# Time Value OF Money

TCH302

Bodie & Merton (2000), *Finance,* Chapter 4

# Learning objectives

Understand…

1. Meaning of time value of money
2. Compounding and future value
3. Discounting and present value
4. Multiple cash flows
5. Special multiple cash flows: Annuities
6. Applications

# Time value of money

Time value of money (TVM) refers to the fact that money (a dollar, a mark, a VND) in hand today is worth more than the expectation of the same amount to be received in the future.

***Why?***

- **Reasons**:

-Investment opportunity

-Risk

-Inflation

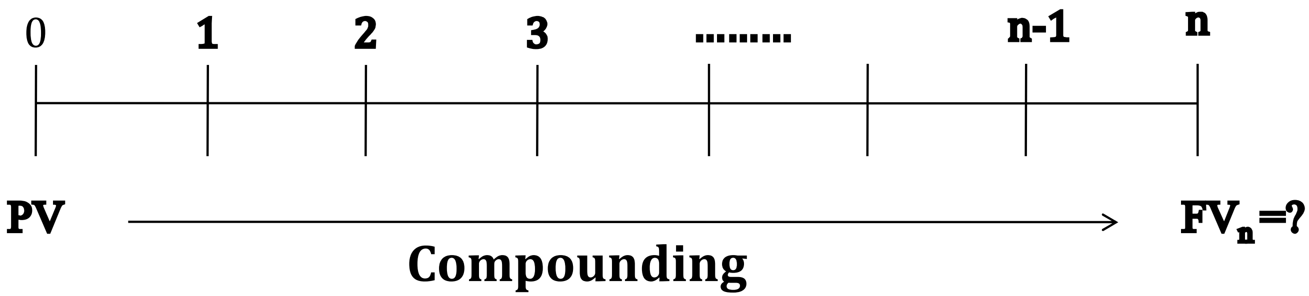
Risk Free Rate

Risk Premium

Inflation Premium

# Compounding and future value

***Compounding*** is the process of going from today's value, or present value (PV), to future value (FV)




Future value is the amount of money that an investment will grow to at some date in the future by earning interest at some compound rates

# Compounding and future value

CS1: Putting \$1,000 into an account earning interest at a rate of 10% per annum.

⇒ The amount you will have in two years, *assuming you take nothing out of the account before then*, is called the future value of \$1,000 at an interest rate of 10% per annum of two years.

| | |
|---|---|
| PV: Present value (beginning amount in your account) | 1,000 |
| FV: Future value at the end of n years | |
| n: Number of years the account will earn interest | 2 |
| i: Interest rate, usually expressed in percent per year | 10% |

# Compounding and future value

Ex: Putting $1.000 into an account earning interest at a rate of 10% per annum.

0 | Year 1 | Year 2

PV = $1,000 → $1,000 * (1+10%) = $1,100

FV = $1,100 * (1+10%)

= $1,000 * (1+10%)^2

Formula:

$$\boxed{FV_n = PV(1+i)^n}$$

Where:

- FV: future value
- PV: present value
- i: interest rate
- n: number of periods

# Simple interest versus compound interest

$\Rightarrow$**The original principal**: \$1.000

$\Rightarrow$**Total interest:**

= 1.210 – 1.000 = 210

$\Rightarrow$**Simple interest**: (the interest on the original principal)

= 10% * 1.000 * 2 = 200

$\Rightarrow$**Compound interest:**

(the interest on the interest earned in the first year)

= 210 – 200 = 10

# Practice

CS2: Calculating the future value of $1,000 at an interest rate of 10% per annum for three years.

PV= $1,000
i= 10%
n= 3 years
⇒FV=?

$$FV =$$

CS3: Calculating the future value of $1,000 at an interest rate of 1% monthly for three years.

PV= $1,000
i= 1%
n= 36 months
⇒FV=?

$$FV =$$

# Frequency of compounding and Effective annual rate

The frequency of compounding (m): the number of compounding periods per year.

Interest rate on loans and saving accounts are usually stated in the form of ***an annual percentage rate (APR)*** (e.g., 14% per annum) with a certain frequency of compounding (e.g., monthly)

⇒***EFF or EAF: effective annual rate*** defined as the equivalent interest rate if compounding were only once per year

$$\text{EFF} = \left(1 + \frac{APR}{m}\right)^{m} - 1$$

$$FV_n = PV\left(1 + \frac{APR}{m}\right)^{m \times n}$$

Where:

- FV: future value
- PV: present value
- APR: annual percentage rate
- n: number of years
- m: frequency of compounding

# Practice

CS4: You take out a loan at an APR of 12% with monthly/quarterly/semiannually/annually compounding. What is the effective annual rate on your loan in each case?

| Compounding frequency | m | Effective annual rate |
|---|---|---|
| Annually | | |
| Semiannually | | |
| Quarterly | | |
| Monthly | | |

# Practice

CS5: If you put \$200 into an account earning interest at interest rate of 8% per annum, quarterly compounding, how much will you have after (a)Two years? (b) a half-year?

