}) = .1$. $P(\text{G}|\text{F}) = .15$, $P(\text{UG}|\text{F}) = .4$, $P(\text{HS}|\text{F}) = .45$

a. $P(\text{M}\cap\text{HS}) = P(\text{HS}|\text{M})P(\text{M}) = (.6)(.8) = .48$
b. $P(\text{G}) = P(\text{M}\cap\text{G}) + P(\text{F}\cap\text{G}) = P(\text{G}|\text{M})P(\text{M}) + P(\text{G}|\text{F})P(\text{F}) = (.10)(.8) + (.15)(.2) = .11$
c. $P(\text{M}|\text{G}) = P(\text{M}\cap\text{G})/P(\text{G}) = .08/.11 = .7273$
d. No, since $P(\text{M}\cap\text{G})$ which is $.8 \neq .088$ which is $P(\text{M})P(\text{G})$
e. $P(\text{F}|\overline{G}) = P(\text{F}\cap \overline{G})/P(\overline{G}) = [P(\text{F}) - P(\text{F}\cap \text{G})]/P(\overline{G}) = (.2 - (.15)(.2))/.89 = .191$

3.102 Let NS – night shift worker, F – women, M – men, FP – favored plan. P(NS) = .5, P(F) = .3, P(M) = .7, P(F|NS) = .2, P(M|NS) = .8, P(FP|NS) = .65, P(FP|F) = .4. Therefore, $P(\text{NS} \cap \text{F})$ = (.5)(.2) = .1, P(FP) = P(FP|M)P(M) + P(FP|F)P(F) = (.5)(.7) + (.4)(.3) = .47, $P(\text{NS} \cap \text{FP})$ = P(FP|NS)P(NS) = (.65)(.5) = .325

a. $P(\text{FP}\cap\text{F}) = P(\text{FP|F})P(\text{F})$ = (.4)(.3) = .12
b. $P(\text{NS}\cup\text{F}) = P(\text{NS}) + P(\text{F}) - P(\text{NS}\cap\text{F})$ = .5 + .3 - .1 = .7
c. No, since $P(\text{NS}\cap\text{F})$ which is .1 $\neq$ .15 which is P(NS)P(F)
d. $P(\text{NS|F}) = P(\text{NS}\cap\text{F})/P(\text{F})$ = .1/.3 = .3333
e. $P(\overline{NS} \cap \overline{FP}) = 1 - P(\text{NS}\cup\text{FP}) = 1 - P(\text{NS}) - P(\text{FP}) + P(\text{NS}\cap\text{FP})$ =
1-.5 -.47 + .325 = .355

3.103 a. $C_{12}^{16} = 16!/4!12! = 1{,}820$

b. P(number of men $\geq$ 7) = P((selection contains 8 men and 4 women)or(selection contains 7 men and 5 women))

$$= \frac{(C_8^8 \times C_4^8) + (C_7^8 \times C_5^8)}{C_{12}^{16}} = \frac{(1)(70) + (8)(56)}{1820} = 0.2846$$

3.104 a. $C_2^{12} = 12!/2!10! = 66$

b. P(faulty) = C_1^{11}/66 = 11/66 = .1667

3.105 Let A_1: stocks that are up after being held for 2 years
A_2: stocks that are not up after being held for 2 years
B: receiving first-year bonus
P(A_1) = .4, P(A_2) = .6, P(B|A_1) = .6, P(B|A_2) = .4

Probability that a stock will be up after two years given that Mr. Roberts received a first-year bonus is P(A1|B)

$$P(A_1 | B) = \frac{P(B | A_1)P(A_1)}{P(B | A_1)P(A_1) + P(B | A_2)P(A_2)} = \frac{.6(.4)}{.6(.4) + .4(.6)} = .5$$

3.106 Let T – treatment , C – patient was cured. P(T) = 10/100 = .1, P(C) = .5, P(C|T) = .75

a. $P(\text{C} \cap \text{T}) = P(\text{C|T})P(\text{T})$ = (.75)(.1) = .075
b. $P(\text{T|C}) = P(\text{C}\cap\text{T})/P(\text{C})$ = .075/.5 = 0.15
c. The probability will be 1 / [C_{10}^{100} = 1/ 100!/10!90! = 10!90!/100!

