CHAPTER 4

THE TIME VALUE OF MONEY AND DISCOUNTED CASH FLOW ANALYSIS

Objectives

- To explain the concepts of compounding and discounting, future value and present value.
- To show how these concepts are applied to making financial decisions.

Outline

Summary

- Compounding is the process of going from present value (*PV*) to future value (*FV*). The future value of $1 earning interest at rate i per period for n periods is $(1+i)^n$.
- Discounting is finding the present value of some future amount. The present value of $1 discounted at rate i per period for n periods is $1/(1+i)^n$.
- One can make financial decisions by comparing the present values of streams of expected future cash flows resulting from alternative courses of action. The present value of cash inflows less the present value of cash outflows is called net present value (*NPV*). If a course of action has a positive *NPV*, it is worth undertaking.
- In any time value of money calculation, the cash flows and the interest rate must be denominated in the same currency.
- Never use a nominal interest rate when discounting real cash flows or a real interest rate when discounting nominal cash flows.

How to Do TVM Calculations in MS Excel

Assume you have the following cash flows set up in a spreadsheet:

	A	B
1	t	CF
2	0	-100
3	1	50
4	2	60
5	3	70
6	NPV	
7	IRR	

Move the cursor to cell **B6** in the spreadsheet. Click the function wizard f_x in the tool bar and when a menu appears, select **financial** and then **NPV**. Then follow the instructions for inputting the discount rate and cash flows. You can input the column of cash flows by selecting and moving it with your mouse. Ultimately cell **B6** should contain the following:

=NPV(0.1,B3:B5)+B2

The first variable in parenthesis is the discount rate. Make sure to input the discount rate as a decimal fraction (i.e., 10% is .1). Note that the NPV function in **Excel** treats the cash flows as occurring at the end of each period, and therefore the initial cash flow of 100 in cell **B2** is added after the closing parenthesis. When you hit the ENTER key, the result should be $47.63.

Now move the cursor to cell **B7** to compute IRR. This time select IRR from the list of financial functions appearing in the menu. Ultimately cell **B7** should contain the following:

=IRR(B2:B5)

When you hit the ENTER key, the result should be 34%.

Your spreadsheet should look like this when you have finished:

	A	B
1	t	CF
2	0	-100
3	1	50
4	2	60
5	3	70
6	NPV	47.63
7	IRR	34%

Solutions to Problems at End of Chapter

1. If you invest $1000 today at an interest rate of 10% per year, how much will you have 20 years from now, assuming no withdrawals in the interim?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
20	10	1000	?	0	*FV =6,727.50*

2. a. If you invest $100 every year for the next 20 years, starting one year from today and you earn interest of 10% per year, how much will you have at the end of the 20 years?
b. How much must you invest each year if you want to have $50,000 at the end of the 20 years?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 20	10	0	?	100	*FV = 5,727.50*
b. 20	10	0	50,000	?	*PMT = 872.98*

3. What is the present value of the following cash flows at an interest rate of 10% per year?
a. $100 received five years from now.
b. $100 received 60 years from now.
c. $100 received each year beginning one year from now and ending 10 years from now.
d. $100 received each year for 10 years beginning now.
e. $100 each year beginning one year from now and continuing forever.

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 5	10	?	100	0	*PV* = $62.09
b. 60	10	?	100	0	*PV = $.3284*
c. 10	10	?	0	100 ordinary	*PV = $614.46*
d. 10	10	?	0	100 immediate	*PV = $675.90*
e. Perpetuity	10	?	0	100 ordinary	*See below*

e. $PV = \frac{\$100}{.10} = \$1{,}000$

4. You want to establish a "wasting" fund which will provide you with $1000 per year for four years, at which time the fund will be exhausted. How much must you put in the fund now if you can earn 10% interest per year?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
4	10	?	0	1,000	***PV*** =$3,169.87

5. You take a one-year installment loan of $1000 at an interest rate of 12% per year (1% per month) to be repaid in 12 equal monthly payments.
a. What is the monthly payment?
b. What is the total amount of interest paid over the 12-month term of the loan?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
12	1	1,000	0	?	***PMT*** = $88.85

a. ***PMT*** = $88.85
b. 12 x $88.85 - $1,000 = $66.20

6. You are taking out a $100,000 mortgage loan to be repaid over 25 years in 300 monthly payments.
a. If the interest rate is 16% per year what is the amount of the monthly payment?
b. If you can only afford to pay $1000 per month, how large a loan could you take?
c. If you can afford to pay $1500 per month and need to borrow $100,000, how many months would it take to pay off the mortgage?
d. If you can pay $1500 per month, need to borrow $100,000, and want a 25 year mortgage, what is the highest interest rate you can pay?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 300	16/12	100,000	0	?	***PMT*** =*$1358.89*
b. 300	16/12	?	0	1,000	*PV = $73,590*
c. ?	16/12	100,000	0	1,500	*n = 166*
d. 300	?	100,000	0	1,500	*i* = 1.482% per month

a. *Note*: Do not round off the interest rate when computing the monthly rate or you will not get the same answer reported here. Divide 16 by 12 and then press the *i* key.
b. *Note*: You must input *PMT* and *PV* with opposite signs.
c. *Note*: You must input *PMT* and *PV* with opposite signs.

