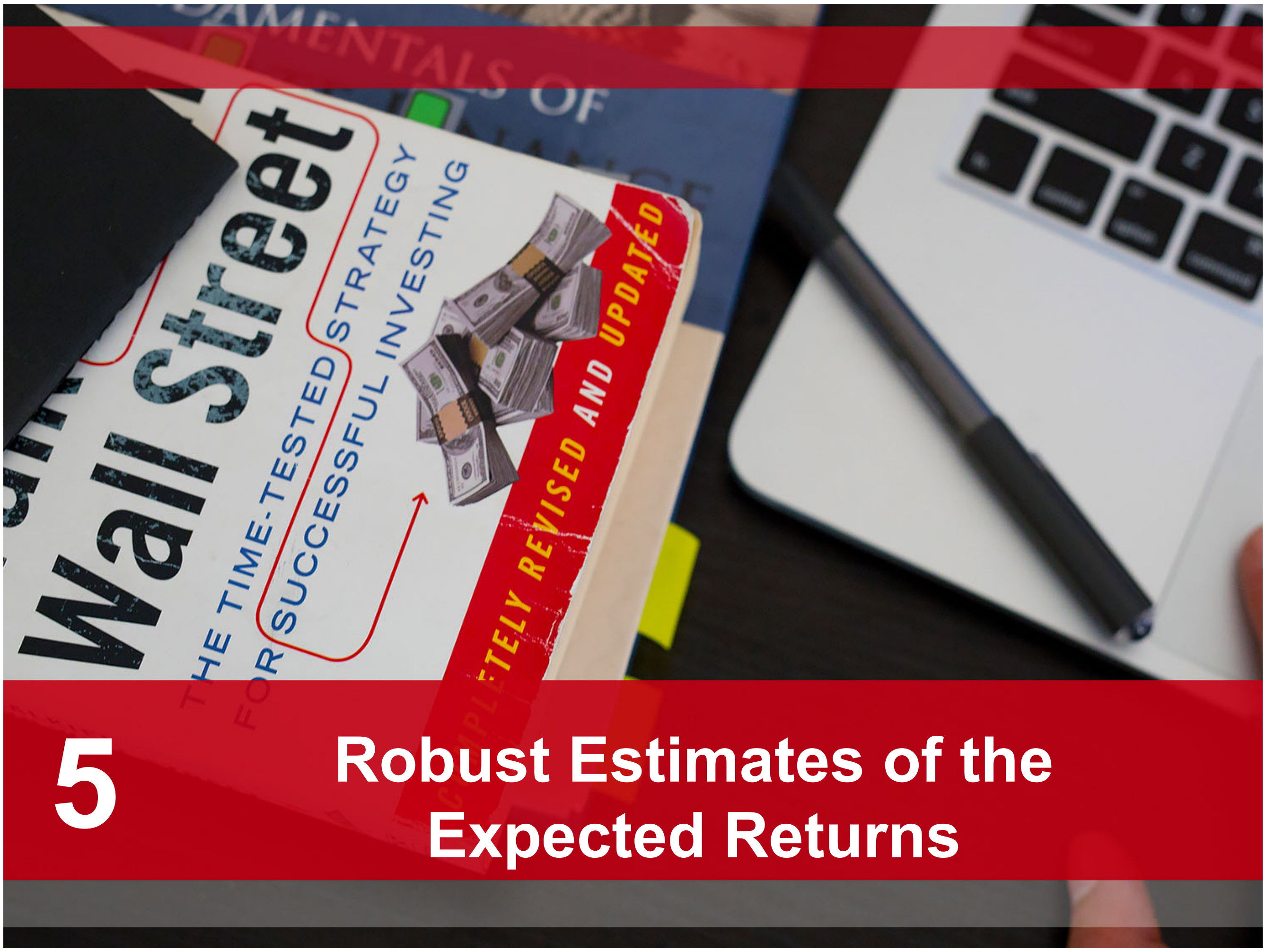


5 Robust Estimates of the Expected Returns

Portfolio management: The shortcomings of portfolio optimisation

- A problem with portfolio management is that implementation of the mean-variance optimisation can lead to estimation of optimal portfolios with extreme weights
- This is because we do not know the true inputs to the portfolio optimisation procedure, but must instead estimate them on the basis of historical data
- Under-estimating expected returns for some stocks will lead to large negative portfolio weights; this is compounded by the fact that if the correlations between the assets are high (as they commonly are in practice), optimisation will favour low expected return assets that have lower correlations
- The problem is so bad that in practice, naïve equally weighted portfolios of assets will in many cases outperform those based on portfolio optimisation. Note that it is not the theory at fault; it is simply that it is very difficult to estimate the inputs that the theory requires

Lack of Robustness of Expected Return Estimates

- For example, suppose that we have collected five years of monthly data for ten stocks in order to estimate the optimal portfolio; these stocks are recorded in the spreadsheet below, together with their market capitalisation

DTU406 - Portfolio Management [Compatibility Mode] - Microsoft Excel

PRICE AND MARKET CAP DATA FOR 10 COMPANIES

	General Motors GM	Home Depot HD	International Paper IP	Hewlett-Packard HPQ	Altria MO	American Express AXP	Alcoa Aluminum AA	DuPont DD	Merck MRK	MMM