3.107 a. $P(R_J) = P(R_J|G_J)P(G_J) + P(R_J|PR_J)P(PR_J) + P(R_J|DM_J)P(DM_J) + P(R_J|SS_J)P(SS_J)$ = (.81)(.08) + (.79)(.41) + (.6)(.06) + (.21)(.45) = .5192

b. $P(R_F) = P(R_F|G_F)P(G_F) + P(R_F|PR_F)P(PR_F) + P(R_F|DM_F)P(DM_F) + P(R_F|SS_F)P(SS_F)$ = (.8)(.1) + (.76)(.57) + (.51)(.24) + (.14)(.09) = .6482

c. The probability of renewals in February have increased; however, the conditional probability of renewal fell in each of the categories

Let T – passenger is identified by TPS, I – the passenger is carrying an illegal amount of liquor. $P(\text{I|T}) = P(\text{T}\cap\text{I})/P(\text{T}) = P(\text{T|I})P(\text{I})/[P(\text{T|I})P(\text{I}) + P(\text{T}|\overline{I})P(\overline{I})]$ = (.8)(.2)/[(.8)(.2) + (.2)(.8)] = .

5. There is a 50% chance that the passenger carrying an illegal amount of liquor is identified by TPS.

3.109 Let D – inhabitants have contracted a particular disease, P – test is positive. $P(\text{D}|\text{P}) = P(\text{P}\cap\text{D})/P(\text{P}) = P(\text{P}|\text{D})P(\text{D})/[P(\text{P}|\text{D})P(\text{D}) + P(\text{P}|\bar{D})P(\bar{D})] = (.8)(.08)/[(.8)(.08) + (.2)(.92)] = .2581$

3.110 Let P – people who already own policies, S – sales made by salesman. $P(\text{S}|\text{P}) = P(\text{S}\cap\text{P})/P(\text{P}) = P(\text{P}|\text{S})P(\text{S})/[P(\text{P}|\text{S})P(\text{S}) + P(\text{P}|\bar{S})P(\bar{S})] = (.7)(.4)/[(.7)(.4) + (.5)(.6)] = .4828$

3.111 Let Af – students obtain a final grade of A, Am – students obtained an A on the midterm examination. $P(\text{Af}|\text{Am}) = P(\text{Af}\cap\text{Am})/P(\text{Am}) = P(\text{Am}|\text{Af})P(\text{Af})/[P(\text{Am}|\text{Af})P(\text{Af}) + P(\text{Am}|\bar{A}\text{f})P(\bar{A}\text{f}) = (.7)(.2)/[(.7)(.2) + (.1)(.8)] = .6364$

3.112 a. $P(\text{W}|\text{FW}) = P(\text{W}\cap\text{FW}) / P(\text{FW}) = .149/.29 = .5138$
b. $P(\bar{I}|\text{FI}) = P(\bar{I}\cap\text{FI})/P(\text{FI}) = .181/.391 = .4629$

3.113 Let G – eventually graduates, F – entering freshmen, JC – enters as community college transfer.
a. $P(\text{G}\cap\text{F}) = P(\text{G}|\text{F})P(\text{F}) = (.62)(.73) = .4526$
b. $P(\text{G}) = P(\text{G}|\text{F})P(\text{F}) + P(\text{G}|\text{JC})P(\text{JC}) = .4526 + (.78)(.27) = .6632$
c. $P(\text{G} \cup \text{F}) = P(\text{G}) + P(\text{F}) – P(\text{G} \cap\text{F}) = .6632 + .73 - .4526 = .9406$
d. No, since $P(\text{G}\cap\text{JC})$ which is $(.78)(.27) = .2106 \neq .1791$ which is $P(\text{G})P(\text{JC}) = (.6632)(.27)$

3.114 a. $P(\text{G}) = P(\text{G}|\text{S})P(\text{S}) + P(\text{G}|\bar{S})P(\bar{S}) = (.7)(.6) + (.2)(.4) = .5$
b. $P(\text{S}|\text{G}) = P(\text{G}\cap\text{S})/P(\text{G}) = .42/.5 = .84$
c. No, since $P(\text{G}\cap\text{S})$ which is $.42 \neq .3$ which is $P(\text{G})P(\text{S})$
d. $P(\text{S} \geq 1) = 1 – P(\text{S} = 0) = 1 - .4^5 = .9898$

3.115 Let W – customer orders Wine, R – regular customer, O – occasional customer, N– new customer
a. $P(\text{W}) = P(\text{W}|\text{R})P(\text{R}) + P(\text{W}|\text{O})P(\text{O}) + P(\text{W}|\text{N})P(\text{N}) = (.7)(.5) + (.5)(.4) + (.3)(.1) = .58$
b. $P(\text{R}|\text{W}) = P(\text{R}\cap\text{W})/P(\text{W}) = .35/.58 = .6034$
c. $P(\text{O}|\text{W}) = P(\text{O}\cap\text{W})/P(\text{W}) = .2/.58 = .3448$

