3 Factor Investing and Multifactor Models

Introduction to Factor Investing

Changing Asset Management Paradigm

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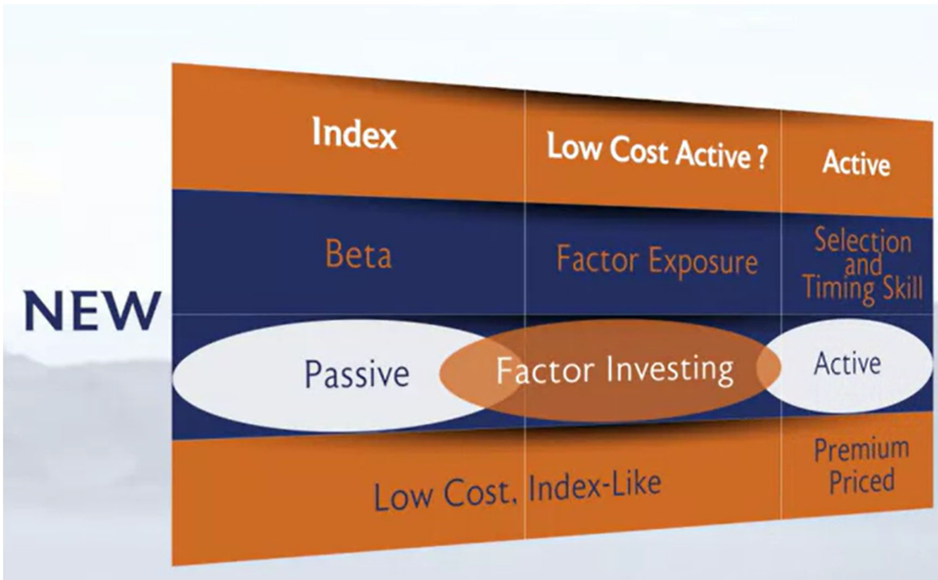

Introduction to Factor Investing

Investor's View of a Portfolio

Introduction to Factor Investing

Asset Allocator's View of a Portfolio

Introduction to Factor Investing

NUTRIENT ANALOGY

Macronutrients				
	Male	Female	Child	Examples of Food
Water	3.7 L/day	2.7 L/day	1.7 L/day	
Carbohydrates	130 g/day	130 g/day	130 g/day	Bread, Beans, Potato Rice
Protein	56 g/day	46 g/day	19 g/day	Cheese, milk, Fish, Soya bean
Fiber	38 g/day	25 g/day	25 g/day	Peas, Wheat, Rice
Fat	20-35% of calories		25-35% of calories	Oily fish, Peanuts, Animal fat
Source: Food and Nutritrion Board, National Academies, 2004				

NUTRIENTS = FACTORS: FOODS ARE DIFFERENT WAYS TO ACCESS NUTRIENTS

FOODS = ASSETS: PEAS, WHEAT, RICE ALL HAVE FIBER JUST LIKE CERTAIN SOVEREIGN BONDS, CORPORATE BONDS, STOCKS, CDS ALL HAVE CREDIT RISK

