Sample test 1

Q1: A : men

B : women

x : graduate

y : undergraduate

z : high school

$P(A) = 0,8$

$P(B) = 0,2$

$P(x|A) = 0,1$

$P(y|A) = 0,3$

$P(z|A) = 0,6$

$P(x|B) = 0,15$

$P(y|B) = 0,4$

$P(z|B) = 0,45$

a) $P(x) = P(A).P(x|A) + P(B).P(x|B)$

$= 0,11$

b) $P(A|x) = \dfrac{P(A \cap x)}{P(x)} = \dfrac{P(x|A).P(A)}{P(x)}$

$= 0,73$

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Q2

a)

	44	45	46	47	48	49	50
$F(x)$	0,04	0,17	0,38	0,67	0,87	0,97	1

b) $P(45 < x < 49) = P(x=46) + P(x=47) + P(x=48)$

$= 0,7$

c) $\mu = \sum_x x\,P(x) = 46,9$

$\sigma = \sqrt{\sum_x (x-\mu)^2 P(x)} = 1,4$

Q3 : $\lambda = 3,2$

$P(x \geq 4) = 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$

$- P(x=4)$

$= 1 - \frac{e^{-\lambda}\lambda^0}{0!} - \frac{e^{-\lambda}\lambda^1}{1!} - \frac{e^{-\lambda}\lambda^2}{2!} - \frac{e^{-\lambda}\lambda^3}{3!} - \frac{e^{-\lambda}\lambda^4}{4!}$

$= 0,22$

Q4 : a) ~~$P(0{,}2 \leq X \leq 0{,}8) =$~~

$$\cancel{\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{0{,}2} f(x)\,dx + \int_{0{,}2}^{0{,}8} f(x)\,dx}$$

$$\cancel{+ \int_{0{,}8}^{1} f(x)\,dx + \int_{1}^{+\infty} f(x)\,dx}$$

$$\cancel{= \int_{0}^{0{,}2}}$$

Q4 : a) $P(0{,}2 \leq X \leq 0{,}8) = \int_{0{,}2}^{0{,}8} f(x)\,dx = \int_{0{,}2}^{0{,}8} 1\,dx$

$$= 0{,}6$$

b)

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{+\infty} x f(x)\,dx$$

$$= \int_{0}^{1} x\,dx = 0{,}5$$

Q5 : $\mu = 700$ $\quad X \sim N(700, 120^2)$

$\sigma = 120$

a) $P(730 \leq x \leq 820) \Leftrightarrow P\left(\frac{730-\mu}{\sigma} \leq Z \leq \frac{820-\mu}{\sigma}\right)$

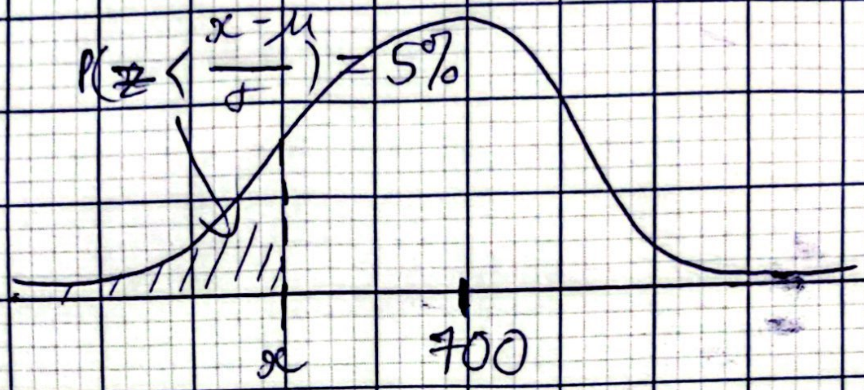
$$\Leftrightarrow P(0{,}25 \leq Z \leq 1)$$

$P(0,25 \le z < 1) = P(z \le 1) - P(z \le 0,25)$

$= 0,8413 - 0,5987$

$= 0,2426$

b)



$\Rightarrow \frac{x-\mu}{\sigma} = -1,645$ (Table)

$\Rightarrow x = 502,6 \approx 503$

Q6: Box 1: 4 defects, 6 non-defects

Box 2: 5 defects, 10 non-defects

Box 3: 5 defects, 15 non-defects

Take a box and take a product randomly

$\Omega = C_3^1 . C_{10}^1 + C_3^1 . C_{15}^1 + C_3^1 . C_{20}^1$

$= 135$

$A = C_3^1 . C_4^1 + C_3^1 . C_5^1 + C_3^1 . C_5^1 = 42$

$\Rightarrow P(\text{defect}) = \frac{A}{\Omega} = 0,31$

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Sample test 2

Q1:

A : weather conditions accidents

B : bodily injury accidents

$P(A) = 0,3$

$P(B) = 0,2$

$P(A|B) = 0,4$

a)

$$P(A \cap B) = P(A|B).P(B)$$
$$= 0,08$$

b)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = 0,27$$

Q2.