After two years

PV=\$200
i(APR)= 8%
n= 2
m=4

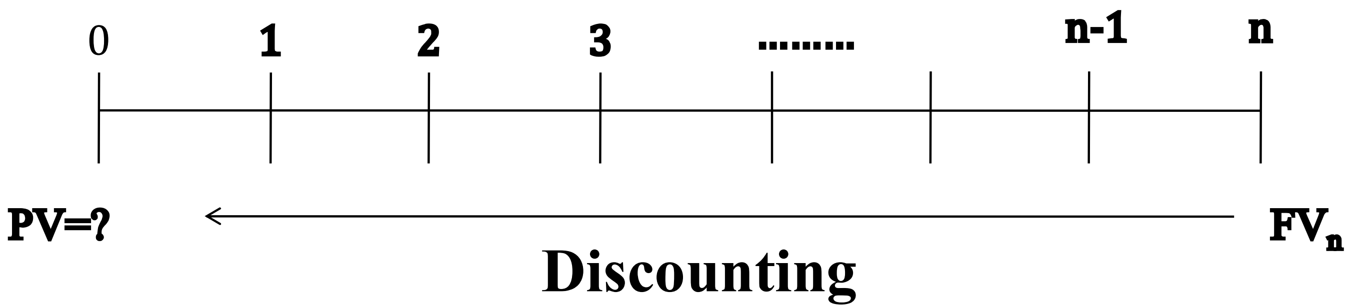
$FV_2$=

After a half-year

PV=\$200
i(APR)= 8%
n= 1/2
m=4

$FV_{1/2}$=

# Discounting and Present value




**Discounting** is finding the present value of some future amount at discount rate

$$PV = \frac{FV_n}{(1+i)^n}$$

or

$$PV = \frac{FV_n}{(1+\frac{APR}{m})^{n \times m}}$$

# Practice

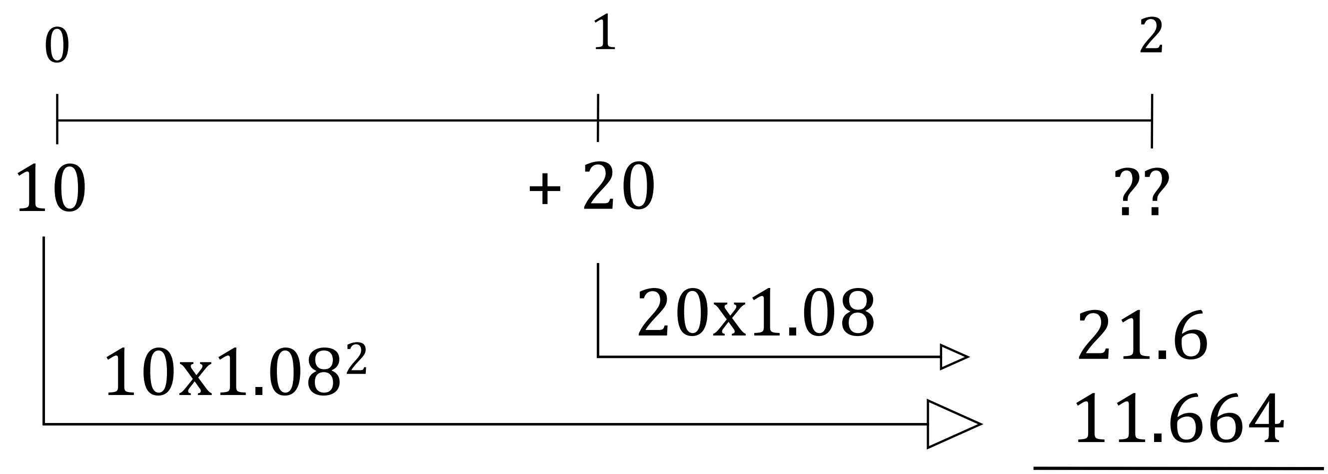
CS5: Suppose Mr. A promises to give you 500 million VNDs ten years later. How much is it really worth today if you can earn 12% interest per annum?

FV= 500 million VNDs
i= 12%
n= 10
$\Rightarrow$PV=?

$$PV =$$

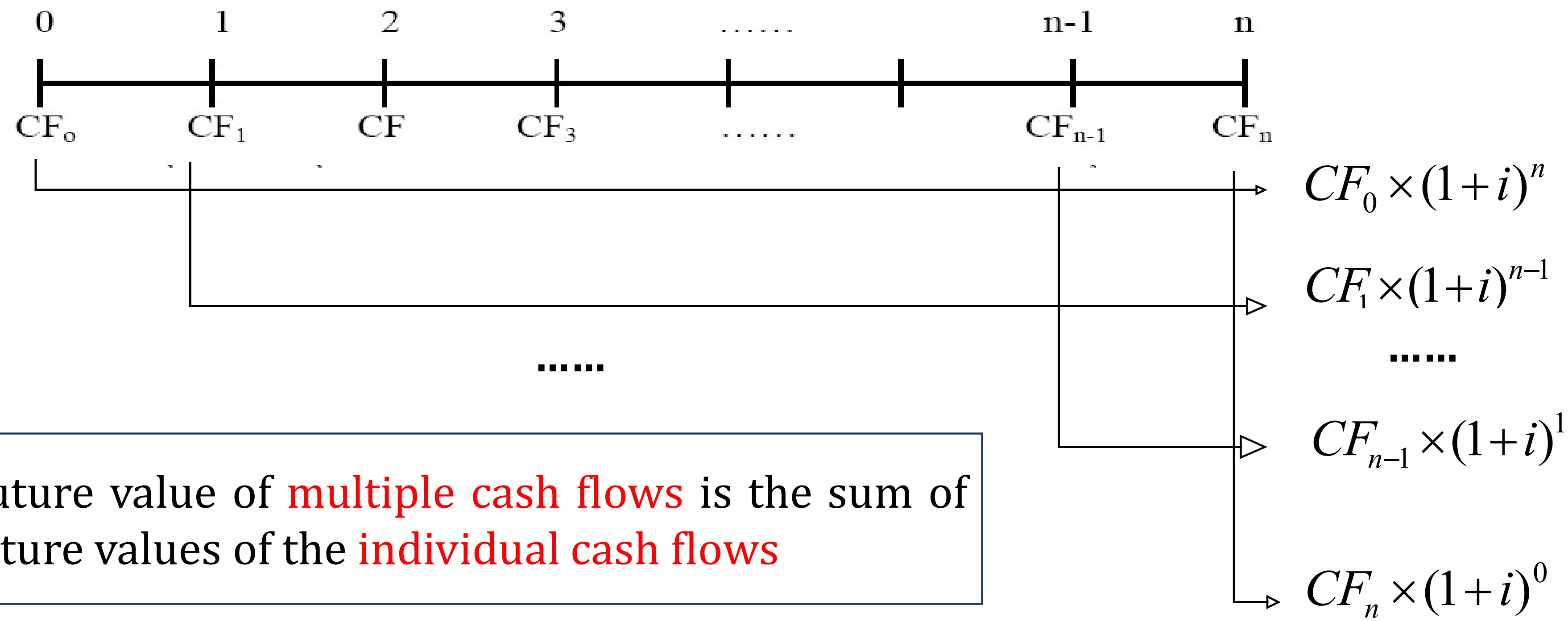
# Practice

CS6: Ms. B deposits \$10 now and then \$20 a year from now, how much will she have two years from now, if the interest rate is 8% per annum with annually compounding?