Market capitalization (billion $)	**16.85**	**73.98**	**15.92**	**88.37**	**153.33**	**65.66**	**28.16**	**38.32**	**79.51**	**60.9**
Benchmark proportion	2.71%	11.91%	2.56%	14.23%	24.69%	10.57%	4.53%	6.17%	12.80%	9.81%

=B3/SUM($B$3:$K$3)

Lack of Robustness of Expected Return Estimates

- Now we estimate the expected return vector and variance-covariance matrix for the ten stocks

DTU406 - Portfolio Management [Compatibility Mode] - Microsoft Excel

=AVERAGE(U10:U69)

	GM	HD	IP	HPQ	MO	AXP	AA	DD	MRK	MMM
Mean Return	**-0.87%**	**-0.39%**	0.06%	0.30%	1.07%	0.82%	**-0.18%**	0.03%	**-0.55%**	0.75%
Return σ	10.88%	8.48%	6.29%	10.90%	8.78%	6.48%	9.62%	6.18%	8.13%	5.58%

=MMULT(TRANSPOSE(L10:U69-X7:AG7),L10:U69-X7:AG7)/(COUNT(U10:U69)-1)

=STDEV(S10:S69)

VCV Matrix

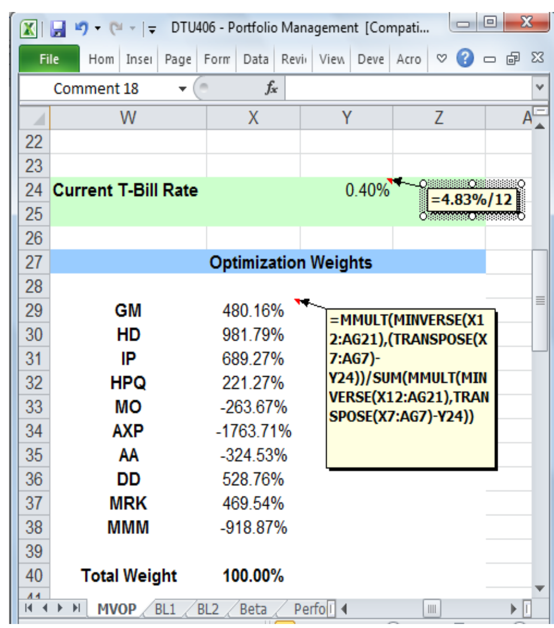
	GM	HD	IP	HPQ	MO	AXP	AA	DD	MRK	MMM
GM	0.0118	0.0031	0.0024	0.0042	0.0014	0.0033	0.0046	0.0018	0.0010	0.0014
HD	0.0031	0.0072	0.0019	0.0043	0.0022	0.0033	0.0046	0.0020	0.0002	0.0018
IP	0.0024	0.0019	0.0040	0.0031	0.0001	0.0024	0.0043	0.0021	0.0012	0.0016
HPQ	0.0042	0.0043	0.0031	0.0119	0.0026	0.0049	0.0061	0.0033	0.0020	0.0022
MO	0.0014	0.0022	0.0001	0.0026	0.0077	0.0016	0.0018	0.0009	0.0007	0.0008
AXP	0.0033	0.0033	0.0024	0.0049	0.0016	0.0042	0.0038	0.0019	0.0011	0.0014
AA	0.0046	0.0046	0.0043	0.0061	0.0018	0.0038	0.0093	0.0041	0.0018	0.0024
DD	0.0018	0.0020	0.0021	0.0033	0.0009	0.0019	0.0041	0.0038	0.0017	0.0019
MRK	0.0010	0.0002	0.0012	0.0020	0.0007	0.0011	0.0018	0.0017	0.0066	0.0005
MMM	0.0014	0.0018	0.0016	0.0022	0.0008	0.0014	0.0024	0.0019	0.0005	0.0031