a)

X	0	1	2
P(X)	1/3	8/15	2/15

b) $\mu = \sum_{0}^{2} X.P(X) = 0,8$

$$\sigma^2 = \sqrt{\sum_{0} (x-\mu)^2 P(x)} = 0,6532$$

Q3

a) $P(x \geq 1) = 1 - P(x = 0)$

$= 1 - C_5^0 \cdot 0{,}25^0 \cdot 0{,}75^5$

$= 0{,}7627$

b) $P(x = 5) = C_5^5 \cdot 0{,}25^5 \cdot 0{,}75^0$

$= 0{,}001$

Q4:

y : payment

x : total sales

$\Rightarrow \quad y = 1{,}5x + 10000$

$\mu = y(30000) = 1{,}5 \cdot 30000 + 10000$

$= 55000$

$\sigma = \sqrt{1{,}5^2 \cdot 8000^2} = 12000$

Q5 a)

$P(460000 \leq x \leq 540000) = P(-0{,}8 \leq z \leq 0{,}8)$

$= P(z \leq 0{,}8) - P(z \geq -0{,}8)$

$= P(z \leq 0{,}8) - \{1 - [1 - P(z \leq 0{,}8$

$= P(z \leq 0{,}8) - 1 + [1 - P(z \leq 0{,}8)$

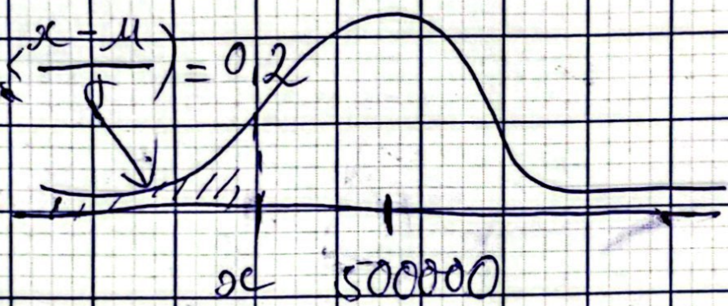
$=$

Q5 : a)

$$P(460000 \leq x \leq 540000) = P(-0,8 \leq z \leq 0,8)$$
$$= P(z \leq 0,8) - P(z \leq -0,8)$$
$$= P(z \leq 0,8) - [1 - P(z \leq 0,8)]$$
$$= 2P(z \leq 0,8) - 1$$
$$= 2 . 0,7881 - 1$$
$$= 0,5762$$

b)

$$P\left(z \leq \frac{x-\mu}{\sigma}\right) = 0,2$$



$$\Rightarrow \frac{x-\mu}{\sigma} = -0,84 \Rightarrow x = 458000$$

Q6:

$$P = P_1 . P_2 \ldots P_{n-1} P_n$$
$$= 0,8^{n-1} . 0,2$$

Sample test 3

Q1:

A: MBA

B: $\geq 3{,}5$

$P(A) = 0{,}35$

$P(B) = 0{,}4$

$P(B|A) = 0{,}3$

a)

$$P(B \cap \overline{A}) = P(B) - P(A \cap B)$$
$$= P(B) - \cancel{P(B|A)}\, P(B|A).P(A)$$
$$= 0{,}295$$

b) ~~P~~

$P(B) \neq P(B|A) \quad (0{,}4 \neq 0{,}3)$

=> The events are not independent

c) $P(A \cap B) \neq 0$

=> The events are not mutually exclusive

Q2:

a)

x	20	21	22	23	24	25	26	2[illegible]
$F(x)$	0,02	0,14	0,37	0,68	0,87	0,95	0,98	1

b) $P(x \geq 24) = 0{,}19 + 0{,}08 + 0{,}03 + 0{,}02$
$= 0{,}32$

c) y: total payments

$\Rightarrow y = 1{,}5x$

$\Rightarrow \mu_y = 1{,}5\,E(x)$

$= 1{,}5 . \sum x P(x)$

$= 34{,}485$

$\sigma_y = 1{,}5\sqrt{E(x^2) - E^2(x)}$

$= 2{,}1159$

Q3:

$\lambda = 2$

$P(x > 2) = 1 - P(x=0) - P(x=1) - P(x=2)$

$= 1 - \frac{e^{-\lambda} . \lambda^0}{0!} - \frac{e^{-\lambda} . \lambda^1}{1!} - \frac{e^{-\lambda} . \lambda^2}{2!}$

$= 0{,}3233$

Q4 a)

$P(x \geq 0{,}5) = \int_{0{,}5}^{1} 5(1-x)^4\,dx$

$= 0{,}03125$

b) $E(x) = \int_{-\infty}^{+\infty} x f(x) = \int_0^1 x . 5(1-x)^4\,dx = 0{,}1667$