Total future value 33.264

# 4.1.Future value of multiple cash flows

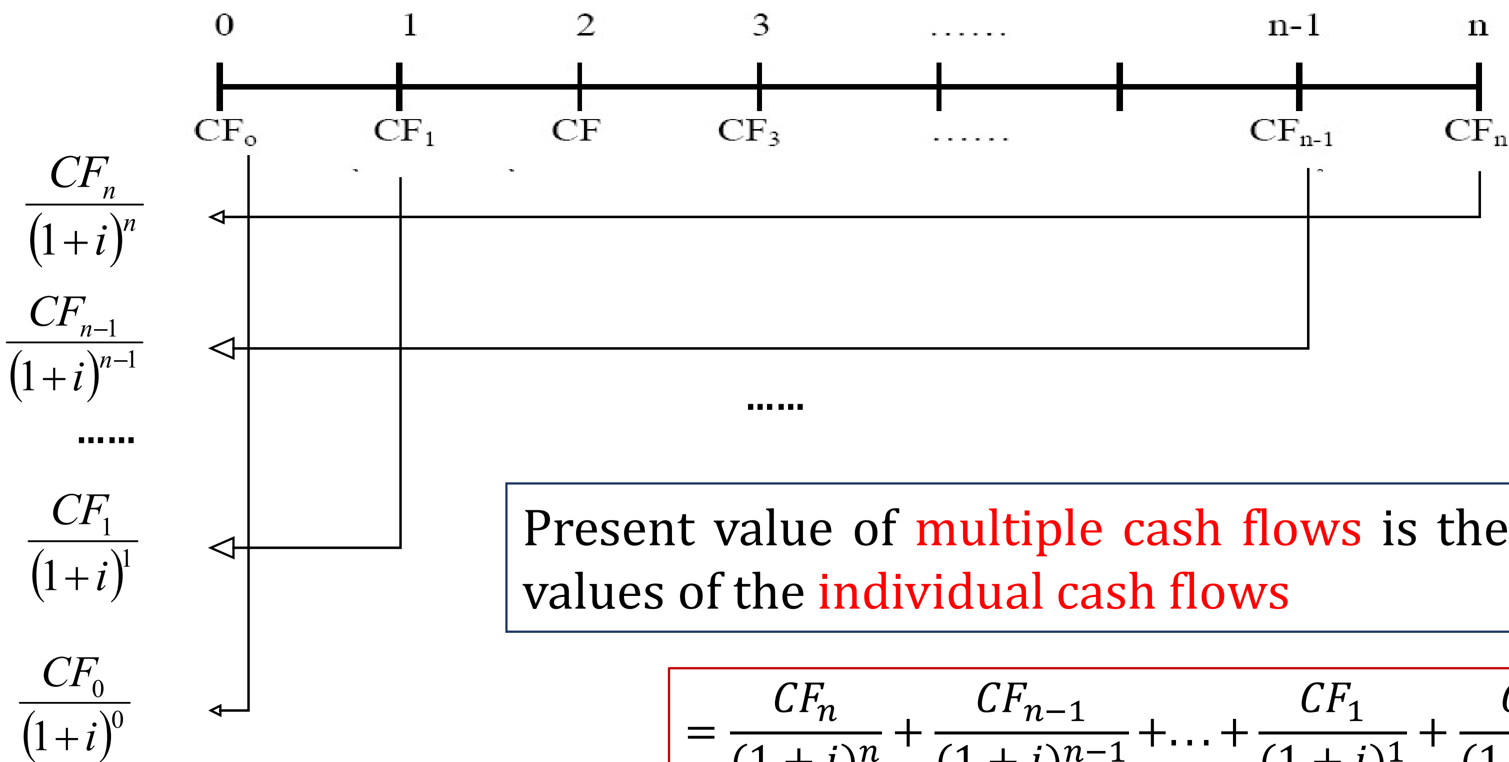



Future value of multiple cash flows is the sum of future values of the individual cash flows

$$= CF_0(1+i)^n + CF_1(1+i)^{n-1} + \ldots + CF_{n-1}(1+i)^1 + CF_n(1+i)^0$$

$$= \sum_{t=0}^{n} CF_t \times (1+i)^{n-t}$$

# 4.2. Present value of multiple cash flows



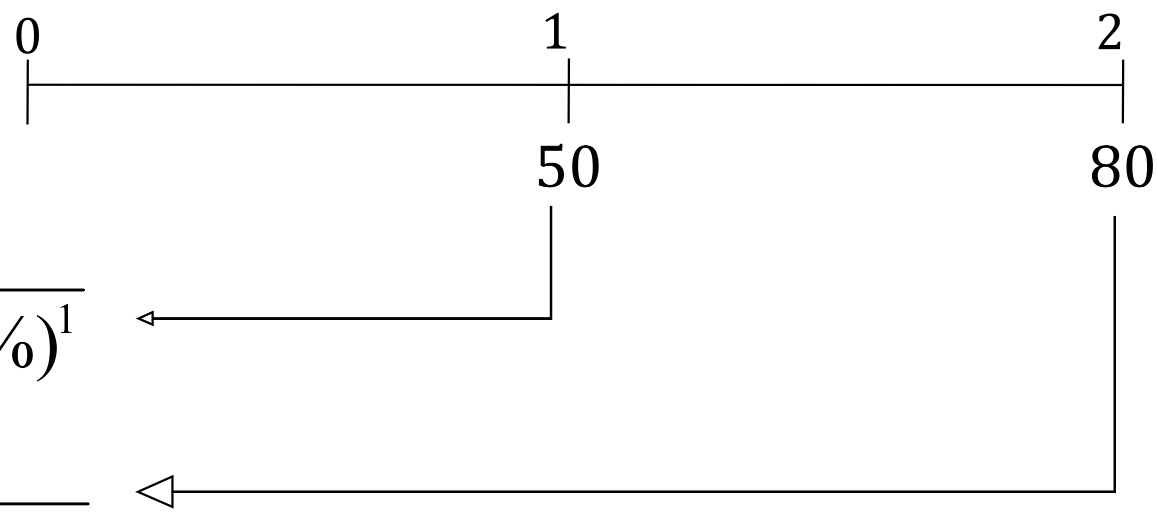

Present value of multiple cash flows is the sum of present values of the individual cash flows

$$= \frac{CF_n}{(1+i)^n} + \frac{CF_{n-1}}{(1+i)^{n-1}} + \ldots + \frac{CF_1}{(1+i)^1} + \frac{CF_0}{(1+i)^0}$$

$$= \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}$$

# Practice

CS6: Mr. C Minh wants to have $50 one year from now and then $80 in two years. If the interest rate is 8% per year, how much would Minh have to put into an account today in order to satisfy her requirement?




