REFRESHER READING

2024 CFA® PROGRAM • LEVEL 2

Portfolio Management

Using Multifactor Models

by Jerald E. Pinto, PhD, CFA, and Eugene L. Podkaminer, CFA.

Jerald E. Pinto, PhD, CFA, is at CFA Institute (USA). Eugene L. Podkaminer, CFA, is at Franklin Templeton Investments (USA).

LEARNING OUTCOMES

Mastery	*The candidate should be able to:*
☐	describe arbitrage pricing theory (APT), including its underlying assumptions and its relation to multifactor models
☐	define arbitrage opportunity and determine whether an arbitrage opportunity exists
☐	calculate the expected return on an asset given an asset's factor sensitivities and the factor risk premiums
☐	describe and compare macroeconomic factor models, fundamental factor models, and statistical factor models
☐	describe uses of multifactor models and interpret the output of analyses based on multifactor models
☐	describe the potential benefits for investors in considering multiple risk dimensions when modeling asset returns
☐	explain sources of active risk and interpret tracking risk and the information ratio

1 BACKGROUND AND USES

As used in investments, a **factor** is a variable or a characteristic with which individual asset returns are correlated. Models using multiple factors are used by asset owners, asset managers, investment consultants, and risk managers for a variety of portfolio construction, portfolio management, risk management, and general analytical purposes. In comparison to single-factor models (typically based on a market risk factor), multifactor models offer increased explanatory power and flexibility. These comparative strengths of multifactor models allow practitioners to

- build portfolios that replicate or modify in a desired way the characteristics of a particular index;

CFA Institute would like to thank Yin Luo, CPA, PStat, CFA, and Sheng Wang, both of Wolfe Research LLC (USA), for contributing the case "Constructing Multifactor Portfolios" in Section 8.

- establish desired exposures to one or more risk factors, including those that express specific macro expectations (such as views on inflation or economic growth), in portfolios;
- perform granular risk and return attribution on actively managed portfolios;
- understand the comparative risk exposures of equity, fixed-income, and other asset class returns;
- identify active decisions relative to a benchmark and measure the sizing of those decisions; and
- ensure that an investor's aggregate portfolio is meeting active risk and return objectives commensurate with active fees.

Multifactor models have come to dominate investment practice, having demonstrated their value in helping asset managers and asset owners address practical tasks in measuring and controlling risk. We explain and illustrate the various practical uses of multifactor models.

We first describe the modern portfolio theory background of multifactor models. We then describe arbitrage pricing theory and provide a general expression for multifactor models. We subsequently explore the types of multifactor models and certain applications. Lastly, we summarize major points.

Multifactor Models and Modern Portfolio Theory

In 1952, Markowitz introduced a framework for constructing portfolios of securities by quantitatively considering each investment in the context of a portfolio rather than in isolation; that framework is widely known today as modern portfolio theory (MPT). Markowitz simplified modeling asset returns using a multivariate normal distribution, which completely defines the distribution of returns in terms of mean returns, return variances, and return correlations. One of the key insights of MPT is that any value of correlation among asset returns of less than one offers the potential for risk reduction by means of diversification.

In 1964, Sharpe introduced the capital asset pricing model (CAPM), a model for the expected return of assets in equilibrium based on a mean–variance foundation. The CAPM and the literature that developed around it has provided investors with useful and influential concepts—such as alpha, beta, and systematic risk—for thinking about investing. The concept of systematic risk, for example, is critical to understanding multifactor models: An investment may be subject to many different types of risks, but they are generally not equally important so far as investment valuation is concerned. Risk that can be avoided by holding an asset in a portfolio, where the risk might be offset by the various risks of other assets, should not be compensated by higher expected return, according to theory. By contrast, investors would expect compensation for bearing an asset's non-diversifiable risk: **systematic risk**. Theory indicates that only systematic risk should be **priced risk**. In the CAPM, an asset's systematic risk is a positive function of its beta, which measures the sensitivity of an asset's return to the market's return. According to the CAPM, differences in mean return are explained by a single factor: market portfolio return. Greater risk with respect to the market factor, represented by higher beta, is expected to be associated with higher return.

The accumulation of evidence from the equity markets during the decades following the CAPM's development have provided clear indications that the CAPM provides an incomplete description of risk and that models incorporating multiple sources of systematic risk more effectively model asset returns. Bodie, Kane, and Marcus (2017) provide an introduction to the empirical evidence. There are, however, various perspectives in practice on how to model risk in the context of multifactor models. We will examine some of these—focusing on macroeconomic factor models and fundamental factor models—in subsequent sections.

ARBITRAGE PRICING THEORY AND MULTIFACTOR MODELS 2

- ☐ describe arbitrage pricing theory (APT), including its underlying assumptions and its relation to multifactor models
- ☐ define arbitrage opportunity and determine whether an arbitrage opportunity exists
- ☐ calculate the expected return on an asset given an asset's factor sensitivities and the factor risk premiums

In the 1970s, Ross (1976) developed the arbitrage pricing theory (APT) as an alternative to the CAPM. APT introduced a framework that explains the expected return of an asset (or portfolio) in equilibrium as a linear function of the risk of the asset (or portfolio) with respect to a set of factors capturing systematic risk. Unlike the CAPM, the APT does not indicate the identity or even the number of risk factors. Rather, for any multifactor model assumed to generate returns ("return-generating process"), the theory gives the associated expression for the asset's expected return.

Suppose that K factors are assumed to generate returns. Then the simplest expression for a multifactor model for the return of asset i is given by

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots + b_{iK}I_K + \varepsilon_{i}, \tag{1}$$

where

R_i = the return to asset i

a_i = an intercept term

I_k = the return to factor k, $k = 1, 2, \ldots, K$

b_{ik} = the sensitivity of the return on asset i to the return to factor k, $k = 1, 2, \ldots, K$

ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model

The intercept term a_i is the expected return of asset i given that all the factors take on a value of zero. Equation 1 presents a multifactor return-generating process (a time-series model for returns). In any given period, the model may not account fully for the asset's return, as indicated by the error term. But error is assumed to average to zero. Another common formulation subtracts the risk-free rate from both sides of Equation 1 so that the dependent variable is the return in excess of the risk-free rate and one of the explanatory variables is a factor return in excess of the risk-free rate. (The Carhart model described next is an example.)

Based on Equation 1, the APT provides an expression for the expected return of asset i assuming that financial markets are in equilibrium. The APT is similar to the CAPM, but the APT makes less strong assumptions than the CAPM. The APT makes just three key assumptions:

1. A factor model describes asset returns.
2. With many assets to choose from, investors can form well-diversified portfolios that eliminate asset-specific risk.
3. No arbitrage opportunities exist among well-diversified portfolios.

Arbitrage is a risk-free operation that requires no net investment of money but earns an expected positive net profit. (Note that "arbitrage," or the phrase "risk arbitrage," is also sometimes used in practice to describe investment operations in which significant risk is present). An **arbitrage opportunity** is an opportunity to conduct an arbitrage—an opportunity to earn an expected positive net profit without risk and with no net investment of money.

In the first assumption, the number of factors is not specified. The second assumption allows investors to form portfolios with factor risk but without asset-specific risk. The third assumption is the condition of financial market equilibrium.

Empirical evidence indicates that Assumption 2 is reasonable (Fabozzi, 2008). When a portfolio contains many stocks, the asset-specific or non-systematic risk of individual stocks makes almost no contribution to the variance of portfolio returns.

According to the APT, if these three assumptions hold, the following equation holds:

$$E(R_p) = R_F + \lambda_1\beta_{p,1} + \ldots + \lambda_K\beta_{p,K}, \tag{2}$$

where

$E(R_p)$ = the expected return to portfolio p

R_F = the risk-free rate

λ_j = the expected reward for bearing the risk of factor j

$\beta_{p,j}$ = the sensitivity of the portfolio to factor j

K = the number of factors

The APT equation, Equation 2, says that the expected return on any well-diversified portfolio is linearly related to the factor sensitivities of that portfolio. The equation assumes that a risk-free rate exists. If no risk-free asset exists, in place of R_F we write λ_0 to represent the expected return on a risky portfolio with zero sensitivity to all the factors. The number of factors is not specified but must be much lower than the number of assets, a condition fulfilled in practice.

The **factor risk premium** (or **factor price**), λ_j, represents the expected reward for bearing the risk of a portfolio with a sensitivity of 1 to factor j and a sensitivity of 0 to all other factors. The exact interpretation of "expected reward" depends on the multifactor model that is the basis for Equation 2. For example, in the Carhart four-factor model, shown later in Equation 3 and Equation 4, the risk premium for the market factor is the expected return of the market in excess of the risk-free rate. Then, the factor risk premiums for the other three factors are the mean returns of the specific portfolios held long (e.g., the portfolio of small-cap stocks for the "small minus big" factor) minus the mean return for a related but opposite portfolio (e.g., a portfolio of large-cap stocks, in the case of that factor). A portfolio with a sensitivity of 1 to factor j and a sensitivity of 0 to all other factors is called a **pure factor portfolio** for factor j (or simply the **factor portfolio** for factor j).

For example, suppose we have a portfolio with a sensitivity of 1 with respect to Factor 1 and a sensitivity of 0 to all other factors. Using Equation 2, the expected return on this portfolio is $E_1 = R_F + \lambda_1 \times 1$. If $E_1 = 0.12$ and $R_F = 0.04$, then the risk premium for Factor 1 is

$$0.12 = 0.04 + \lambda_1 \times 1.$$

$$\lambda_1 = 0.12 - 0.04 = 0.08, \text{ or } 8\%.$$

EXAMPLE 1

Determining the Parameters in a One-Factor APT Model

Suppose we have three well-diversified portfolios that are each sensitive to the same single factor. Exhibit 1 shows the expected returns and factor sensitivities of these portfolios. Assume that the expected returns reflect a one-year investment horizon. To keep the analysis simple, all investors are assumed to agree upon the expected returns of the three portfolios as shown in the exhibit.

Exhibit 1: Sample Portfolios for a One-Factor Model

Portfolio	Expected Return	Factor Sensitivity
A	0.075	0.5
B	0.150	2.0
C	0.070	0.4

We can use these data to determine the parameters of the APT equation. According to Equation 2, for any well-diversified portfolio and assuming a single factor explains returns, we have $E(R_p) = R_F + \lambda_1\beta_{p,1}$. The factor sensitivities and expected returns are known; thus there are two unknowns, the parameters R_F and λ_1. Because two points define a straight line, we need to set up only two equations. Selecting Portfolios A and B, we have

$$E(R_A) = 0.075 = R_F + 0.5\lambda_1$$

and

$$E(R_B) = 0.150 = R_F + 2\lambda_1.$$

From the equation for Portfolio A, we have $R_F = 0.075 - 0.5\lambda_1$. Substituting this expression for the risk-free rate into the equation for Portfolio B gives

$$0.15 = 0.075 - 0.5\lambda_1 + 2\lambda_1.$$

$$0.15 = 0.075 + 1.5\lambda_1.$$

So, we have $\lambda_1 = (0.15 - 0.075)/1.5 = 0.05$. Substituting this value for λ_1 back into the equation for the expected return to Portfolio A yields

$$0.075 = R_F + 0.05 \times 0.5.$$

$$R_F = 0.05.$$

So, the risk-free rate is 0.05 or 5%, and the factor premium for the common factor is also 0.05 or 5%. The APT equation is

$$E(R_p) = 0.05 + 0.05\beta_{p,1}.$$

From Exhibit 1, Portfolio C has a factor sensitivity of 0.4. Therefore, according to the APT, the expected return of Portfolio C should be

$$E(R_B) = 0.05 + (0.05 \times 0.4) = 0.07,$$

which is consistent with the expected return for Portfolio C given in Exhibit 1.

EXAMPLE 2

Checking Whether Portfolio Returns Are Consistent with No Arbitrage

In this example, we examine how to tell whether expected returns and factor sensitivities for a set of well-diversified portfolios may indicate the presence of an arbitrage opportunity. Exhibit 2 provides data on four hypothetical portfolios. The data for Portfolios A, B, and C are repeated from Exhibit 1. Portfolio D is a new portfolio. The factor sensitivities given relate to the one-factor APT model $E(R_p) = 0.05 + 0.05\beta_{p,1}$ derived in Example 1. As in Example 1, all investors are assumed to agree upon the expected returns of the portfolios. The question raised by the addition of this new Portfolio D is whether the addition of this portfolio created an arbitrage opportunity. If a portfolio can be formed from Portfolios A, B, and C that has the same factor sensitivity as Portfolio D but a different expected return, then an arbitrage opportunity exists: Portfolio D would be either undervalued (if it offers a relatively high expected return) or overvalued (if it offers a relatively low expected return).