7. In 1626 Peter Minuit purchased Manhattan Island from the Native Americans for about $24 worth of trinkets. If the tribe had taken cash instead and invested it to earn 6% per year compounded annually, how much would the Indians have had in 1986, 360 years later?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
360	6	24	?	0	***FV*** $= 3.09 \times 10^{10}$

FV = 30,925,930,000

8. You win a $1 million lottery which pays you $50,000 per year for 20 years, beginning one year from now. How much is your prize really worth assuming an interest rate of 8% per year?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
20	8	?	0	50,000	*PV = $490,907*

9. Your great-aunt left you $20,000 when she died. You can invest the money to earn 12% per year. If you spend $3,540 per year out of this inheritance, how long will the money last?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
?	12	20,000	0	3,540	*n = 10 years*

10. You borrow $100,000 from a bank for 30 years at an APR of 10.5%. What is the monthly payment? If you must pay two points up front, meaning that you only get $98,000 from the bank, what is the true APR on the mortgage loan?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
360	.875	100,000	0	?	*PMT = $914.74*

If you must pay 2 points up front, the bank is in effect lending you only $98,000. Keying in 98000 as PV and computing i, we get:

n	*i*	*PV*	*FV*	*PMT*	*Result*
360	?	98,000	0	*914.74*	*i = .89575*

i =.89575% per month; APR = $12 \times .89575 \rightarrow 10.75\%$

11. Suppose that the mortgage loan described in question 10 is a one-year adjustable rate mortgage (ARM), which means that the 10.5% interest applies for only the first year. If the interest rate goes up to 12% in the second year of the loan, what will your new monthly payment be?

SOLUTION:
Step 1 is to compute the remaining balance after the first 12 payments:

n	*i*	*PV*	*FV*	*PMT*	*Result*
348	.875	?	0	*914.74*	*PV = $ 99499.57*

Step 2 is to compute the new monthly payment at an interest rate of 1% per month:

n	*i*	*PV*	*FV*	*PMT*	*Result*
348	1	99499.57	0	*?*	*PMT = $1,027.19*

12. You just received a gift of $500 from your grandmother and you are thinking about saving this money for graduation which is four years away. You have your choice between Bank A which is paying 7% for one-year deposits and Bank B which is paying 6% on one-year deposits. Each bank compounds interest annually. What is the future value of your savings one year from today if you save your money in Bank A? Bank B? Which is the better decision? What savings decision will most individuals make? What likely reaction will Bank B have?

SOLUTION:
Future Value in Bank A:

n	*i*	*PV*	*FV*	*PMT*
1	7	- $500	*Solve*	0
			$535	

Formula:
$500 x (1.07) = $535

Future Value in Bank B:

n	*i*	*PV*	*FV*	*PMT*
1	6	- $500	*Solve*	
			$530	

Formula:
$500 x (1.06) = $530

a. You will decide to save your money in Bank A because you will have more money at the end of the year. You made an extra $5 because of your savings decision. That is an increase in value of 1%. Because interest compounded only once per year and your money was left in the account for only one year, the increase in value is strictly due to the 1% difference in interest rates.
b. Most individuals will make the same decision and eventually Bank B will have to raise its rates. However, it is also possible that Bank A is paying a high rate just to attract depositors even though this rate is not profitable for the bank. Eventually Bank A will have to lower its rate to Bank B's rate in order to make money.

13. Sue Consultant has just been given a bonus of $2,500 by her employer. She is thinking about using the money to start saving for the future. She can invest to earn an annual rate of interest of 10%.

a. According to the Rule of 72, approximately how long will it take for Sue to increase her wealth to $5,000?

b. Exactly how long does it actually take?

SOLUTION:

a. According to the Rule of 72: n = 72/10 = 7.2 years
It will take approximately 7.2 years for Sue's $2,500 to double to $5,000 at 10% interest.

b. At 10% interest

n	*i*	*PV*	*FV*	*PMT*
Solve	10	- $2,500	$5,000	
7.27 Years				

Formula:
$2,500 x $(1.10)^n$ = $5,000
Hence, $(1.10)^n$ = 2.0
n log 1.10 = log 2.0
n = .693147 = 7.27 Years
.095310

14. Larry's bank account has a "floating" interest rate on certain deposits. Every year the interest rate is adjusted. Larry deposited $20,000 three years ago, when interest rates were 7% (annual compounding). Last year the rate was only 6%, and this year the rate fell again to 5%. How much will be in his account at the end of this year?

