Valuation of financial assets

TCH302

Bodie (2000), Chapter 7, 8, 9

Financial Mathematics

- A quick review from previous session

$$PV_0 = \sum_{t=0}^{n} \boldsymbol{CF_t}\,(1+i)^{-t}$$

- Some small modifications to the notation for this lecture:
  1. Calculating a price today, P_0
  2. Future cash flows noted as CF_t
  3. Start from t=1, because if you purchase a stock or bond today you are generally not entitled to CF_0.
  4. The discount rate will be noted as r, k_e or k_d.

$$\boldsymbol{P_0} = \sum_{\boldsymbol{t=1}}^{\boldsymbol{n}} \boldsymbol{CF_t}\,(\boldsymbol{1}+\boldsymbol{r})^{-\boldsymbol{t}}$$

Valuation fundamentals

- **Valuation** is the process that links risk and return to estimate the worth of an asset or a firm.

  Book value is the accounting value of a firm or an asset. Be an historical value rather than a current value. Reported in the financial statement.
  Market value is the price that the owner can receive from selling an asset in the market place. The key determinant of market value is supply and demand for the asset.
  Intrinsic value, also called **fundamental value**, is a measure of the theoretical value of an asset. Because determining the intrinsic value ***requires estimates***, we may never know the "actual" or "true" intrinsic value of some financial assets. Nonetheless, intrinsic value serves as a basis for determining whether to buy or sell a financial asset when compared to its market value or price.

  - **In efficient capital market, the fundamental value = its market value.**

Discounted Cash Flow Valuation

Valuation is the process that links risk and return to estimate the worth of an asset or a firm.

The valuation of any financial asset is a function of the:

- amount of the expected cash flows (returns) generated by the asset over its life;
- timing of the cash flows; and
- riskiness associated with these cash flows as measured by the required rate of return.

3 step:

1. Estimate the future cash flows expected over the life of the asset.
2. Determine the appropriate required rate of return on the asset.
3. Calculate the present value of the estimated cash flows using the required rate of return as the discount rate.

•Discounted Cash Flow Valuation:

Estimating the required rate of return

The required rate of return consists of three major components:

- **Real risk-free rate of interest** (RRFR) is a default free rate in which investors know the expected returns with certainty.
- **Expected inflation rate premium** (INF) is an adjustment to the real risk-free rate to compensate investors for expected inflation. The **nominal risk-free rate** (NRFR) is the combination of the real risk-free rate and the expected inflation rate premium.
- **Risk premium** (RP)

Discounted Cash Flow Valuation

Standard financial securities entitle the owner to a stream of payments in the future. The value/price of such a security is the present value of its future payments (cash flows).

- Debt: Interest/Coupon (C) + Repayment of Face Value (FV)

$$P_0 = \sum_{t=1}^{n} C_t\,(1+r)^{-t} + FV_n(1+r)^{-n} = \sum_{\boldsymbol{t=1}}^{\boldsymbol{n}} \boldsymbol{CF_t}\,(\mathbf{1}+\boldsymbol{r})^{-\boldsymbol{t}}$$

- Equity: Dividends

$$P_0 = \sum_{t=1}^{\infty} D_t\,(1+r)^{-t} = \sum_{\boldsymbol{t=1}}^{\boldsymbol{\infty}} \boldsymbol{CF_t}\,(\mathbf{1}+\boldsymbol{r})^{-\boldsymbol{t}}$$

Capital market instruments – Bond

Bonds are long-term debt securities:

- issued by corporations and government agencies to support their operations
- which the issuers promise to repaid to bondholders in some date in the future

Maturity: The period from the date of issuing to expired date

Par value/ face value: the amount that the issuer must pay to maturity

By issuer: Corporate bonds & Treasury bonds

By interest payment: Coupon bonds & Discount bonds

By coupon rate: Plain – vanilla bonds & floating rate bonds

Capital market instruments – Coupon bond

Coupon bond: bond that the issuer promises:

- to make periodic payments of interest – called ***coupon payments*** * – to the bondholder for the life of the bond,
- to pay the face value of the bond when the bond matures