$$\frac{50}{(1+8\%)^1}$$

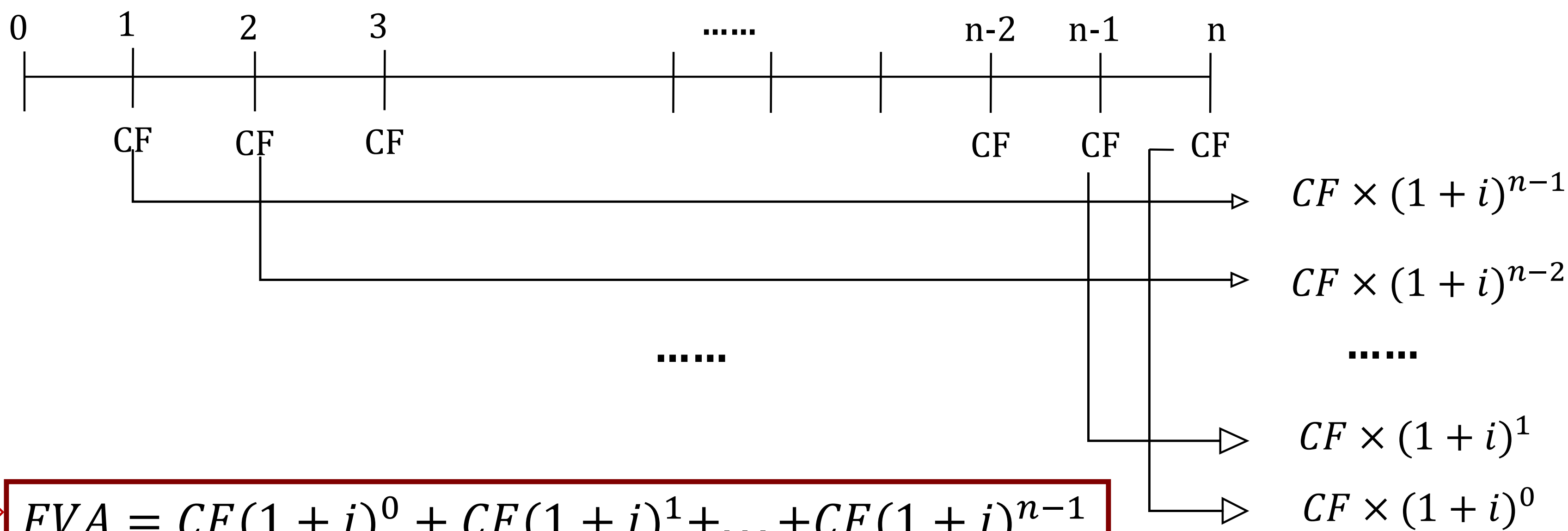
$$\frac{80}{(1+8\%)^2}$$

# 5. Special multiple cash flows: ANNUITIES

**Annuity** is a stream of equal cash flows.

- **Ordinary annuity**: the cash flows start at the end of the current period.
- **Immediate annuity/ annuity due**: the cash flows start immediately.
- **Perpetual annuity/ perpetuity**: a stream of cash flows that continues forever.
- **Growth perpetuity:** a stream of cash flows that grow over time.

# 5.1. Ordinary annuities




$$\boxed{FVA = CF(1+i)^0 + CF(1+i)^1 + \ldots + CF(1+i)^{n-1}}$$

$$\boxed{FVA_n = \sum_{t=0}^{n-1} CF(1+i)^t = CF\left[\frac{(1+i)^n - 1}{i}\right]}$$

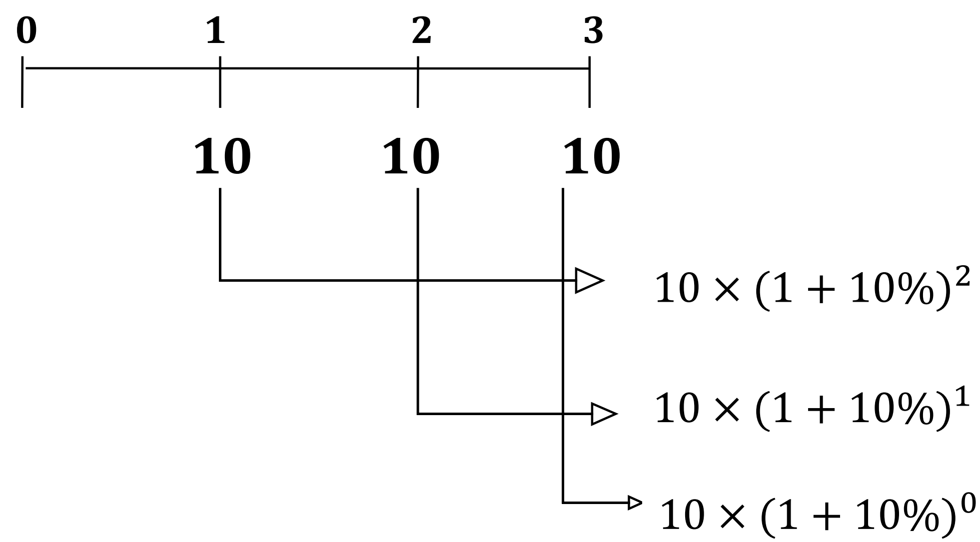
Where:

FVA: Future value of ordinary annuity
i: interest rate
n: the period (maybe: the number of year)
CF: the annuity

# Practice

CS7: Suppose that Ms. D intends to save \$10 each year for the next three years, starting one year from now. How much will she have accumulated at the end of that time if the interest rate is 10% per annum?

CF= \$10
i= 10%
n= 3
$\Rightarrow FVA_3 = ?/TotalFV = ?$

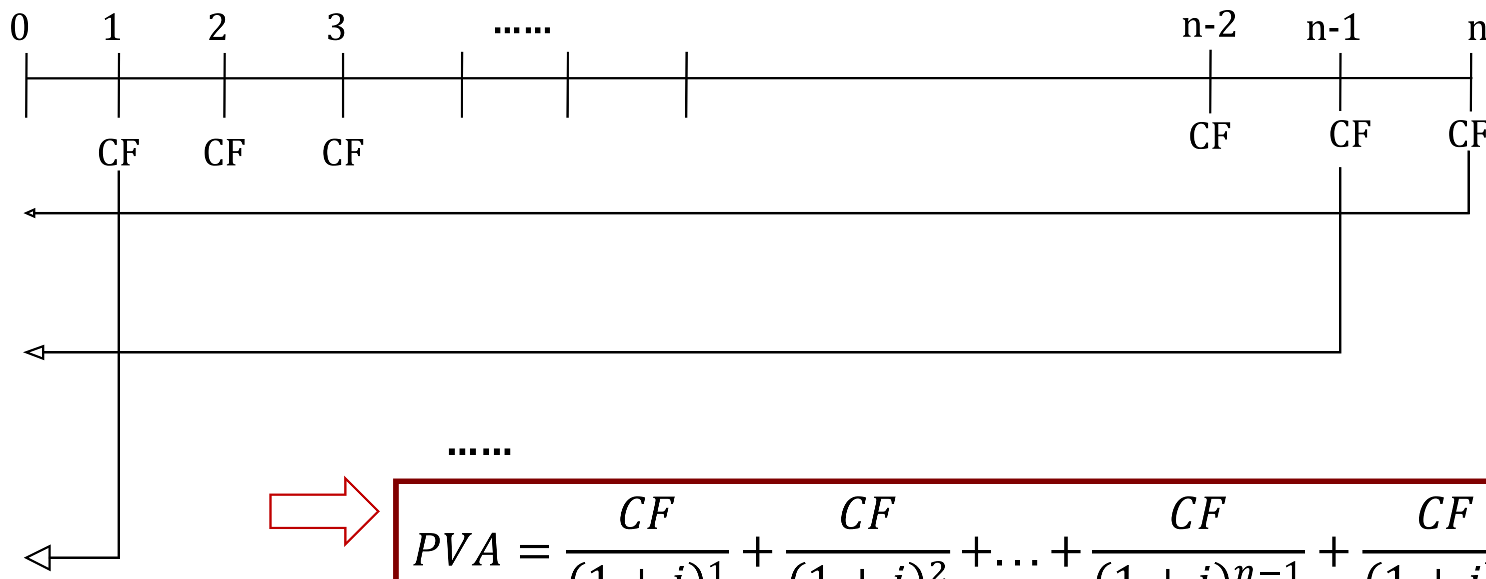



We have:

$FVA_3 =$

$$\boxed{\Rightarrow FVA_3 =}$$

# 5.1. Ordinary annuities




$$\frac{CF}{(1+i)^n}$$

$$\frac{CF}{(1+i)^{n-1}}$$

……

$$\frac{CF}{(1+i)^1}$$

……

$$\boxed{PVA = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \ldots + \frac{CF}{(1+i)^{n-1}} + \frac{CF}{(1+i)^n}}$$

$$\boxed{PVA_n = \sum_{t=1}^{n} \frac{CF}{(1+i)^t} = \frac{CF}{i}\left[1 - \frac{1}{(1+i)^n}\right]}$$

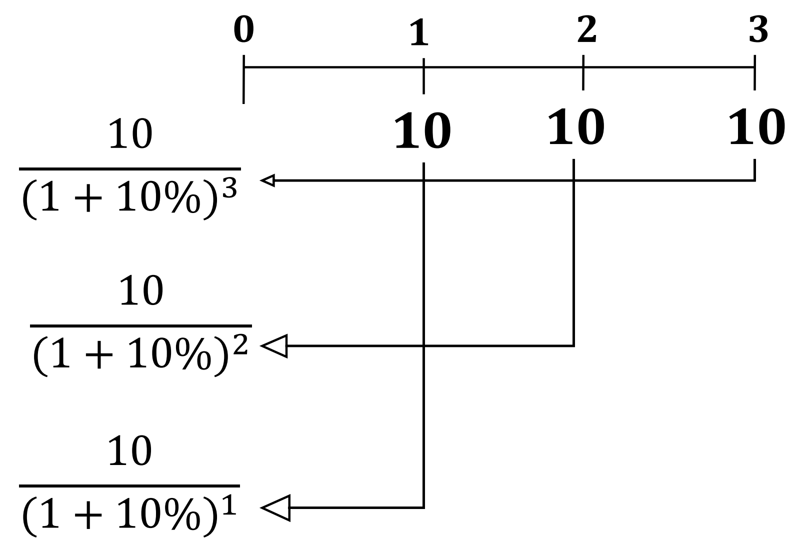
Where:

PVA: Present value of ordinary annuity
i: interest rate
n: the period (maybe: the number of year)
CF: the annuity

# Practice

CS8: You want to take out \$10 per year for next three years. How much would you have to put into a bank account earning 10% interest per year?

CF= \$10
i= 10%
n= 3
$\Rightarrow PVA_3 = ?/TotalPV = ?$

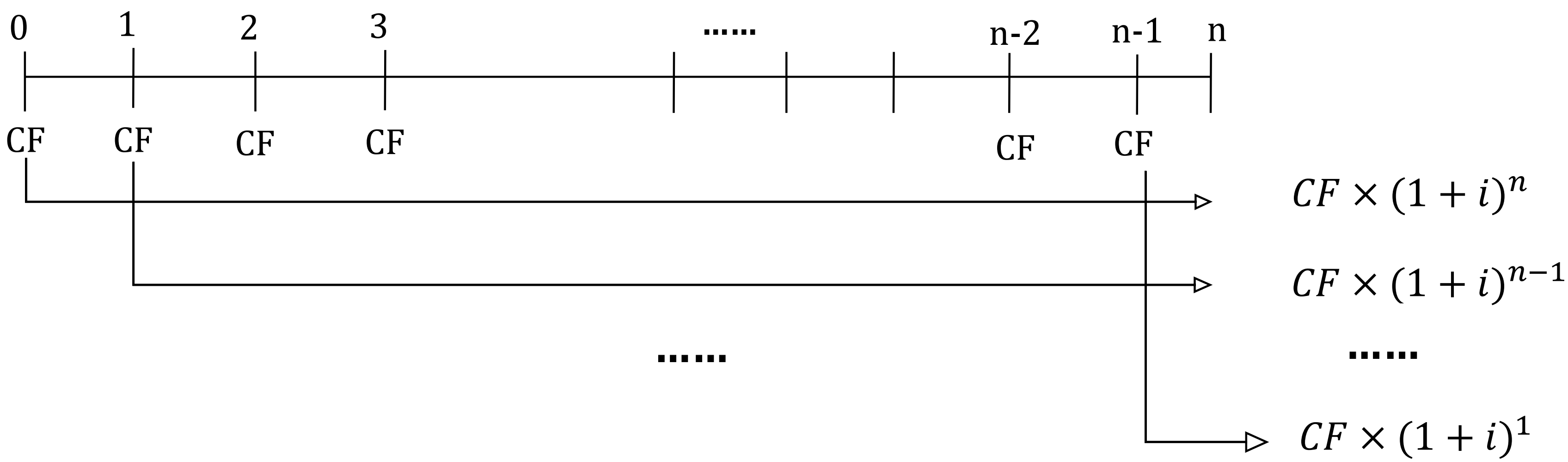



We have:

$PVA_3 =$

$$\boxed{\Rightarrow PVA_3 = \$24.87}$$

# 5.2. Immediate annuities




$$\boxed{FVAD = CF(1+i)^n + CF(1+i)^{n-1} + \ldots + CF(1+i)^1}$$

$$\boxed{FVAD_n = \sum_{t=1}^{n} CF(1+i)^t = CF\left[\frac{(1+i)^n - 1}{i}\right](1+i)}$$

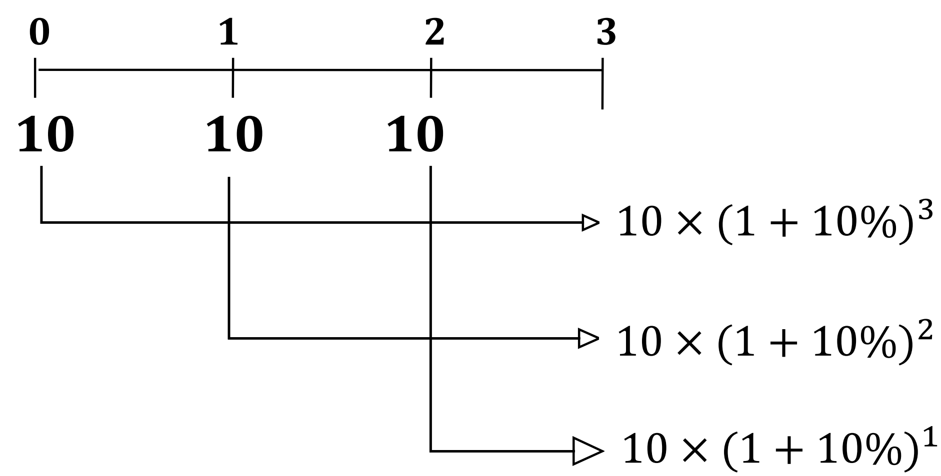
Where:

FVAD: Future value of annuity due
i: interest rate
n: the period
CF: the annuity

# Practice

CS9: Suppose that Mr. E intends to save \$10 each year for three years, starting from now. How much will he have accumulated at the end of that time if the interest rate is 10% per annum?

