Time Value OF Money- Application

TCH302

Bodie & Merton (2000), *Finance,* Chapter 4

Applications

1. Alternative discounted-cash-flow decision rules

    1.1. Net present value (NPV)

    1.2. Internal rate of return (IRR)

    1.3 Payback period/ Discount payback period

2. Amortization schedule

3. Valuation

1. Alternative discounted-cash-flow decision rules

1.1. Net present value (NPV)

Net present value of an investment is the present value of its expected cash inflows minus the present value of its expected cash outflows.

Formula:

$$\boxed{NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}}$$

Where:

CF_t : the expected net cash flow/ net benefits at time t
i: the discount rate/ required interest rate/ opportunity cost of capital
n: the investment's projected life

The NPV rule:

No alternative projects are under consideration (for example in cases where we need to make "accept/go" or "reject/do not proceed" decisions.

a project is potentially worthwhile doing if its NPV is POSITIVE

Mutually exclusive projects are under consideration

Exclusive projects offer alternative solutions to a single problem

The project that has the HIGHEST & POSITIVE value of NPV should be selected.

Practice

CS12: An investment project with an initial cost of \$500,000 and positive cash flows of \$160,000 at the end of year 1; \$240,000 at the end of year 2; \$280,000 at the end of year 3. Should you undertake it if the opportunity cost is 12%?

0	1	2	3
- 500,000	160,000	240,000	280,000

The NPV of this project is the sum of the PVs of the project's individual cash flows and is determined as follows:

$$NPV = -500{,}000 + \frac{160{,}000}{1+12\%} + \frac{240{,}000}{(1+12\%)^2} + \frac{280{,}000}{(1+12\%)^3}$$

$$= \$33.48 \qquad > 0$$

$\Rightarrow$Should undertake it

NPV in more complicated decision-making scenarios

- There is a **budget constraint** in a given year that precludes the financing of all the projects that yield a positive NPV.
- Projects with **different lives**.
- Projects are complementary.

Accommodating budget constraints

Where budget constraints limit the number of projects which can be undertaken, the appropriate decision rule is to choose that **subset of available projects** which maximises total net present value.

CS13: Suppose that a department has a capital outlay budget of $8 million, which project/s would you undertake?

Project A costs $2 million and has a NPV of $140,000
Project B costs $6 million and has a NPV of $500,000
Project C costs $4 million and has a NPV of $320,000
Project D costs $4 million and has a NPV of $450,000

=> With the combined net present value of 770,000 (the highest NPV),
project C&D should be undertaken

Projects with Different Lifetimes

CS14: The road services could be provided by a series of projects with short lives such as gravel surface, or by ones with longer lives such as paved surface. If considered 15 years life:

Alternative Investment Projects:	Duration of Road
A: Gravel surfaced road	3 years
B: Gravel-Tar surfaced road	5 years
C: Asphalt surface road	15 years

⇒Bringing different projects to the same time length and then comparing NPV rules

Projects with Different Lifetimes

CS15: if one project is seven years in length and the other eleven years

$\Rightarrow$Bringing different projects to the same time length:

$\Rightarrow$we need to extend to 7x11 = 77 years length

$\Rightarrow$not practical. A lengthy time period necessarily needs to assume that technology and other conditions remain the same; otherwise, a comparison over the extended time period becomes unrealistic.

$\Rightarrow$The alternative approach is to use the Equivalent Annual Annuity Approach.

Equivalent Annual Annuity Approach

Equivalent Annual Annuity defined as

$$\text{EAA} = \frac{\text{NPV}}{\text{Annuity Factor}} = \frac{\text{NPV} * i}{1-(1+\text{i})^{-n}}$$

CS16: Considering two projects by using EAA approach with the 6% discount rate.

Project A: seven-year term and a NPV of $100,000.

Project B: nine-year term and a NPV of $120,000.