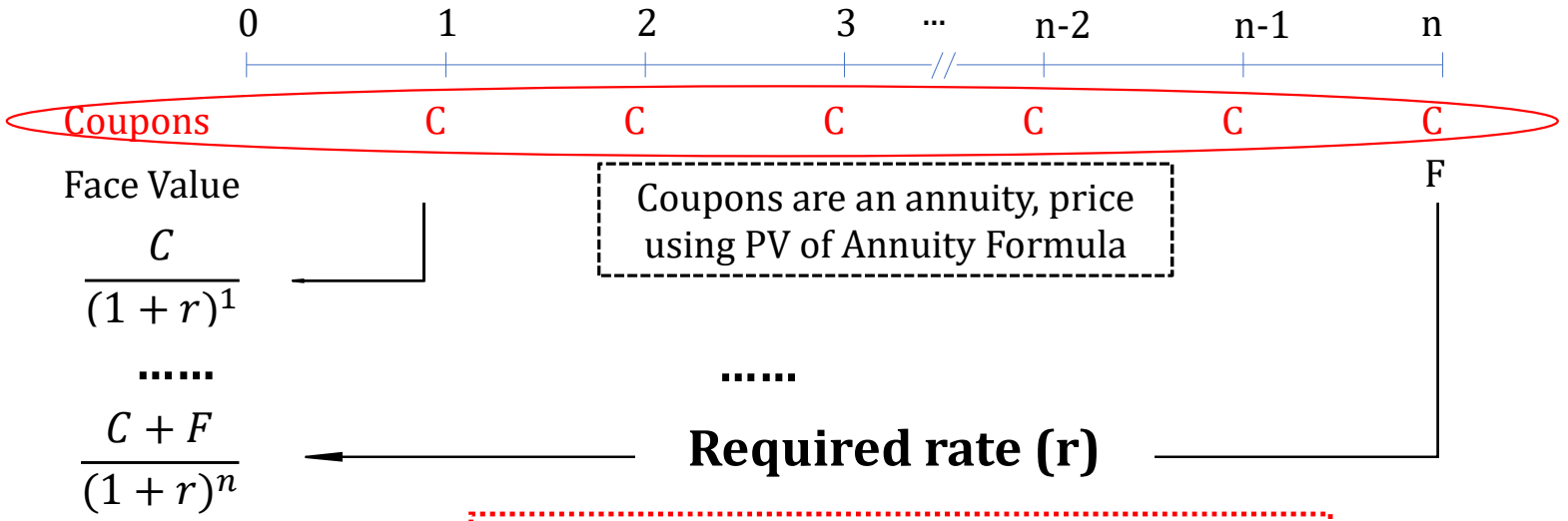
** Coupon payments: at **coupon rate –** the interest rate applied to the face value*

E.g. A bond with a face value of $100 that makes annual coupon payment at a coupon rate of 10% and its maturity is 10 years.

⇒ The issuer has to pay (every year)

⇒ At the end of 10 years, the issuer pays

Coupon Bond

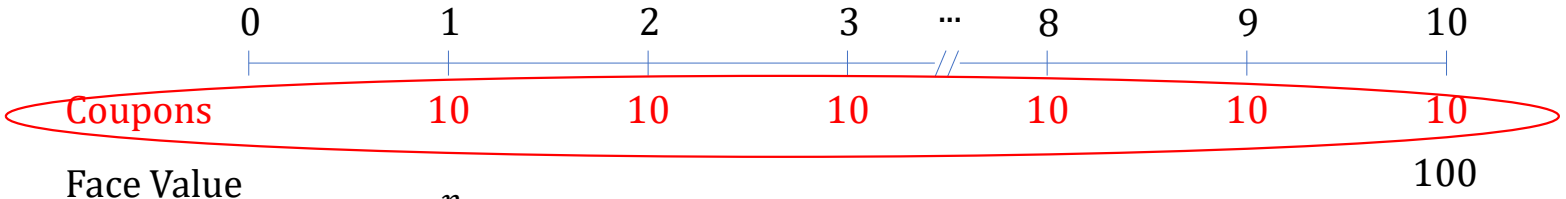


$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^n} + \frac{F}{(1+r)^n}$$

$$P = C\left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{F}{(1+r)^n}$$

•Coupon Bond

Example 1:
$100 bond, 10% annual coupon and 10 years to maturity. Value of bond with k_d = 8%



$$P_0 = \sum_{t=1}^{n} CF_t\,(1+k_d)^{-t} = \boldsymbol{C\left[1-\frac{1}{(1+k_d)^n}\right]/k_d + FV_n(1+k_d)^{-n}}$$

$$= 10(1+0.08)^{-1} + 10(1+0.08)^{-1} + \cdots + 110(1+0.08)^{-10}$$

$$= 10\left[1-\frac{1}{(1+0.08)^{10}}\right]/0.08 + 100(1+0.08)^{-10}$$

$$= 113.420$$

Coupon Bond

Example 2:

Corporation A issued an annual-pay bond with a 10 percent coupon rate and a $1,000 par value 25 years ago. The bond now has 20 years remaining until maturity. Due to changing interest rates and market conditions, the required rate of return on this bond is 8 percent. That is, the discount rate on the bond is 8 percent. What is the intrinsic value of the annual-pay bond?

