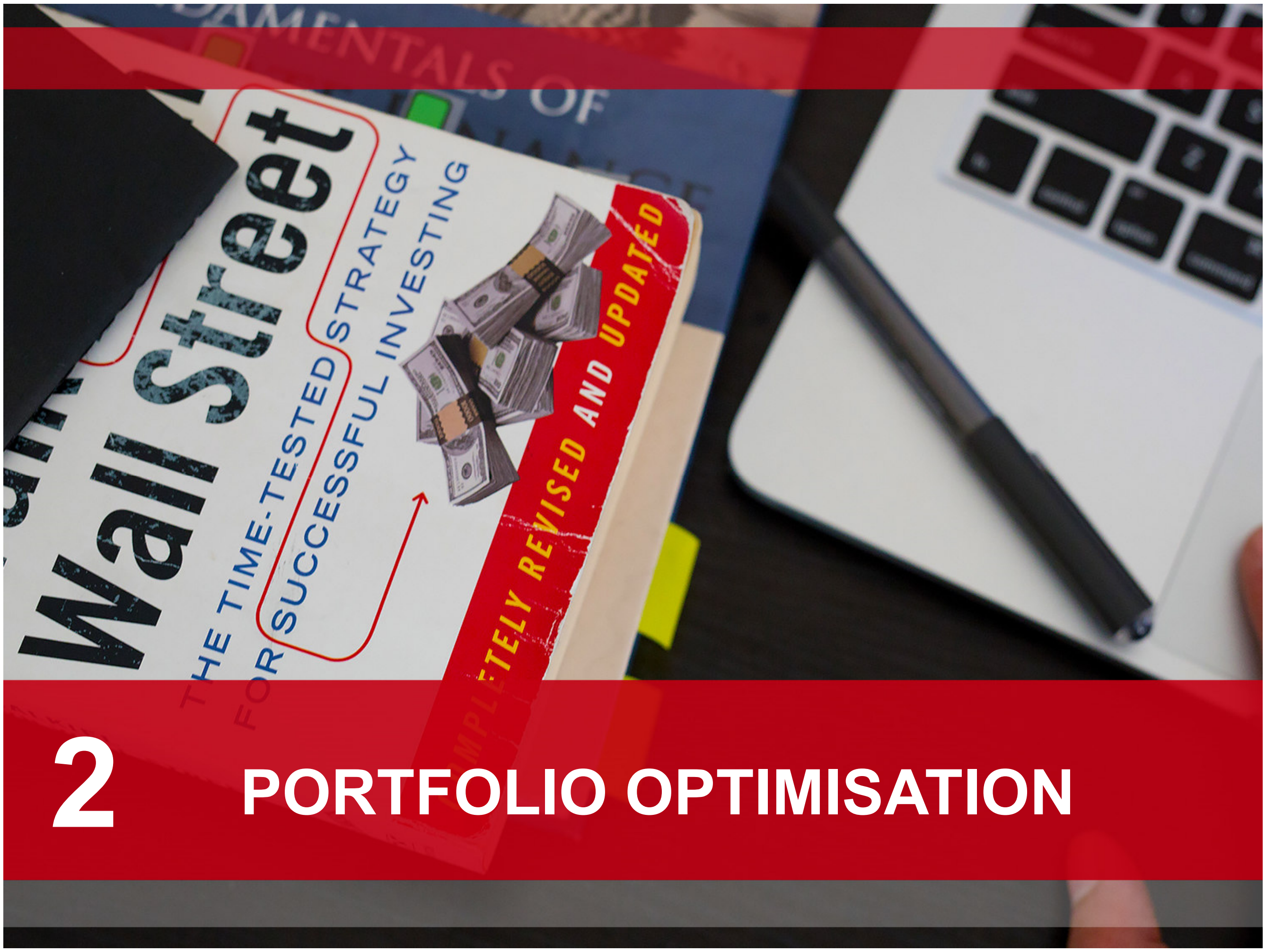


# 2 PORTFOLIO OPTIMISATION

## Risk aversion and portfolio selection

- Portfolio theory assumes that investors are ***risk averse***, meaning that given a choice between two assets with equal rates of return, they will select the asset with the lowest level of risk
  - Most people may be risk loving over small investments (such as the cost of a weekly lottery ticket), but risk averse over larger investments (such as the value of their house or car)
- It is reasonable to assume that investors are generally risk averse over the typical size of investments made in capital markets; this is borne out by empirical evidence
  - Many investors purchase insurance for: Life, Automobile, Health, and Disability Income.
  - Yield on bonds increases with risk classifications