Exhibit 2: Sample Portfolios for a One-Factor Model

Portfolio	Expected Return	Factor Sensitivity
A	0.0750	0.50
B	0.1500	2.00
C	0.0700	0.40
D	0.0800	0.45
0.5A + 0.5C	0.0725	0.45

Exhibit 2 gives data for an equally weighted portfolio of A and C. The expected return and factor sensitivity of this new portfolio are calculated as weighted averages of the expected returns and factor sensitivities of A and C. Expected return is thus (0.50)(0.0750) + (0.50)(0.07) = 0.0725, or 7.25%. The factor sensitivity is (0.50)(0.50) + (0.50)(0.40) = 0.45. Note that the factor sensitivity of 0.45 matches the factor sensitivity of Portfolio D. In this case, the configuration of expected returns in relation to factor risk presents an arbitrage opportunity involving Portfolios A, C, and D. Portfolio D offers, at 8%, an expected return that is too high given its factor sensitivity. According to the assumed APT model, the expected return on Portfolio D should be $E(R_D) = 0.05 + 0.05\beta_{D,1} = 0.05 + (0.05 \times 0.45) = 0.0725$, or 7.25%. Portfolio D is undervalued relative to its factor risk. We will buy D (hold it long) in the portfolio that exploits the arbitrage opportunity (the **arbitrage portfolio**). We purchase D using the proceeds from selling short an equally weighted portfolio of A and C with exactly the same 0.45 factor sensitivity as D.

The arbitrage thus involves the following strategy: Invest $10,000 in Portfolio D and fund that investment by selling short an equally weighted portfolio of Portfolios A and C; then close out the investment position at the end of one year (the investment horizon for expected returns). Exhibit 3 demonstrates the arbitrage profits to the arbitrage strategy. The final row of the exhibit shows the net cash flow to the arbitrage portfolio.

Exhibit 3: Arbitrage Opportunity within Sample Portfolios

	Initial Cash Flow	Final Cash Flow	Factor Sensitivity
Portfolio D	−\$10,000.00	\$10,800.00	0.45
Portfolios A and C	\$10,000.00	−\$10,725.00	−0.45
Sum	\$0.00	\$75.00	0.00

As Exhibit 3 shows, if we buy \$10,000 of Portfolio D and sell \$10,000 of an equally weighted portfolio of Portfolios A and C, we have an initial net cash flow of \$0. The expected value of our investment in Portfolio D at the end of one year is \$10,000(1 + 0.08) = \$10,800. The expected value of our short position in Portfolios A and C at the end of one year is −\$10,000(1.0725) = −\$10,725. So, the combined expected cash flow from our investment position in one year is \$75.

What about the risk? Exhibit 3 shows that the factor risk has been eliminated: Purchasing D and selling short an equally weighted portfolio of A and C creates a portfolio with a factor sensitivity of 0.45 − 0.45 = 0. The portfolios are well diversified, and we assume any asset-specific risk is negligible.

Because an arbitrage is possible, Portfolios A, C, and D cannot all be consistent with the same equilibrium. If Portfolio D actually had an expected return of 8%, investors would bid up its price until the expected return fell and the arbitrage opportunity vanished. Thus, arbitrage restores equilibrium relationships among expected returns.

The Carhart four-factor model, also known as the four-factor model or simply the Carhart model, is a frequently referenced multifactor model in current equity portfolio management practice. Presented in Carhart (1997), it is an extension of the three-factor model developed by Fama and French (1992) to include a momentum factor. According to the model, three groups of stocks tend to have higher returns than those predicted solely by their sensitivity to the market return:

- Small-capitalization stocks
- Low price-to-book stocks, commonly referred to as "value" stocks
- Stocks whose prices have been rising, commonly referred to as "momentum" stocks

On the basis of that evidence, the Carhart model posits the existence of three systematic risk factors beyond the market risk factor. They are named, in the same order as above, the following:

- Small minus big (SMB)
- High minus low (HML)
- Winners minus losers (WML)

Equation 3 is the Carhart model, in which the excess return on the portfolio is explained as a function of the portfolio's sensitivity to a market index (RMRF), a market capitalization factor (SMB), a book-to-market factor (HML), which is essentially the reciprocal of the aforementioned price-to-book ratio, and a momentum factor (WML).

$$R_p - R_F = a_p + b_{p1}\mathrm{RMRF} + b_{p2}\mathrm{SMB} + b_{p3}\mathrm{HML} + b_{p4}\mathrm{WML} + \varepsilon_p, \tag{3}$$

where

R_p and R_F = the return on the portfolio and the risk-free rate of return, respectively

a_p = "alpha" or return in excess of that expected given the portfolio's level of systematic risk (assuming the four factors capture all systematic risk)

b_p = the sensitivity of the portfolio to the given factor

RMRF = the return on a value-weighted equity index in excess of the one-month T-bill rate

SMB = small minus big, a size (market capitalization) factor; SMB is the average return on three small-cap portfolios minus the average return on three large-cap portfolios

HML = high minus low, the average return on two high book-to-market portfolios minus the average return on two low book-to-market portfolios

WML = winners minus losers, a momentum factor; WML is the return on a portfolio of the past year's winners minus the return on a portfolio of the past year's losers. (Note that WML is an equally weighted average of the stocks with the highest 30% 11-month returns lagged 1 month minus the equally weighted average of the stocks with the lowest 30% 11-month returns lagged 1 month.)

ε_p = an error term that represents the portion of the return to the portfolio, *p*, not explained by the model

Following Equation 2, the Carhart model can be stated as giving equilibrium expected return as

$$E(R_p) = R_F + \beta_{p,1}\text{RMRF} + \beta_{p,2}\text{SMB} + \beta_{p,3}\text{HML} + \beta_{p,4}\text{WML} \quad (4)$$

because the expected value of alpha is zero.

The Carhart model can be viewed as a multifactor extension of the CAPM that explicitly incorporates drivers of differences in expected returns among assets variables that are viewed as anomalies from a pure CAPM perspective. (The term "anomaly" in this context refers to an observed capital market regularity that is not explained by, or contradicts, a theory of asset pricing.) From the perspective of the CAPM, there are size, value, and momentum anomalies. From the perspective of the Carhart model, however, size, value, and momentum represent systematic risk factors; exposure to them is expected to be compensated in the marketplace in the form of differences in mean return.

Size, value, and momentum are common themes in equity portfolio construction, and all three factors continue to have robust uses in active management risk decomposition and return attribution.

TYPES OF MULTIFACTOR MODELS

3

- ☐ describe and compare macroeconomic factor models, fundamental factor models, and statistical factor models
- ☐ describe uses of multifactor models and interpret the output of analyses based on multifactor models

Having introduced the APT, it is appropriate to examine the diversity of multifactor models in current use.

In the following sections, we explain the basic principles of multifactor models and discuss various types of models and their application. We also expand on the APT, which relates the expected return of investments to their risk with respect to a set of factors.

Factors and Types of Multifactor Models

Many varieties of multifactor models have been proposed and researched. We can categorize most of them into three main groups according to the type of factor used:

- In a **macroeconomic factor model**, the factors are surprises in macroeconomic variables that significantly explain returns. In the example of equities, the factors can be understood as affecting either the expected future cash flows of companies or the interest rate used to discount these cash flows back to the present. Among macroeconomic factors that have been used are interest rates, inflation risk, business cycle risk, and credit spreads.
- In a **fundamental factor model**, the factors are attributes of stocks or companies that are important in explaining cross-sectional differences in stock prices. Among the fundamental factors that have been used are the book-value-to-price ratio, market capitalization, the price-to-earnings ratio, and financial leverage.
- In a **statistical factor model**, statistical methods are applied to historical returns of a group of securities to extract factors that can explain the observed returns of securities in the group. In statistical factor models, the factors are actually portfolios of the securities in the group under study and are therefore defined by portfolio weights. Two major types of factor models are factor analysis models and principal components models. In factor analysis models, the factors are the portfolios of securities that best explain (reproduce) historical *return covariances*. In principal components models, the factors are portfolios of securities that best explain (reproduce) the historical *return variances*.

A potential advantage of statistical factor models is that they make minimal assumptions. But the interpretation of statistical factors is generally difficult in contrast to macroeconomic and fundamental factors. A statistical factor that is a portfolio with weights that are similar to market index weights might be interpreted as "the market factor," for example. But in general, associating a statistical factor with economic meaning may not be possible. Because understanding statistical factor models requires substantial preparation in quantitative methods, a detailed discussion of statistical factor models is outside the scope of our coverage.

Our discussion concentrates on macroeconomic factor models and fundamental factor models. Industry use has generally favored fundamental and macroeconomic models, perhaps because such models are much more easily interpreted and rely less on data-mining approaches. Nevertheless, statistical factor models have proponents and are also used in practical applications.

The Structure of Fundamental Factor Models

We earlier gave the equation of a macroeconomic factor model as

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{iK}F_K + \varepsilon_i.$$

We can also represent the structure of fundamental factor models with this equation, but we need to interpret the terms differently.

In fundamental factor models, the factors are stated as *returns* rather than return *surprises* in relation to predicted values, so they do not generally have expected values of zero. This approach changes the meaning of the intercept, which is no longer interpreted as the expected return. Note that if the coefficients were not standardized, as described in the following paragraph, the intercept could be interpreted as the risk-free rate because it would be the return to an asset with no factor risk (zero factor betas) and no asset-specific risk (with standardized coefficients, the intercept is not interpreted beyond being an intercept in a regression included so that the expected asset-specific risk equals zero).

Factor sensitivities are also interpreted differently in most fundamental factor models. In fundamental factor models, the factor sensitivities are attributes of the security. An asset's sensitivity to a factor is expressed using a **standardized beta**: the value of the attribute for the asset minus the average value of the attribute across all stocks divided by the standard deviation of the attribute's values across all stocks.

$$b_{ik} = \frac{\text{Value of attribute } k \text{ for asset } i - \text{Average value of attribute } k}{\sigma(\text{Values of attribute } k)}. \tag{5}$$

Consider a fundamental model for equities that uses a dividend yield factor. After standardization, a stock with an average dividend yield will have a factor sensitivity of 0; a stock with a dividend yield one standard deviation above the average will have a factor sensitivity of 1; and a stock with a dividend yield one standard deviation below the average will have a factor sensitivity of −1. Suppose, for example, that an investment has a dividend yield of 3.5% and that the average dividend yield across all stocks being considered is 2.5%. Further, suppose that the standard deviation of dividend yields across all stocks is 2%. The investment's sensitivity to dividend yield is (3.5% − 2.5%)/2% = 0.50, or one-half standard deviation above average. The scaling permits all factor sensitivities to be interpreted similarly, despite differences in units of measure and scale in the variables. The exception to this interpretation is factors for binary variables, such as industry membership. A company either participates in an industry or does not. The industry factor is represented by dummy variables: The value of the variable is 1 if the stock belongs to the industry and 0 if it does not.

A second distinction between macroeconomic multifactor models and fundamental factor models is that with the former, we develop the factor (surprise) series first and then estimate the factor sensitivities through regressions. With the latter, we generally specify the factor sensitivities (attributes) first and then estimate the factor returns through regressions.

Financial analysts use fundamental factor models for a variety of purposes, including portfolio performance attribution and risk analysis. (*Performance attribution* consists of return attribution and risk attribution. *Return attribution* is a set of techniques used to identify the sources of the excess return of a portfolio against its benchmark. *Risk attribution* addresses the sources of risk, identifying the sources of portfolio volatility for absolute mandates and the sources of tracking risk for relative mandates.) Fundamental factor models focus on explaining the returns to individual stocks using observable fundamental factors that describe either attributes of the securities themselves or attributes of the securities' issuers. Industry membership, price-to-earnings ratio, book-value-to-price ratio, size, and financial leverage are examples of fundamental factors.

Example 4 discusses a study that examined macroeconomic, fundamental, and statistical factor models.

We encounter a range of distinct representations of risk in the fundamental models that are currently used in practical applications. Diversity exists in both the identity and exact definition of factors as well as in the underlying functional form and estimation procedures. Despite the diversity, we can place the factors of most fundamental factor models for equities into three broad groups:

- **Company fundamental factors**. These are factors related to the company's internal performance. Examples are factors relating to earnings growth, earnings variability, earnings momentum, and financial leverage.
- **Company share-related factors**. These factors include valuation measures and other factors related to share price or the trading characteristics of the shares. In contrast to the previous category, these factors directly incorporate investors' expectations concerning the company. Examples include price multiples, such as earnings yield, dividend yield, and book to market. Market capitalization falls under this heading. Various models incorporate variables relating to share price momentum, share price volatility, and trading activity that fall in this category.
- **Macroeconomic factors**. Sector or industry membership factors fall under this heading. Various models include such factors as CAPM beta, other similar measures of systematic risk, and yield curve level sensitivity—all of which can be placed in this category.

For global factor models, in particular, a classification of country, industry, and style factors is often used. In that classification, country and industry factors are dummy variables for country and industry membership, respectively. Style factors include those related to earnings, risk, and valuation that define types of securities typical of various styles of investing.

Fixed-Income Multifactor Models

While the previous discussion focuses on equity applications, similar approaches are equally suited to fixed income. In addition, some of the same broad factor groupings are relevant for bonds.

Macroeconomic Multifactor Models

Macroeconomic models, as discussed earlier, are easily translatable to fixed-income investing. For instance, surprises to economic growth, interest rates, and inflation will impact bond pricing, often mechanically.