Lack of Robustness of Expected Return Estimates

- We derive the optimal portfolio with a risk free asset; we use a risk free rate of 4.83% per annum (i.e. 0.40% per month)
- We assume that short selling is allowed although the resulting portfolio should, by construction, comprise only long positions (to the extent that it is representative of the market portfolio whose weights are all positive)
- The resulting portfolio is clearly impracticable: most mutual funds are not allowed to short sell stocks, and even funds that can short sell stocks will find it difficult to short sell 17.63 times the fund value in AXP or 9.19 times the fund value in MMM; the enormous long positions that result from these short-sale positions are similarly impracticable

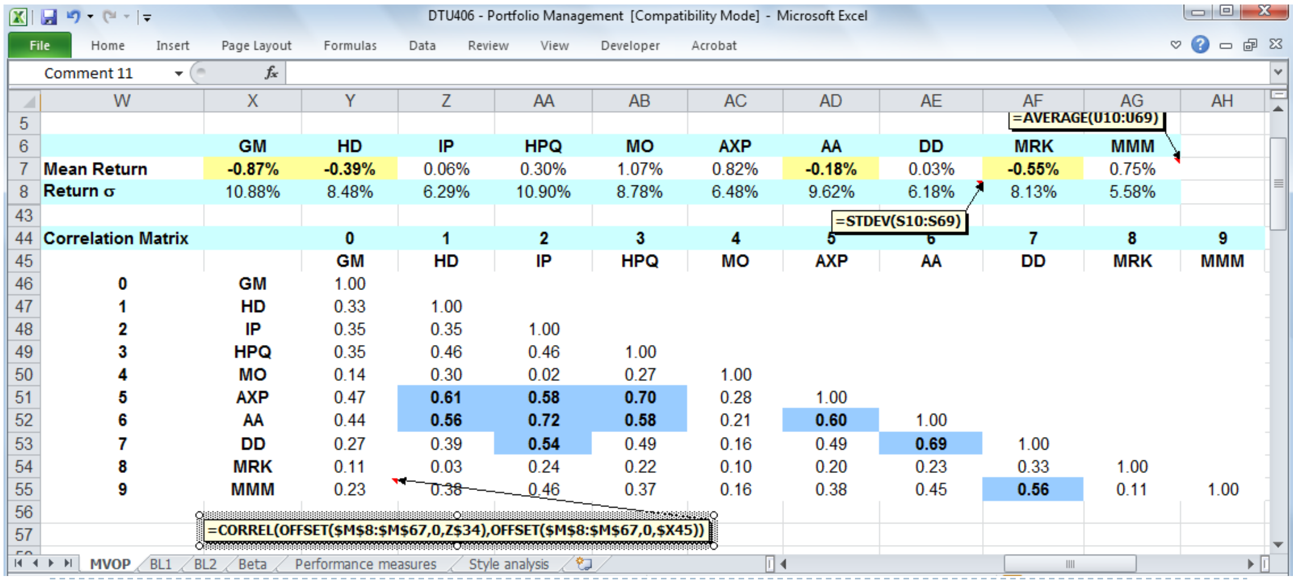
Lack of Robustness of Expected Return Estimates