# Modern Portfolio Theory

- In this lecture, we consider what happens when we combine individual assets into a **portfolio**
- The model that we use to do this is known as **Markowitz portfolio theory**, which was developed in the 1950s
- Markowitz mean–variance portfolio theory makes certain assumptions and then from these, derives the *optimal combination* of risky assets given their characteristics
- Markowitz portfolio theory, the oldest and perhaps most accepted part of modern portfolio theory, provides the theoretical foundation for examining the roles of risk and return in portfolio selection
- Markowitz portfolio theory forms the basis of virtually all professional fund management today



# The feasible set of two risky assets

Lecture 01 - Portfolio return and risk.xlsx - Excel

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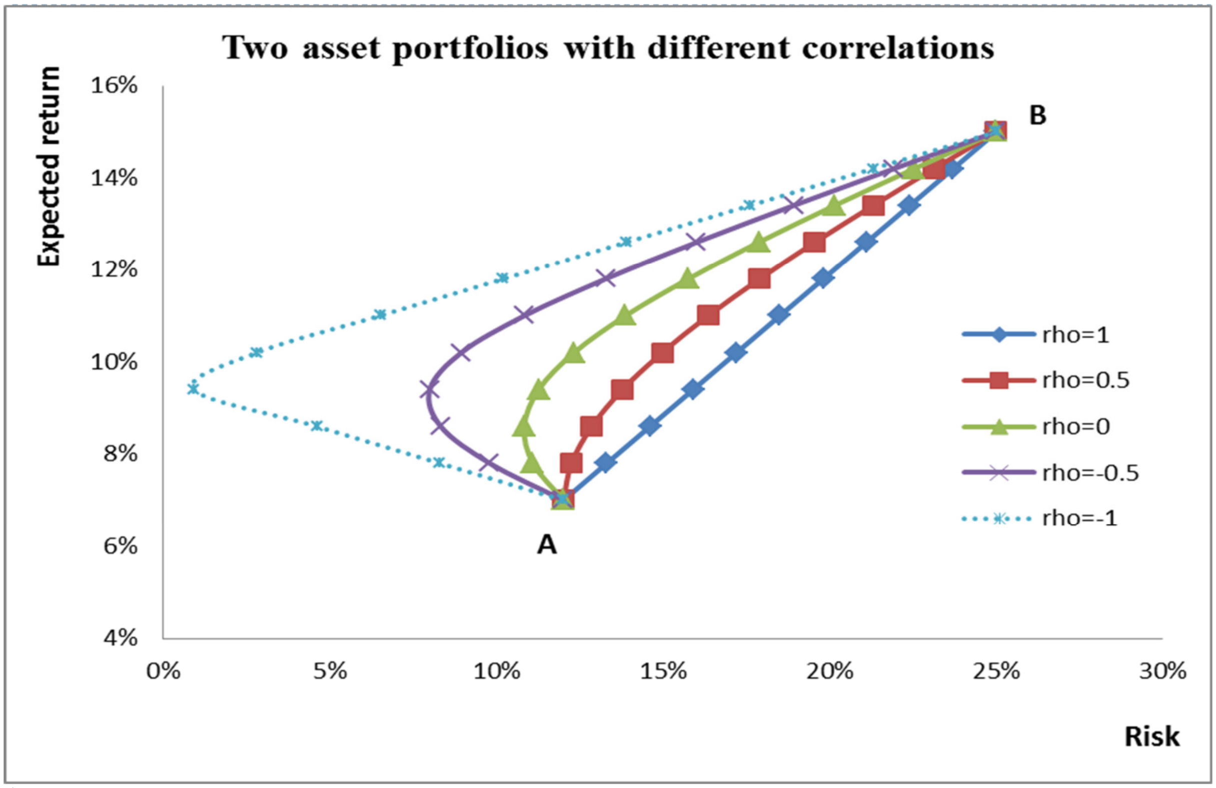
| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | Stock A | Stock B | | | | | |
| 2 | Expected return | 7% | 15% | | | | | |
| 3 | Risk | 12% | 25% | | | | | |
| 4 | Weights | 50% | 50% | | | | | |
| 5 | Correlation | 0.50 | | | | | | |
| 6 | | | | | | | | |
| 7 | Portfolio return | 11.00% | =B4*B2+C4*C2 | | | | | |
| 8 | Portfolio risk | 16.35% | =SQRT((B4*B3)^2+(C4*C3)^2+2*B4*C4*B3*C3*B5) | | | | | |
| 9 | | | | | | | | |
| 10 | | | | | | | | |
| 11 | | Portfolio return | | | Portfolio risk with correlation of | | | |
| 12 | Weight in Stock A | 11.00% | | 16.35% | 1.00 | 0.50 | 0.00 | -0.50 |
| 13 | 0% | 15.00% | | 0.00% | 25.00% | 25.00% | 25.00% | 25.00% |
| 14 | 10% | 14.20% | | 10.00% | 23.70% | 23.12% | 22.53% | 21.92% |
| 15 | 20% | 13.40% | | 20.00% | 22.40% | 21.30% | 20.14% | 18.91% |
| 16 | 30% | 12.60% | | 30.00% | 21.10% | 19.55% | 17.87% | 16.01% |
| 17 | 40% | 11.80% | | 40.00% | 19.80% | 17.89% | 15.75% | 13.27% |
| 18 | 50% | 11.00% | | 50.00% | 18.50% | 16.35% | 13.87% | 10.83% |
| 19 | 60% | 10.20% | | 60.00% | 17.20% | 14.96% | 12.32% | 8.94% |
| 20 | 70% | 9.40% | | 70.00% | 15.90% | 13.78% | 11.26% | 7.99% |