MALE/FEMALE/CHILD = INVESTORS: EACH INVESTOR HAS AN OPTIMAL MIX OF FACTORS WITH DIFFERENT LIABILITIES AND RISK AVERSIONS

Introduction to Factor Investing

A Factor Investor’s View of a Portfolio

Nutrition Facts	Serving Size (g)	Calories	Calories from fat	Total fat (g)	Saturated Fat (g)	Trans Fat (g)	Chol (mg)	Sodium (mg)	Total Carb (g)	Dietary Fiber (g)	Total Sugar (g)	Protein (g)
WHOPPER® SANDWICHES												
VHOPPER® Sandwich	270	660	360	40	12	1.5	90	980	49	2	11	28
VHOPPER® Sandwich with Cheese	292	740	420	46	16	2	115	1340	50	2	11	32
acon & Cheese WHOPPER® Sandwich	303	790	460	51	17	2	125	1560	50	2	11	35
OUBLE WHOPPER® Sandwich	354	900	520	58	20	3	175	1050	49	2	11	48
OUBLE WHOPPER® Sandwich with Cheese	377	980	580	64	24	3	195	1410	50	2	11	52
RIPLE WHOPPER® Sandwich	438	1130	680	75	28	4	255	1120	49	2	11	67
RIPLE WHOPPER® Sandwich with Cheese	461	1220	740	82	32	4.5	280	1470	50	2	11	71
VHOPPER JR.® Sandwich	134	310	160	18	5	0.5	40	390	27	1	7	13
FLAME BROILED BURGERS												
ACON KING™ Sandwich	356	1150	710	79	31	3.5	240	2150	49	2	10	61
heddar BACON KING™ Sandwich	366	1190	750	84	33	3.5	235	1930	50	2	11	64
ingle Quarter Pound KING™ Sandwich	231	580	260	29	13	1.5	105	1310	49	2	10	32
ouble Quarter Pound KING™ Sandwich	338	900	480	54	25	3	210	1740	50	2	11	56

Introduction to Factor Investing

A FACTOR IS A VARIABLE WHICH INFLUENCES THE RETURNS OF ASSETS

IT REPRESENTS **COMMONALITY** IN THE RETURNS
SOMETHING OUTSIDE OF THE INDIVIDUAL ASSET

EXPOSURE TO FACTOR RISK OVER THE LONG RUN YIELDS A REWARD, THE RISK PREMIUM

TYPES OF FACTORS

MACRO FACTORS: GROWTH, INFLATION, ...

STATISTICAL FACTORS: SOMETHING EXTRACTED FROM THE DATA THAT MAY OR MAY NOT BE IDENTIFIABLE

INTRINSIC FACTORS OR STYLE FACTORS:
VALUE-GROWTH, MOMENTUM, LOW VOLATILITY, ...

Multifactor Models and Modern Portfolio Theory

- Our analysis of risky securities and the construction of portfolio's leads us to several important conclusions:
  - When a security is included in a portfolio part of its risk is diversified away (its **diversifiable risk**) while part of its risk contributes to the risk of the portfolio (its **non-diversifiable risk**)
- The non-diversifiable risk of a security is clearly related to the correlation of the security's return and the portfolio's return; the lower the correlation, the more diversification of risk

Multifactor Models and Modern Portfolio Theory

- The concept of *systematic risk* is critical to under standing multifactor models: An investment may be subject to many different types of risks, but they are generally not equally important so far as investment valuation is concerned. Risk that can be avoided by holding an asset in a portfolio, where the risk might be offset by the various risks of other assets, should not be compensated by higher expected return, according to theory
- Hence it should be the *non-diversifiable* risk that they are compensated for, not the diversifiable risk

Multifactor models

- CAPM is criticized because of
  - Many unrealistic assumptions
  - Difficulties in selecting a proxy for the market portfolio as a benchmark
- Alternative pricing theory with fewer assumptions was developed:
  - Arbitrage Pricing Theory (APT)
  - Macroeconomic factor models
  - …

Multifactor models

Multifactor models

- A **factor** is a common or underlying element with which several variables are correlated. For example, the market factor is an underlying element with which individual share returns are correlated.
- We search for **systematic factors**, which affect the average returns of a large number of different assets. These factors represent **priced risk**, risk for which investors require an additional return for bearing. Systematic factors should thus help explain returns.

Arbitrage Pricing Theory and the Multifactor Models

- In the 1970s, Ross (1976) developed the arbitrage pricing theory (APT) as an alternative to the CAPM

Suppose that K factors are assumed to generate returns. Then the simplest expression for a multifactor model for the return of asset i is given by

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots + b_{iK}I_K + \varepsilon_{i}, \tag{1}$$

where

R_i = the return to asset i

a_i = an intercept term

I_k = the return to factor k, $k = 1, 2, \ldots, K$

b_{ik} = the sensitivity of the return on asset i to the return to factor k, $k = 1, 2, \ldots, K$

ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model

Arbitrage Pricing Theory

Three Major Assumptions:

1. A factor model describes asset returns.
2. There are many assets, so investors can form well-diversified portfolios that eliminate asset-specific risk.
3. No arbitrage opportunities exist among well-diversified portfolios.