- The reason of this odd ‘optimised portfolio’ can be explained as:
- A number of the historical mean returns are negative; if we ignore the effects of correlations, a negative expected return should imply a short position in the stock
- The correlations between asset returns are in some cases very large; large correlations for a particular stock can lead us to prefer other stock with smaller returns but more moderate correlations



Lack of Robustness of Expected Return Estimates

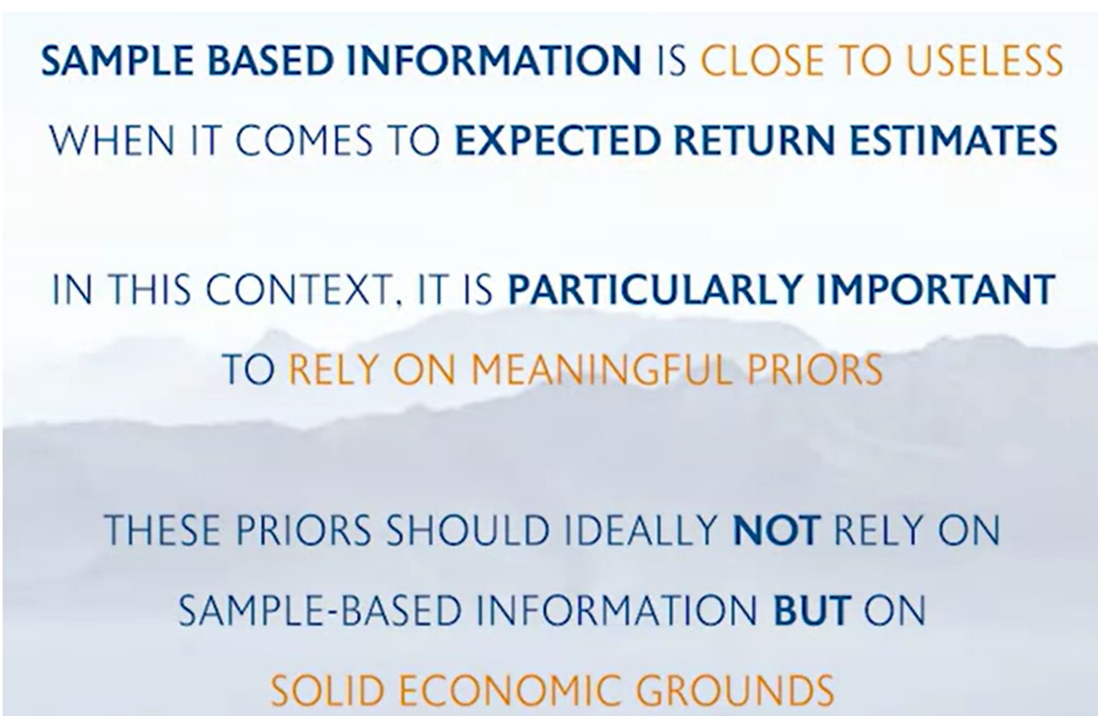
- Here we highlight the stocks with negative historical returns and stocks whose correlations are greater than 0.5



	GM	HD	IP	HPQ	MO	AXP	AA	DD	MRK	MMM
Mean Return	-0.87%	-0.39%	0.06%	0.30%	1.07%	0.82%	-0.18%	0.03%	-0.55%	0.75%
Return σ	10.88%	8.48%	6.29%	10.90%	8.78%	6.48%	9.62%	6.18%	8.13%	5.58%

Correlation Matrix		0	1	2	3	4	5	6	7	8	9
		GM	HD	IP	HPQ	MO	AXP	AA	DD	MRK	MMM
0	GM	1.00									
1	HD	0.33	1.00								
2	IP	0.35	0.35	1.00							
3	HPQ	0.35	0.46	0.46	1.00						
4	MO	0.14	0.30	0.02	0.27	1.00					
5	AXP	0.47	0.61	0.58	0.70	0.28	1.00				
6	AA	0.44	0.56	0.72	0.58	0.21	0.60	1.00			
7	DD	0.27	0.39	0.54	0.49	0.16	0.49	0.69	1.00		
8	MRK	0.11	0.03	0.24	0.22	0.10	0.20	0.23	0.33	1.00	
9	MMM	0.23	0.38	0.46	0.37	0.16	0.38	0.45	0.56	0.11	1.00