CF= \$10
i= 10%
n= 3
$\Rightarrow FVAD_3 = ?/TotalFV = ?$

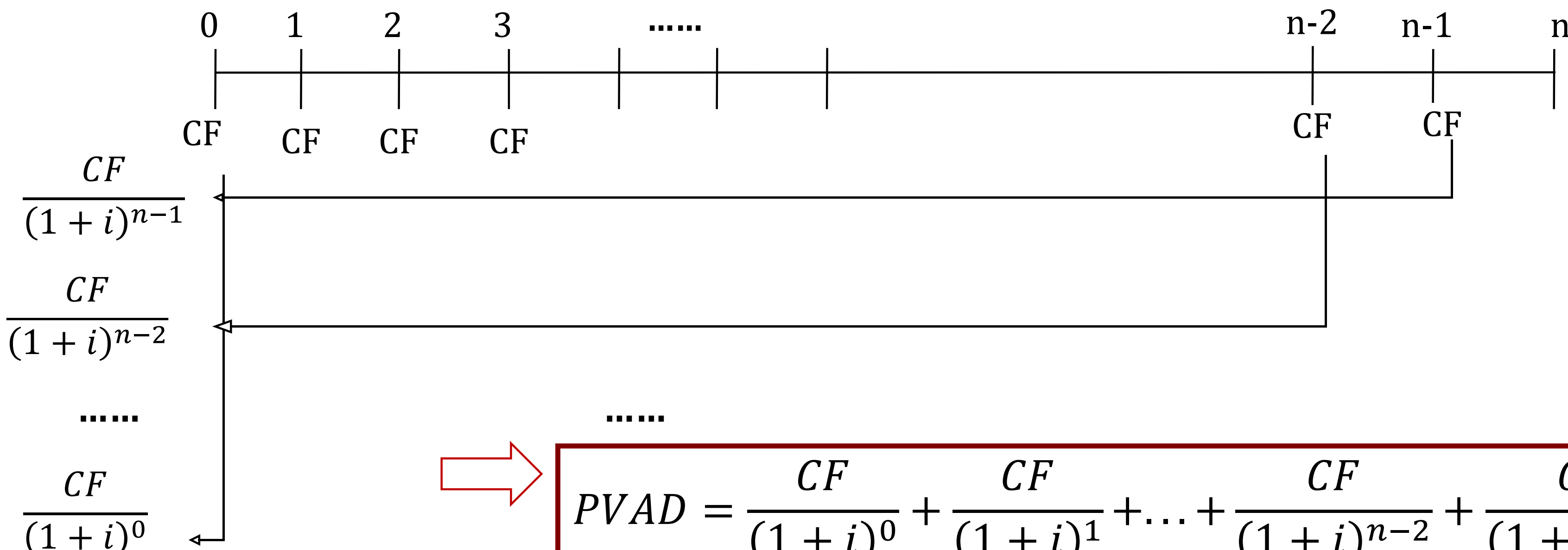



We have:

$FVAD_3 =$

$\boxed{\Rightarrow FVAD_3 =}$

# 5.2. Immediate annuities




$$\boxed{PVAD = \frac{CF}{(1+i)^0} + \frac{CF}{(1+i)^1} + \ldots + \frac{CF}{(1+i)^{n-2}} + \frac{CF}{(1+i)^{n-1}}}$$

$$\boxed{PVAD_n = \sum_{t=1}^{n} \frac{CF}{(1+i)^t} = \frac{CF}{i}\left[1 - \frac{1}{(1+i)^n}\right](1+i)}$$

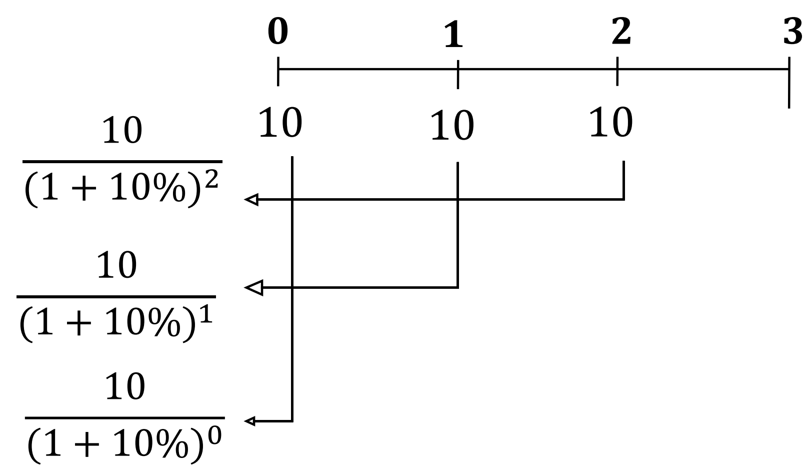
Where:

PVAD: Present value of annuity due
i: interest rate
n: the period
CF: the annuity

# Practice

CS10: Ms. F has to pay you \$10 at the beginning of each year for three years. She wants to pay off all at the beginning of the first year. How much would she pay you if the interest rate is 10% per annum.