3.116 a. $P(\text{P}) = P(\text{P}|\text{HS})P(\text{HS}) + P(\text{P}|\text{C})P(\text{C}) + P(\text{P}|\text{O})P(\text{O}) = (.2)(.3)+(.6)(.5) + (.8)(.2) = .52$
b. $P(\text{HS}|\text{P}) = P(\text{HS}\cap\text{P})/P(\text{P}) = .06/.52 = .1154$

3.117 The sample space is the number of combinations from $C_5^{16} = 16!/5!11! = 4{,}368$. To obtain all five Fs for male students: $[C_5^8 = 8!/5!3!][C_0^8 = 8!/0!8!] = [56][1] = 56$. Probability $= 56/4368 = .0128$.

3.118
a. 0.16+0.1 = 0.26
b. 1- (0.16+0.1+0.21) = 0.53
c. 0.2
d. 1-0.33=0.67

3.119 Let F – failure (from any source), D – disk error. $P(F) = (.5)(.6)+(.3)(.7)+(.2)(.3)=.57$

$$P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{(.5)(.6)}{(.5)(.6)+(.3)(.7)+(.2)(.3)} = \frac{.3}{.57} = .5263$$

3.120 Let G_1 – Sales will grow, G_2 – Sales will not grow. Let N_1 – new operating system, N_2 – no new operating system. $P(G_1) = 0.70$, $P(G_2) = 0.30$ and $P(N_1|G_1) = .30$, $P(N_1|G_2) = .10$, $P(N_2|G_1) = .70$, $P(N_2|G_2) = .90$

$$P(G_1 \mid N_1) = \frac{P(N_1 \mid G_1)P(G_1)}{P(N_1 \mid G_1)P(G_1) + P(N_1 \mid G_2)P(G_2)} = \frac{(.30)(.70)}{(.30)(.70)+(.10)(.30)} = 0.875$$

3.121 Let D – defect lumber pile, C – clear lumber pile, NH – Northern Hardwoods, MT – Mountain Top, SV – Spring Valley
$P(D) = .2$, $P(C) = .8$, $P(NH|D) = .3$, $P(MT|D) = .5$, $P(SV|D) = .2$
$P(NH|C) = .4$, $P(SV|C) = .4$, $P(MT|C) = .2$

Percent of clear lumber from Northern Hardwoods,

$$P(C \mid NH) = \frac{P(NH \mid C)P(C)}{P(NH \mid C)P(C) + P(NH \mid D)P(D)} = \frac{(.4)(.8)}{(.4)(.8)+(.3)(.2)} = .8421$$

Percent of clear lumber from Mountain Top,

$$P(C \mid MT) = \frac{P(MT \mid C)P(C)}{P(MT \mid C)P(C) + P(MT \mid D)P(D)} = \frac{(.2)(.8)}{(.2)(.8)+(.5)(.2)} = .6154$$

Percent of clear lumber from Spring Valley

$$P(C \mid SV) = \frac{P(SV \mid C)P(C)}{P(SV \mid C)P(C) + P(SV \mid D)P(D)} = \frac{(.4)(.8)}{(.4)(.8)+(.2)(.2)} = .8889$$

Percent of lumber from each of the three suppliers,

$P(NH) = P(D \cap NH) + P(C \cap NH) = (.3)(.2) + (.4)(.8) = .3800$

$P(MT) = P(D \cap MT) + P(C \cap MT) = (.5)(.2) + (.2)(.8) = .2600$

$P(SV) = P(D \cap SV) + P(C \cap SV) = (.2)(.2) + (.4)(.8) = .3600$

3.122 Let $\boldsymbol{P}_1$ – regular plowing, $\boldsymbol{P}_2$ – minimal plowing, $\boldsymbol{H}$ – high yield, $\boldsymbol{L}$ – low yield. $P(\boldsymbol{P}_1) = .4$, $P(\boldsymbol{P}_2) = .6$, $P(\boldsymbol{H}|\boldsymbol{P}_2) = .5$, $P(\boldsymbol{L}|\boldsymbol{P}_1) = .4$

a. $P(H|P_1) = 1 - P(L|P_1) = 1 - .4 = .6$

b. $$P(P_1 \mid H) = \frac{P(H \mid P_1)P(P_1)}{P(H \mid P_1)P(P_1) + P(H \mid P_2)P(P_2)}$$
$$= \frac{(.6)(.4)}{(.6)(.4) + (.5)(.6)} = .4444$$