NPV: summary

NPV is commonly used and there some rules of thumbs:

- Rule 1: Only project with a **positive** NPV should be considered
- Rule 2: When there is **no budget constraint** but only one project should be considered (from a set of mutually exclusive alternatives), one should choose the alternative that generates the **largest NPV**.
- Rule 3: Within a **fixed budget constraint**, choose the subset of the available projects that **together maximises the total NPV.**
- **In a more complicated situation, consider other criteria.**

1. Alternative discounted-cash-flow decision rules

1.2. Internal rate of return (IRR)

The internal rate of return (IRR) is defined as the rate of return that equates the PV of an investment's expected benefits with the PV of its costs; or the discount rate for which the NPV of an investment is zero

Formula:

$$\boxed{0 = \sum_{t=0}^{n} \frac{CF_t}{(1 + IRR)^t}}$$

Where:

CF_t : the expected net cash flow at time t
n: the investment's projected life

The IRR rule:

Accept projects with an IRR that is **greater** than the firm's (investor's) required rate of return

Reject projects with an IRR that is **less** than the firm's (investor's) required rate of return

Practice

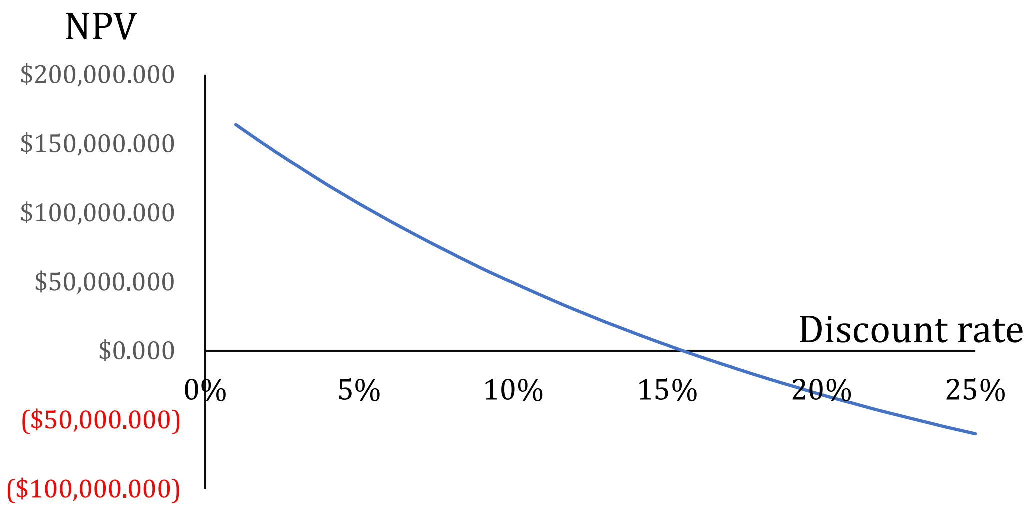
CS13: An investment project with an initial cost of \$500,000 and positive cash flows of \$160,000 at the end of year 1; \$240,000 at the end of year 2; \$280,000 at the end of year 3. What is its IRR?

0	1	2	3
- 500,000	160,000	240,000	280,000

IRR is the results of following equation:

$$0 = -500000 + \frac{160000}{1 + IRR} + \frac{240000}{(1 + IRR)^2} + \frac{280000}{(1 + IRR)^3} \quad \Rightarrow IRR \approx 0.1552 = 15.52\%$$