Lack of Robustness of Expected Return Estimates



Using Factor Models to Estimate Expected Returns

- The CAPM model
- The multifactor model

The Black – Litterman Model

- The Black-Litterman model is a traditional asset allocation model, which was developed in 1990 by Fisher Black and Robert Litterman at Goldman Sachs
- It provides investors with a tool to calculate optimal portfolio weights under specified parameters of unintuitive results from the mean variance optimisation (MVO)
- The model combines both passive input for expected returns using and investor forecasts of expected returns (i.e. unique active views)
- It is more intuitive compared to the Markowitz mean variance model and prevents heavy changes in portfolio weightings

The Black – Litterman Model

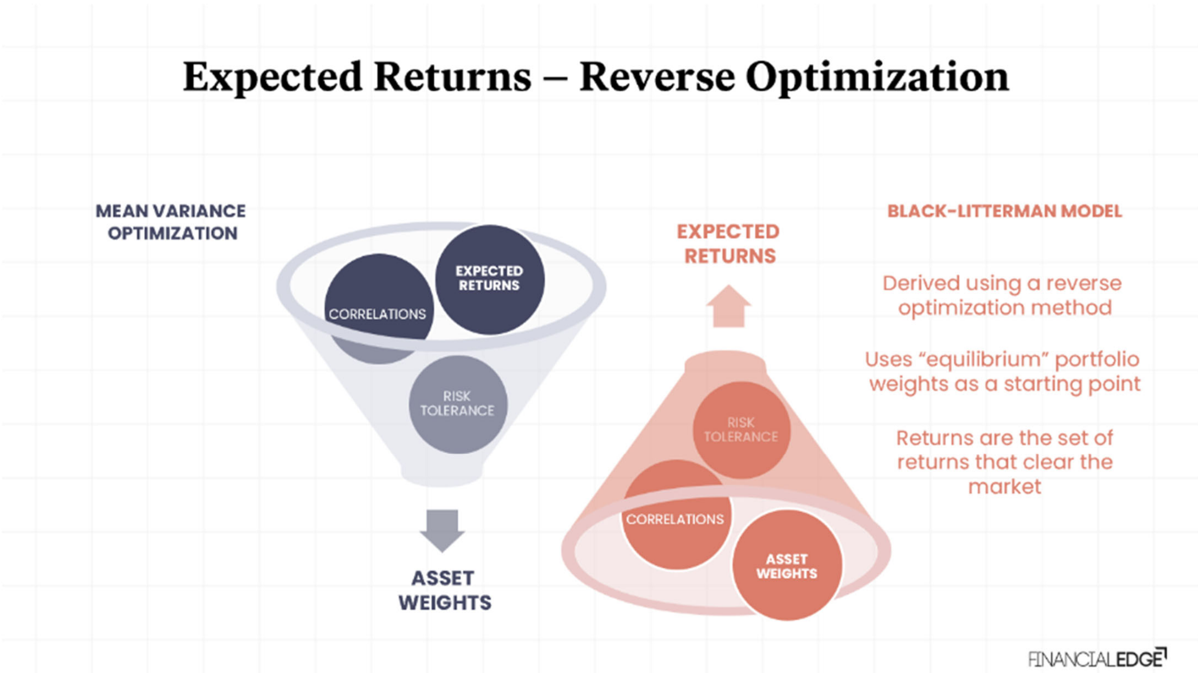
- The B&L model is an application of Bayesian analysis to portfolio construction.
- It leads to intuitive portfolios with sensible portfolio weights.
- Black-Litterman Model takes the Markowitz Model one step further
  - Incorporates an investor's own views in determining asset allocations