Capital market instruments – Discount bond (zero-coupon bond)

(Pure) discount bonds (zero – coupon bonds): bonds that promise a single payment of cash at some date in the future, called the maturity date.

$\Rightarrow$ The interest earned by investors:

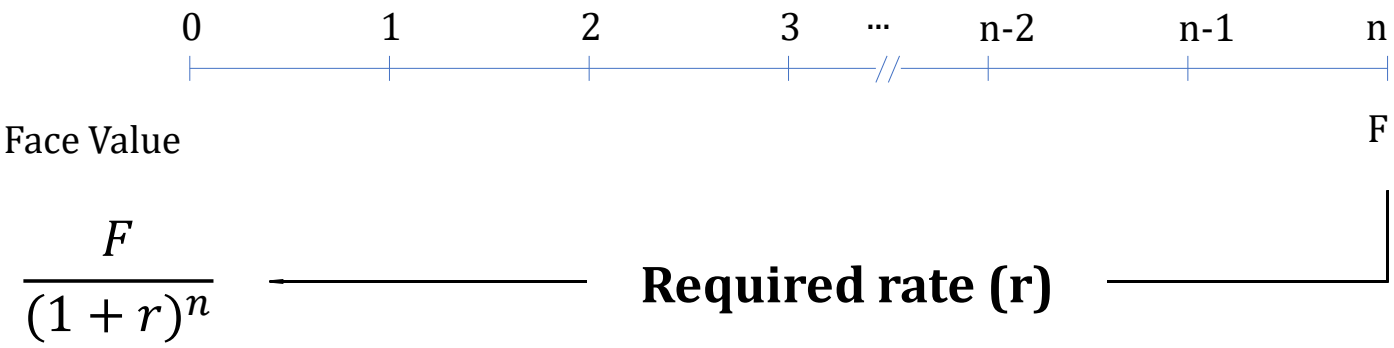
the face value – the price

E.g. A pure discount bond with a face value of \$1000 maturing in one year and a purchase price of \$950

$\Rightarrow$ During this 1 year, it does not make any payment

$\Rightarrow$ At the end of 1 year, the holder receives the amount of \$1000 (face value)

Zero-coupon Bond



$$P = \frac{F}{(1+r)^n}$$

r or k_d represents the market required rate of return given the current economic cost of funds, default risk, time-to-maturity and coupon of a particular bond.

Zero-coupon Bond

Example 3:

Suppose Corporation A has a 20-year zero coupon bond with a maturity value of $1,000. If investors require an 8 percent rate of return, what is the value of this zero-coupon bond?

Bond pricing relationships

Example 4:

Par value	Coupon rate	Maturity	Required rate/ Yield		
			8%	10%	12%
1,000	10%	10			

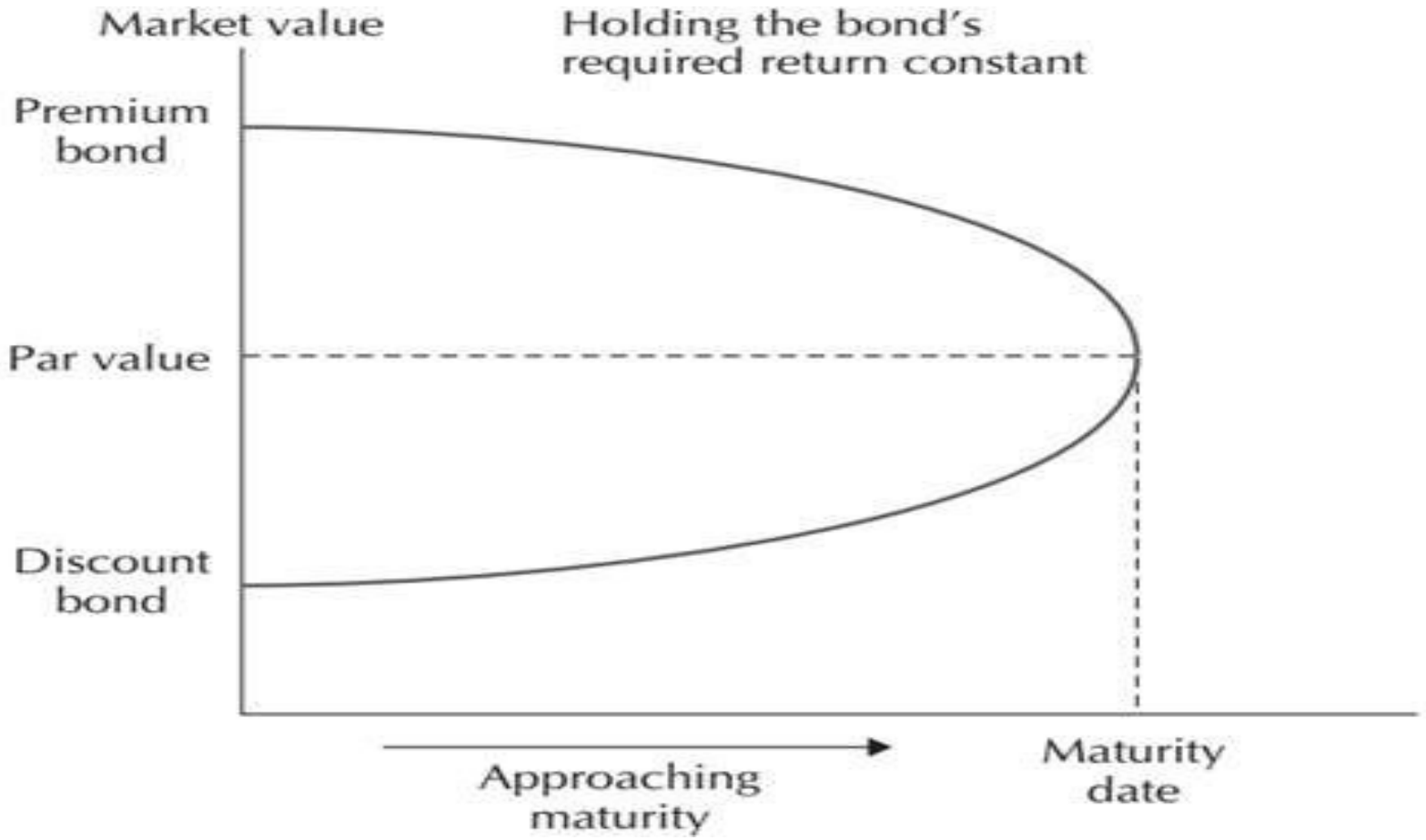
- Something to note about bond prices.
  - if $P_0 < FV$, bond is selling at a discount,
  - if $P_0 = FV$, bond is selling at a par,
  - if $P_0 > FV$, bond is selling at a premium,

- Bond pricing relationships

Example 5:

Par value	Coupon rate	Maturity	Required rate		
			8%	10%	12%
1,000	10%	10			
1,000	10%	8			
1,000	10%	6			
1,000	10%	4			

- Bond pricing relationships



Bond yields

- **Yield to maturity** (YTM), also known as **promised yield**, is the rate of return that investors expect to earn if they buy a bond at its market price and hold it until maturity.

Yield to maturity involves three embedded assumptions:

1. Investors will hold the bond to maturity.
2. Investors will receive all coupon payments in a prompt and timely fashion.
3. Investors will reinvest all coupons to maturity at a rate of return that equals the bond's YTM.

Rate of return

Bond yields

Example 6:
Suppose a 15-year, $1,000 par value, 10 percent annual-pay bond is currently trading at a price of $1,100. What is the bond's yield to maturity? If an investor requires a 5.5 percent return, would this bond be attractive?

Example 7:
A 5- year, 10% coupon bond with a face value of 1000 USD, payment annually is selling for 965 USD. Calculate the YTM ?

Example 8:
A discount bond with a face value of 1000 USD is currently selling for 850 USD, maturing in 2 years. Calculate the yield to maturity?

Example 9:

A 10-year, 9%-coupon bond with a face value of 10 million VND is currently bought for 9.5 million VND and you intend to sell one year later for the price P'.

a. Write down a formula used to calculate yield to maturity.
b. Use a calculator or Excel to calculate the yield?
c. P'= 9 million VND, YTM and the rate of return?
d. P'= 8.5 million VND, YTM and the rate of return?

Interest rate risk

- **Interest rate risk**, which is the sensitivity of bond prices to changes in interest rates