Arbitrage Pricing Theory

- Arbitrage is a risk-free operation that requires no net investment of money but earns an expected positive net profit.
- An arbitrage opportunity is an opportunity to conduct an arbitrage—an opportunity to earn an expected positive net profit without risk and with no net investment of money.
- Since no investment is required, investors can create large positions to obtain large profits.

Arbitrage Pricing Theory

Does not assume:

- Normally distributed security returns
- Quadratic utility function
- A mean-variance efficient market portfolio

Arbitrage Pricing Theory

- The APT Model

$$E(R_i)=\lambda_0+ \lambda_1 b_{i1}+ \lambda_2 b_{i2}+\ldots+ \lambda_k b_{ik}$$

where:

λ_0=the expected return on an asset with zero systematic risk

λ_j=the risk premium related to the *j* th common risk factor

b_{ij}=the pricing relationship between the risk premium and the asset; that is, how responsive asset *i* is to the *j* th common factor

Arbitrage Pricing Theory

- The APT equation says that the expected return on any well- diversified portfolio is linearly related to the factor sensitivities of that portfolio
- The factor risk premium (or factor price) represents the expected return in excess of the risk-free rate for a portfolio with a sensitivity of 1 to factor *i* and a sensitivity of 0 to all other factors. Such a portfolio is called a **pure factor portfolio** for factor *i*.

Arbitrage Pricing Theory

- To use the APT equation, we need to estimate its parameters.
- The parameters of the APT equation are the risk-free rate and the factor risk-premiums (the factor sensitivities are specific to individual investments). The following example shows how the expected returns and factor sensitivities of a set of portfolios can determine the parameters of the APT model.

Arbitrage Pricing Theory

EXAMPLE 1

Determining the Parameters in a One-Factor APT Model

Suppose we have three well-diversified portfolios that are each sensitive to the same single factor. Exhibit 1 shows the expected returns and factor sensitivities of these portfolios. Assume that the expected returns reflect a one-year investment horizon. To keep the analysis simple, all investors are assumed to agree upon the expected returns of the three portfolios as shown in the exhibit.

Exhibit 1: Sample Portfolios for a One-Factor Model

Portfolio	Expected Return	Factor Sensitivity
A	0.075	0.5
B	0.150	2.0
C	0.070	0.4

Arbitrage Pricing Theory

- Suppose we have three well-diversified portfolios that are each sensitive to the same single factor

	A	B	C	D	E
1	Portfolio	Expected return	Factor sensitivity		
2	A	0.075	0.5		
3	B	0.150	2.0		
4	C	0.070	0.4		
5					
6	Selecting portfolios A and B, we have				
7	$E(R_A) = 0.075 = R_f + 0.5\ \lambda_1$			Rf	0.05
8	$E(R_B) = 0.1505 = R_f + 2\ \lambda_1$			λ1	0.05
9					
10	The APT equation:		$E(R_p) = 0.05 + 0.05\ \beta_{p,1}$		
11	Check with portfolio C -> No arbitrage opportunity exists				

Arbitrage Pricing Theory

- In this example, we demonstrate how to tell whether a set of expected returns for well-diversified portfolios is consistent with the APT by testing whether an arbitrage opportunity exists

	A	B	C	D	E	F
14	Portfolio	Expected return	Factor sensitivity			
15	A	0.075	0.50			
16	B	0.150	2.00			
17	C	0.070	0.40			
18	D	0.080	0.45			
19	0.5A + 0.5C	0.725	0.45			
20	Portfolio D offers too high an expected rate of return given its					
21	factor sensitivity, or Portfolio D is undervalued relative to its factor risk.					
22	We will buy D (hold it long) in the portfolio that exploits the arbitrage					
23	opportunity (the arbitrage portfolio). We purchase D using the proceeds					
24	from selling short a portfolio consisting of A and C with exactly the same					
25	0.45 factor sensitivity as D.					