The Black – Litterman Model: Basic Idea

1. Find implied returns
2. Formulate investor views
3. Determine what the expected returns are
4. Find the asset allocation for the optimal portfolio

The Black – Litterman Model

- **Extracting Implied Expected Returns**

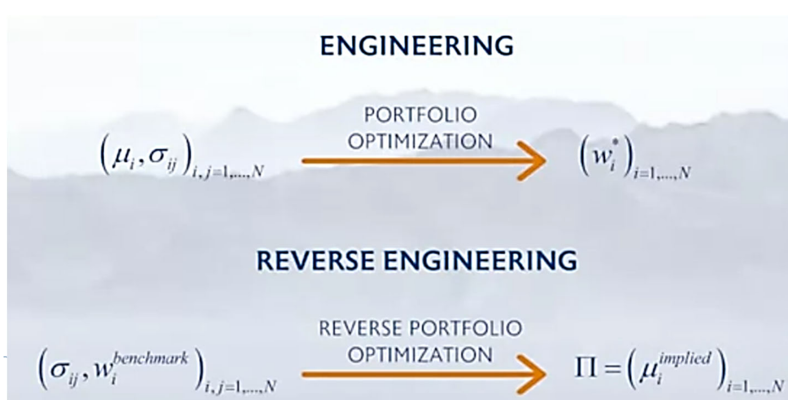


The Black – Litterman Model

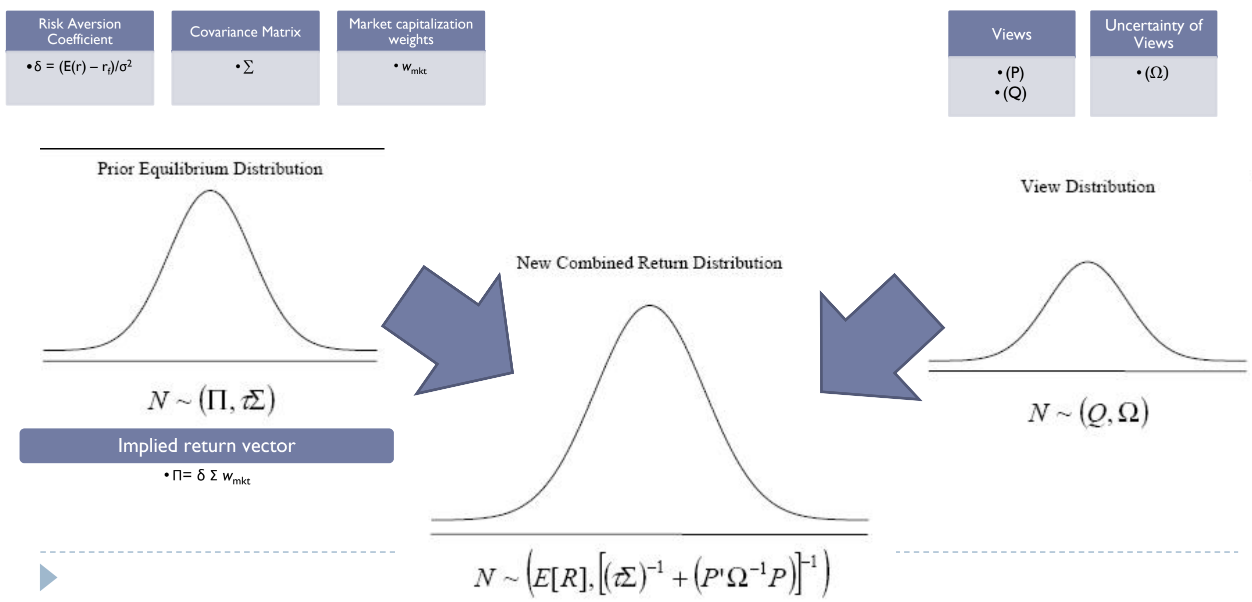
- **Extracting Implied Expected Returns**

The Black-Litterman model uses the market portfolio which is the *true optimal portfolio* according to CAPM as an anchor point.

The neutral prior distribution is obtained by reverse engineering assuming market or benchmark is the optimal portfolio.