Practice



- Discount rate = 10%

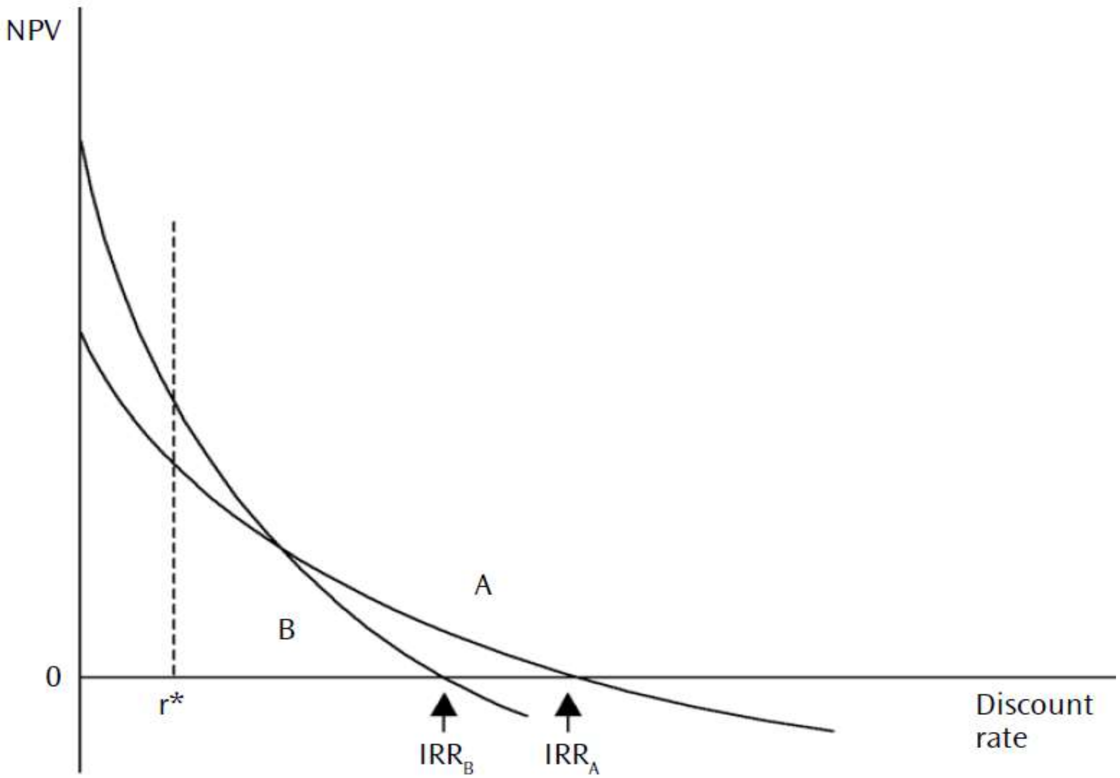
  NPV=$49,245.27
- Discount rate= 20%:

  NPV= - $31,635.8

IRR could be misleading

- The IRR should only be used to complement the NPV where it is useful.
- The IRR can be misleading when it is used for ranking projects. Especially when
  - Choosing between mutually exclusive options
  - Projects differ in scale and length of life or
  - Projects with early returns or negative net benefits

IRR could be misleading



Consider two projects A and B

- Plotting NPVs against discount rates.
- IRR (A) is higher IRR (B)
- But if we use discount rate of r*,

NPV (B) > NPV (A)

IRR could be misleading

Year	0	1	2	3	4		n
Project A	-1000	+ 300	+ 300	+ 300	+ 300		+ 300
Project B	-5000	+ 1000	+ 1000	+ 1000	+ 1000		+ 1000

Periods 5 onwards in perpetuity

PV=CF/i

- Two projects are mutually exclusive:
  - IRRs are found to be 30% for Project A and 20% for Project B. So using the IRR, A is recommended.
  - At a 10% discount rate, the NPV of project A = $2000, vs. the NPV of project B = $5000
  - Why?