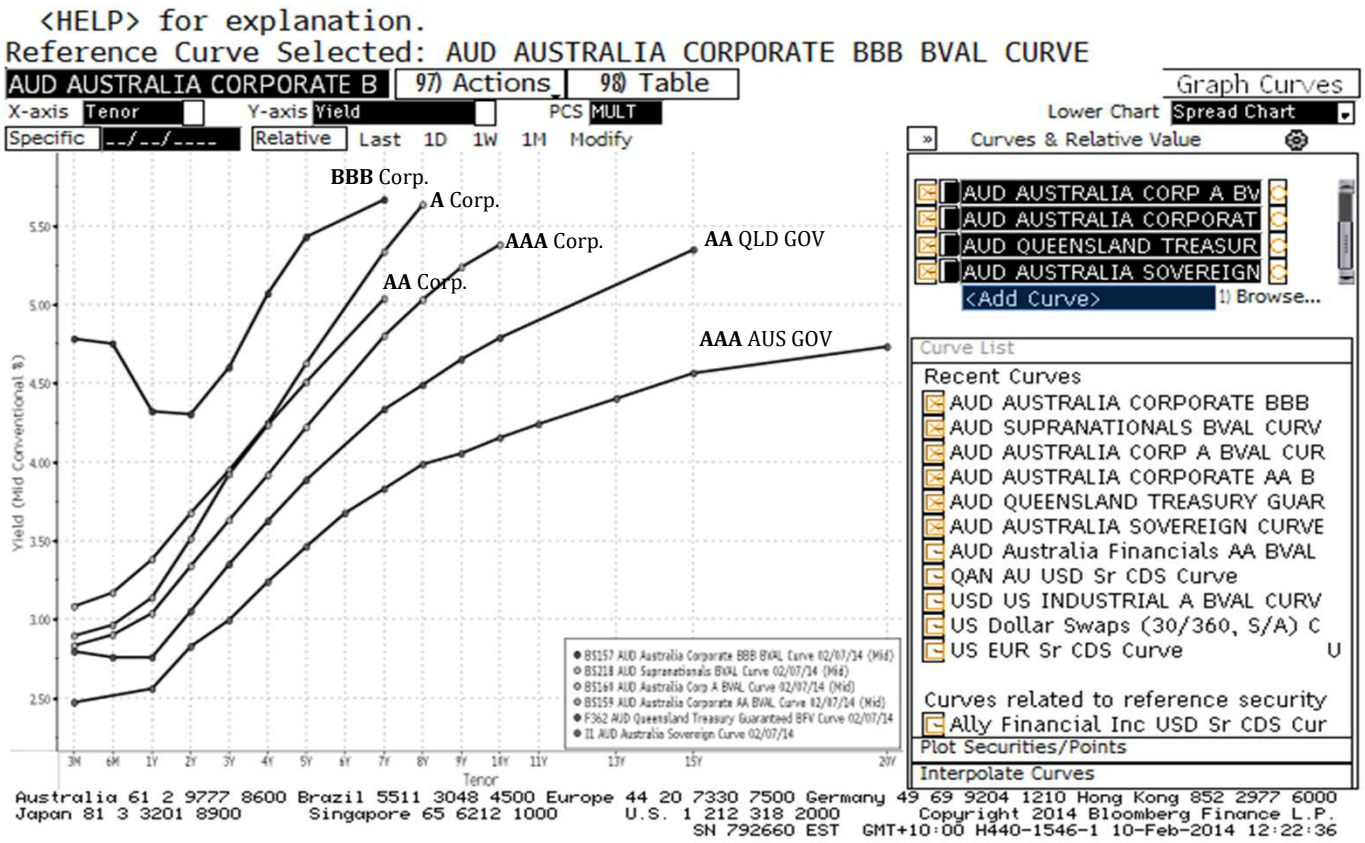
Par value	Coupon rate	Maturity	Required rate		
			8%	10%	12%
1,000	10%	10			
1,000	10%	4			

Par value	Coupon rate	Maturity	Required rate		
			8%	10%	12%
1,000	10%	10			
1,000	5%	10			