SOLUTION:
$20,000 x 1.07 x 1.06 x 1.05 = $23,818.20

15. You have your choice between investing in a bank savings account which pays 8% compounded annually (BankAnnual) and one which pays 7.5% compounded daily (BankDaily).

a. Based on effective annual rates, which bank would you prefer?

b. Suppose BankAnnual is only offering one-year Certificates of Deposit and if you withdraw your money early you lose all interest. How would you evaluate this additional piece of information when making your decision?

SOLUTION:

a. Effective Annual Rate: BankAnnual = 8%.
Effective Annual Rate BankDaily = $[1 + .075/365]^{365} - 1 = .07788 = 7.788\%$
Based on effective annual rates, you would prefer BankAnnual (you will earn more money.)

b. If BankAnnual's 8% annual return is conditioned upon leaving the money in for one full year, I would need to be sure that I did not need my money within the one year period. If I were unsure of when I might need the money, it might be safer to go for BankDaily. The option to withdraw my money whenever I might need it will cost me the potential difference in interest:
FV (BankAnnual) = $1,000 x 1.08 = $1,080
FV (BankDaily) = $1,000 x 1.07788 = $1,077.88
Difference = $2.12.

16. What are the effective annual rates of the following:
a. 12% APR compounded monthly?
b. 10% APR compounded annually?
c. 6% APR compounded daily?

SOLUTION:
Effective Annual Rate (EFF) = $[1 + \frac{APR}{m}]^{m} - 1$

a. $(1 + \frac{.12}{12})^{12} - 1 = .1268 = 12.68\%$
b. $(1 + \frac{.10}{1}) - 1 = .10 = 10\%$
c. $(1 + \frac{.06}{365})^{365} - 1 = .0618 = 6.18\%$

17. Harry promises that an investment in his firm will double in six years. Interest is assumed to be paid quarterly and reinvested. What effective annual yield does this represent?

SOLUTION:

N	*i*	*PV*	*FV*	*PMT*	*Result*
24	?	-$100	$200	0	*=2.9302%*

EAR=$(1.029302)^4$-1=12.25%

18. Suppose you know that you will need $2,500 two years from now in order to make a down payment on a car.
a. BankOne is offering 4% interest (compounded annually) for two-year accounts, and BankTwo is offering 4.5% (compounded annually) for two-year accounts. If you know you need $2,500 two years from today, how much will you need to invest in BankOne to reach your goal? Alternatively, how much will you need to invest in BankTwo? Which Bank account do you prefer?
b. Now suppose you do not need the money for three years, how much will you need to deposit today in BankOne? BankTwo?

SOLUTION:
a. Present Value of Deposit in BankOne:

n	*i*	*PV*	*FV*	*PMT*	*Result*
2	4	*?*	$2,500	0	PV= $2, *311.39*

Formula:

$$PV = \frac{\$2,500}{(1.04)^2} = \$2,311.39$$

Present Value of Deposit in BankTwo:

n	*i*	*PV*	*FV*	*PMT*	*Result*
2	4.5	*?*	$2,500	0	*PV = $2,289.32*

Formula:

$$PV = \frac{\$2,500}{(1.045)^2} = \$2,289.32$$

You would prefer BankTwo because you earn more; therefore, you can deposit fewer dollars today in order to reach your goal of $2,500 two years from today.

b.
Present Value of Deposit in BankOne:

n	*i*	*PV*	*FV*	*PMT*	*Result*
3	4	*?*	$2,500	0	*PV= $2,222.49*

Formula:

$$PV = \frac{\$2,500}{(1.04)^3} = \$2,222.49$$

Present Value of Deposit in BankTwo:

n	*i*	*PV*	*FV*	*PMT*	*Result*
3	4.5	*?*	$2,500	0	*PV= $2,190.74*

Formula:

$$PV = \frac{\$2,500}{(1.045)^3} = \$2,190.74$$

Again, you would prefer BankTwo because you earn more; therefore, you can deposit fewer dollars today in order to reach your goal of $2,500 three years from today.

19. Lucky Lynn has a choice between receiving $1,000 from her great-uncle one year from today or $900 from her great-aunt today. She believes she could invest the $900 at a one-year return of 12%.

a. What is the future value of the gift from her great-uncle upon receipt? From her great-aunt?
b. Which gift should she choose?
c. How does your answer change if you believed she could invest the $900 from her great-aunt at only 10%? At what rate is she indifferent?