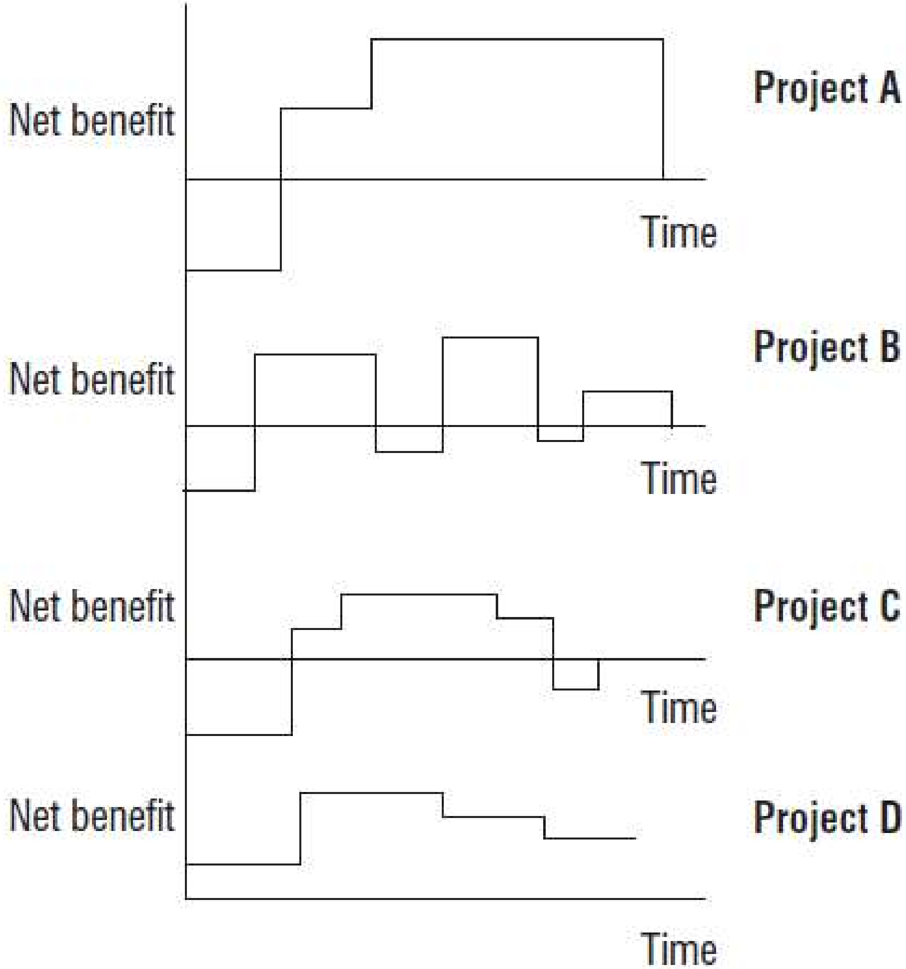
IRRofA=300/1000=30%
IRRofB=1000/5000=20%

IRR could be misleading

- IRRs cannot be **aggregated**, they cannot be used to choose an optimal package of projects.

- Also if a project has negative net benefits more than once during the project period, it will usually not be possible to determine a unique rate of return (*multiple IRR problem*)

IRR could be misleading



- Typically, Project A has one IRR solution
- Projects B and C has multiple IRR solutions.
- Project D does not yield any IRR solution.

Payback period

Organisations sometimes require that the initial outlay on a project be **recoverable** within some specified cut-off period.

- The ‘payback period’ of a project is found by counting the number of years it takes before cumulative forecasted cashflows equal the initial investment.
- The pay-out or pay-back period measures the number of years it will take for the net **cash flows** to repay the capital investment.
- In its simplest form the pay-out period measures the number of years it will take for the undiscounted net cash flows to repay the investment.
- A more sophisticated version compares the discounted benefits over a number of years with the discounted investment costs.
- ***Project with the shortest pay-back period is preferred.***

Cashflows ($)							
Year	0	1	2	3	4	NPV (8%)	Payback Period (years)
Projects							
A	-3000	+1000	+1000	+1000	+4000	2331	3
B	-3000			+3000	+4000	2150	3
C	-3000			+1000	+10,000	4763	4

Payback period versus NPV

Cashflows ($)							
Year	0	1	2	3	4	NPV (8%)	Payback Period (years)
Projects							
A	-3000	+1000	+1000	+1000	+4000	2331	3
B	-3000			+3000	+4000	2150	3
C	-3000			+1000	+10,000	4763	4

- The “payback period” rule suggests that A and B are better projects as they repay faster than Project C.
- However, the NPV rule suggests that C is the best option as it has the largest NPV

6.2. Amortization schedule

Loan amortization is the process of paying off a loan with a series of periodic loan payments whereby a portion of the outstanding loan amount is paid off, or amortized, with each payment.

Periodic payment

- Interest on the outstanding balance of loan
- The rest: repayment of principal

Ex: Home mortgage loans, repaid in equal periodic installments

Practice

CS14: Assume that Mr. G takes a \$100000 home mortgage loan at an interest rate of 9%/ year to be repaid with interest in three annual installments. How much would he pay each year? Show the exact amount of the principal and interest components of each loan payment.