An example: The Carhart 4 factor model

- The Carhart four-factor model, also known as the four-factor model or simply the Carhart model, is a frequently referenced multifactor model in current equity portfolio management practice. Presented in Carhart (1997), it is an extension of the three-factor model developed by Fama and French (1992) to include a momentum factor.
- On the basis of that evidence, the Carhart model posits the existence of three systematic risk factors beyond the market risk factor. They are named, in the same order as above, the following:
  - Small minus big (SMB)
  - High minus low (HML)
  - Winners minus losers (WML)

An example: The Carhart 4 factor model

$$R_p - R_F = a_p + b_{p1}\text{RMRF} + b_{p2}\text{SMB} + b_{p3}\text{HML} + b_{p4}\text{WML} + \varepsilon_p,$$

RMRF = the return on a value-weighted equity index in excess of the one-month T-bill rate

SMB = small minus big, a size (market capitalization) factor; SMB is the average return on three small-cap portfolios minus the average return on three large-cap portfolios

HML = high minus low, the average return on two high book-to-market portfolios minus the average return on two low book-to-market portfolios

WML = winners minus losers, a momentum factor; WML is the return on a portfolio of the past year's winners minus the return on a portfolio of the past year's losers. (Note that WML is an equally weighted average of the stocks with the highest 30% 11-month returns lagged 1 month minus the equally weighted average of the stocks with the lowest 30% 11-month returns lagged 1 month.)

Comparing the CAPM & APT Models

	CAPM	APT
Form of Equation	Linear	Linear
Number of Risk Factors	1	K (≥ 1)
Factor Risk Premium	$[E(RM) - RFR]$	$\{\lambda_j\}$
Factor Risk Sensitivity	β_i	$\{b_{ij}\}$
"Zero-Beta" Return	RFR	λ_0

Unlike CAPM that is a one-factor model, APT is a multifactor pricing model

APT and CAPM

APT

- Equilibrium means no arbitrage opportunities.
- APT equilibrium is quickly restored even if only a few investors recognize an arbitrage opportunity.
- The expected return–beta relationship can be derived without using the true market portfolio.

CAPM

- Model is based on an inherently unobservable "market" portfolio.
- Rests on mean-variance efficiency. The actions of many small investors restore CAPM equilibrium.
- CAPM describes equilibrium for all assets.

Comparing the CAPM & APT Models

- However, unlike CAPM that identifies the market portfolio return as the factor, APT model does not specifically identify these risk factors in application
- The APT is largely silent on where to look for priced sources of risk
- These multiple factors may include
  - Inflation
  - Growth in GNP
  - Major political upheavals
  - Changes in interest rates

Multifactor APT

- Use of more than a single systematic factor
- Requires formation of factor portfolios
- What factors?
  - Factors that are important to performance of the general economy
  - What about firm characteristics?

Where Should We Look for Factors?

- Need important systematic risk factors
  - Chen, Roll, and Ross used industrial production, expected inflation, unanticipated inflation, excess return on corporate bonds, and excess return on government bonds.
  - Fama and French used firm characteristics that proxy for systematic risk factors.

Types of Multifactor models

- In *macroeconomic factor* models, the factors *are surprises in macroeconomic variables* that significantly explain equity returns. The factors can be understood as affecting either the expected future cash flows of companies or the interest rate used to discount these cash flows back to the present.
- In *fundamental factor* models, the factors are *attributes of stocks or companies* that are important in explaining cross-sectional differences in stock prices. Among the fundamental factors that have been used are the book-value-to-price ratio, market capitalization, the price–earnings ratio, and financial leverage.

Multifactor models

- In *statistical factor* models, statistical methods are applied to a set of historical returns to determine portfolios that explain historical returns in one of two senses.
  - In *factor analysis models*, the factors are the portfolios that best explain (reproduce) historical return covariances.
  - In *principal-components models*, the factors are portfolios that best explain (reproduce) the historical return variances.