Returns | Feasible set | Sheet1

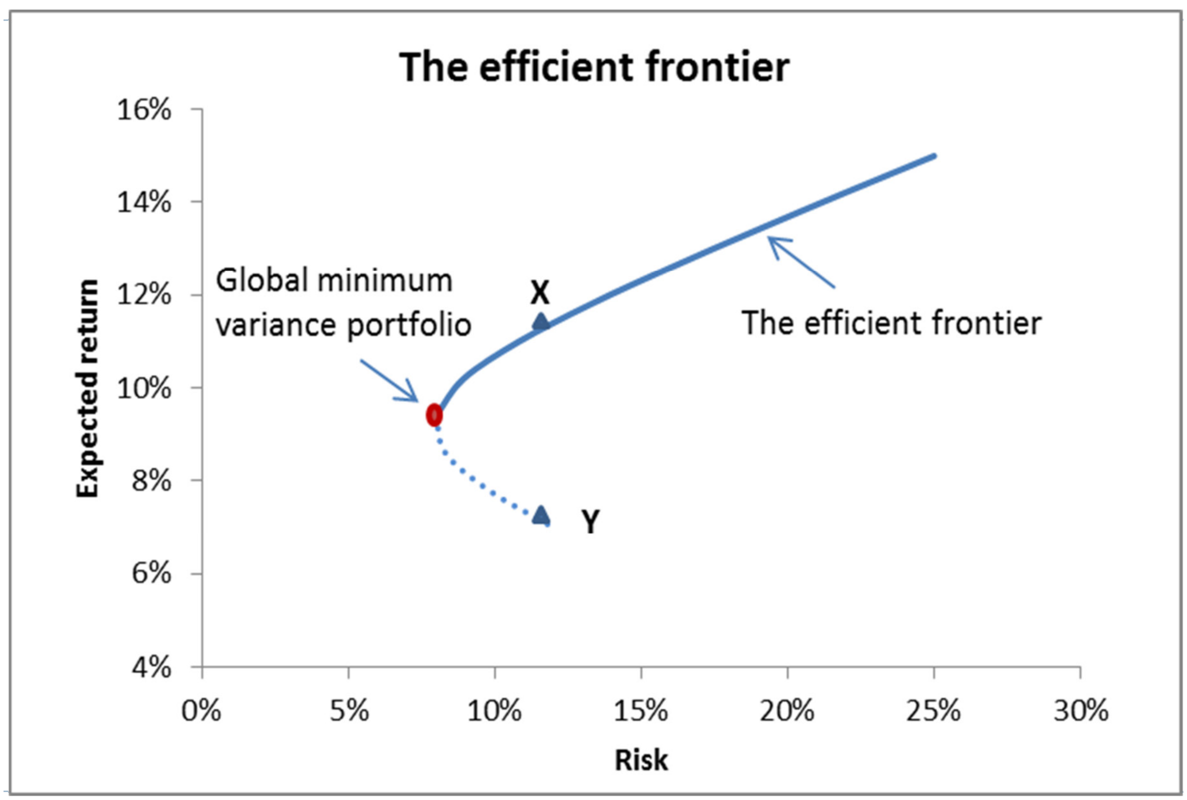
Ready 100%

# The feasible set of two risky assets






# The efficient frontier



## LAB Section: The Efficient Frontier

- Use the file:

  lab_107_Efficient Frontier 1.ipynb

- Data files:

  ind30_m_vw_rets.csv



## **Matrix Form Expressions**

Suppose there are $N$ assets; a portfolio $P$ is defined as a combination of these $N$ assets with portfolio weights $\boldsymbol{w}$ given by

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

In matrix notation, the expected returns of the N assets are given by

$$E(R) = \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{bmatrix}$$

## Matrix Form Expressions

The expected return of the portfolio is given by

$$E(r_P) = \sum_{i=1}^{N} w_i E(r_i)$$

$$= \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix} \begin{bmatrix} E(r_1) \\ E(r_2) \\ \vdots \\ E(r_N) \end{bmatrix}$$

$$= \mathbf{w}^T E(R)$$



## Matrix Form Expressions

As we have seen, the variance-covariance matrix of N assets is the N x N matrix is estimated as

$$\hat{V} = \frac{1}{T-1} \begin{bmatrix} r_{11} - \overline{r}_1 & \cdots & r_{1T} - \overline{r}_1 \\ \vdots & \ddots & \vdots \\ r_{N1} - \overline{r}_N & \cdots & r_{NT} - \overline{r}_N \end{bmatrix} \begin{bmatrix} r_{11} - \overline{r}_1 & \cdots & r_{N1} - \overline{r}_N \\ \vdots & \ddots & \vdots \\ r_{1T} - \overline{r}_1 & \cdots & r_{NT} - \overline{r}_N \end{bmatrix}$$

$$= \frac{1}{T-1} \left(R - \overline{R}\right)^T \left(R - \overline{R}\right)$$

## Matrix Form Expressions

We can write the portfolio variance using matrix algebra as

$$\sigma_p^2 = \begin{bmatrix} w_1 & \cdots & w_N \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_N^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}$$

$$= \mathbf{w}^T \mathbf{V} \mathbf{w}$$



## LAB Section: The Efficient Frontier

- Use the file:

  lab_108_Efficient Frontier 2.ipynb

  lab_109_Efficient Frontier 3.ipynb

- Data files:

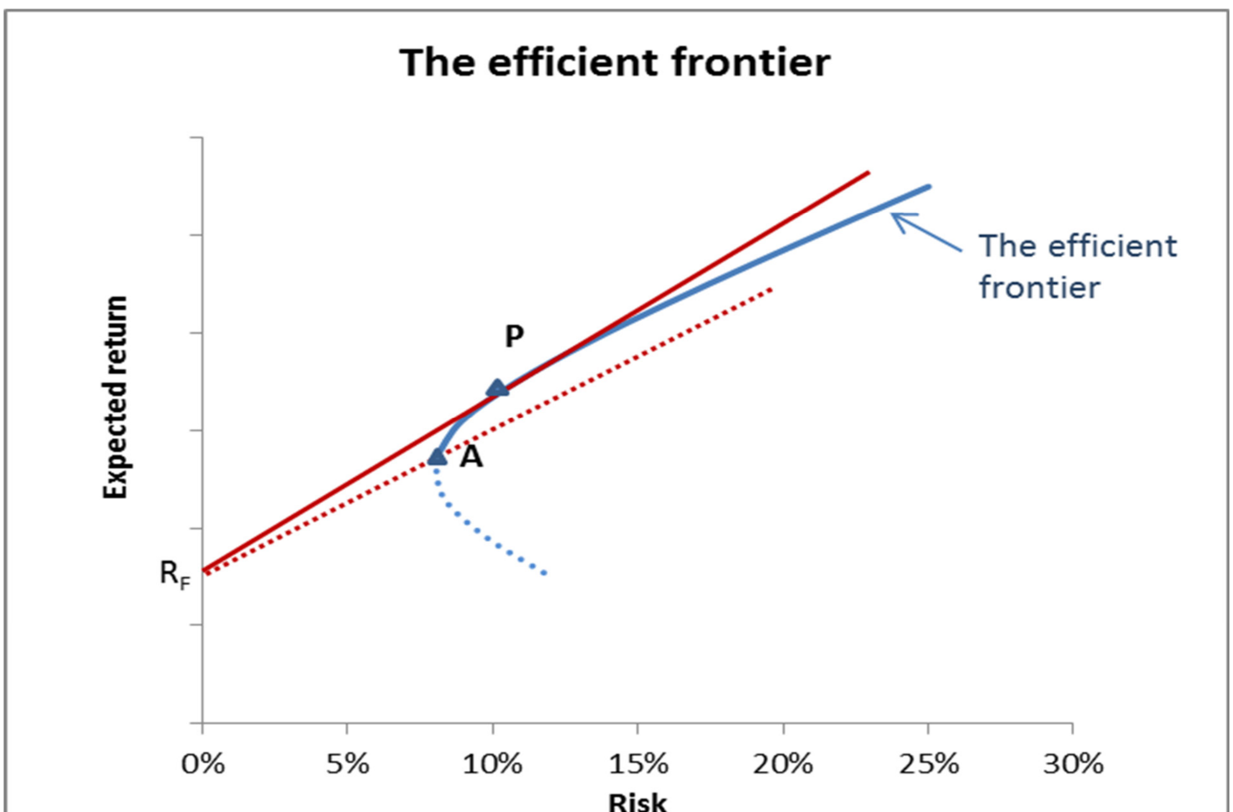
  ind30_m_vw_rets.csv

# Adding a risk free asset to the feasible set

- Suppose that we have a risk free asset, $F$
- Note that, being a risk free asset, $F$ offers a certain return, $R_F$, and so its variance, $\sigma_F^2$ (and hence its standard deviation, $\sigma_F$), is zero and its covariance, $\sigma_{Fi}$ (and hence its correlation, $\rho_{Fi}$), with any other asset is equal to zero
- We can plot the risk free asset in mean-standard deviation space as follows
- Now consider the set of all possible investment opportunities available to an investor. In addition to the feasible set of risky assets that we have already derived, an investor can combine any risky asset or portfolio with the risk free asset to create a new portfolio



# Adding a risk free asset to the feasible set



# Adding a risk free asset to the feasible set

- Suppose that the investor invests a fraction $w_F$ of his/her wealth in the risk free asset, and a fraction $(1-w_F)$ in a risky portfolio, $P$, with expected return $E(R_p)$ and variance $\sigma_p$

- The resulting portfolio will have the following expected return and standard deviation

$$E(R_Q) = w_F R_F + (1-w_F)E(R_P) \qquad \sigma_Q = (1-w_F)\sigma_P$$

- Thus the new portfolio will lie on a ***straight*** line connecting the risk free asset, $F$, to the risky portfolio, $P$ – *the Capital Allocation Line (CAL)*

- The CAL represents the portfolios available to an investor. The equation for this line can be derived from the above two equations:

$$E(R_Q) = \left(1 - \frac{\sigma_Q}{\sigma_P}\right)R_F + \frac{\sigma_Q}{\sigma_P}E(R_P) = R_F + \frac{(E(R_P) - R_F)}{\sigma_P}\sigma_Q$$

- The line is straight because the equation is linear

- The slope of the CAL equals incremental return per incremental risk and is called the Sharpe ratio.



# Adding a risk free asset to the feasible set

- The best risky portfolio to combine the risk free asset with is the portfolio that marks the point of tangency, *P**, of a ray emanating from the risk free asset, with the feasible set of risky portfolios