SOLUTION:

a. Future Value of gift from great-uncle is simply equal to what she will receive one year from today ($1000). She earns no interest as she doesn't receive the money until next year.
b. Future Value of gift from great-aunt: $900 x (1.12) = $1,008.
c. She should choose the gift from her great-aunt because it has future value of $1008 one year from today. The gift from her great-uncle has a future value of $1,000. This assumes that she will able to earn 12% interest on the $900 deposited at the bank today.
d. If she could invest the money at only 10%, the future value of her investment from her great-aunt would only be $990: $900 x (1.10) = $990. Therefore she would choose the $1,000 one year from today. Lucky Lynn would be indifferent at an annual interest rate of 11.11%:

$$\frac{\$1000}{(1+i)} = \$900 \text{ or } (1+i) = \frac{1,000}{900} = 1.1111$$

$$i = .1111 = 11.11\%$$

20. As manager of short-term projects, you are trying to decide whether or not to invest in a short-term project that pays one cash flow of $1,000 one year from today. The total cost of the project is $950. Your alternative investment is to deposit the money in a one-year bank Certificate of Deposit which will pay 4% compounded annually.

a. Assuming the cash flow of $1,000 is guaranteed (there is no risk you will not receive it) what would be a logical discount rate to use to determine the present value of the cash flows of the project?

b. What is the present value of the project if you discount the cash flow at 4% per year? What is the net present value of that investment? Should you invest in the project?

c. What would you do if the bank increases its quoted rate on one-year CDs to 5.5%?

d. At what bank one-year CD rate would you be indifferent between the two investments?

SOLUTION:

a. Because alternative investments are earning 4%, a logical choice would be to discount the project's cash flows at 4%. This is because 4% can be considered as your opportunity cost for taking the project; hence, it is your cost of funds.

b. Present Value of Project Cash Flows:

$$PV = \frac{\$1,000}{(1.04)} = \$961.54$$

The net present value of the project = $961.54 - $950 (cost) = $11.54

The net present value is positive so you should go ahead and invest in the project.

c. If the bank increased its one-year CD rate to 5.5%, then the present value changes to:

$$PV = \frac{\$1,000}{(1.055)} = \$947.87$$

Now the net present value is negative: $947.87 - $950 = - $2.13. Therefore you would not want to invest in the project.

d. You would be indifferent between the two investments when the bank is paying the following one-year interest rate:

$$\frac{\$1,000}{(1+i)} = \$950 \text{ hence} \qquad i = 5.26\%$$

21. Calculate the net present value of the following cash flows: you invest $2,000 today and receive $200 one year from now, $800 two years from now, and $1,000 a year for 10 years starting four years from now. Assume that the interest rate is 8%.

SOLUTION:

Since there are a number of different cash flows, it is easiest to do this problem using cash flow keys on the calculator:

Time	Input	Key
0	-$2,000	Cf_i
1	$200	Cf_i
2	$800	Cf_i
3	$0	Cf_i
Refer to note	10	2^{nd} F N_i
4	1,000	Cf_i
	8	i
		NPV
		=$4,197.74

Note: This enables you to instruct the calculator that the next cash flow occurs for N times (here N =10).

22. Your cousin has asked for your advice on whether or not to buy a bond for $995 which will make one payment of $1,200 five years from today or invest in a local bank account.

a. What is the internal rate of return on the bond's cash flows? What additional information do you need to make a choice?

b. What advice would you give her if you learned the bank is paying 3.5% per year for five years (compounded annually?)

c. How would your advice change if the bank were paying 5% annually for five years? If the price of the bond were $900 and the bank pays 5% annually?

SOLUTION:

a. Internal Rate of Return:

n	*i*	*PV*	*FV*	*PMT*	*Result*
5	?	-$995	$1200	0	*i =3.82%*

Formula:

$\$995 \times (1+i)^5 = \$1,200.$

$(1+i)^5 = \frac{\$1,200}{\$995}$

Take 5th root of both sides:

$(1+i) = 1.0382$

$i = .0382 = 3.82\%$

In order to make a choice, you need to know what interest rate is being offered by the local bank.

b. Upon learning that the bank is paying 3.5%, you would tell her to choose the bond because it is earning a higher rate of return of 3.82% .

c. If the bank were paying 5% per year, you would tell her to deposit her money in the bank. She would earn a higher rate of return.

If the price of the bond were $900, its IRR would be the following:

n	*i*	*PV*	*FV*	*PMT*	*Result*
5	?	-$900	$1200	0	*i =5.92%*

5.92% is higher than the rate the bank is paying (5%); hence, she should choose to buy the bond.

23. You and your sister have just inherited $300 and a US savings bond from your great-grandfather who had left them in a safe deposit box. Because you are the oldest, you get to choose whether you want the cash or the bond. The bond has only four years left to maturity at which time it will pay the holder $500.

a. If you took the $300 today and invested it at an interest rate 6% per year, how long (in years) would it take for your $300 to grow to $500? (Hint: you want to solve for *n* or number of periods. Given these circumstances, which are you going to choose?

b. Would your answer change if you could invest the $300 at 10% per year? At 15% per year? What other Decision Rules could you use to analyze this decision?