Macroeconomic Factor Models

- The representation of returns in macroeconomic factor models assumes that the returns to each asset are correlated with only the **surprises** in some factors related to the aggregate economy, such as inflation or real output.
- We can define **surprise** in general as *the actual value minus predicted (or expected) value*. A factor's surprise is the component of the factor's return that was unexpected, and the factor surprises constitute the model's independent variables. This idea contrasts to the representation of independent variables as returns (as opposed to the surprise in returns) in fundamental factor models, or for that matter in the market model.

Macroeconomic Factor Models

Suppose that K macro factors explain asset returns. Then in a macroeconomic factor model, the return of asset *i* can be expressed as:

$$R_i = a_i + [b_{i1}F_1 + b_{i2}\,F_2 + \ldots + b_{iK}\,F_K] + e_i$$

where:

a_i = the expected return to stock i

F_i=the surprise in the factor k, $k = 1, 2, \ldots, K$

b_{ik} = the sensitivity of the return on asset i to a surprise in factor k, $k = 1, 2, \ldots, K$

Macroeconomic Factor Models

- The macroeconomic factor model structure analyses the return to an asset into three components: the asset's expected return, its unexpected return resulting from new information about the factors, and an error term.
- Consider a factor model in which the returns to each asset are correlated with two factors. For example, we might assume that the returns for a particular stock are correlated with surprises in interest rates and surprises in GDP growth. For stock *i*, the return to the stock can be modelled as

$$R_i = a_i + b_{i1}F_{INT} + b_{i2}F_{GDP} + \varepsilon_i$$

Exhibit 6: Growth and Inflation Factor Matrix

	Inflation	
Growth	**Low Inflation/Low Growth** ▪ Cash ▪ Government bonds	**High Inflation/Low Growth** ▪ Inflation-linked bonds ▪ Commodities ▪ Infrastructure
	Low Inflation/High Growth ▪ Equity ▪ Corporate debt	**High Inflation/High Growth** ▪ Real assets (real estate, timberland, farmland, energy)

Note: Entries are assets likely to benefit from the specified combination of growth and inflation.

Macroeconomic Factor Models

- In macroeconomic factor models, the time series of factor surprises are constructed first. Regression analysis is then used to estimate assets' sensitivities to the factors.
- In practice, estimated sensitivities and intercepts are often acquired from one of the many consulting companies that specialize in factor models.
- When we have the parameters for the individual assets in a portfolio, we can calculate the portfolio's parameters as a weighted average of the parameters of individual assets. An individual asset's weight in that calculation is the proportion of the total market value of the portfolio that the individual asset represents.

An example

Estimating Returns for a Two-Stock Portfolio Given Factor Sensitivities

Suppose that stock returns are affected by two common factors: surprises in inflation and surprises in GDP growth. A portfolio manager is analyzing the returns on a portfolio of two stocks, Manumatic (MANM) and Nextech (NXT). The following equations describe the returns for those stocks, where the factors F_{INFL} and F_{GDP} represent the surprise in inflation and GDP growth, respectively:

$$R_{MANM} = 0.09 - 1F_{INFL} + 1F_{GDP} + \varepsilon_{MANM}.$$

$$R_{NXT} = 0.12 + 2F_{INFL} + 4F_{GDP} + \varepsilon_{NXT}.$$

One-third of the portfolio is invested in Manumatic stock, and two-thirds is invested in Nextech stock.

In evaluating the equations for surprises in inflation and GDP, convert amounts stated in percentage terms to decimal form.