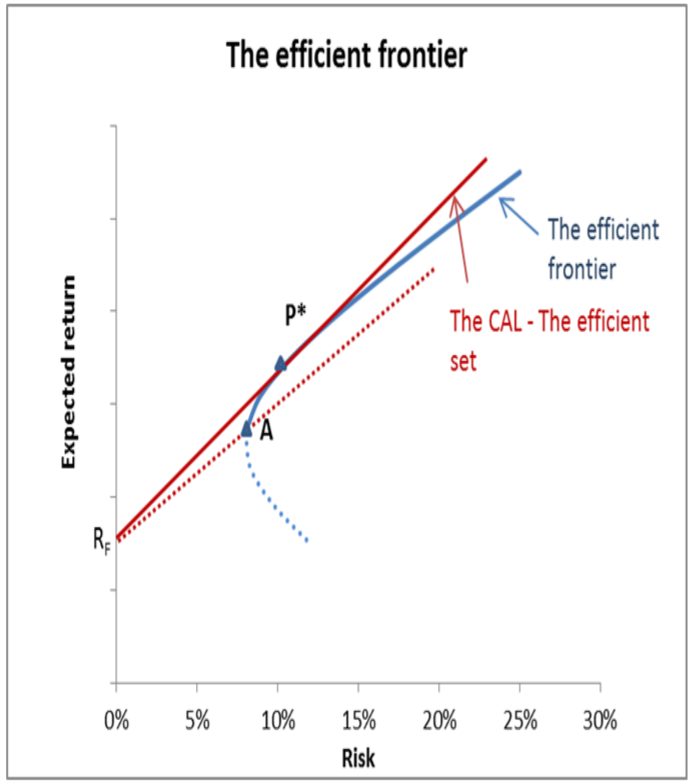
- There are four possibilities

  (1) Investing at point *F* represents lending 100% of your wealth

  (2) Investing between *F* and *P** represents lending some of your wealth and investing some in risky assets

  (3) Investing at point *P** represents investing 100% of your wealth in risky assets

  (4) Investing beyond point *P** represents borrowing and investing more than 100% of your wealth in risky assets




## LAB Section: Adding a risk free asset

- Use the file:

  lab_110_Max SR Portfolio.ipynb

- Data files:

  ind30_m_vw_rets.csv

# Criticisms of the mean-variance analysis

- **GIGO**: The quality of the output from the MVO (portfolio allocations) is highly sensitive to the quality of the inputs (i.e., expected returns, variances, and correlations). In other settings, this is often called the "garbage-in-garbage-out" (GIGO) problem. Although all three inputs are a source of estimation error in MVO, expected returns are particularly problematic, so we focus here on addressing the quality of the expected return inputs.
- **Concentrated asset class allocations**: MVO often identifies efficient portfolios that are highly concentrated in a subset of asset classes, with zero allocation to others; in other words, lowest calculated standard deviation is not the same thing as practical diversification.



# Criticisms of the mean-variance analysis

- **Ignores liabilities**: MVO also does not account for the fact that investors create portfolios as a source of cash to pay for something in the future: individual investors are looking to fund their consumption spending in retirement, for example, while pension funds are focused on funding the pension liability and repaying employees the retirement benefits promised to them. A more robust approach needs to account for the factors that affect these liabilities and the correlations between changes in value of the liabilities and returns on the asset portfolio.
- **Single-period framework**: MVO is a single-period framework that does not take into account interim cash flows or the serial correlation of asset returns from one time period to the next. This means it ignores the potential costs and benefits of rebalancing a portfolio as capital market conditions change and asset allocations drift away from their optimal starting point

## Criticisms of the mean-variance analysis

- **Skewness and kurtosis**: MVO analysis, by definition, only looks at the first two moments of the return distribution: expected return and variance; it does not take into account skewness or kurtosis. But empirical evidence suggests quite strongly that asset returns are not normally distributed: there is significant skewness and kurtosis in actual returns.
- **Risk diversification**: MVO identifies an asset allocation diversified across asset classes but not necessarily the sources of risk. For example, equities and fixed-income securities are two different asset classes, but they are driven by some common risk factors, and diversifying across the two classes won't necessarily diversify those risk factors.



## Criticisms of the mean-variance analysis

- **Quadratic Utility Function**: the Nobel Prize winners Kahneman and Tversky have actually looked at people in the laboratory, and shown that the utility function is actually not symmetric for most investors

## Practical issues in mean-variance analysis

- Markowitz optimisation treats expected returns, variances and covariances as deterministic. However, in practice, these moments of returns are unobservable and must be estimated.
- We now discuss practical issues that arise in the application of mean–variance analysis in choosing portfolios. The two areas of focus are:
  - estimating inputs for mean–variance optimization, and
  - the instability of the minimum-variance frontier, which results from the optimisation process's sensitivity to the inputs.



## LAB Section:
## Lack of Robustness and GMV Portfolio

- Use the file:

  lab_111_Robustness and GMV.ipynb

- Data files:

  ind30_m_vw_rets.csv