Consider a bond factor model in which the returns are correlated with two factors. Following our earlier discussion, returns for bonds are assumed to be correlated with surprises in inflation rates and surprises in GDP growth. The return to *bond i*, R_i, can be modeled as

$$R_i = a_i + b_{i1}F_{INFL} + b_{i2}F_{GDP} + \varepsilon_i,$$

where

R_i = the return to bond i

a_i = the expected return to bond i

b_{i1} = the sensitivity of the return on bond i to inflation rate surprises

F_{INFL} = the surprise in inflation rates

b_{i2} = the sensitivity of the return on bond i to GDP growth surprises

F_{GDP} = the surprise in GDP growth (assumed to be uncorrelated with F_{INFL})

ε_i = an error term with a zero mean that represents the portion of the return to bond i not explained by the factor model

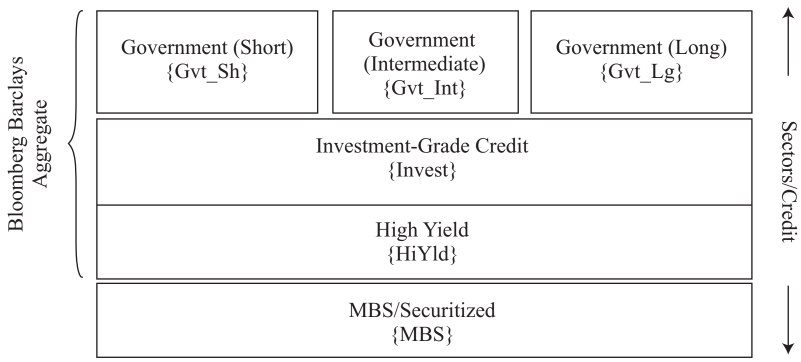
Fundamental Multifactor Models

Fundamental factor approaches have been developed to address the unique aspects of fixed income by using, for example, the following categories:

- Duration (ranging from cash to long-dated bonds)
- Credit (ranging from government securities to high yield)
- Currency (ranging from home currency to foreign developed and emerging market currencies)
- Geography (specific developed and emerging markets)

A simplified structure, shown in Exhibit 4, divides the US Barclays Bloomberg Aggregate index, a standard bond benchmark, into sectors, where each has such unique factor exposures as spread or duration. This factor model was developed by Dopfel (2004), and the factors have been chosen to cover three macro sectors plus high yield. The government sector is further broken down into three maturity buckets to help explain duration exposures.

Exhibit 4: A Simple Fixed-Income Fundamental Framework



Source: Dopfel (2004).

These components can be thought of as both macroeconomic and fundamental. They are macroeconomically oriented because spread, or expected return above similar duration government bonds, is closely related to the growth factor and is sometimes expressed as simply credit spread. Fundamentally, duration can also be thought of as a factor. This simplistic approach can be extended to encompass global fixed-income markets or adapted to a specific country's market:

$$R_i = a_i + b_{i1}F_{Gvt\_Sh} + b_{i2}F_{Gvt\_Int} + b_{i3}F_{Gvt\_Lg} + b_{i4}F_{Invest} + b_{i5}F_{HiYld} + b_{i6}F_{MBS} + \varepsilon_i, \text{where}$$

R_i = the return to bond i

a_i = the expected return to bond i

b_{ik} = the sensitivity of the return on bond i to factor k

F_k = factor k, where k represents "Gov't (Short)," "Gov't (Long)," and so on

ε_i = an error term with a zero mean that represents the portion of the return to bond i not explained by the factor model

The historic style factor weights, b_{ik}, are determined by a constrained regression (the constraint being that the total "weights" add up to 100%) of the portfolio returns against the listed style factors.

This framework lends itself readily to performance and risk attribution, along with portfolio construction. When evaluating a fixed-income manager, such characteristics as spread, duration, yield, and quality can be incorporated. This type of framework can also be extended to ESG (environmental, social, and governance) considerations as these should be generally unrelated to the basic duration and spread foundation presented. For instance, each box in Exhibit 4 could also contain E, S, and G scores, which after the initial disaggregation of a fixed-income return stream into duration and spread components could be used to model the overall portfolio's aggregate scores. For forward-looking portfolio construction purposes, a desired loading on duration, spread, and ESG scores could be handled with a quantitative objective function.

Risk and Style Multifactor Models

Another category of multifactor approach incorporates risk, or style, factors, several of which can thematically apply across asset classes. Examples of such factors include momentum, value, carry, and volatility. Many of these are similar in construction to those commonly used in equity portfolios. Examples include defining value as real (inflation-adjusted) yield, momentum as the previous 12-month excess return, and carry as the term spread. An illustrative example of risk factor approaches, in this case across asset classes, can be found in Exhibit 5.

Exhibit 5: An Illustration of Factor Approaches across Asset Classes

	Factor/Asset Class	Equity	Credit	Treasury	Commodities	Currency
Macro	Economic Growth	xx	x			
	Rates		x	xx		
	Inflation			x	xx	x

	Factor/Asset Class	Equity	Credit	Treasury	Commodities	Currency
Style	Value	xx	x		x	x
	Size	xx				
	Momentum	xx	xx	xx	xx	xx
	Carry	x	xx	xx	xx	xx
	Low-Volume	xx	x			

Note: Double check marks denote strong alignment between risk factor and asset class; single check marks denote moderate alignment.
*Source:*Podkaminer (2017).

Of the three types of multifactor models (macroeconomic, fundamental, and statistical), statistical models can be most easily applied to various asset classes, including fixed income, as no asset-class-specific tuning is required given the minimal required assumption set. This is in contrast to macroeconomic and fundamental models, which both require adjustments and repurposing to ensure the frameworks are fit for the specifics of bond investing. Example 3 shows how expected return could be expressed.

EXAMPLE 3

Calculating Factor-Based Expected Returns at the Portfolio Level

1. A fixed-income portfolio has the following estimated exposures: 35% intermediate government bonds, 40% investment-grade credit, 5% securitized, and 20% high yield. The expected component returns are
   - **A.** Short government bonds: 0.25%
   - **B.** Intermediate government bonds: 1.50%
   - **C.** Long government bonds: 3.00%
   - **D.** Investment-grade credit: 4.25%
   - **E.** MBS/Securitized: 1.75%
   - **F.** High yield: 5.75%
   - **G.** Express the expected return of the portfolio.

Solution

Expected return could be expressed as

$E(R) = 3.46\%$

$= (0.35)(1.50\%) + (0.40)(4.25\%) + (0.05)(1.75\%) + (0.20)(5.75\%).$

EXAMPLE 4

Reconciling Bond Portfolio Characteristics Using Style Factors

Talia Ayalon is evaluating intermediate duration (between 5 and 7 years) investment-grade fixed-income strategies using the framework presented in Exhibit 4. One of the strategies has the following sector attribution (totaling to 100%):

Gov't (Short) 2%	Gov't (Intermediate) 4%	Gov't (Long) 14%
Investment-Grade Credit 56%		
MBS/Securitized 6%		
High Yield 18%		

Are these sector exposures consistent with an intermediate duration investment-grade approach? Why or why not?

Suggested answer:

No, the sector exposures are inconsistent with the stated approach for two reasons: 1) The 18% exposure to high yield constitutes a significant amount of below investment-grade exposure. A true investment-grade portfolio would, for example, not have exposure to high yield. 2) The loading to longer duration sectors implies a longer-than-intermediate duration for the portfolio.

MACROECONOMIC FACTOR MODELS

4

- ☐ calculate the expected return on an asset given an asset's factor sensitivities and the factor risk premiums
- ☐ describe and compare macroeconomic factor models, fundamental factor models, and statistical factor models

The representation of returns in macroeconomic factor models assumes that the returns to each asset are correlated with only the surprises in some factors related to the aggregate economy, such as inflation or real output. We can define *surprise* in general as the actual value minus predicted (or expected) value. A factor's surprise is the component of the factor's return that was unexpected, and the factor surprises constitute the model's independent variables. This idea contrasts with the representation of independent variables as returns in Equation 2, reflecting the fact that how the independent variables are represented varies across different types of models.

Suppose that K macro factors explain asset returns. Then in a macroeconomic factor model, Equation 6 expresses the return of asset i:

$$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{iK}F_K + \varepsilon_i, \quad (6)$$

where

R_i = the return to asset i

a_i = the expected return to asset i

b_{ik} = the sensitivity of the return on asset i to a surprise in factor k, $k = 1, 2, \ldots, K$

F_k = the surprise in the factor k, $k = 1, 2, \ldots, K$

ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model

Surprise in a macroeconomic factor can be illustrated as follows: Suppose we are analyzing monthly returns for stocks. At the beginning of each month, we have a prediction of inflation for the month. The prediction may come from an econometric model or a professional economic forecaster, for example. Suppose our forecast at the beginning of the month is that inflation will be 0.4% during the month. At the end of the month, we find that inflation was actually 0.5% during the month. During any month,

Actual inflation = Predicted inflation + Surprise inflation.

In this case, actual inflation was 0.5% and predicted inflation was 0.4%. Therefore, the surprise in inflation was 0.5% − 0.4% = 0.1%.

What is the effect of defining the factors in terms of surprises? Suppose we believe that inflation and gross domestic product (GDP) growth are two factors that carry risk premiums; that is, inflation and GDP represent priced risk. (GDP is a money measure of the goods and services produced within a country's borders.) We do not use the predicted values of these variables because the predicted values should already be reflected in stock prices and thus in their expected returns. The intercept a_i, the expected return to asset i, reflects the effect of the predicted values of the macroeconomic variables on expected stock returns. The surprise in the macroeconomic variables during the month, however, contains new information about the variable. As a result, this model structure analyzes the return to an asset in three components: the asset's expected return, its unexpected return resulting from new information about the factors, and an error term.

Consider a factor model in which the returns to each asset are correlated with two factors. For example, we might assume that the returns for a particular stock are correlated with surprises in inflation rates and surprises in GDP growth. For stock i, the return to the stock can be modeled as

$$R_i = a_i + b_{i1}F_{INFL} + b_{i2}F_{GDP} + \varepsilon_i,$$

where

R_i = the return to stock i

a_i = the expected return to stock i

b_{i1} = the sensitivity of the return on stock i to inflation rate surprises

F_{INFL} = the surprise in inflation rates

b_{i2} = the sensitivity of the return on stock i to GDP growth surprises

F_{GDP} = the surprise in GDP growth (assumed to be uncorrelated with F_{INFL})

ε_i = an error term with a zero mean that represents the portion of the return to asset i not explained by the factor model

Consider first how to interpret b_{i1}. The factor model predicts that a 1 percentage point surprise in inflation rates will contribute b_{i1} percentage points to the return to stock i. The slope coefficient b_{i2} has a similar interpretation relative to the GDP growth factor. Thus, slope coefficients are naturally interpreted as the factor sensitivities of the asset. A *factor sensitivity* is a measure of the response of return to each unit of increase in a factor, holding all other factors constant. (Factor sensitivities are sometimes called *factor betas* or *factor loadings*.)

Now consider how to interpret the intercept a_i. Recall that the error term has a mean or average value of zero. If the surprises in both inflation rates and GDP growth are zero, the factor model predicts that the return to asset i will be a_i. Thus, a_i is the expected value of the return to stock i.

Finally, consider the error term, ε_i. The intercept a_i represents the asset's expected return. The term $(b_{i1}F_{INFL} + b_{i2}F_{GDP})$ represents the return resulting from factor surprises, and we have interpreted these as the sources of risk shared with other assets. The term ε_i is the part of return that is unexplained by expected return or the factor surprises. If we have adequately represented the sources of common risk (the factors), then ε_i must represent an asset-specific risk. For a stock, it might represent the return from an unanticipated company-specific event.

The risk premium for the GDP growth factor is typically positive. The risk premium for the inflation factor, however, is typically negative. Thus, an asset with a positive sensitivity to the inflation factor—an asset with returns that tend to be positive in response to unexpectedly high inflation—would have a lower required return than if its inflation sensitivity were negative; an asset with positive sensitivity to inflation would be in demand for its inflation-hedging ability.

This discussion has broader applications. It can be used for various asset classes, including fixed income and commodities. It can also be used in asset allocation, where asset classes can be examined in relation to inflation and GDP growth, as illustrated in the following exhibit. In Exhibit 6, each quadrant reflects a unique mix of inflation and economic growth expectations. Certain asset classes or securities can be expected to perform differently in various inflation and GDP growth regimes and can be plotted in the appropriate quadrant, thus forming a concrete illustration of a two-factor model.

Exhibit 6: Growth and Inflation Factor Matrix

	Inflation	
	Low Inflation/Low Growth ▪ Cash ▪ Government bonds	**High Inflation/Low Growth** ▪ Inflation-linked bonds ▪ Commodities ▪ Infrastructure
Growth	**Low Inflation/High Growth** ▪ Equity ▪ Corporate debt	**High Inflation/High Growth** ▪ Real assets (real estate, timberland, farmland, energy)

Note: Entries are assets likely to benefit from the specified combination of growth and inflation.

In macroeconomic factor models, the time series of factor surprises are constructed first. Regression analysis is then used to estimate assets' sensitivities to the factors. In practice, estimated sensitivities and intercepts are often acquired from one of the many consulting companies that specialize in factor models. When we have the parameters for the individual assets in a portfolio, we can calculate the portfolio's parameters as a weighted average of the parameters of individual assets. An individual asset's weight in that calculation is the proportion of the total market value of the portfolio that the individual asset represents.