Implied Returns + Investor Views = Expected Returns



Bayesian Theory

- Traditionally, personal views are used for the prior distribution
- Then observed data is used to generate a posterior distribution
- The Black-Litterman Model assumes implied returns as the prior distribution and personal views alter it

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Expected Returns: Inputs

$$\Pi = \delta \, \Sigma \, w_{mkt}$$

- Π = The equilibrium risk premium over the risk free rate (Nx1 vector)
- $\delta = (E(r) - r_f)/\sigma^2$, risk aversion coefficient
- Σ = A covariance matrix of the assets (NxN matrix)

Expected Returns

$$E(R) = [(\tau \, \Sigma)^{-1} + P^T \, \Omega^{-1} P]^{-1} \, [(\tau \, \Sigma)^{-1} \, \Pi + P^T \, \Omega^{-1} \, Q]$$

- Assuming there are N-assets in the portfolio, this formula computes E(R), the expected new return.
- τ = A scalar number indicating the uncertainty of the CAPM distribution (0.025-0.05)

Expected Returns

- P = A matrix with investors views; each row a specific view of the market and each entry of the row represents the portfolio weights of each assets (KxN matrix)
- Ω = A diagonal covariance matrix with entries of the uncertainty within each view (KxK matrix)
- Q = The expected returns of the portfolios from the views described in matrix P (Kx1 vector)

Expected Returns

Understanding the Formula

- Consider the second part first:

$$[(\tau \Sigma)^{-1} \Pi + P^T \Omega^{-1} Q]$$

- We are combining implied excess returns with our own views on excess returns.
  - A weighted average.
- **What are the weights?**
  - How confident the investor is about his/her views relative to the implied excess returns.

Expected Returns

$$E(R) = [(\tau \Sigma)^{-1} + P^T \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P^T \Omega^{-1} Q]$$

- **What does the first part do?**
- The first term is there to ensure that the weights assigned to implied excess returns and our views add up to 1.
- The formula is just a weighted average!

An example

- **Consider 1 stock, AMZN.**
- The implied excess returns are 0.74% per month and the variance is $2.015\%^2$.
- We predict excess returns of 2% per month. The uncertainty surrounding this view is reflected by a variance of $0.50\%^2$.
- Assume $\tau = 1$, $P = 1$.

Another example

Table 1 Expected Excess Return Vectors

Asset Class	Historical μ_{Hist}	CAPM GSMI μ_{GSMI}	CAPM Portfolio μ_P	Implied Equilibrium Return Vector Π
US Bonds	3.15%	0.02%	0.08%	0.08%
Int'l Bonds	1.75%	0.18%	0.67%	0.67%
US Large Growth	-6.39%	5.57%	6.41%	6.41%
US Large Value	-2.86%	3.39%	4.08%	4.08%
US Small Growth	-6.75%	6.59%	7.43%	7.43%
US Small Value	-0.54%	3.16%	3.70%	3.70%
Int'l Dev. Equity	-6.75%	3.92%	4.80%	4.80%
Int'l Emerg. Equity	-5.26%	5.60%	6.60%	6.60%
Weighted Average	-1.97%	2.41%	3.00%	3.00%
Standard Deviation	3.73%	2.28%	2.53%	2.53%
High	3.15%	6.59%	7.43%	7.43%
Low	-6.75%	0.02%	0.08%	0.08%

** All four estimates are based on 60 months of excess returns over the risk-free rate. The two CAPM estimates are based on a risk premium of 3. Dividing the risk premium by the variance of the market (or benchmark) excess returns (σ^2) results in a risk-aversion coefficient (λ) of approximately 3.07.*