SOLUTION:

a. Time it takes to grow $300 to $500:

n	*i*	*PV*	*FV*	*PMT*	*Result*
?	6	-$300	$500	0	*n = 8.77 Years*

Formula:

$\$300 \times (1.06)^n = \$500$

$(1.06)^n = 1.6667$

$n \log 1.06 = \log 1.6667$

$n = .510845 = 8.77$ Years

.0582689

You would choose the bond because it will increase in value to $500 in 4 years. If you took the $300 today, it would take more than 8 years to grow to $500.

b. Investing the $300 at 10%:

n	*i*	*PV*	*FV*	*PMT*	*Result*
?	10	-$300	$500	0	*n = 5.36 Years*

At 10% you would still choose the bond. (It pays out $500 faster).

Investing the $300 at 15%:

n	*i*	*PV*	*FV*	*PMT*	*Result*
?	15	-$300	$500	0	*= 3.65 Years*

At 15% you would choose the $300. (It grows to $500 faster).

You could also analyze this decision by computing the NPV of the bond investment at the different interest rates:

n	*i*	*PV*	*FV*	*PMT*	*Result*
4	6	?	$500	0	*PV= $396.05; NPV= $96.05>0*
4	10	?	$500	0	*PV= $341.51; NPV= $41.51>0*
4	15	?	$500	0	*PV= $285.88; NPV= -$14.12<0*

In the calculations of the NPV, $300 can be considered your "cost" for acquiring the bond since you will give up $300 in cash by choosing the bond. Note that the first two interest rates give positive NPVs for the bond, i.e. you should go for the bond, while the last NPV is negative, hence choose the cash instead. These results confirm the previous method's results.

24. Suppose you have three personal loans outstanding to your friend Elizabeth. A payment of $1,000 is due today, a $500 payment is due one year from now and a $250 payment is due two years from now. You would like to consolidate the three loans into one, with 36 equal monthly payments, beginning one month from today. Assume the agreed interest rate is 8% (*effective annual rate*) per year.

a. What is the annual percentage rate you will be paying?

b. How large will the new monthly payment be?

SOLUTION:

a. To find the APR, you must first compute the monthly interest rate that corresponds to an effective annual rate of 8% and then multiply it by 12:

$1.08 = (1+ i)^{12}$

Take 12th root of both sides:

$1.006434 = 1+ i$

$i = .006434$ or .6434% per month

Or using the financial calculator:

n	*i*	*PV*	*FV*	*PMT*	*Result*
12	?	-1	1.08	0	*i = .6434%*

APR = 12 x .6434% = 7.72% per year

b. The method is to first compute the PV of the 3 loans and then compute a 36 month annuity payment with the same PV. Most financial calculators have keys which allow you to enter several cash flows at once. This approach will give the user the PV of the 3 loans.

Time	$ or i	Key
0	$1,000	Cf_i
1	$500	Cf_i
2	$250	Cf_i
Refer to note	8	i
		NPV
		=$1,677.30

Note: The APR used to discount the cash flows is the effective rate in this case, because this method is assuming annual compounding.

n	*i*	*PV*	*FV*	*PMT*	*Result*
36	.6434	1,677.30	0	?	*PMT = 52.34*

25. As CEO of ToysRFun, you are offered the chance to participate, without initial charge, in a project that produces cash flows of $5,000 at the end of the first period, $4,000 at the end of the next period and a loss of $11,000 at the end of the third and final year.

a. What is the net present value if the relevant discount rate (the company's cost of capital) is 10%?

b. Would you accept the offer?

c. What is the internal rate of return? Can you explain why you would reject a project which has an internal rate of return greater than its cost of capital?

SOLUTION:

At 10% discount rate:

$$\text{Net Present Value} = -0 + \frac{\$5{,}000}{(1.10)} + \frac{\$4{,}000}{(1.10)^2} - \frac{\$11{,}000}{(1.10)^3} = -413.22$$

Alternatively,

Time	$ or i	Key
0	$0	Cfi
1	$5,000	Cfi
2	$4,000	Cfi
3	-$11,000	Cfi
	10	i
		NPV
		-$413.22

b. Reject the project (it has negative NPV).

c.

Time	$ or i	Key
0	$0	Cfi
1	$5,000	Cfi
2	$4,000	Cfi
3	-$11,000	Cfi
		IRR
		13.6%

This example is a project with cash flows that begin positive and then turn negative--it is like a loan. The 13.6% IRR is therefore like an interest rate on that loan. The opportunity to take a loan at 13.6% when the cost of capital is only 10% is not worthwhile.