Q5:

a) $P(400 \leq x \leq 480) = P(-0{,}25 \leq z \leq 0{,}75)$

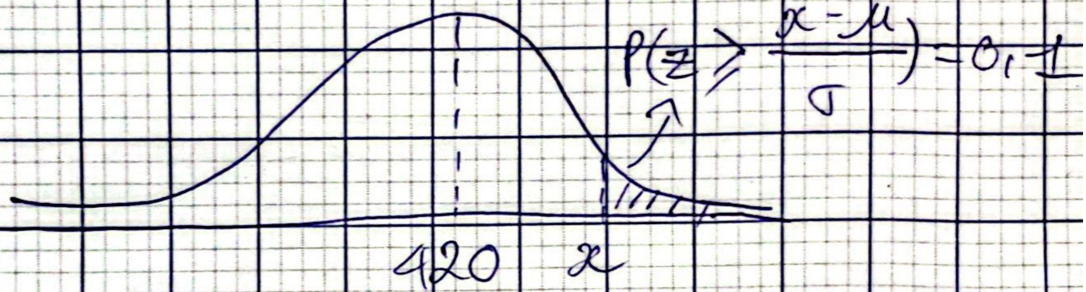
$= P(z \leq 0{,}75) - P(z \leq -0{,}25)$

$= P(z \leq 0{,}75) - 1 + P(z \leq 0{,}25)$

$= 0{,}7734 - 1 + 0{,}5987$

$= 0{,}3721$

b)



$\Rightarrow P\left(z \leq \frac{x-\mu}{\sigma}\right) = 0{,}9$

$\Rightarrow \frac{x-\mu}{\sigma} = 1{,}28 \Rightarrow x = 522{,}4$

Q6: * First take

$P(3 \text{ new}) = \frac{C_9^3 \cdot C_6^0}{C_{15}^3} = \frac{12}{65}$

* Second take

$P(1 \text{ new}) = \frac{C_6^1 \cdot C_9^2}{C_{15}^3} = \frac{216}{455}$

$\Rightarrow P(1 \text{ new in second draw}) = \frac{12}{65} \cdot \frac{216}{455}$

$= 0,0876$

Sample test 4

Q1:

A : regular   a : wine

B : occasional

C : new

$P(A) = 0,5$ $P(a|A) = 0,7$

$P(B) = 0,4$ $P(a|B) = 0,5$

$P(C) = 0,1$ $P(a|C) = 0,3$

a)

$P(a) = P(a|A) \cdot P(A) + P(a|B) \cdot P(B) + P(a|C) \cdot P(C)$

$= 0,58$

b)

$P(B|a) = \frac{P(a|B) \cdot P(B)}{P(a)} = 0,3448$

Q2: a)

$\mu = \sum x P(x) = 1{,}82$

$\cancel{\sigma = \sqrt{[E(x^2) - E^2(x)]}}$

$\cancel{=}$

$\sigma = \sqrt{(x-\mu)^2 P(x)} = 1{,}3986$

b) y: breakdown costs

$\Rightarrow y = 1500x$

$\mu_y = 1500\, E(x) = 2730$

$\sigma_y = \sqrt{1500^2 \cdot \sigma^2} = 2097{,}9$

Q3:

$P(\text{accept}) = P(x=15) + P(x=16)$

$= C_{16}^{15} \cdot 0{,}85^{15} \cdot 0{,}15^{1} + C_{16}^{16} \cdot 0{,}85^{16} \cdot 0{,}15^{0}$

$= 0{,}2839$

Q4: a) $P(x < 0{,}25) = \int_0^{0{,}25} 4x^3\,dx = 0{,}0039$

b) $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^1 x \cdot 4x^3\,dx = 0{,}8$

Q5

a) $P(x < 60) = P(z < -0{,}75)$

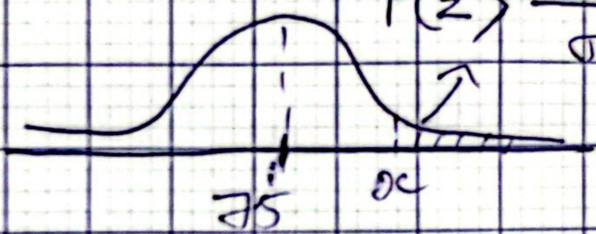
$= 1 - P(z < 0{,}75)$

$= 1 - 0{,}7734$

$= 0{,}2266$

b)

$P\left(z > \frac{x-\mu}{\sigma}\right) = 0{,}1$



$\Rightarrow P\left(z < \frac{x-\mu}{\sigma}\right) = 0{,}9$

$\Rightarrow \frac{x-\mu}{\sigma} = 1{,}28 \Rightarrow x = 100{,}6$

Q6 :

$$P(1\text{ point}) = \frac{C_4^1 \cdot C_4^0 \cdot C_6^2 + C_4^2 \cdot C_4^1 \cdot C_6^0}{C_{14}^3} = 0{,}2308$$