1. Formulate an expression for the return on the portfolio.

An example

2. State the expected return on the portfolio.
3. Calculate the return on the portfolio given that the surprises in inflation and GDP growth are 1% and 0%, respectively, assuming that the error terms for MANM and NXT both equal 0.5%

Multifactor Model Equation

$$r_i = E(r_i) + \beta_{iGDP} GDP + \beta_{iIR} IR + e_i$$

r_i = Return for security i

β_{GDP} = Factor sensitivity for GDP

β_{IR} = Factor sensitivity for Interest Rate

e_i = Firm specific events

Multifactor SML Models

$$E(r_i) = r_f + \beta_{iGDP} RP_{GDP} + \beta_{iIR} RP_{IR}$$

$\beta_{i_{GDP}}$ = Factor sensitivity for GDP

RP_{GDP} = Risk premium for GDP

$\beta_{i\,IR}$ = Factor sensitivity for Interest Rate

RP_{IR} = Risk premium for Interest Rate

Interpretation

The expected return on a security is the sum of:

1. The risk-free rate
2. The sensitivity to GDP times the risk premium for bearing GDP risk
3. The sensitivity to interest rate risk times the risk premium for bearing interest rate risk

Macroeconomic Factor Models

- Security returns are governed by a set of broad economic influences in the following fashion by Chen, Roll, and Ross in 1986 modeled by the following equation:
- 1) inflation, including unanticipated inflation and changes in expected inflation,
- 2) a factor related to the term structure of interest rates, represented by long-term government bond returns minus one-month Treasury-bill rates,
- 3) a factor reflecting changes in market risk and investors' risk aversion, represented by the difference between the returns on low-rated and high-rated bonds, and
- 4) changes in industrial production.

Macroeconomic Factor Models

Exhibit 7.15 Estimating a Multifactor Model with Macroeconomic Risk Factors

Period	Constant	R_M	MP	DEI	UI	UPR	UTS
1958-1984	10.71	−2.40	11.76	−0.12	−0.80	8.27	−5.91
	(2.76)	(−0.63)	(3.05)	(−1.60)	(−2.38)	(2.97)	(−1.88)
1958-1967	9.53	1.36	12.39	0.01	−0.21	5.20	−0.09
	(1.98)	(0.28)	(1.79)	(0.06)	(−0.42)	(1.82)	(−0.04)
1968-1977	8.58	−5.27	13.47	−0.26	−1.42	12.90	−11.71
	(1.17)	(−0.72)	(2.04)	(−3.24)	(−3.11)	(2.96)	(−2.30)
1978-1984	15.45	−3.68	8.40	−0.12	−0.74	6.06	−5.93
	(1.87)	(−0.49)	(1.43)	(−0.46)	(−0.87)	(0.78)	(−0.64)

Source: Nai-fu Chen, Richard Roll, and Stephen A. Ross, "Economic Forces and the Stock Market," *Journal of Business* 59, no. 3 (April 1986).

$$R_{it} = a_i + [b_{i1}R_{mt} + b_{i2}MP_t + b_{i3}DEI_t + b_{i4}UI_t + b_{i5}UPR_t + b_{i6}UTS_t] + e_{it}$$

Macroeconomic Factor Models

- Burmeister, Roll, and Ross (1994)
  - Analyzed the predictive ability of a model based on the following set of macroeconomic factors.
    - Confidence risk
    - Time horizon risk
    - Inflation risk
    - Business cycle risk
    - Market timing risk

Exercise

- Suppose that two factors, surprise in inflation (Factor 1) and surprise in GDP growth (Factor 2), explain returns. According to the APT, an arbitrage opportunity exists unless

$$E\left(R_p\right) = R_f + \lambda_1 \beta_{p,1} + \lambda_2 \beta_{p,2}$$

- Estimate the three parameters of the model with the following assumptions on the three well-diversified portfolios, J, K, and L:

Portfolio	Expected return	Sensitivity to inflation factor	Sensitivity to GDP factor
J	0.14	1.0	1.5
K	0.12	0.5	1.0
L	0.11	1.3	1.1

Fundamental Factor Models

- In fundamental factor models, the factors are stated as returns rather than return surprises in relation to predicted values, so they do not generally have expected values of zero. This approach changes the meaning of the intercept, which is no longer interpreted as the expected return.
- Factor sensitivities are also interpreted differently in most fundamental factor models. In fundamental factor models, the factor sensitivities are attributes of the security. An asset's sensitivity to a factor is expressed using a standardized beta:

$$b_{ik} = \frac{\text{Value of attribute } k \text{ for asset } i - \text{Average value of attribute } k}{\sigma(\text{Values of attribute } k)}$$