Coupon bond has: F= 1,000

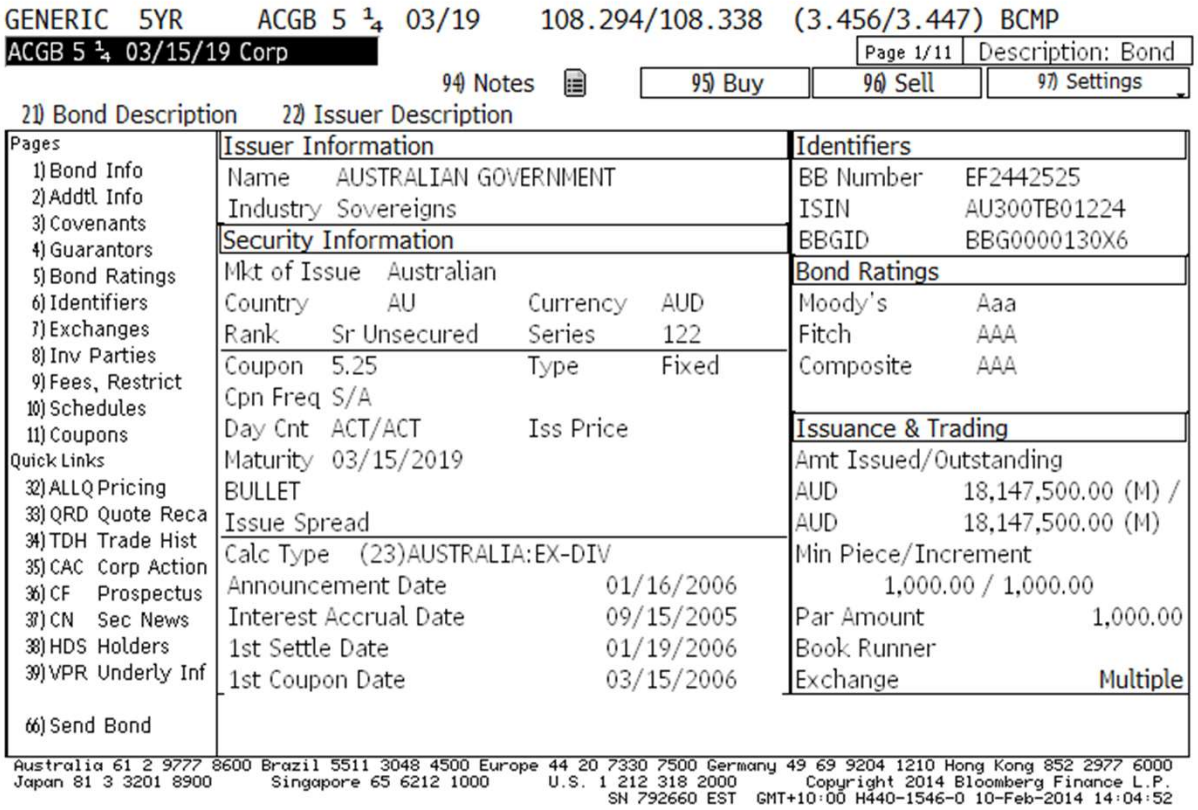
n	YTM_0	P_0	YTM_1	P_1	R
20	10%	1,000	15%		
15	10%	1,000	15%		
5	10%	1,000	15%		
1	10%	1,000	15%		
15	10%	1,000	8%		
5	10%	1,000	8%		
1	10%	1,000	8%		

•Coupon Bond



(Sean Wu , 2022)

•Coupon Bond



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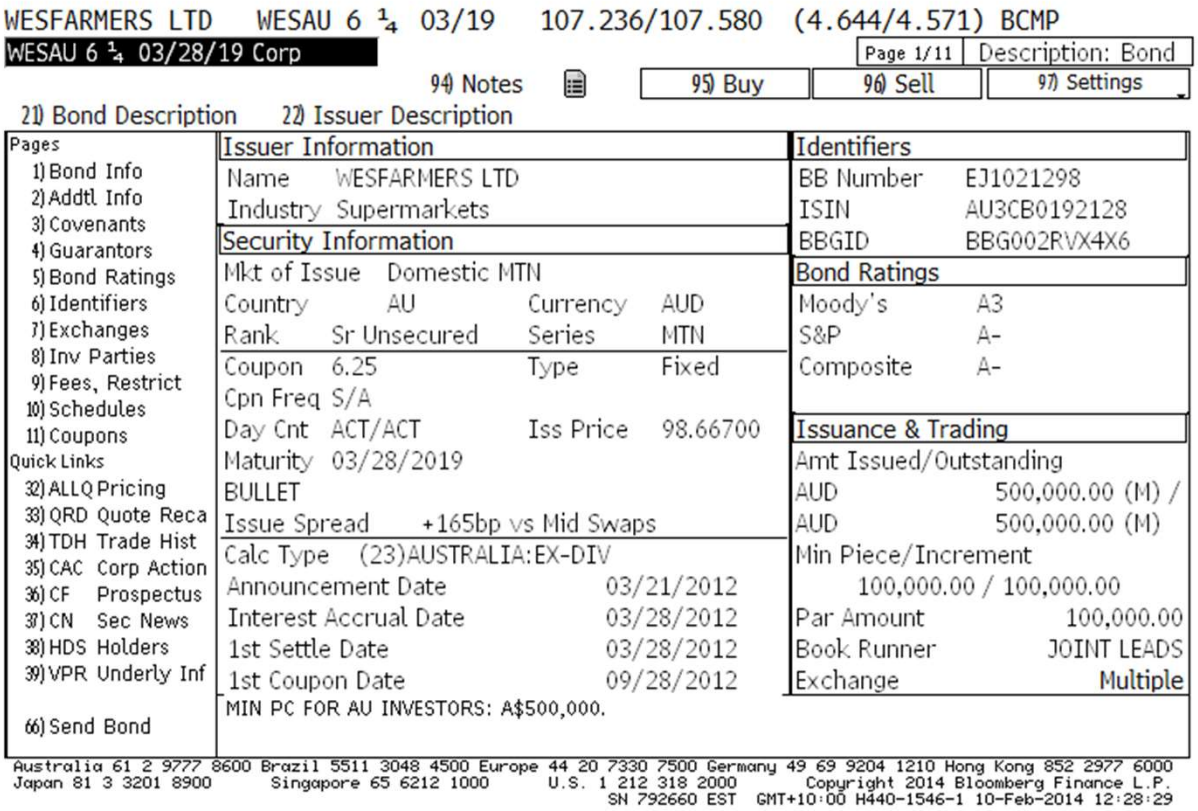
•Coupon Bond