CF= \$10
i= 10%
n= 3
$\Rightarrow PVAD_3 = ?/TotalPV = ?$

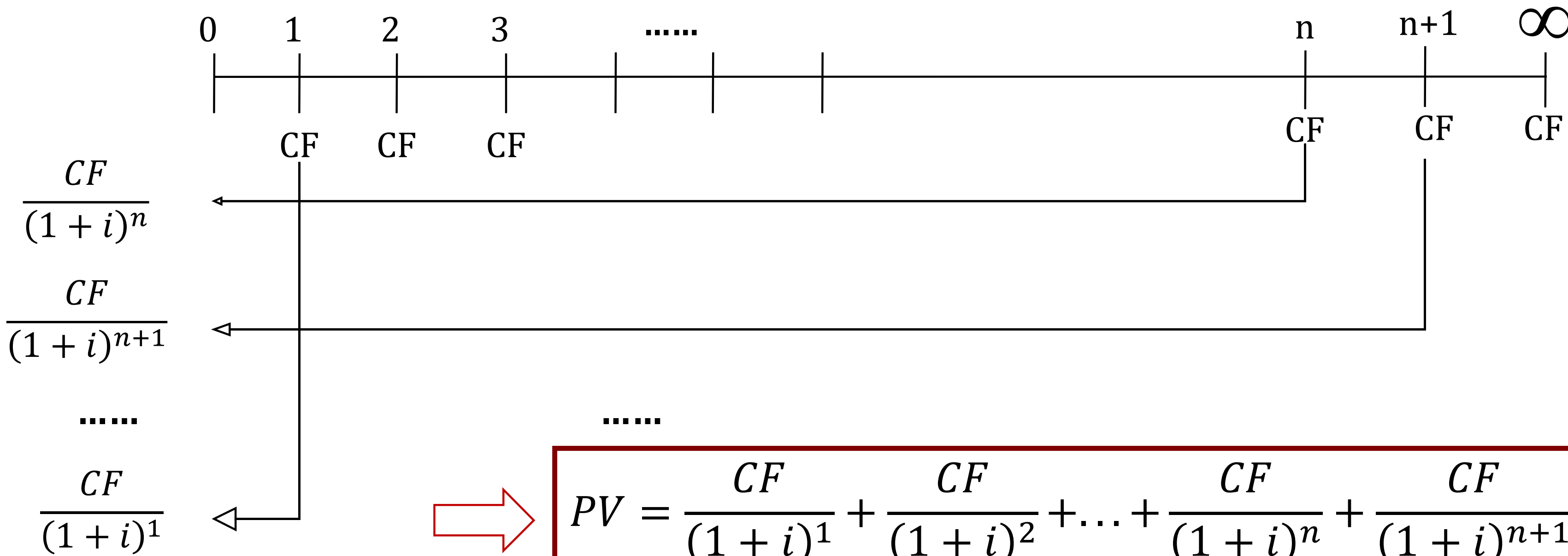



We have:

$PVAD_3 =$

$$\boxed{\Rightarrow PVAD_3 = \$27.36}$$

# 5.3. Perpetual annuities




$$\boxed{PV = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \ldots + \frac{CF}{(1+i)^n} + \frac{CF}{(1+i)^{n+1}} + \cdots}$$

$$\boxed{PV_\infty = \sum_{t=1}^{\infty} \frac{CF}{(1+i)^t} = \frac{CF}{i}}$$

Where:

$PV_\infty$ : Present value of perpetuity
i: interest rate
CF: the annuity

# Practice

CS11: How much money would you have to put into a bank account offering interest of 10%/year in order to be able to take out $100 each year forever?

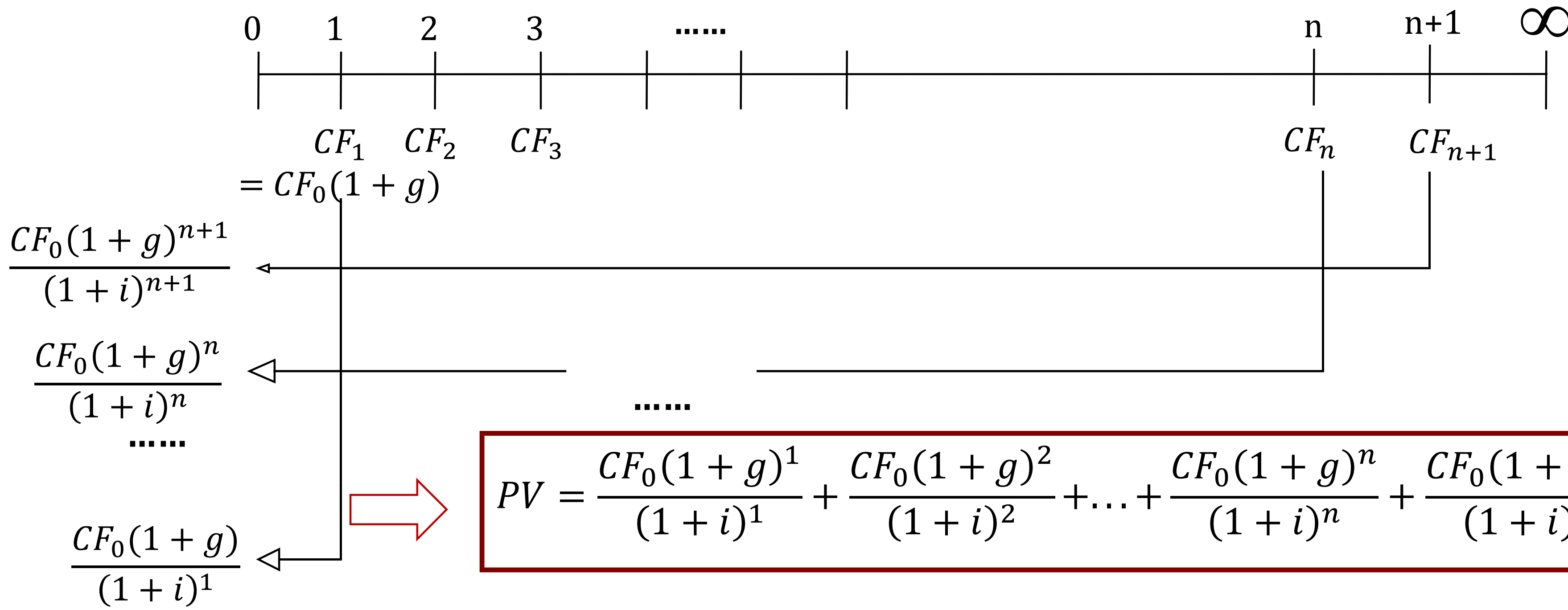
CF= $100
i= 10%
$n \to \infty$
$\Rightarrow PV_\infty = ?$

We have:

$$PV_\infty = \sum_{t=1}^{\infty} \frac{100}{(1+10\%)^t} = \frac{100}{10\%}$$

$$\boxed{\Rightarrow PV_\infty = \$1{,}000}$$

# 5.4. Growth perpetuity




$$PV = \frac{CF_0(1+g)^1}{(1+i)^1} + \frac{CF_0(1+g)^2}{(1+i)^2} + \ldots + \frac{CF_0(1+g)^n}{(1+i)^n} + \frac{CF_0(1+g)^{n+1}}{(1+i)^{n+1}} + \cdots$$

Where:

$$PV_\infty = \sum_{t=1}^{\infty} \frac{CF_0(1+g)^t}{(1+i)^t} = \frac{CF_0(1+g)}{i-g} = \frac{CF_1}{i-g}$$

i: interest rate
g: growth rate
$CF_1$: the first year's cash flow

# 6. Applications

6.1. Alternative discounted-cash-flow decision rules

6.1.1. Net present value (NPV)

6.1.2. Internal rate of return (IRR)

6.2. Amortization schedule

6.3. Valuation

# 6.1. Alternative discounted-cash-flow decision rules

## 6.1.1. Net present value (NPV)

**Net present value** of an investment is the present value of its expected cash inflows minus the present value of its expected cash outflows.

Formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+i)^t}$$

Where:

$CF_t$ : the expected net cash flow at time t
i: the discount rate/ required interest rate/ opportunity cost of capital
n: the investment's projected life

**The NPV rule**:

If the investment's NPV is positive, an investor should undertake it;
If the investment's NPV is negative, an investor should not undertake it;
If an investor has two candidates for investment but only invest in one, the investor should choose the candidate with the higher positive NPV.

# Practice

CS12: An investment project with an initial cost of \$500,000 and positive cash flows of \$160,000 at the end of year 1; \$240,000 at the end of year 2; \$280,000 at the end of year 3. Should you undertake it if the opportunity cost is 12%?

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| - 500,000 | 160,000 | 240,000 | 280,000 |

The NPV of this project is the sum of the PVs of the project's individual cash flows and is determined as follows:

$$NPV =$$

$$= \$33.48 \quad > 0$$

$\Rightarrow$Should undertake it

# 6.1. Alternative discounted-cash-flow decision rules

## 6.1.2. Internal rate of return (IRR)

The internal rate of return (IRR) is defined as the rate of return that equates the PV of an investment's expected benefits with the PV of its costs; or the discount rate for which the NPV of an investment is zero

Formula:

$$\boxed{0 = \sum_{t=0}^{n} \frac{CF_t}{(1 + IRR)^t}}$$

Where:

$CF_t$ : the expected net cash flow at time t
n: the investment's projected life

**The IRR rule**:

**Accept** projects with an IRR that is **greater** than the firm's (investor's) required rate of return

**Reject** projects with an IRR that is **less** than the firm's (investor's) required rate of return

# Practice

CS13: An investment project with an initial cost of \$500,000 and positive cash flows of \$160,000 at the end of year 1; \$240,000 at the end of year 2; \$280,000 at the end of year 3. What is its IRR?

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| - 500,000 | 160,000 | 240,000 | 280,000 |

IRR is the results of following equation:

$$0 = -500000 + \frac{160000}{1 + IRR} + \frac{240000}{(1 + IRR)^2} + \frac{280000}{(1 + IRR)^3} \quad \Rightarrow IRR \approx 0.1552 = 15.52\%$$

$> 0$

$\Rightarrow$Should undertake it

# 6.2. Amortization schedule

Loan amortization is the process of paying off a loan with a series of periodic loan payments whereby a portion of the outstanding loan amount is paid off, or amortized, with each payment.

Periodic payment

Interest on the outstanding balance of loan

The rest: repayment of principal

Ex: Home mortgage loans, repaid in equal periodic installments

# Practice

CS14: Assume that Mr. G takes a \$100000 home mortgage loan at an interest rate of 9%/ year to be repaid with interest in three annual installments. How much would he pay each year? Show the exact amount of the principal and interest components of each loan payment.

Periodic payment/installment: CF

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 100,000 | -CF | -CF | -CF |

Present value of installments should be equal to the loan principle.

$$\Rightarrow 100{,}000 = \frac{CF}{(1+i)^1} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} = \sum_{t=1}^{3} \frac{CF}{(1+i)^t} \quad \text{or} \quad 100{,}000 = CF\left[\frac{1-(1+i)^{-n}}{i}\right]$$

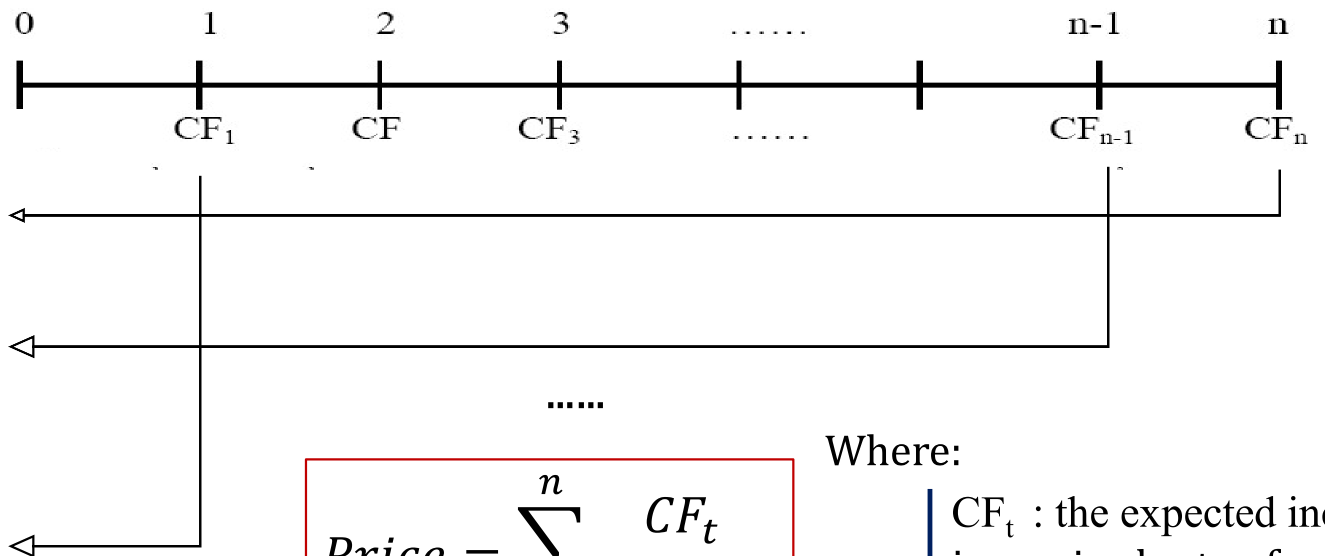
$$\Rightarrow CF = \frac{100{,}000 \times i}{1-(1+i)^{-n}} = \frac{100000 \times 9\%}{1-(1+9\%)^{-3}} = \$39505.48$$

## 6.2. Amortization schedule

| Year | Beginning balance | Total payment | Interest paid | Principal paid | Remaining balance |
|---|---|---|---|---|---|
| 1 | 100000 | 39505.48 | 9000 | 30505.48 | 69494.52 |
| 2 | 69494.52 | 39505.48 | 6254.51 | 33250.97 | 36243.55 |
| 3 | 36243.55 | 39505.48 | 3261.92 | 36243.56 | $\approx 0$ |

# 6.3. Valuation

The price of an asset/ security is equal to the present value of its expected income stream




$$\frac{CF_n}{(1+i)^n}$$

$$\frac{CF_{n-1}}{(1+i)^{n-1}}$$

......

$$\frac{CF_1}{(1+i)^1}$$

$$Price = \sum_{t=1}^{n} \frac{CF_t}{(1+i)^t}$$

Where:

$CF_t$ : the expected income at time t
i: required rate of return
n: the investment's projected life