Fundamental Factor Models

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Fundamental Factor Models

- A second distinction between macroeconomic multifactor models and fundamental factor models is that with the former, we develop the factor (surprise) series first and then estimate the factor sensitivities through regressions. With the latter, we generally specify the factor sensitivities (attributes) first and then estimate the factor returns through regressions.
- Financial analysts use fundamental factor models for a variety of purposes, including portfolio performance attribution and risk analysis.

Fama-French Three-Factor Model

- SMB = Small Minus Big (firm size)
- HML = High Minus Low (book-to-market ratio - Value)
- Are these firm characteristics correlated with actual (but currently unknown) systematic risk factors?

$$r_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it}$$

Factor Models in Return Attribution

EXAMPLE 7

Four-Factor Model Active Return Decomposition

As an equity analyst at a pension fund sponsor, Ronald Service uses the Carhart four-factor multifactor model of Equation 3a to evaluate US equity portfolios:

$$R_p - R_F = a_p + b_{p1}\text{RMRF} + b_{p2}\text{SMB} + b_{p3}\text{HML} + b_{p4}\text{WML} + \varepsilon_p.$$

Service's current task is to evaluate the performance of the most recently hired US equity manager. That manager's benchmark is an index representing the performance of the 1,000 largest US stocks by market value. The manager describes himself as a "stock picker" and points to his performance in beating the benchmark as evidence that he is successful. Exhibit 9 presents an analysis based on the Carhart model of the sources of that manager's active return during the year, given an assumed set of factor returns. In Exhibit 9, the entry "A. Return from Factor Tilts = 2.1241%" is the sum of the four numbers above it. The entry "B. Security Selection" gives security selection as equal to −0.05%. "C. Active Return" is found as the sum of these two components: 2.1241% + (−0.05%) = 2.0741%.

$$\text{Active return} = \sum_{k=1}^{K}\left[(\text{Portfolio sensitivity})_k - (\text{Benchmark sensitivity})_k\right] \times (\text{Factor return})_k + \text{Security selection}.$$

Exhibit 9: Active Return Decomposition

	Factor Sensitivity				Contribution to Active Return	
Factor	**Portfolio (1)**	**Benchmark (2)**	**Difference (3) = (1) – (2)**	**Factor Return (4)**	**Absolute (3) × (4)**	**Proportion of Total Active**
RMRF	0.95	1.00	−0.05	5.52%	−0.2760%	−13.3%
SMB	−1.05	−1.00	−0.05	−3.35%	0.1675%	8.1%
HML	0.40	0.00	0.40	5.10%	2.0400%	98.4%
WML	0.05	0.03	0.02	9.63%	0.1926%	9.3%
				A. Return from Factor Tilts =	2.1241%	102.4%
				B. Security Selection =	−0.0500%	−2.4%
				C. Active Return (A + B) =	2.0741%	100.0%

1. Determine the manager's investment mandate and his actual investment style.
2. Evaluate the sources of the manager's active return for the year
3. What concerns might Service discuss with the manager as a result of the return decomposition?

Factor Models in Risk Attribution

- **Active risk** can be represented by the standard deviation of active returns. A traditional term for that standard deviation is **tracking error** (TE).
- **Tracking risk** is a synonym for **tracking error** that is often used in the CFA Program curriculum. We will use the abbreviation TE for the concept of active risk and refer to it usually as tracking error:

$$TE = s(R_p - R_B).$$

- Information Ratio:

$$IR = \frac{\bar{R}_p - \bar{R}_B}{s\left(R_p - R_B\right)}.$$

Factor Models in Risk Attribution

A Comparison of Active Risk

Richard Gray is comparing the risk of four US equity managers who share the same benchmark. He uses a fundamental factor model, the BARRA US-E4 model, which incorporates 12 style factors and a set of 60 industry factors. The style factors measure various fundamental aspects of companies and their shares, such as size, liquidity, leverage, and dividend yield. In the model, companies have non-zero exposures to all industries in which the company operates. Exhibit 10 presents Gray's analysis of the active risk squared of the four managers, based on Equation 12 (note that there is a covariance term in active factor risk, reflecting the correlation of industry membership and the risk indexes, which we assume is negligible in this example). In Exhibit 10, the column labeled "Industry" gives the portfolio's active factor risk associated with the industry exposures of its holdings; the "Style Factor" column gives the portfolio's active factor risk associated with the exposures of its holdings to the 12 style factors.

Active risk squared = Active factor risk + Active specific risk.

Exhibit 10: Active Risk Squared Decomposition

	Active Factor				
Portfolio	**Industry**	**Style Factor**	**Total Factor**	**Active Specific**	**Active Risk Squared**
A	12.25	17.15	29.40	19.60	49
B	1.25	13.75	15.00	10.00	25
C	1.25	17.50	18.75	6.25	25
D	0.03	0.47	0.50	0.50	1

Note: Entries are in % squared.

Using the information in Exhibit 10, address the following:

1. Contrast the active risk decomposition of Portfolios A and B.

Exhibit 11: Active Risk Decomposition (restated)

Portfolio	Active Factor (% of total active)			Active Specific (% of total active)	Active Risk
	Industry	Style Factor	Total Factor		
A	25%	35%	60%	40%	7%
B	5%	55%	60%	40%	5%
C	5%	70%	75%	25%	5%
D	3%	47%	50%	50%	1%

Portfolio A has assumed a higher level of active risk than B (7% versus 5%). Portfolios A and B assumed the same proportions of active factor and active specific risk, but a sharp contrast exists between the two in the types of active factor risk exposure. Portfolio A assumed substantial active industry risk, whereas Portfolio B was approximately industry neutral relative to the benchmark. By contrast, Portfolio B had higher active bets on the style factors representing company and share characteristics.

2. Contrast the active risk decomposition of Portfolios B and C.

Solution:

Portfolios B and C were similar in their absolute amounts of active risk. Furthermore, both Portfolios B and C were both approximately industry neutral relative to the benchmark. Portfolio C assumed more active factor risk related to the style factors, but B assumed more active specific risk. It is also possible to infer from the greater level of B's active specific risk that B is somewhat less diversified than C.

3. Characterize the investment approach of Portfolio D.

Solution:

Portfolio D appears to be a passively managed portfolio, judging by its negligible level of active risk. Referring to Exhibit 11, Portfolio D's active factor risk of 0.50, equal to 0.707% expressed as a standard deviation, indicates that the portfolio's risk exposures very closely match the benchmark.

Factor Models in Portfolio Construction

- **Passive management**. In managing a fund that seeks to track an index with many component securities, portfolio managers may need to select a sample of securities from the index. Analysts can use multifactor models to replicate an index fund's factor exposures, mirroring those of the index tracked.
- **Active management**. Many quantitative investment managers rely on multifactor models in predicting alpha (excess risk-adjusted returns) or relative return (the return on one asset or asset class relative to that of another) as part of a variety of active investment strategies. In constructing portfolios, analysts use multifactor models to establish desired risk profiles.

Factor Models in Portfolio Construction

- **Rules-based active management** (alternative indexes or factor investing). These strategies routinely tilt toward such factors as size, value, quality, or momentum when constructing portfolios. As such, alternative index approaches aim to capture some systematic exposure traditionally attributed to manager skill, or "alpha," in a transparent, mechanical, rules-based manner at low cost. Alternative index strategies rely heavily on factor models to introduce intentional factor and style biases versus capitalization-weighted indexes.

LAB Section: Factor Analysis using the CAPM and Fama-French Factor models

- Use the file:

  lab_201_CAPM FF.ipynb

  lab_202_Style Analysis.ipynb

- Data files:

  ind30_m_vw_rets.csv