EXAMPLE 5

Estimating Returns for a Two-Stock Portfolio Given Factor Sensitivities

Suppose that stock returns are affected by two common factors: surprises in inflation and surprises in GDP growth. A portfolio manager is analyzing the returns on a portfolio of two stocks, Manumatic (MANM) and Nextech (NXT). The following equations describe the returns for those stocks, where the factors F_{INFL} and F_{GDP} represent the surprise in inflation and GDP growth, respectively:

$$R_{MANM} = 0.09 - 1F_{INFL} + 1F_{GDP} + \varepsilon_{MANM}.$$

$$R_{NXT} = 0.12 + 2F_{INFL} + 4F_{GDP} + \varepsilon_{NXT}.$$

One-third of the portfolio is invested in Manumatic stock, and two-thirds is invested in Nextech stock.

In evaluating the equations for surprises in inflation and GDP, convert amounts stated in percentage terms to decimal form.

1. Formulate an expression for the return on the portfolio.

Solution to 1:

The portfolio's return is the following weighted average of the returns to the two stocks:

$$R_P = (1/3)(0.09) + (2/3)(0.12) + [(1/3)(-1) + (2/3)(2)]F_{INFL} + [(1/3)(1) + (2/3)(4)] F_{GDP} + (1/3)\varepsilon_{MANM} + (2/3)\varepsilon_{NXT}$$

$$= 0.11 + 1F_{INFL} + 3F_{GDP} + (1/3)\varepsilon_{MANM} + (2/3)\varepsilon_{NXT}.$$

2. State the expected return on the portfolio.

Solution to 2:

The expected return on the portfolio is 11%, the value of the intercept in the expression obtained in the solution to 1.

3. Calculate the return on the portfolio given that the surprises in inflation and GDP growth are 1% and 0%, respectively, assuming that the error terms for MANM and NXT both equal 0.5%.

Solution to 3:

$$R_P = 0.11 + 1F_{INFL} + 3F_{GDP} + (1/3)\varepsilon_{MANM} + (2/3)\varepsilon_{NXT}$$

$$= 0.11 + 1(0.01) + 3(0) + (1/3)(0.005) + (2/3)(0.005)$$

$$= 0.125, \text{ or } 12.5\%.$$

FUNDAMENTAL FACTOR MODELS

5

- ☐ calculate the expected return on an asset given an asset's factor sensitivities and the factor risk premiums
- ☐ describe and compare macroeconomic factor models, fundamental factor models, and statistical factor models

EXAMPLE 6

Comparing Types of Factor Models

Connor (1995) contrasted a macroeconomic factor model with a fundamental factor model to compare how well the models explain stock returns.

Connor reported the results of applying a macroeconomic factor model to the returns for 779 large-cap US stocks based on monthly data from January 1985 through December 1993. Using five macroeconomic factors, Connor was able to explain approximately 11% of the variance of return on these stocks. Exhibit 7 shows his results.

Exhibit 7: The Explanatory Power of the Macroeconomic Factors

Factor	Explanatory Power from Using Each Factor Alone	Increase in Explanatory Power from Adding Each Factor to All the Others
Inflation	1.3%	0.0%
Term structure	1.1%	7.7%
Industrial production	0.5%	0.3%
Default premium	2.4%	8.1%
Unemployment	–0.3%	0.1%
All factors (total explanatory power)		10.9%

Notes: The explanatory power of a given model was computed as 1 – [(Average asset –Specific variance of return across stocks)/(Average total variance of return across stocks)]. The variance estimates were corrected for degrees of freedom, so the marginal contribution of a factor to explanatory power can be zero or negative. Explanatory power captures the proportion of the total variance of return that a given model explains for the average stock.

Source: Connor (1995).

Connor also reported a fundamental factor analysis of the same companies. The factor model employed was the BARRA US-E2 model (as of 2019, the current version is E4). Exhibit 8 shows these results. In the exhibit, "variability in markets" represents the stock's volatility, "success" is a price momentum variable, "trade activity" distinguishes stocks by how often their shares trade, and "growth" distinguishes stocks by past and anticipated earnings growth (explanations of variables are from Grinold and Kahn 1994).

Exhibit 8: The Explanatory Power of the Fundamental Factors

Factor	Explanatory Power from Using Each Factor Alone	Increase in Explanatory Power from Adding Each Factor to All the Others
Industries	16.3%	18.0%
Variability in markets	4.3%	0.9%
Success	2.8%	0.8%
Size	1.4%	0.6%
Trade activity	1.4%	0.5%
Growth	3.0%	0.4%
Earnings to price	2.2%	0.6%
Book to price	1.5%	0.6%
Earnings variability	2.5%	0.4%
Financial leverage	0.9%	0.5%
Foreign investment	0.7%	0.4%
Labor intensity	2.2%	0.5%
Dividend yield	2.9%	0.4%
All factors (total explanatory power)		42.6%

Source: Connor (1995).

As Exhibit 8 shows, the most important fundamental factor is "industries," represented by 55 industry dummy variables. The fundamental factor model explained approximately 43% of the variation in stock returns, compared with approximately 11% for the macroeconomic factor model. Because "industries" must sum to the market and the market portfolio is not incorporated in the macroeconomic factor model, some advantage to the explanatory power of the fundamental factor may be built into the specific models being compared. Connor's article also does not provide tests of the statistical significance of the various factors in either model; however, Connor's research is strong evidence for the usefulness of fundamental factor models. Moreover, this evidence is mirrored by the wide use of those models in the investment community. For example, fundamental factor models are frequently used in portfolio performance attribution. Typically, fundamental factor models employ many more factors than macroeconomic factor models, giving a more detailed picture of the sources of an investment manager's returns.

We cannot conclude from this study, however, that fundamental factor models are inherently superior to macroeconomic factor models. Each major type of model has its uses. The factors in various macroeconomic factor models are individually backed by statistical evidence that they represent systematic risk (i.e., risk that cannot be diversified away). The same may not be true of each factor in a fundamental factor model. For example, a portfolio manager can easily construct a portfolio that excludes a particular industry, so exposure to a particular industry is not systematic risk.

The two types of factors, macroeconomic and fundamental, have different implications for measuring and managing risk, in general. The macroeconomic factor set is parsimonious (five variables in the model studied) and allows a portfolio manager to incorporate economic views into portfolio construction by adjustments to portfolio exposures to macro factors. The fundamental factor set examined by Connor is large (67 variables, including the 55 industry dummy variables); at the expense of greater complexity, it can give a more detailed picture of risk in terms that are easily related to company and security characteristics. Connor found that the macroeconomic factor model had no marginal explanatory power when added to the fundamental factor model, implying that the fundamental risk attributes capture all the risk characteristics represented by the macroeconomic factor betas. Because the fundamental factors supply such a detailed description of the characteristics of a stock and its issuer, however, this finding is not necessarily surprising.

FACTOR MODELS IN RETURN ATTRIBUTION

6

- ☐ calculate the expected return on an asset given an asset's factor sensitivities and the factor risk premiums
- ☐ describe uses of multifactor models and interpret the output of analyses based on multifactor models
- ☐ describe the potential benefits for investors in considering multiple risk dimensions when modeling asset returns

The following sections present selected applications of multifactor models in investment practice. The applications discussed are return attribution, risk attribution, portfolio construction, and strategic portfolio decisions. We begin by discussing portfolio return attribution and risk attribution, focusing on the analysis of benchmark-relative returns. After discussing performance attribution and risk analysis, we explain the use of multifactor models in creating a portfolio with a desired set of risk exposures.

Additionally, multifactor models can be used for asset allocation purposes. Some large, sophisticated asset owners have chosen to define their asset allocation opportunity sets in terms of macroeconomic or thematic factors and aggregate factor exposures (represented by pure factor portfolios as defined earlier). Many others are examining their traditionally derived asset allocation policies using factor models to map asset class exposure to factor sensitivities. The trend toward factor-based asset allocation has two chief causes: First is the increasing availability of sophisticated factor models (like the BARRA models used in the following examples); second is the more intense focus by asset owners on the many dimensions of risk.

Factor Models in Return Attribution

Multifactor models can help us understand in detail the sources of a manager's returns relative to a benchmark. For simplicity, in this section we analyze the sources of the returns of a portfolio fully invested in the equities of a single national equity market, which allows us to ignore the roles of country selection, asset allocation, market timing, and currency hedging. The same methodology can, however, be applied across asset classes and geographies.

Analysts often favor fundamental multifactor models in decomposing (separating into basic elements) the sources of returns. In contrast to statistical factor models, fundamental factor models allow the sources of portfolio performance to be described using commonly understood terms. Fundamental factors are also thematically understandable and can be incorporated into simple narratives for clients concerning return or risk attribution.

Also, in contrast to macroeconomic factor models, fundamental models express investment style choices and security characteristics more directly and often in greater detail.

We first need to understand the objectives of active managers. As mentioned previously, managers are commonly evaluated relative to a specified benchmark. Active portfolio managers hold securities in different-from-benchmark weights in an attempt to add value to their portfolios relative to a passive investment approach. Securities held in different-from-benchmark weights reflect portfolio manager expectations that differ from consensus expectations. For an equity manager, those expectations may relate to common factors driving equity returns or to considerations unique to a company. Thus, when we evaluate an active manager, we want to ask such questions as, Did the manager have insights that were effectively translated into returns in excess of those that were available from a passive alternative? Analyzing the sources of returns using multifactor models can help answer these questions.

The return on a portfolio, R_p, can be viewed as the sum of the benchmark's return, R_B, and the **active return** (portfolio return minus benchmark return):

$$\text{Active return} = R_p - R_B. \quad (7)$$

With the help of a factor model, we can analyze a portfolio manager's active return as the sum of two components. The first component is the product of the portfolio manager's factor tilts (over- or underweights relative to the benchmark factor sensitivities) and the factor returns; we call this component the return from factor tilts. The second component of active return reflects the manager's skill in individual asset selection (ability to overweight securities that outperform the benchmark or underweight securities that underperform the benchmark); we call this component security selection. Equation 8 shows the decomposition of active return into those two components, where k represents the factor or factors represented in the benchmark portfolio:

$$\begin{aligned}\text{Active return} &= \sum_{k=1}^{K}\left[(\text{Portfolio sensitivity})_k - (\text{Benchmark sensitivity})_k\right] \\ &\times \quad (\text{Factor return})_k + \text{Security selection}\end{aligned}. \quad (8)$$

In Equation 8, the portfolio's and benchmark's sensitivities to each factor are calculated as of the beginning of the evaluation period.

EXAMPLE 7

Four-Factor Model Active Return Decomposition

As an equity analyst at a pension fund sponsor, Ronald Service uses the Carhart four-factor multifactor model of Equation 3a to evaluate US equity portfolios:

$$R_p - R_F = a_p + b_{p1}\text{RMRF} + b_{p2}\text{SMB} + b_{p3}\text{HML} + b_{p4}\text{WML} + \varepsilon_p.$$

Service's current task is to evaluate the performance of the most recently hired US equity manager. That manager's benchmark is an index representing the performance of the 1,000 largest US stocks by market value. The manager describes himself as a "stock picker" and points to his performance in beating the benchmark as evidence that he is successful. Exhibit 9 presents an analysis based on the Carhart model of the sources of that manager's active return during the year, given an assumed set of factor returns. In Exhibit 9, the entry "A. Return from Factor Tilts = 2.1241%" is the sum of the four numbers above it. The entry "B. Security Selection" gives security selection as equal to −0.05%. "C. Active Return" is found as the sum of these two components: 2.1241% + (−0.05%) = 2.0741%.

Exhibit 9: Active Return Decomposition

	Factor Sensitivity				Contribution to Active Return	
Factor	Portfolio (1)	Benchmark (2)	Difference (3) = (1) − (2)	Factor Return (4)	Absolute (3) × (4)	Proportion of Total Active
RMRF	0.95	1.00	−0.05	5.52%	−0.2760%	−13.3%
SMB	−1.05	−1.00	−0.05	−3.35%	0.1675%	8.1%
HML	0.40	0.00	0.40	5.10%	2.0400%	98.4%
WML	0.05	0.03	0.02	9.63%	0.1926%	9.3%
				A. Return from Factor Tilts =	2.1241%	102.4%
				B. Security Selection =	−0.0500%	−2.4%
				C. Active Return (A + B) =	2.0741%	100.0%

From his previous work, Service knows that the returns to growth-style portfolios often have a positive sensitivity to the momentum factor (WML). By contrast, the returns to certain value-style portfolios, in particular those following a contrarian strategy, often have a negative sensitivity to the momentum factor. Using the information given, address the following questions (assume the benchmark chosen for the manager is appropriate):

1. Determine the manager's investment mandate and his actual investment style.

Solution:

The benchmarks chosen for the manager should reflect the baseline risk characteristics of the manager's investment opportunity set and his mandate. We can ascertain whether the manager's actual style follows the mandate by examining the portfolio's actual factor exposures:

- The sensitivities of the benchmark are consistent with the description in the text. The sensitivity to RMRF of 1 indicates that the assigned benchmark has average market risk, consistent with it being a broad-based index; the negative sensitivity to SMB indicates a large-cap orientation. The mandate might be described as large-cap without a value/growth bias (HML is zero) or a momentum bias (WML is close to zero).
- Stocks with high book-to-market ratios are generally viewed as value stocks. Because the equity manager has a positive sensitivity to HML (0.40), it appears that the manager has a value orientation. The manager is approximately neutral to the momentum factor, so the equity manager is not a momentum investor and probably not a contrarian value investor. In summary, these considerations suggest that the manager has a large-cap value orientation.