26. You must pay a creditor $6,000 one year from now, $5,000 two years from now, $4,000 three years from now, $2,000 four years from now, and a final $1,000 five years from now. You would like to restructure the loan into five equal annual payments due at the end of each year. If the agreed interest rate is 6% compounded annually, what is the payment?

SOLUTION:
Since there are a number of different cash flows, it is easiest to do the first step of this problem using cash flow keys on the calculator. To find the present value of the current loan payments:

Time	$ or i	Key
0	$0	Cf_i
1	6,000	Cf_i
2	5,000	Cf_i
3	4,000	Cf_i
4	2,000	Cf_i
5	1,000	Cf_i
	6	i
		NPV=-$15,800.28

Step 2: To find the 5 new equal payments beginning at the end of each year:

n	*i*	*PV*	*FV*	*PMT*	*Result*
5	6	-$15,800.28	0	?	*PMT =$3,750.93*

27. Find the future value of the following ordinary annuities (payments begin one year from today and all interest rates compound annually):

a. $100 per year for 10 years at 9%.
b. $500 per year for 8 years at 15%.
c. $800 per year for 20 years at 7%.
d. $1,000 per year for 5 years at 0%.
e. Now find the present values of the annuities in a-d.
f. What is the relationship between present values and future values?

SOLUTION:
Future Value of Annuity:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 10	9	0	?	100	*FV =$1,519.29*

n	*i*	*PV*	*FV*	*PMT*	*Result*
b. 8	15	0	?	500	*FV = $6,863.41*

n	*i*	*PV*	*FV*	*PMT*	*Result*
c. 20	7	0	?	800	*FV =$32,796.39*

n	*i*	*PV*	*FV*	*PMT*	*Result*
d. 5	0	0	?	1000	*FV =$5,000*

e.

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 10	9	?	0	100	*PV =$641.77*
b. 8	15	?	0	500	*PV = $2,243.66*
c. 20	7	?	0	800	*PV =$8,475.21*
d. 5	0	?	0	1000	*PV = $5,000*

f. The relationship between present value and future value is the following:
$FV = PV \times (1+i)^n$

28. Suppose you will need $50,000 ten years from now. You plan to make seven equal annual deposits beginning three years from today in an account that yields 11% compounded annually. How large should the annual deposit be?

SOLUTION:
You will be making 7 payments beginning 3 years from today. So, we need to find the value of an *immediate* annuity with 7 payments whose FV is $50,000:

n	*i*	*PV*	*FV*	*PMT*	*Result*
7	11	0	50000	? (BGN)	*PMT = $4,604.29*

29. Suppose an investment offers $100 per year for five years at 5% beginning one year from today.

a. What is the present value? How does the present value calculation change if one additional payment is added today?

b. What is the future value of this ordinary annuity? How does the future value change if one additional payment is added today?

SOLUTION:

a. Present Value of $100 Ordinary Annuity:

n	*i*	*PV*	*FV*	*PMT*	*Result*
5	5	?	0	100	*PV =$432.95*

If you added a payment of $100 *today,* the present value would increase by $100 to $532.95.

b. Future Value of $100 Ordinary Annuity:

n	*i*	*PV*	*FV*	*PMT*	*Result*
5	5	0	?	100	*FV =$552.56*

Formula:

$$\$100 \times \frac{[(1.05)^5] - 1}{.05} = \$552.56$$

If you were to add one additional payment of $100 today, the future value would increase by:
$100 x $(1.05)^5$ = $127.63. Total future value = $552.56 + $127.63 = $680.19.
Another way to do it would be to use the BGN mode for 5 payments of $100 at 5%, find the future value of that, and then add $100. The same $680.19 is obtained.

30. You are buying a $20,000 car. The dealer offers you two alternatives: (1) pay the full $20,000 purchase price and finance it with a loan at 4.0% APR over 3 years or (2) receive $1,500 cash back and finance the rest at a bank rate of 9.5% APR. Both loans have monthly payments over three years. Which should you choose?

SOLUTION:
You will choose the alternative which results in the lowest monthly payment.

n	*i*	*PV*	*FV*	*PMT*	*Result*
(1) 36	4/12	20,000	0	?	*PMT = $590.48*
(2) 36	9.5/12	18,500	0	?	*PMT = $592.61*

So alternative (1), the 4% APR loan, is better.

31. You are looking to buy a sports car costing $23,000. One dealer is offering a special reduced financing rate of 2.9% APR on new car purchases for three year loans, with monthly payments. A second dealer is offering a cash rebate. Any customer taking the cash rebate would of course be ineligible for the special loan rate and would have to borrow the balance of the purchase price from the local bank at the 9% annual rate. How large must the cash rebate be on this $23,000 car to entice a customer away from the dealer who is offering the special 2.9% financing?