Another example

Table 2 Recommended Portfolio Weights

Asset Class	Weight Based on Historical w_{Hist}	Weight Based on CAPM GSMI w_{GSMI}	Weight Based on Implied Equilibrium Return Vector Π	Market Capitalization Weight w_{mkt}
US Bonds	1144.32%	21.33%	19.34%	19.34%
Int'l Bonds	-104.59%	5.19%	26.13%	26.13%
US Large Growth	54.99%	10.80%	12.09%	12.09%
US Large Value	-5.29%	10.82%	12.09%	12.09%
US Small Growth	-60.52%	3.73%	1.34%	1.34%
US Small Value	81.47%	-0.49%	1.34%	1.34%
Int'l Dev. Equity	-104.36%	17.10%	24.18%	24.18%
Int'l Emerg. Equity	14.59%	2.14%	3.49%	3.49%
High	1144.32%	21.33%	26.13%	26.13%
Low	-104.59%	-0.49%	1.34%	1.34%

View 1: International Developed Equity will have an absolute excess return of 5.25% (Confidence of View = 25%).

View 2: International Bonds will outperform US Bonds by 25 basis points (Confidence of View = 50%).

View 3: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 2% (Confidence of View = 65%).

General Case:

$$Q+\varepsilon = \begin{bmatrix} Q_1 \\ \vdots \\ Q_k \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

Example:

$$Q+\varepsilon = \begin{bmatrix} 5.25 \\ 0.25 \\ 2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix}$$

where

$$\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_k \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \omega_{11} & \cdots & \omega_{1k} \\ \vdots & \ddots & \vdots \\ \omega_{k1} & \cdots & \omega_{kk} \end{bmatrix} \right) \leftarrow \boxed{\Omega}$$

Another example

General Case:

$$P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{k,1} & \cdots & p_{k,n} \end{bmatrix}$$

Example (Based on Satchell and Scowcroft (2000)): (6)

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .5 & -.5 & .5 & -.5 & 0 & 0 \end{bmatrix}$$

Table 4 Variance of the View Portfolios

View	Formula	Variance
1	$p_1 \Sigma p_1'$	2.836%
2	$p_2 \Sigma p_2'$	0.563%
3	$p_3 \Sigma p_3'$	3.462%

Another example

General Case:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}$$

The easiest way to calibrate the Black-Litterman model is to make an assumption about the value of the scalar (τ). He and Litterman (1999) calibrate the confidence of a view so that the ratio of ω/τ is equal to the variance of the view portfolio ($p_k \Sigma p_k^{'}$). Assuming τ = 0.025 and using the individual variances of the view portfolios ($p_k \Sigma p_k^{'}$) from Table 4, the covariance matrix of the error term (Ω) has the following form:

General Case: Example: (8)

$$\Omega = \begin{bmatrix} (p_1 \Sigma p_1^{'}) * \tau & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & (p_k \Sigma p_k^{'}) * \tau \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0.000709 & 0 & 0 \\ 0 & 0.000141 & 0 \\ 0 & 0 & 0.000866 \end{bmatrix}$$

Another example

General Case:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_k \end{bmatrix}$$

There is no best way to calculate Ω. It will depend on how confident you are of your predictions.

Black and Litterman recommend:

$$\Omega = \tau P S P^T$$

Another example

Table 6 Return Vectors and Resulting Portfolio Weights

Asset Class	New Combined Return Vector $E[R]$	Implied Equilibrium Return Vector Π	Difference $E[R]-\Pi$	New Weight $\hat{w}$	Market Capitalization Weight w_{mkt}	Difference $\hat{w}-w_{mkt}$
US Bonds	0.07%	0.08%	-0.02%	29.88%	19.34%	10.54%
Int'l Bonds	0.50%	0.67%	-0.17%	15.59%	26.13%	-10.54%
US Large Growth	6.50%	6.41%	0.08%	9.35%	12.09%	-2.73%
US Large Value	4.32%	4.08%	0.24%	14.82%	12.09%	2.73%
US Small Growth	7.59%	7.43%	0.16%	1.04%	1.34%	-0.30%
US Small Value	3.94%	3.70%	0.23%	1.65%	1.34%	0.30%
Int'l Dev. Equity	4.93%	4.80%	0.13%	27.81%	24.18%	3.63%
Int'l Emerg. Equity	6.84%	6.60%	0.24%	3.49%	3.49%	0.00%
			Sum	103.63%	100.00%	3.63%

LAB Section:

- Use the file:

  lab_23_BlackLitterman.ipynb