2. Evaluate the sources of the manager's active return for the year.

Solution:

The dominant source of the manager's positive active return was his positive active exposure to the HML factor. The bet contributed approximately 98% of the realized active return of about 2.07%. The manager's active exposure to the overall market (RMRF) was unprofitable, but his active exposures to small stocks (SMB) and to momentum (WML) were profitable. The magnitudes of the manager's active exposures to RMRF, SMB, and WML were relatively small, however, so the effects of those bets on active return were minor compared with his large and successful bet on HML.

3. What concerns might Service discuss with the manager as a result of the return decomposition?

Solution:

Although the manager is a self-described "stock picker," his active return from security selection in this period was actually negative. His positive active return resulted from the concurrence of a large active bet on HML and a high return to that factor during the period. If the market had favored growth rather than value without the manager doing better in individual security selection, the manager's performance would have been unsatisfactory. Service's conversations with the manager should focus on evidence that he can predict changes in returns to the HML factor and on the manager's stock selection discipline.

7 FACTOR MODELS IN RISK ATTRIBUTION

- ☐ explain sources of active risk and interpret tracking risk and the information ratio
- ☐ describe uses of multifactor models and interpret the output of analyses based on multifactor models
- ☐ describe the potential benefits for investors in considering multiple risk dimensions when modeling asset returns

Building on the discussion of active returns, this section explores the analysis of active risk. A few key terms are important to the understanding of how factor models are used to build an understanding of a portfolio manager's risk exposures. We will describe them briefly before moving on to the detailed discussion of risk attribution.

Active risk can be represented by the standard deviation of active returns. A traditional term for that standard deviation is **tracking error** (TE). **Tracking risk** is a synonym for tracking error that is often used in the CFA Program curriculum. We will use the abbreviation TE for the concept of active risk and refer to it usually as tracking error:

$$\text{TE} = s(R_p - R_B). \quad (9)$$

In Equation 9, $s(R_p - R_B)$ indicates that we take the sample standard deviation (indicated by s) of the time series of differences between the portfolio return, R_p, and the benchmark return, R_B. We should be careful that active return and tracking error are stated on the same time basis. As an approximation assuming returns are serially uncorrelated, to annualize a daily TE based on daily returns, we multiply daily TE by $(250)^{1/2}$ based on 250 trading days in a year. To annualize a monthly TE based on monthly returns, we multiply monthly TE by $(12)^{1/2}$.

As a broad indication of the range for tracking error, in US equity markets a well-executed passive investment strategy can often achieve a tracking error on the order of 0.10% or less per year. A low-risk active or enhanced index investment strategy, which makes tightly controlled use of managers' expectations, often has a tracking error goal of 2% per year. A diversified active large-cap equity strategy that might be benchmarked to the S&P 500 Index would commonly have a tracking error in the range of 2%–6% per year. An aggressive active equity manager might have a tracking error in the range of 6%–10% or more.

Somewhat analogous to the use of the traditional Sharpe measure in evaluating absolute returns, the **information ratio** (IR) is a tool for evaluating mean active returns per unit of active risk. The historical or *ex post* IR is expressed as follows:

$$\mathrm{IR} = \frac{\bar{R}_p - \bar{R}_B}{s\left(R_p - R_B\right)}. \tag{10}$$

In the numerator of Equation 10, $\bar{R}_p$ and $\bar{R}_B$ stand for the sample mean return on the portfolio and the sample mean return on the benchmark, respectively. The equation assumes that the portfolio being evaluated has the same systematic risk as its benchmark. To illustrate the calculation, if a portfolio achieved a mean return of 9% during the same period that its benchmark earned a mean return of 7.5% and the portfolio's tracking error (the denominator) was 6%, we would calculate an information ratio of (9% − 7.5%)/6% = 0.25. Setting guidelines for acceptable active risk or tracking error is one of the methods that some investors use to ensure that the overall risk and style characteristics of their investments are in line with their chosen benchmark.

Note that in addition to focusing exclusively on *active* risk, multifactor models can also be used to decompose and attribute sources of *total* risk. For instance, a multi-asset class multi-strategy long/short fund can be evaluated with an appropriate multifactor model to reveal insights on sources of total risk.

EXAMPLE 8

Creating Active Manager Guidelines

The framework of active return and active risk is appealing to investors who want to manage the risk of investments. The benchmark serves as a known and continuously observable reference standard in relation to which quantitative risk and return objectives may be stated and communicated. For example, a US public employee retirement system invited investment managers to submit proposals to manage a "low-active-risk US large-cap equity fund" that would be subject to the following constraints:

- Shares must be components of the S&P 500.
- The portfolio should have a minimum of 200 issues. At time of purchase, the maximum amount that may be invested in any one issuer is 5% of the portfolio at market value or 150% of the issuers' weight within the S&P 500, whichever is greater.
- The portfolio must have a minimum information ratio of 0.30 either since inception or over the last seven years.
- The portfolio must also have tracking risk of less than 3% with respect to the S&P 500 either since inception or over the last seven years.

Once a suitable active manager is found and hired, these requirements can be written into the manager's guidelines. The retirement system's individual mandates would be set such that the sum of mandates across managers would equal the desired risk exposures.

Analysts use multifactor models to understand a portfolio manager's risk exposures in detail. By decomposing active risk, the analyst's objective is to measure the portfolio's active exposure along each dimension of risk—in other words, to understand the sources of tracking error. This can even be done at the level of individual holdings. Among the questions analysts will want to answer are the following:

- What active exposures contributed most to the manager's tracking error?
- Was the portfolio manager aware of the nature of his active exposures, and if so, can he articulate a rationale for assuming them?
- Are the portfolio's active risk exposures consistent with the manager's stated investment philosophy?
- Which active bets earned adequate returns for the level of active risk taken?

In addressing these questions, analysts often choose fundamental factor models because they can be used to relate active risk exposures to a manager's portfolio decisions in a fairly direct and intuitive way. In this section, we explain how to decompose or explain a portfolio's active risk using a multifactor model.

We previously addressed the decomposition of active return; now we address the decomposition of active risk. In analyzing risk, it is more convenient to use variances rather than standard deviations because the variances of uncorrelated variables are additive. We refer to the variance of active return as **active risk squared**:

$$\text{Active risk squared} = s^2(R_p - R_B). \tag{11}$$

We can separate a portfolio's active risk squared into two components:

- **Active factor risk** is the contribution to active risk squared resulting from the portfolio's different-from-benchmark exposures relative to factors specified in the risk model.
- **Active specific risk** or **security selection risk** measures the active non-factor or residual risk assumed by the manager. Portfolio managers attempt to provide a positive average return from security selection as compensation for assuming active specific risk.

As we use the terms, "active specific risk" and "active factor risk" refer to variances rather than standard deviations. When applied to an investment in a single asset class, active risk squared has two components:

$$\text{Active risk squared} = \text{Active factor risk} + \text{Active specific risk}. \tag{12}$$

Active factor risk represents the part of active risk squared explained by the portfolio's active factor exposures. Active factor risk can be found indirectly as the risk remaining after active specific risk is deducted from active risk squared. Active specific risk can be expressed as

$$\text{Active specific risk} = \sum_{i=1}^{n} \left(w_i^a\right)^2 \sigma_{\varepsilon_i}^2,$$

where w_i^a is the ith asset's active weight in the portfolio (that is, the difference between the asset's weight in the portfolio and its weight in the benchmark) and $\sigma_{\varepsilon_i}^2$ is the residual risk of the ith asset (the variance of the ith asset's returns left unexplained by the factors).

The direct procedure for calculating active factor risk is as follows. A portfolio's active factor exposure to a given factor j, b_j^a, is found by weighting each asset's sensitivity to factor j by its active weight and summing the terms:

$$b_j^a = \sum_{i=1}^{n} w_i^a b_{ji}.$$

Then active factor risk equals

$$\sum_{i=1}^{K}\sum_{j=1}^{K} b_i^a b_j^a \operatorname{cov}\left(F_i, F_j\right).$$

EXAMPLE 9

A Comparison of Active Risk

Richard Gray is comparing the risk of four US equity managers who share the same benchmark. He uses a fundamental factor model, the BARRA US-E4 model, which incorporates 12 style factors and a set of 60 industry factors. The style factors measure various fundamental aspects of companies and their shares, such as size, liquidity, leverage, and dividend yield. In the model, companies have non-zero exposures to all industries in which the company operates. Exhibit 10 presents Gray's analysis of the active risk squared of the four managers, based on Equation 12 (note that there is a covariance term in active factor risk, reflecting the correlation of industry membership and the risk indexes, which we assume is negligible in this example). In Exhibit 10, the column labeled "Industry" gives the portfolio's active factor risk associated with the industry exposures of its holdings; the "Style Factor" column gives the portfolio's active factor risk associated with the exposures of its holdings to the 12 style factors.

Exhibit 10: Active Risk Squared Decomposition

	Active Factor				
Portfolio	Industry	Style Factor	Total Factor	Active Specific	Active Risk Squared
A	12.25	17.15	29.40	19.60	49
B	1.25	13.75	15.00	10.00	25
C	1.25	17.50	18.75	6.25	25
D	0.03	0.47	0.50	0.50	1

Note: Entries are in % squared.

Using the information in Exhibit 10, address the following:

1. Contrast the active risk decomposition of Portfolios A and B.

Solution:

Exhibit 11 restates the information in Exhibit 10 to show the proportional contributions of the various sources of active risk. (e.g., Portfolio A's active risk related to industry exposures is 25% of active risk squared, calculated as 12.25/49 = 0.25, or 25%).

The last column of Exhibit 11 now shows the square root of active risk squared—that is, active risk or tracking error.

Exhibit 11: Active Risk Decomposition (restated)

	Active Factor (% of total active)				
Portfolio	**Industry**	**Style Factor**	**Total Factor**	**Active Specific (% of total active)**	**Active Risk**
A	25%	35%	60%	40%	7%
B	5%	55%	60%	40%	5%
C	5%	70%	75%	25%	5%
D	3%	47%	50%	50%	1%

Portfolio A has assumed a higher level of active risk than B (7% versus 5%). Portfolios A and B assumed the same proportions of active factor and active specific risk, but a sharp contrast exists between the two in the types of active factor risk exposure. Portfolio A assumed substantial active industry risk, whereas Portfolio B was approximately industry neutral relative to the benchmark. By contrast, Portfolio B had higher active bets on the style factors representing company and share characteristics.

2. Contrast the active risk decomposition of Portfolios B and C.

Solution:

Portfolios B and C were similar in their absolute amounts of active risk. Furthermore, both Portfolios B and C were both approximately industry neutral relative to the benchmark. Portfolio C assumed more active factor risk related to the style factors, but B assumed more active specific risk. It is also possible to infer from the greater level of B's active specific risk that B is somewhat less diversified than C.

3. Characterize the investment approach of Portfolio D.

Solution:

Portfolio D appears to be a passively managed portfolio, judging by its negligible level of active risk. Referring to Exhibit 11, Portfolio D's active factor risk of 0.50, equal to 0.707% expressed as a standard deviation, indicates that the portfolio's risk exposures very closely match the benchmark.

The discussion of performance attribution and risk analysis has used examples related to common stock portfolios. Multifactor models have also been effectively used in similar roles for portfolios of bonds and other asset classes. For example, such factors as duration and spread can be used to decompose the risk and return of a fixed-income manager.

FACTOR MODELS IN PORTFOLIO CONSTRUCTION

8

- ☐ describe uses of multifactor models and interpret the output of analyses based on multifactor models
- ☐ describe the potential benefits for investors in considering multiple risk dimensions when modeling asset returns

Equally as important to the use of multifactor models in analyzing a portfolio's active returns and active risk is the use of such multifactor models in portfolio construction. At this stage of the portfolio management process, multifactor models permit the portfolio manager to make focused bets or to control portfolio risk relative to the benchmark's risk. This greater level of detail in modeling risk that multifactor models afford is useful in both passive and active management.

- *Passive management*. In managing a fund that seeks to track an index with many component securities, portfolio managers may need to select a sample of securities from the index. Analysts can use multifactor models to replicate an index fund's factor exposures, mirroring those of the index tracked.
- *Active management*. Many quantitative investment managers rely on multifactor models in predicting alpha (excess risk-adjusted returns) or relative return (the return on one asset or asset class relative to that of another) as part of a variety of active investment strategies. In constructing portfolios, analysts use multifactor models to establish desired risk profiles.
- *Rules-based active management (alternative indexes)*. These strategies routinely tilt toward such factors as size, value, quality, or momentum when constructing portfolios. As such, alternative index approaches aim to capture some systematic exposure traditionally attributed to manager skill, or "alpha," in a transparent, mechanical, rules-based manner at low cost. Alternative index strategies rely heavily on factor models to introduce intentional factor and style biases versus capitalization-weighted indexes.