SOLUTION:
Step 1: Find the monthly payment on alternative 1:

n	*i*	*PV*	*FV*	*PMT*	*Result*
36	2.9/12	23,000	0	?	*PMT = $667.85*

Step 2: Find the present value of the above payments at a 9% APR to see how much lower it is:

n	*i*	*PV*	*FV*	*PMT*	*Result*
36	9/12	?	0	667.85	*PV = $21,001.75*

Therefore, you would want at least $1,998.25 in cash back ($23,000 - $21,001.75) to choose the cash rebate instead of the 2.9% financing.

32. Show proof that investing $475.48 today at 10% allows you to withdraw $150 at the end of each of the next 4 years and have nothing remaining.

SOLUTION:
You deposit $475.48 and earn 10% interest after one year. Then you withdraw $150. The table shows what happens each year.

Amount at beginning of year		End of year
$475.48	x 1.10 =	$ 523.03
		- 150
$373.03	x 1.10 =	$ 410.33
		- 150
$260.33	x 1.10 =	$ 286.36
		- 150
$136.36	x 1.10 =	$ 150
		- 150
$0		

Another way to do it is simply to compute the PV of the $150 annual withdrawals at 10% : it turns out to be exactly $475.48, hence both amounts are equal.

33. As a pension manager, you are considering investing in a preferred stock which pays $5,000,000 per year <u>forever</u> beginning one year from now. If your alternative investment choice is yielding 10% per year, what is the present value of this investment? What is the highest price you would be willing to pay for this investment? If you paid this price, what would be the dividend yield on this investment?

SOLUTION:
Present Value of Investment:
PV = $\frac{\$5,000,000}{.10} = \$50,000,000$
Highest price you would be willing to pay is $50,000,000.
Dividend yield = $\frac{\$5,000,000}{\$50,000,000} = 10\%$.

34. A new lottery game offers a choice for the grand prize winner. You can receive either a lump sum of $1,000,000 immediately or a perpetuity of $100,000 per year forever, with the first payment *today*. (If you die, your estate will still continue to receive payments). If the relevant interest rate is 9.5% compounded annually, what is the difference in value between the two prizes?

SOLUTION:
The present value of the perpetuity assuming that payments begin at the *end* of the year is:
$100,000/.095 = $1,052,631.58
If the payments begin immediately, you need to add the first payment. $100,000 + 1,052,632 = $1,152,632.
So the annuity has a PV which is greater than the lump sum by $152,632.

35. Find the future value of a $1,000 lump sum investment under the following compounding assumptions:
a. 7% compounded annually for 10 years
b. 7% compounded semiannually for 10 years
c. 7% compounded monthly for 10 years
d. 7% compounded daily for 10 years
e. 7% compounded continuously for 10 years

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 10	7	1,000	?	0	*FV =$1,967.15*
b. 20	7/2	1,000	?	0	*FV = $1,989.79*
c. 120	7/12	1,000	?	0	*FV = $2,009.66*
d. 3650	7/365	1,000	?	0	*FV = $2,013.62*

Formulas:
a. $\$1,000 \times (1.07)^{10} = \$1,967.15$
b. $\$1,000 \times (1.035)^{20} = \$1,989.79$
c. $\$1,000 \times (1.0058)^{120} = \$2,009.66$
d. $\$1,000 \times (1.0019178)^{3650} = \$2,013.62$
e. $\$1,000 \times e^{.07 \times 10} = \$2,013.75$

36. Sammy Jo charged $1,000 worth of merchandise one year ago on her MasterCard which has a stated interest rate of 18% APR compounded monthly. She made 12 regular monthly payments of $50, at the end of each month, and refrained from using the card for the past year. How much does she still owe?

SOLUTION:
Sammy Jo has taken a $1,000 loan at 1.5% per month and is paying it off in monthly installments of $50. We could work out the amortization schedule to find out how much she still owes after 12 payments, but a shortcut on the financial calculator is to solve for FV as follows:

n	*i*	*PV*	*FV*	*PMT*	*Result*
12	1.5	1,000	?	-50	*FV =$543.56*

37. Suppose you are considering borrowing $120,000 to finance your dream house. The annual percentage rate is 9% and payments are made monthly,

a. If the mortgage has a 30 year amortization schedule, what are the monthly payments?

b. What effective annual rate would you be paying?

c. How do your answers to parts a and b change if the loan amortizes over 15 years rather than 30?

SOLUTION:

n	*i*	*PV*	*FV*	*PMT*	*Result*
a. 360	9/12	120,000	0	?	*PMT = $965.55*

b. EFF = $[1 + \frac{.09}{12}]^{12} - 1 = .0938 = 9.38\%$

n	*i*	*PV*	*FV*	*PMT*	*Result*
c. 180	9/12	120,000	0	?	*PMT = $1,217*

The effective annual rate is the same as in part b.