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Stock - Shares

- An owner of a share is entitled to future dividend payments.
- Share value = present value of these dividends

$$P_0 = \sum_{t=1}^{\infty} D_t \, (1 + k_e)^{-t}$$

where k_e = cost of equity capital

- We start at t = 1
  - If you purchase a stock that will pay a dividend soon (let's call it D_0), you are not entitled to that dividend if you purchased the stock on or after the 'ex-dividend date', which is generally about a month before the actual dividend is paid

Stock - Shares

- Remember the perpetuity formula $$PV_0 = \frac{C}{i}$$

- We can apply it (and some variants) to shares
  - We'll need to make some assumptions, watch timing of cash flows and come up with an appropriate discount rate.

- Basic models
  - Perpetuity (constant dividend)
  - Constant growth (dividend that grows at a constant rate)
  - Standard model (non-constant dividends at the start)

Stock - Shares

- Perpetuity
  - Assume dividends are constant

$$\boldsymbol{P_0 = \frac{D}{k_e}}$$

Where D = constant dividend (to be received in one-period's time)
"k_e" cost of equity capital

- This approach can realistically be applied to ***non-redeemable preference shares***

•Stock - Shares

Example 11

What is price of a non-redeemable preference share that pays a constant dividend of 90 cents per year if the discount rate (required rate of return) is 15%? The dividend has just been paid.

Stock - Shares

- Constant Growth Model
  - Assume dividend grows at a constant rate (g)

$$P_0 = \frac{D_1}{k_e - g} = \frac{D_0 \times (1 + g)}{k_e - g}$$

where g = constant growth rate of dividends to ∞

  - Advantages
    - potentially more realistic than constant dividend
    - relatively easy to calculate
    - is an important part of the standard model (covered next)
  - Limitation
    - unlikely to find shares with growth pattern demanded by the model

Stock - Shares

Example 12

What is the value of a share that just paid a 90-cent dividend, where dividends are expected to grow at 10% p.a. forever and the required rate is 15%?

•Stock – Shares – ***Standard Model***

•Assume particular dividend values until a certain date and from then make one of the previous two simplifying assumptions

$$\boldsymbol{P_0} = \sum_{\boldsymbol{t=1}}^{\boldsymbol{n}} \frac{\boldsymbol{D_t}}{(\mathbf{1}+\boldsymbol{k_e})^{\boldsymbol{t}}} + \frac{\boldsymbol{P_n}}{(\mathbf{1}+\boldsymbol{k_e})^{\boldsymbol{n}}} \quad \text{where} \quad P_n = \frac{D_{n+1}}{k_e - g}$$

Valuing non-constant dividends

Valuing constant dividends

•Advantage - More flexible

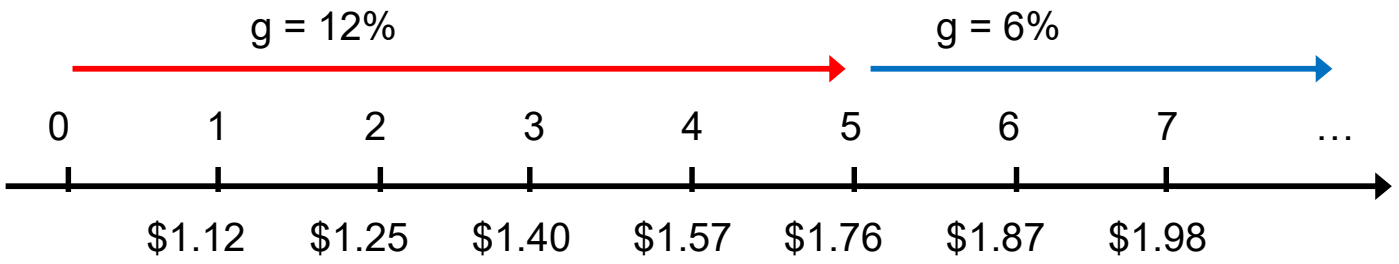
- •During the business cycle you find that firms grow at different rates during the cycle.
- •Typically, new firms will grow rapidly in early years then level out to some lower growth rate as their market matures.