In the following, we explore some of these uses in more detail. As indicated, an important use of multifactor models is to establish a specific desired risk profile for a portfolio. In the simplest instance, the portfolio manager may want to create a portfolio with sensitivity to a single factor. This particular (pure) factor portfolio would have a sensitivity of 1 for that factor and a sensitivity (or weight) of 0 for all other factors. It is thus a portfolio with exposure to only one risk factor and exactly represents the risk of that factor. As a pure bet on a source of risk, factor portfolios are of interest to a portfolio manager who wants to hedge that risk (offset it) or speculate on it. This simple case can be expanded to multiple factors where a factor replication portfolio can be built based either on an existing target portfolio or on a set of desired exposures. Example 10 illustrates the use of factor portfolios.

EXAMPLE 10

Factor Portfolios

Analyst Wanda Smithfield has constructed six portfolios for possible use by portfolio managers in her firm. The portfolios are labeled A, B, C, D, E, and F in Exhibit 12. Smithfield adapts a macroeconomic factor model based on research presented in Burmeister, Roll, and Ross (1994). The model includes five factors:

- Confidence risk, based on the yield spread between corporate bonds and government bonds. A positive surprise in the spread suggests that investors are willing to accept a smaller reward for bearing default risk and so that confidence is high.
- Time horizon risk, based on the yield spread between 20-year government bonds and 30-day Treasury bills. A positive surprise indicates increased investor willingness to invest for the long term.
- Inflation risk, measured by the unanticipated change in the inflation rate.
- Business cycle risk, measured by the unexpected change in the level of real business activity.
- Market timing risk, measured as the portion of the return on a broad-based equity index that is unexplained by the first four risk factors.

Exhibit 12: Factor Portfolios

	Portfolios					
Risk Factor	**A**	**B**	**C**	**D**	**E**	**F**
Confidence risk	0.50	0.00	1.00	0.00	0.00	0.80
Time horizon risk	1.92	0.00	1.00	1.00	1.00	1.00
Inflation risk	0.00	0.00	1.00	0.00	0.00	−1.05
Business cycle risk	1.00	1.00	0.00	0.00	1.00	0.30
Market timing risk	0.90	0.00	1.00	0.00	0.00	0.75

Note: Entries are factor sensitivities.

1. A portfolio manager wants to place a bet that real business activity will increase.

   Determine and justify the portfolio among the six given that would be most useful to the manager.

Solution:

Portfolio B is the most appropriate choice. Portfolio B is the factor portfolio for business cycle risk because it has a sensitivity of 1 to business cycle risk and a sensitivity of 0 to all other risk factors. Portfolio B is thus efficient for placing a pure bet on an increase in real business activity.

2. Would the manager take a long or short position in the portfolio chosen in Part A?

Solution:

The manager would take a long position in Portfolio B to place a bet on an increase in real business activity.

3. A portfolio manager wants to hedge an existing positive (long) exposure to time horizon risk.

Determine and justify the portfolio among the six given that would be most useful to the manager.

Solution:

Portfolio D is the appropriate choice. Portfolio D is the factor portfolio for time horizon risk because it has a sensitivity of 1 to time horizon risk and a sensitivity of 0 to all other risk factors. Portfolio D is thus efficient for hedging an existing positive exposure to time horizon risk.

4. What type of position would the manager take in the portfolio chosen in Part A?

Solution:

The manager would take a short position in Portfolio D to hedge the positive exposure to time horizon risk.

CONSTRUCTING MULTIFACTOR PORTFOLIOS

In practice, most stock selection models use some common multifactor structure. Here, we describe constructing two types of multifactor portfolios—a benchmark portfolio and a risk parity portfolio—that target desired risk exposures to eight fundamental factors. The benchmark portfolio equally weights the pure factors, whereas the risk parity portfolio weights the pure factors based on equal risk contribution. We focus on the benchmark and risk parity portfolios because their factor weighting schemes are clear and objective.

Setting the Scene: Pure Factor Portfolios

For demonstration purposes, we use fundamental factor models and choose common company- and company share–related factors from each main investment style (i.e., value, growth, price momentum, analyst sentiment, and quality):

1. *Defensive value*: Trailing earnings yield—companies with high earnings yield are preferred.
2. *Cyclical value*: Book-to-market ratio—companies with high book-to-market ratios (i.e., cheap stock valuations) are bought.
3. *Growth*: Consensus FY1/FY0 EPS growth—companies with high expected earnings growth are preferred.
4. *Price momentum*: 12M total return excluding the most recent month—companies with positive price momentum are preferred.
5. *Analyst sentiment*: 3M EPS revision—companies with positive earnings revisions are bought.

6. *Profitability*: Return on equity (ROE)—companies with high ROEs are bought.
7. *Leverage*: Debt/equity ratio—companies with low financial leverage are preferred.
8. *Earnings quality*: Non-cash earnings—companies with low accruals are bought. Research suggests that net income with low levels of non-cash items (i.e., accruals) is less likely to be manipulated.

The stock universe for this demonstration consists of the Russell 3000 Index (US), the S&P/TSX Composite Index (Canada), the MSCI China A Index (China), and the S&P Global Broad Market Index (all other countries). A pure factor portfolio is formed for each of the eight factors by buying the top 20% of stocks and shorting the bottom 20% of stocks ranked by the factor. Stocks held long and short are equally weighted, and the eight factor portfolios are each rebalanced monthly. Note that this demonstration does not account for transaction costs or other portfolio constraints. Other methods for forming pure factor portfolios include ranking stocks by Pearson IC (correlation between prior period factor scores and current period stock returns) or by Spearman Rank IC (correlation between prior period ranked factor scores and current period ranked stock returns), as well as ranking by other univariate regression methods. However, for simplicity, we follow the long–short portfolio approach.

A straightforward way to combine these pure factor portfolios into a multifactor portfolio is equal weighting. We call the equally weighted multifactor portfolio the "benchmark (BM) portfolio." The experience in practice is that portfolios constructed using this simple weighting scheme typically perform at least as well as those using more sophisticated optimization techniques.

Risk parity is a common alternative portfolio construction technique used in the asset allocation space. Risk parity accounts for the volatility of each factor and the correlations of returns among all factors to be combined into the multifactor portfolio. The objective is for each factor to contribute equally to the overall (or targeted) risk of the portfolio. Thus, a risk parity (RP) multifactor portfolio can be created by equally weighting the risk contribution of each of the eight pure factors mentioned.

Constructing and Backtesting Benchmark and Risk Parity Multifactor Portfolios

To create a successful multifactor portfolio strategy, the investment manager needs to perform backtesting to assess factor performance and effectiveness. In a typical backtest, a manager first forms her investment hypothesis, determines her investment rules and processes, collects the required data, and creates the portfolio, and then she periodically rebalances and evaluates the portfolio.

In the rolling window backtesting methodology, analysts use a rolling window framework, fit factors based on the rolling window, rebalance the portfolio periodically, and then track performance. Thus, backtesting is a proxy for actual investing. As new information arrives, investment managers readjust their models and rebalance their stock positions, typically monthly. Thus, they repeat the same in-sample training/out-of-sample testing process. If the investment strategy's performance in out-of-sample periods is desirable and the strategy makes intuitive sense, then it is deemed successful.

The following exhibit illustrates rolling window backtesting of the defensive value factor from November 2011 to April 2012. On 30 November 2011, we compute each stock's trailing 12-month earnings yield, then buy the 20% of stocks with the highest earnings yield and short the bottom quintile of stocks, and assess performance using returns in the next month, December 2011, the out-of-sample (OOS) period. The process is repeated on 31 December 2011, and so on, and finally, we compute the average monthly return, volatility, Sharpe ratio, and drawdown from the test results of the six OOS periods.

An Example of Rolling Window Backtesting of the Defensive Value Factor

	2010:12	2011:01	2011:02	2011:03	2011:04	2011:05	2011:06	2011:07	2011:08	2011:09	2011:10	2011:11	2011:12	2012:01	2012:02	2012:03	2012:04	2012:05
11/30/2011												In-Sample (Last 12M EPS/Price)	OOS					
12/31/2011													In-Sample (Last 12M EPS/Price)	OOS				
1/31/2012														In-Sample (Last 12M EPS/Price)	OOS			
2/29/2012															In-Sample (Last 12M EPS/Price)	OOS		
3/31/2012																In-Sample (Last 12M EPS/Price)	OOS	
4/30/2012																	In-Sample (Last 12M EPS/Price)	OOS

Source: Wolfe Research Luo's QES.

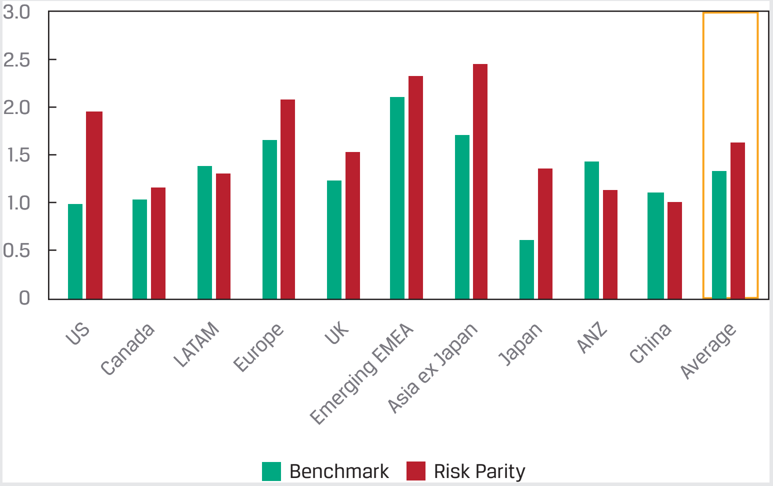
Constructing and backtesting multifactor portfolios is similar to the method just described, except that the rolling window procedure is implemented twice. First, we form the eight pure factor portfolios for each month from 1988 until May 2019 by implementing the rolling window procedure. Then, we combine the underlying factor portfolios into the multifactor portfolios using the two approaches—equally weighting all factors (i.e., benchmark, or BM, allocation) and equally risk weighting all factors (i.e., risk parity, or RP, allocation).

Importantly, the process for creating the multifactor portfolios requires a second implementation of the rolling window procedure to avoid look-ahead bias; note this second rolling window covers the same time span as the first one (i.e., 1988 until May 2019). At each month-end, the previous five years of monthly data are used to estimate the variance–covariance matrix for the eight factor portfolios. Once the covariance matrix is estimated, we optimize and compute the weights for each of the eight pure factor portfolios and then form the RP portfolio. Finally, we compute the returns of the two multifactor portfolios (BM and RP) during this out-of-sample period using the weights at the end of the previous month and the returns of the eight underlying factor portfolios for the current month. This process is repeated every month over the entire horizon of 1988 until May 2019.

We created and backtested the multifactor portfolios using both the equal weighting (BM) scheme and risk parity (RP) scheme for each of 10 markets, including the United States. Both multifactor portfolios are rebalanced monthly to maintain equal factor weights or equal factor risk contributions. As noted previously, the key input to the RP allocation is the monthly variance–covariance matrix for the eight underlying factor portfolios derived from the rolling (five-year) window procedure. To be clear, each of the eight factor portfolios is a long–short portfolio. However, our factor allocation strategies to form the BM and RP multifactor portfolios are long only, meaning the weights allocated to each of the eight factor portfolios are restricted to be non-negative. Therefore, factor weights for the BM and RP portfolios are positive and add to 100%.

In the United States over the period 1993–2019, the weights of the eight factor portfolios in the RP allocation are relatively stable. Interestingly, book-to-market and earnings quality factor portfolios receive the largest allocations, whereas ROE and price momentum factor portfolios have the lowest weights. The RP multifactor portfolio provides a lower cumulative return than does the BM multifactor portfolio; however, the RP portfolio's volatility is substantially lower than that of the BM portfolio. Consequently, in the United States, the RP portfolio's Sharpe ratio is nearly double that of the BM portfolio, as shown in the following exhibit. Outperformance of the RP portfolio in terms of Sharpe ratio is also apparent across most markets examined.

Average Sharpe Ratios for Multifactor Portfolios: Equally Weighted vs. Risk Parity Weighted (1993–2019)



Sources: Bloomberg Finance LLP, FTSE Russell, S&P Capital IQ, Thomson Reuters, Wolfe Research Luo's QES.

The following case was written by Yin Luo, CPA, PStat, CFA, and Sheng Wang, both of Wolfe Research LLC (USA).

9 FACTOR MODELS IN STRATEGIC PORTFOLIO DECISIONS

☐ describe uses of multifactor models and interpret the output of analyses based on multifactor models

☐ describe the potential benefits for investors in considering multiple risk dimensions when modeling asset returns

Multifactor models can help investors recognize considerations that are relevant in making various strategic decisions. For example, given a sound model of the systematic risk factors that affect assets' mean returns, the investor can ask, relative to other investors,

- What types of risk do I have a comparative advantage in bearing?
- What types of risk am I at a comparative disadvantage in bearing?