Periodic payment/installment: CF

0	1	2	3
100,000	-CF	-CF	-CF

Present value of installments should be equal to the loan principle.

$$\Rightarrow 100{,}000 = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} = \sum_{t=1}^{3} \frac{CF}{(1+i)^t} \quad \text{or} \quad 100{,}000 = CF\left[\frac{1-(1+i)^{-n}}{i}\right]$$

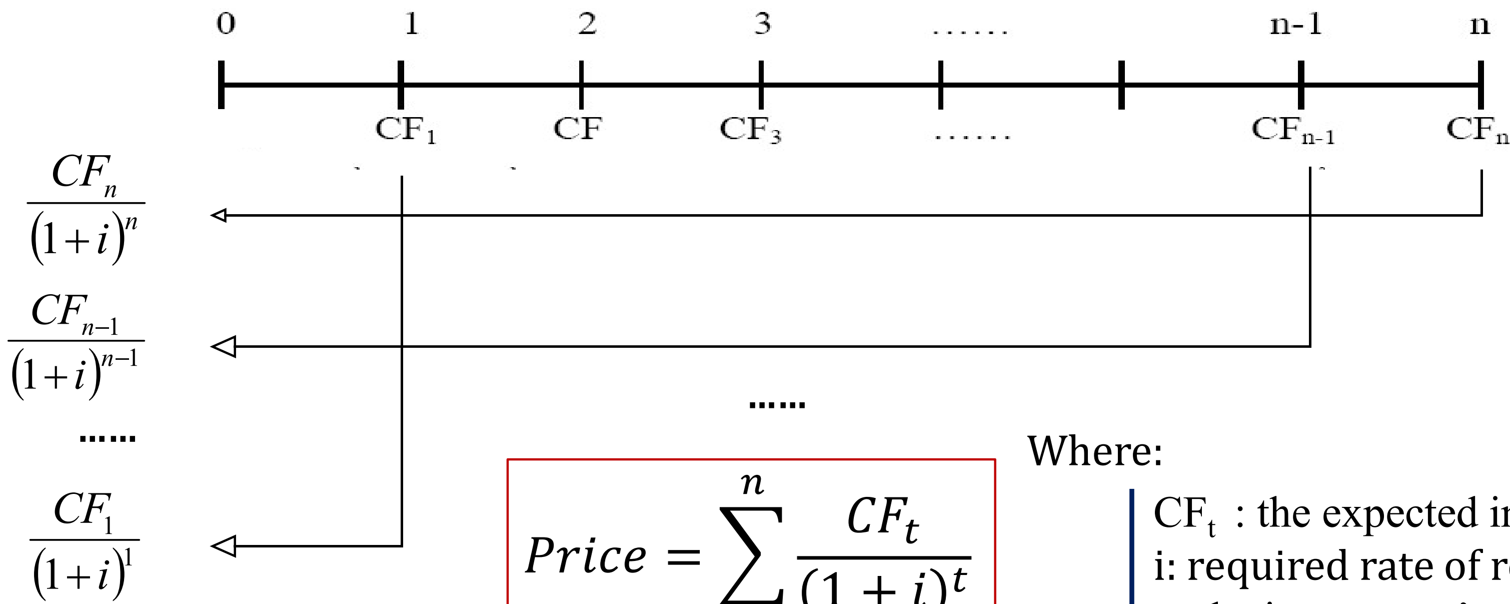
$$\Rightarrow CF = \frac{100{,}000 \times i}{1-(1+i)^{-n}} = \frac{100000 \times 9\%}{1-(1+9\%)^{-3}} = \$39505.48$$

2. Amortization schedule

Year	Beginning balance	Total payment	Interest paid	Principal paid	Remaining balance
1	100000	39505.48	9000	30505.48	69494.52
2	69494.52	39505.48	6254.51	33250.97	36243.55
3	36243.55	39505.48	3261.92	36243.56	≈ 0

3. Valuation

The price of an asset/ security is equal to the present value of its expected income stream



$$Price = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t}$$

Where:

CF_t : the expected income at time t
i: required rate of return
n: the investment's projected life