38. Suppose last year you took out the loan described in problem #37a. Now interest rates have declined to 8% per year. Assume there will be no refinancing fees.

a. What is the remaining balance of your current mortgage after 12 payments?

b. What would be your payment if you refinanced your mortgage at the lower rate for 29 years?

SOLUTION:

a. Find the remaining balance after 12 payments as follows:

n	*i*	*PV*	*FV*	*PMT*	*Result*
12	9/12	120,000	?	*965.55*	*FV = $119,180.13*

b. Find the new monthly payment as follows:

n	*i*	*PV*	*FV*	*PMT*	*Result*
348	8/12	*119,180.13*	0	?	*PMT = $881.87*

Exchange Rates and the Time Value of Money

39. The exchange rate between the pound sterling and the dollar is currently $1.50 per pound, the dollar interest rate is 7% per year, and the pound interest rate is 9% per year. You have $100,000 in a one-year account that allows you to choose between either currency, and it pays the corresponding interest rate.

a. If you expect the dollar/pound exchange rate to be $1.40 per pound a year from now and are indifferent to risk, which currency should you choose?

b. What is the "break-even" value of the dollar/pound exchange rate one year from now?

SOLUTION:

a. You could invest $1 today in dollar-denominated bonds and have $1.07 one year from now. Or you could convert the dollar today into 2/3 (i.e., 1/1.5) of a pound and invest in pound-denominated bonds to have .726667 (i.e., 2/3 x 1.09) pounds one year from now. At an exchange rate of $1.4 per pound, this would yield 0.726667 (1.4) = $1.017 (this is lower than $1.07), so you would choose the dollar currency.

b. For you to break-even the .726667 pounds would have to be worth $1.07 one year from now, so the break-even exchange rate is $1.07/.726667 or $1.4725 per pound. So for exchange rates lower than $1.4725 per pound one year from now, the dollar currency will give a better return.

Real versus Nominal Interest Rates

40. The interest rate on conventional 10-year Treasury bonds is 7% per year and the interest rate on 10-year TIPS (Treasury inflation-protected securities) is 3.5% per year. You have $10,000 to invest in one of them.

a. If you expect the average inflation rate to be 4% per year, which bond offers the higher expected rate of return?

b. Which would you prefer to invest in?

SOLUTION:

a. In real terms, the expected rate of return on the nominal bonds is (7% - 4%)/1.04 = 2.885%, which is less than the 3.5% offered by the TIPS. Or, in other words, TIPS offer a higher expected nominal rate of return. Expected nominal rate of return on TIPS = .035 +.04 + .035x.04 = .0764 or 7.64%. This is higher than the 7% nominal return on conventional Treasury bonds.

b. Since they offer a higher expected return rate of return, you should invest in TIPS.

41. You are 20 years from retirement, and expect to live another 20 years after retirement. If you start saving now, how much will you be able to withdraw each year for every dollar per year that you save assuming an effective annual interest rate of:

a. 0,1%, 2%, 3%, 3.5%, 4%, 6%, 8%, and 10%?

b. How would your answer change if you expect the rate of inflation to be 4% per year?

SOLUTION

a. Step 1 is to compute the FV at year 20 in BGN mode of a $1 annuity for 20 years.
Step 2 is to spread this FV to an annuity for another 20 years, starting at year 20, again using BGN.

n	*i*	*PV*	*FV*	*PMT*	*Result*
20	0%	0	?	1	*FV = $20*
20	1%	0	?	1	*FV = $22.2392*
20	2%	0	?	1	*FV = $24.783*
20	3%	0	?	1	*FV = $27.6765*
20	3.5%	0	?	1	*FV = $29.269*
20	4%	0	?	1	*FV = $30.969*
20	6%	0	?	1	*FV = $38.993*
20	8%	0	?	1	*FV = $49.423*
20	10%	0	?	1	*FV = $63.0025*

20	0%	20	0	?	*PMT = $1*
20	1%	22.2392	0	?	*PMT = $1.220*
20	2%	24.783	0	?	*PMT = $1.486*
20	3%	27.6765	0	?	*PMT= $1.806*
20	3.5%	29.269	0	?	*PMT= $1.9898*
20	4%	30.969	0	?	*PMT= $2.191*
20	6%	38.993	0	?	*PMT= $3.207*
20	8%	49.423	0	?	*PMT= $4.661*
20	10%	63.0025	0	?	*PMT = $6.728*

b. If the rate of inflation is expected to be 4%, then the above results still apply in nominal $: you would still save 1$ in nominal terms, and withdraw the same amounts above in nominal terms. Note however that these numbers do not represent the real purchasing power of your money.