For example, university endowments, because they typically have very long investment horizons, may have a comparative advantage in bearing business cycle risk of traded equities or the liquidity risk associated with many private equity investments. They may tilt their strategic asset allocation or investments within an asset class to capture the associated risk premiums for risks that do not much affect them. However, such investors may be at a comparative disadvantage in bearing inflation risk to the extent that the activities they support have historically been subject to cost increases running above the average rate of inflation.

This is a richer framework than that afforded by the CAPM, according to which all investors optimally should invest in two funds: the market portfolio and a risk-free asset. Practically speaking, a CAPM-oriented investor might hold a money market fund and a portfolio of capitalization-weighted broad market indexes across many asset classes, varying the weights in these two in accordance with risk tolerance. These types of considerations are also relevant to individual investors. An individual investor who depends on income from salary or self-employment is sensitive to business cycle risk, in particular to the effects of recessions. If this investor compared two stocks with the same CAPM beta, given his concern about recessions, he might be very sensitive to receiving an adequate premium for investing in procyclical assets. In contrast, an investor with independent wealth and no job-loss concerns would have a comparative advantage in bearing business cycle risk; his optimal risky asset portfolio might be quite different from that of the investor with job-loss concerns in tilting toward greater-than-average exposure to the business cycle factor, all else being equal. Investors should be aware of which priced risks they face and analyze the extent of their exposure.

A multifactor approach can help investors achieve better-diversified and possibly more-efficient portfolios. For example, the characteristics of a portfolio can be better explained by a combination of SMB, HML, and WML factors in addition to the market factor than by using the market factor alone.

Thus, compared with single-factor models, multifactor models offer a richer context for investors to search for ways to improve portfolio selection.

SUMMARY

In our coverage of multifactor models, we have presented concepts, models, and tools that are key ingredients to quantitative portfolio management and are used to both construct portfolios and to attribute sources of risk and return.

- Multifactor models permit a nuanced view of risk that is more granular than the single-factor approach allows.
- Multifactor models describe the return on an asset in terms of the risk of the asset with respect to a set of factors. Such models generally include systematic factors, which explain the average returns of a large number of risky assets. Such factors represent priced risk—risk for which investors require an additional return for bearing.
- The arbitrage pricing theory (APT) describes the expected return on an asset (or portfolio) as a linear function of the risk of the asset with respect to a set of factors. Like the CAPM, the APT describes a financial market equilibrium; however, the APT makes less strong assumptions.
- The major assumptions of the APT are as follows:
  - Asset returns are described by a factor model.
  - With many assets to choose from, asset-specific risk can be eliminated.
  - Assets are priced such that there are no arbitrage opportunities.
- Multifactor models are broadly categorized according to the type of factor used:
  - Macroeconomic factor models
  - Fundamental factor models
  - Statistical factor models

- In *macroeconomic* factor models, the factors are surprises in macroeconomic variables that significantly explain asset class (equity in our examples) returns. Surprise is defined as actual minus forecasted value and has an expected value of zero. The factors can be understood as affecting either the expected future cash flows of companies or the interest rate used to discount these cash flows back to the present and are meant to be uncorrelated.
- In *fundamental* factor models, the factors are attributes of stocks or companies that are important in explaining cross-sectional differences in stock prices. Among the fundamental factors are book-value-to-price ratio, market capitalization, price-to-earnings ratio, and financial leverage.
- In contrast to macroeconomic factor models, in fundamental models the factors are calculated as returns rather than surprises. In fundamental factor models, we generally specify the factor sensitivities (attributes) first and then estimate the factor returns through regressions. In macroeconomic factor models, however, we first develop the factor (surprise) series and then estimate the factor sensitivities through regressions. The factors of most fundamental factor models may be classified as company fundamental factors, company share-related factors, or macroeconomic factors.
- In *statistical* factor models, statistical methods are applied to a set of historical returns to determine portfolios that explain historical returns in one of two senses. In factor analysis models, the factors are the portfolios that best explain (reproduce) historical return covariances. In principal-components models, the factors are portfolios that best explain (reproduce) the historical return variances.
- Multifactor models have applications to return attribution, risk attribution, portfolio construction, and strategic investment decisions.
- A factor portfolio is a portfolio with unit sensitivity to a factor and zero sensitivity to other factors.
- Active return is the return in excess of the return on the benchmark.
- Active risk is the standard deviation of active returns. Active risk is also called tracking error or tracking risk. Active risk squared can be decomposed as the sum of active factor risk and active specific risk.
- The information ratio (IR) is mean active return divided by active risk (tracking error). The IR measures the increment in mean active return per unit of active risk.
- Factor models have uses in constructing portfolios that track market indexes and in alternative index construction.
- Traditionally, the CAPM approach would allocate assets between the risk-free asset and a broadly diversified index fund. Considering multiple sources of systematic risk may allow investors to improve on that result by tilting away from the market portfolio. Generally, investors would gain from accepting above average (below average) exposures to risks that they have a comparative advantage (comparative disadvantage) in bearing.

REFERENCES

Bodie, Zvi, Alex Kane, Alan J. Marcus. 2017. Investments. 11th ed.New York: McGraw-Hill Education.

Burmeister, Edwin, Richard Roll, Stephen A. Ross. 1994. "A *Practitioner's* Guide to Arbitrage Pricing Theory." In A Practitioner's Guide to Factor Models. Charlottesville, VA: Research Foundation of the Institute of Chartered Financial Analysts.

Carhart, Mark M. 1997. "On Persistence in Mutual Fund Performance." Journal of Finance52 (1): 57–82. 10.1111/j.1540-6261.1997.tb03808.x

Connor, Gregory. 1995. "The Three Types of Factor Models: A Comparison of Their Explanatory Power." Financial Analysts Journal51 (3): 42–46. 10.2469/faj.v51.n3.1904

Dopfel, Frederick E. 2004. "Fixed-Income Style Analysis and Optimal Manager Structure." Journal of Fixed Income14 (2): 32–43. 10.3905/jfi.2004.439835

Fabozzi, Frank J. 2008. Handbook of Finance, Financial Markets and Instruments. Wiley.

Fama, Eugene F., Kenneth R. French. 1992. "The Cross-Section of Expected Stock Returns." Journal of Finance47 (2): 427–65. 10.1111/j.1540-6261.1992.tb04398.x

Grinold, Richard, Ronald N. Kahn. 1994. "Multi-Factor Models for Portfolio Risk." In A Practitioner's Guide to Factor Models. Charlottesville, VA: Research Foundation of the Institute of Chartered Financial Analysts.

Podkaminer, Eugene L. 2017. "Smart Beta Is the Gateway Drug to Risk Factor Investing." Journal of Portfolio Management43 (5): 130–34. 10.3905/jpm.2017.43.5.130

Ross, S. A. 1976. "The Arbitrage Theory of Capital Asset Pricing." Journal of Economic Theory13 (3): 341–60. 10.1016/0022-0531(76)90046-6

PRACTICE PROBLEMS

1. Compare the assumptions of the arbitrage pricing theory (APT) with those of the capital asset pricing model (CAPM).

2. Assume that the following one-factor model describes the expected return for portfolios:

   $E(R_p) = 0.10 + 0.12\beta_{p,1}$

   Also assume that all investors agree on the expected returns and factor sensitivity of the three highly diversified Portfolios A, B, and C given in the following table:

Portfolio	Expected Return	Factor Sensitivity
A	0.196	0.80
B	0.156	1.00
C	0.244	1.20

   Assuming the one-factor model is correct and based on the data provided for Portfolios A, B, and C, determine if an arbitrage opportunity exists and explain how it might be exploited.

The following information relates to questions 3-8

Carlos Altuve is a manager-of-managers at an investment company that uses quantitative models extensively. Altuve seeks to construct a multi-manager portfolio using some of the funds managed by portfolio managers within the firm. Maya Zapata is assisting him.

Altuve uses arbitrage pricing theory (APT) as a basis for evaluating strategies and managing risks. From his earlier analysis, Zapata knows that Funds A and B in Exhibit 1 are well diversified. He has not previously worked with Fund C and is puzzled by the data because it is inconsistent with APT. He asks Zapata gather additional information on Fund C's holdings and to determine if an arbitrage opportunity exists among these three investment alternatives. Her analysis, using the data in Exhibit 1, confirms that an arbitrage opportunity does exist.

Exhibit 1: Expected Returns and Factor Sensitivities (One-Factor Model)

Fund	Expected Return	Factor Sensitivity
A	0.02	0.5
B	0.04	1.5
C	0.03	0.9

Using a two-factor model, Zapata now estimates the three funds' sensitivity to inflation and GDP growth. That information is presented in Exhibit 2. Zapata assumes a zero value for the error terms when working with the selected two-factor model.

Exhibit 2: Expected Returns and Factor Sensitivities (Two-Factor Model)

Fund	Expected Return	Factor Sensitivity	
		Inflation	GDP Growth
A	0.02	0.5	1.0
B	0.04	1.6	0.0
C	0.03	1.0	1.1

Altuve asks Zapata to calculate the return for Portfolio AC, composed of a 60% allocation to Fund A and 40% allocation to Fund C, using the surprises in inflation and GDP growth in Exhibit 3.

Exhibit 3: Selected Data on Factors

Factor	Research Staff Forecast	Actual Value
Inflation	2.0%	2.2%
GDP Growth	1.5%	1.0%

Finally, Altuve asks Zapata about the return sensitivities of Portfolios A, B, and C given the information provided in Exhibit 3.

3. Which of the following is *not* a key assumption of APT, which is used by Altuve to evaluate strategies and manage risks?
   A. A factor model describes asset returns.
   B. Asset-specific risk can be eliminated through diversification.
   C. Arbitrage opportunities exist among well-diversified portfolios.

4. The arbitrage opportunity identified by Zapata can be exploited with:
   A. Strategy 1: Buy $50,000 Fund A and $50,000 Fund B; sell short $100,000 Fund C.
   B. Strategy 2: Buy $60,000 Fund A and $40,000 Fund B; sell short $100,000 Fund C.
   C. Strategy 3: Sell short $60,000 of Fund A and $40,000 of Fund B; buy $100,000 Fund C.

5. The two-factor model Zapata uses is a:
   A. statistical factor model.
   B. fundamental factor model.
   C. macroeconomic factor model.

6. Based on the data in Exhibits 2 and 3, the return for Portfolio AC, given the sur-

prises in inflation and GDP growth, is *closest* to:

A. 2.02%.

B. 2.40%.

C. 4.98%.

7. The surprise in which of the following had the greatest effect on fund returns?

   A. Inflation on Fund B

   B. GDP growth on Fund A

   C. GDP growth on Fund C

8. Based on the data in Exhibit 2, which fund is most sensitive to the combined surprises in inflation and GDP growth in Exhibit 3?

   A. Fund A

   B. Fund B

   C. Fund C

The following information relates to questions 9-14

Hui Cheung, a portfolio manager, asks her assistant, Ronald Lam, to review the macroeconomic factor model currently in use and to consider a fundamental factor model as an alternative.

The current macroeconomic factor model has four factors:

$$R_i = a_i + b_{i1}F_{GDP} + b_{i2}F_{CAP} + b_{i3}F_{CON} + b_{i4}F_{UNEM} + \varepsilon_i,$$

Where F_{GDP}, F_{CAP}, F_{CON}, and F_{UNEM} represent unanticipated changes in four factors: gross domestic product, manufacturing capacity utilization, consumer spending, and the rate of unemployment, respectively. Lam assumes the error term is equal to zero when using this model.

Lam estimates the current model using historical monthly returns for three portfolios for the most recent five years. The inputs used in and estimates derived from the macroeconomic factor model are presented in Exhibit 1. The US Treasury bond rate of 2.5% is used as a proxy for the risk-free rate of interest.

Exhibit 1: Inputs for and Estimates from the Current Macroeconomic Model

	Factor Sensitivities and Intercept Coefficients				
Factor	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Benchmark**	**Factor Surprise (%)**
Intercept (%)	2.58	3.20	4.33		
F_{GDP}	0.75	1.00	0.24	0.50	0.8
F_{CAP}	−0.23	0.00	−1.45	−1.00	0.5

	Factor Sensitivities and Intercept Coefficients				Factor
Factor	Portfolio 1	Portfolio 2	Portfolio 3	Benchmark	Surprise (%)
F_{CON}	1.23	0.00	0.50	1.10	2.5
F_{UNEM}	−0.14	0.00	−0.05	−0.10	1.0
	Annual Returns, Most Recent Year				
Return (%)	6.00	4.00	5.00	4.50	

Lam uses the macroeconomic model to calculate the tracking error and the mean active return for each portfolio. He presents these statistics in Exhibit 2.