•Stock - Shares

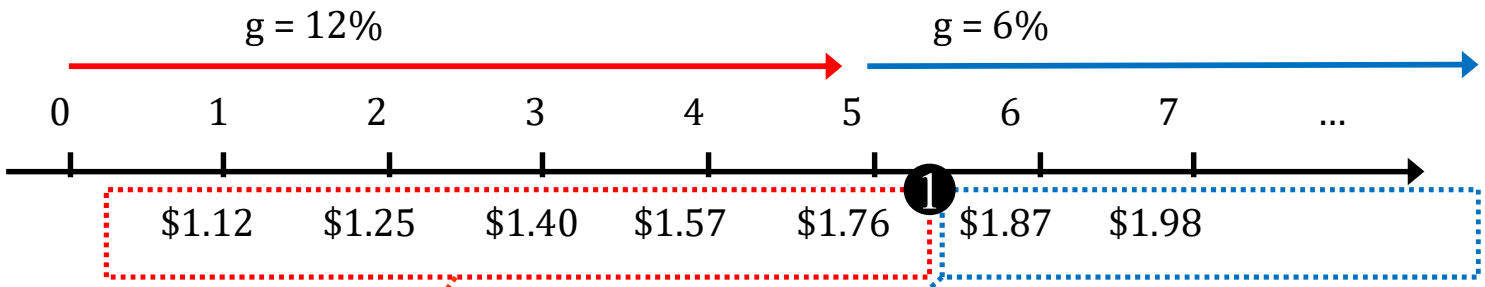
Example 13

A stock's current dividend of $1.00 has just been paid. Dividends are expected to grow by 12% for the next five years and 6% forever after that. The required rate of return is 10%. What is the price of the share?

To start, let's do a timeline of dividends



Stock - Shares



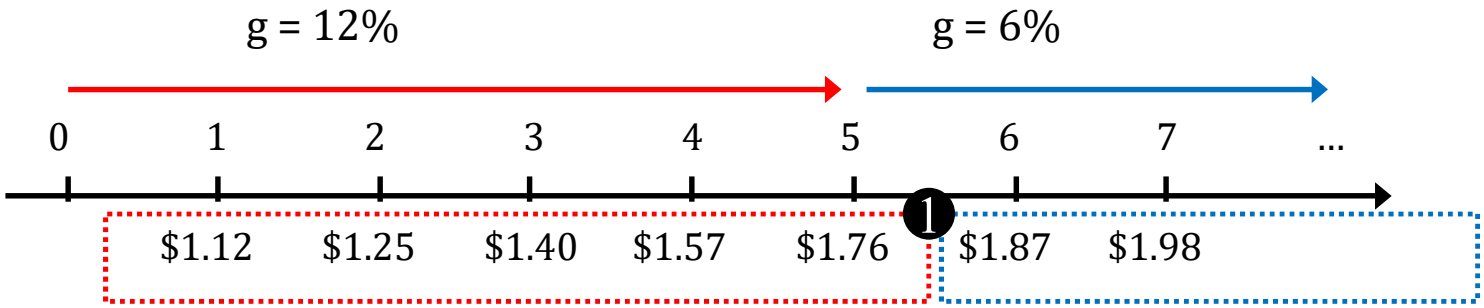
$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t} + \frac{P_n}{(1+k_e)^n}$$

$$P_n = \frac{D_{n+1}}{k_e - g}$$

1. Determine the period of constant dividend growth
2. Calculate PV of dividends in the 1st growth stage
3. Calculate PV of dividends in the constant growth stage at n, PV_n (P_n)
4. Discount P_n to 0 using $(1+k_e)^n$
5. PV_0 = PV of non-constant dividends + PV of constant dividends

Stock - Shares

- Example 13 cont.



② $\widetilde{P_0} = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t}$

$= \frac{1.12}{1.1^1} + \frac{1.25}{1.1^2} + \frac{1.40}{1.1^3} + \frac{1.57}{1.1^4} + \frac{1.76}{1.1^5}$

$= 5.27$

③ $P_5 = \frac{D_6}{k_e - g} = \frac{1.87}{0.1 - 0.06}$

$= 46.75$

④ $\hat{P}_0 = \frac{P_n}{(1+k_e)^n} = \frac{46.75}{1.1^5}$

$= 29.03$

⑤ $P_0 = \widetilde{P_0} + \hat{P}_0 = 5.27 + 29.03 = 34.30$

Stock - Shares

Example 14

A commencing company is expected to pay the following dividends: $0.12 in year 3, $0.18 in year 4 and $0.25 in year 5. All further dividends will grow at 5% indefinitely. Investors require a return of 15%. What do you think that the share price should be?