Exhibit 2: Macroeconomic Factor Model Tracking Error and Mean Active Return

Portfolio	Tracking Error	Mean Active Return
Portfolio 1	1.50%	1.50%
Portfolio 2	1.30%	−0.50%
Portfolio 3	1.00%	0.50%

Lam considers a fundamental factor model with four factors:

$$R_i = a_j + b_{j1}F_{\text{LIQ}} + b_{j2}F_{\text{LEV}} + b_{j3}F_{\text{EGR}} + b_{j4}F_{\text{VAR}} + \varepsilon_j,$$

where F_{LIQ}, F_{LEV}, F_{EGR}, and F_{VAR} represent liquidity, financial leverage, earnings growth, and the variability of revenues, respectively.

Lam and Cheung discuss similarities and differences between macroeconomic factor models and fundamental factor models, and Lam offers a comparison of those models to statistical factor models. Lam makes the following statements.

Statement 1 The factors in fundamental factor models are based on attributes of stocks or companies, whereas the factors in macroeconomic factor models are based on surprises in economic variables.

Statement 2 The factor sensitivities are generally determined first in fundamental factor models, whereas the factor sensitivities are estimated last in macroeconomic factor models.

Lam also tells Cheung:

An advantage of statistical factor models is that they make minimal assumptions, and therefore, statistical factor model estimation lends itself to easier interpretation than macroeconomic and fundamental factor models.

Lam tells Cheung that multifactor models can be useful in active portfolio management, but not in passive management. Cheung disagrees; she tells Lam that multifactor models can be useful in both active and passive management.

9. Based on the information in Exhibit 1, the expected return for Portfolio 1 is *closest* to:

A. 2.58%.

B. 3.42%.

C. 6.00%.

10. Based on Exhibit 1, the active risk for Portfolio 2 is explained by surprises in:
    A. GDP.
    B. consumer spending.
    C. all four model factors.

11. Based on Exhibit 2, which portfolio has the best information ratio?
    A. Portfolio 1
    B. Portfolio 2
    C. Portfolio 3

12. Which of Lam's statements regarding macroeconomic factor models and fundamental factor models is correct?
    A. Only Statement 1
    B. Only Statement 2
    C. Both Statements 1 and 2

13. Is Lam's comment regarding statistical factor models correct?
    A. Yes
    B. No, because he is incorrect with respect to interpretation of the models' results
    C. No, because he is incorrect with respect to the models' assumptions

14. Whose statement regarding the use of multifactor models in active and passive portfolio management is correct?
    A. Lam only
    B. Cheung only
    C. Both Lam and Cheung

---

15. Last year the return on Harry Company stock was 5 percent. The portion of the return on the stock not explained by a two-factor macroeconomic factor model was 3 percent. Using the data given below, calculate Harry Company stock's expected return.

Macroeconomic Factor Model for Harry Company Stock

Variable	Actual Value (%)	Expected Value (%)	Stock's Factor Sensitivity
Change in interest rate	2.0	0.0	–1.5
Growth in GDP	1.0	4.0	2.0

16. Which type of factor model is most directly applicable to an analysis of the style orientation (for example, growth vs. value) of an active equity investment manager? Justify your answer.

17. Suppose an active equity manager has earned an active return of 110 basis points, of which 80 basis points is the result of security selection ability. Explain the likely source of the remaining 30 basis points of active return.

The following information relates to questions 18-19

Address the following questions about the information ratio.

18. What is the information ratio of an index fund that effectively meets its investment objective?

19. What are the two types of risk an active investment manager can assume in seeking to increase his information ratio?

---

20. A wealthy investor has no other source of income beyond her investments and that income is expected to reliably meet all her needs. Her investment advisor recommends that she tilt her portfolio to cyclical stocks and high-yield bonds. Explain the advisor's advice in terms of comparative advantage in bearing risk.

SOLUTIONS

1. APT and the CAPM are both models that describe what the expected return on a risky asset should be in equilibrium given its risk. The CAPM is based on a set of assumptions including the assumption that investors' portfolio decisions can be made considering just returns' means, variances, and correlations. The APT makes three assumptions:

   1. A factor model describes asset returns.
   2. There are many assets, so investors can form well-diversified portfolios that eliminate asset-specific risk.
   3. No arbitrage opportunities exist among well-diversified portfolios.

2. According to the one-factor model for expected returns, the portfolio should have these expected returns if they are correctly priced in terms of their risk:

   Portfolio 1 $E(R_A) = 0.10 + 0.12\beta_{A,1} = 0.10 + (0.12)(0.80) = 0.10 + 0.10 = 0.20$

   Portfolio 2 $E(R_B) = 0.10 + 0.12\beta_{B,1} = 0.10 + (0.12)(1.00) = 0.10 + 0.12 = 0.22$

   Portfolio 3 $E(R_C) = 0.10 + 0.12\beta_{C,1} = 0.10 + (0.12)(1.20) = 0.10 + 0.14 = 0.24$

   In the table below, the column for expected return shows that Portfolios A and C are correctly priced but Portfolio B offers too little expected return for its risk, 0.15 or 15%. By shorting Portfolio B (selling an overvalued portfolio) and using the proceeds to buy a portfolio 50% invested in A and 50% invested in C with a sensitivity of 1 that matches the sensitivity of B, for each monetary unit shorted (say each euro), an arbitrage profit of €0.22 – €0.15 = €0.07 is earned.

Portfolio	Expected Return	Factor Sensitivity
A	0.196	0.80
B	0.156	1.00
C	0.244	1.20
0.5A + 0.5C	0.22	1.00

3. C is correct. Arbitrage pricing theory (APT) is a framework that explains the expected return of a portfolio in equilibrium as a linear function of the risk of the portfolio with respect to a set of factors capturing systematic risk. A key assumption of APT is that, in equilibrium, there are no arbitrage opportunities.

4. C is correct. The expected return and factor sensitivities of a portfolio with a 60% weight in Fund A and a 40% weight in Fund B are calculated as weighted averages of the expected returns and factor sensitivities of Funds A and B:

$$\text{Expected return of Portfolio 60/40} = (0.60)(0.02) + (0.40)(0.04) = 0.028\text{, or } 2.8\%$$

$$\text{Factor sensitivity of Portfolio 60/40} = (0.60)(0.5) + (0.40)(1.5) = 0.9$$

Fund	Expected Return	Factor Sensitivity
A	0.02	0.5
B	0.04	1.5
C	0.03	0.9

Portfolio 60/40		
60%A + 40%B	0.028	0.900

Portfolio 50/50		
50%A + 50%B	0.030	1.000

The factor sensitivity of Portfolio 60/40 is identical to that of Fund C; therefore, this strategy results in no factor risk relative to Portfolio C. However, Fund C's expected return of 3.0% is higher than Portfolio 60/40's expected return of 2.8%. This difference supports Strategy 3: buying Fund C and selling short Portfolio 60/40 to exploit the arbitrage opportunity.

5. C is correct. In a macroeconomic factor model, the factors are surprises in macroeconomic variables, such as inflation risk and GDP growth, that significantly explain returns.

6. A is correct. The macroeconomic two-factor model takes the following form:

   $R_i = a_i + b_{i1}F_{\text{INF}} + b_{i2}F_{\text{GDP}} + \varepsilon_i$,

   where F_{INF} and F_{GDP} represent surprises in inflation and surprises in GDP growth, respectively, and a_i represents the expected return to asset i. Using this model and the data in Exhibit 2, the returns for Fund A and Fund C are represented by the following:

   $R_A = 0.02 + 0.5F_{\text{INF}} + 1.0F_{\text{GDP}} + \varepsilon_A$

   $R_C = 0.03 + 1.0F_{\text{INF}} + 1.1F_{\text{GDP}} + \varepsilon_C$

   Surprise in a macroeconomic model is defined as actual factor minus predicted factor. The surprise in inflation is 0.2% (= 2.2% – 2.0%). The surprise in GDP growth is –0.5% (= 1.0% – 1.5%). The return for Portfolio AC, composed of a 60% allocation to Fund A and 40% allocation to Fund C, is calculated as the following:

   $$R_{AC} = (0.6)(0.02) + (0.4)(0.03) + [(0.6)(0.5) + (0.4)(1.0)](0.002) + [(0.6)(1.0) + (0.4)(1.1)](-0.005) + 0.6(0) + 0.4(0)$$

   $$= 2.02\%$$

7. C is correct. Surprise in a macroeconomic model is defined as actual factor minus predicted factor. For inflation, the surprise factor is 2.2% – 2.0% = 0.2%; for GDP growth, the surprise factor is 1.0% – 1.5% = –0.5%. The effect on returns is the product of the surprise and the factor sensitivity.

	Change in Portfolio Return due to Surprise in	
Fund	Inflation	GDP Growth
A	0.5 × 0.2% = 0.10%	1.0 × −0.5% = −0.50%
B	1.6 × 0.2% = 0.32%	0.0 × −0.5% = 0.00%
C	1.0 × 0.2% = 0.20%	1.1 × −0.5% = −0.55%

The effect of the GDP growth surprise on Fund C was the largest single-factor effect on Fund returns (–0.55%).

8. A is correct. The effect of the surprises in inflation and GDP growth on the returns of the three funds is calculated as the following.

	Change in Portfolio Return Because of Surprise in	
Fund	Inflation	GDP Growth
A	0.5 × 0.2% = 0.10%	1.0 × −0.5% = −0.50%
B	1.6 × 0.2% = 0.32%	0.0 × −0.5% = 0.00%
C	1.0 × 0.2% = 0.20%	1.1 × −0.5% = −0.55%

The combined effects for the three funds are the following.

Fund A: 0.10% + (−0.50%) = −0.40%

Fund B: 0.32% + (0.00%) = 0.32%

Fund C: 0.20% + (−0.55%) = −0.35%

Therefore, Fund A is the most sensitive to the surprises in inflation and GDP growth in Exhibit 3.

9. A is correct. When using a macroeconomic factor model, the expected return is the intercept (when all model factors take on a value of zero). The intercept coefficient for Portfolio 1 in Exhibit 1 is 2.58.

10. C is correct. Active risk, also referred to as tracking risk or tracking error, is the sample standard deviation of the time series of active returns, where the active returns consist of the differences between the portfolio return and the benchmark return. Whereas GDP is the only portfolio non-zero sensitivity for Portfolio 2, the contribution to the portfolio's active return is the sum of the differences between the portfolio's and the benchmark's sensitivities multiplied by the factor return. Because all four of the factor sensitivities of Portfolio 2 are different from the factor sensitivities of the benchmark, all four factors contribute to the portfolio's active return and, therefore, to its active risk.

11. A is correct. Portfolio 1 has the highest information ratio, 1.0, and thus has the best mean active return per unit of active risk:

$$\begin{aligned} \text{IR} &= \frac{\overline{R}_P - \overline{R}_B}{s\left(R_P - R_B\right)} \\ &= \frac{1.50\%}{1.50\%} \\ &= 1.00 \end{aligned}$$

This information ratio exceeds that of Portfolio 2 (–0.38) or Portfolio 3 (0.50).

12. C is correct. In a macroeconomic factor model, the factors are surprises in mac-

roeconomic variables that significantly explain returns. Factor sensitivities are generally specified first in fundamental factor models, whereas factor sensitivities are estimated last in macroeconomic factor models.

13. B is correct. An advantage of statistical factor models is that they make minimal assumptions. However, the interpretation of statistical factors is generally more difficult than the interpretation of macroeconomic and fundamental factor models.

14. B is correct. Analysts can use multifactor models in passively managed portfolios to replicate an index fund's factor exposures.

15. In a macroeconomic factor model, the surprise in a factor equals actual value minus expected value. For the interest rate factor, the surprise was 2 percent; for the GDP factor, the surprise was –3 percent. The intercept represents expected return in this type of model. The portion of the stock's return not explained by the factor model is the model's error term.

    $$\begin{aligned} 5\% &= \text{Expected return} - 1.5(\text{Interest rate surprise}) + 2(\text{GDP surprise}) + \text{Error term} \\ &= \text{Expected return} - 1.5(2\%) + 2(-3\%) + 3\% \\ &= \text{Expected return} - 6\% \end{aligned}$$

    Rearranging terms, the expected return for Harry Company stock equals 5% + 6% = 11%.

16. A fundamental factor model. Such models typically include many factors related to the company (e.g., earnings) and to valuation that are commonly used indicators of a growth orientation. A macroeconomic factor model may provide relevant information as well, but typically indirectly and in less detail.

17. This remainder of 30 basis points would be attributable to the return from factor tilts. A portfolio manager's active return is the sum of two components, factor tilts and security selection. Factor tilt is the product of the portfolio manager's higher or lower factor sensitivities relative to the benchmark's factor sensitivities and the factor returns. Security selection reflects the manager's ability to overweight securities that outperform or underweight securities that underperform.

18. An index fund that effectively meets its investment objective is expected to have an information ratio of zero, because its active return should be zero.

19. The active manager may assume active factor risk and active specific risk (security selection risk) in seeking a higher information ratio.

20. This wealthy investor has a comparative advantage in bearing business cycle risk compared with the average investor who depends on income from employment. Because the average investor is sensitive to the business cycle and in particular the risk of recession, we would expect there to be a risk premium to hold recession-sensitive securities. Cyclical stocks and high-yield bonds are both very sensitive to the risk of recessions. Because the welfare of the wealthy investor is not affected by recessions, she can tilt her portfolio to include cyclical stocks and high yield bonds to attempt to capture the associated risk premiums.