Risk and Return

TCH302

Ceccetti and Schoenholtz (2015), Chapter 5

Learning objectives

Understand…

- Measuring historical return and expected return
- Risk as a measure of uncertainty
- Measures of risk
- The trade-off between risk and return
- Different attitudes towards risk
- Sources of risk and risk reduction

Historical Rates of Return

- Return over A Holding Period
  - Holding Period Yield (HPY) $$HPY = \frac{Ending\ Value\ of\ Investment}{Beginning\ Value\ of\ Investment} - 1$$
  - Annual HPY

$$\text{Annual HPY} = \left[\frac{Ending\ Value\ of\ Investment}{Beginning\ Value\ of\ Investment}\right]^{\frac{1}{n}} - 1$$

where n=number of years of the investment

Historical Rates of Return

A Portfolio of Investments

Portfolio HPY: The mean historical rate of return for a portfolio of investments is measured as the ***weighted average of the HPYs*** for the individual investments in the portfolio, or the overall change in the value of the original portfolio.

The weights used in the computation are the relative beginning market values for each investment, which is often referred to as dollar-weighted or value-weighted mean rate of return.

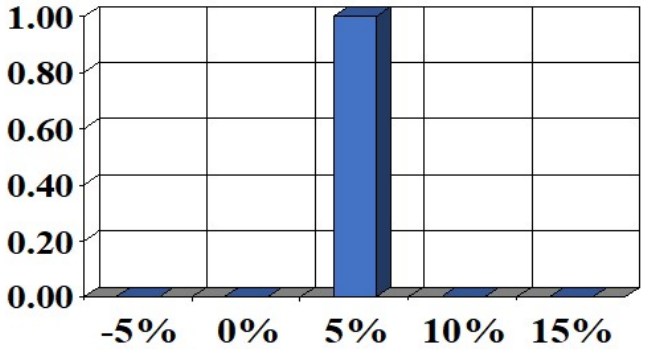
Expected Rates of Return

- In previous examples, we discussed realized historical rates of return.
- In contrast, an investor would be ***more interested in the expected return*** on a ***future risky investment.***
- Risk refers to the uncertainty of the future outcomes of an investment

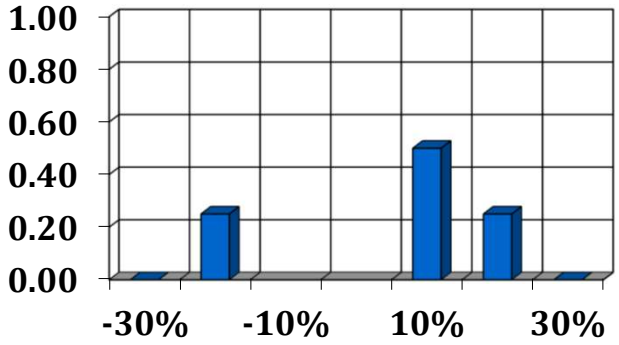
  - There are many possible returns/outcomes/payoffs from an investment due to the uncertainty
  - Probability is the likelihood of an outcome
  - The sum of the probabilities of all the possible outcomes is equal to 1.0.

Probability Distributions – Examples

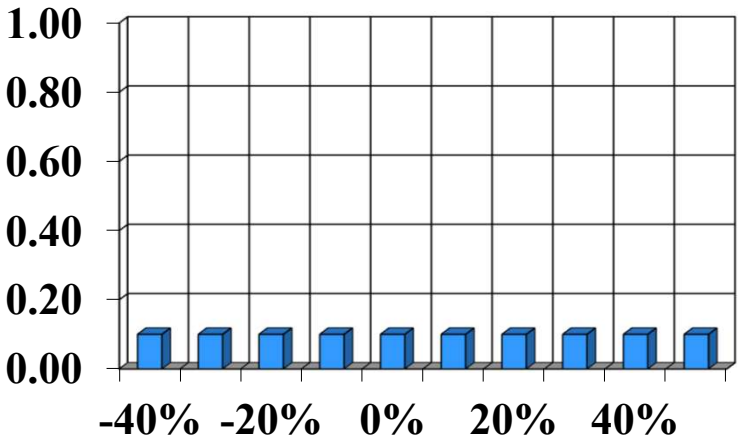
Risk-free Investment



Risky Investment with 3 Possible Returns



Risky investment with ten possible returns



Expected Rates of Return

Computing Expected Rate of Return

$$E(R_i) = \sum_{i=1}^{n} (Probability\ of\ Return_i) \times (PossibleReturn_i)$$

$$= [(P_1)(R_1) + (P_2)(R_2) + \cdots + (P_n)(R_n)]$$

$$\boxed{E(R_i) = \sum_{i=1}^{n} (P_i)(R_i)}$$

Where:

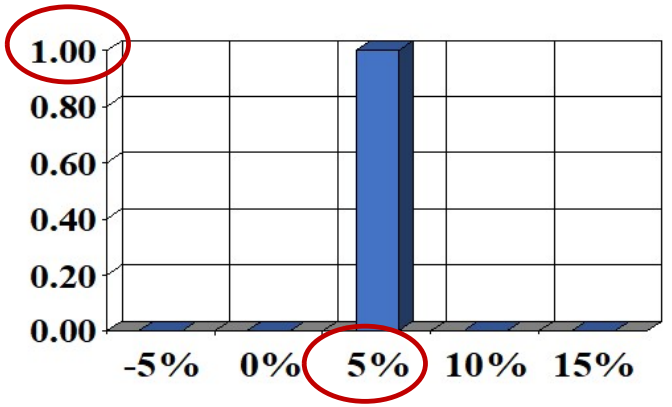
- P_i Probability of return i
- R_i Possible return i

Expected Rates of Return

Example 1: Risk-free Investment

Computing Expected Rate of Return

Risk-free Investment



$$E(R_i) = \sum_{i=1}^{n} (\text{Probability of Return}) \times (\text{Possible Return})$$

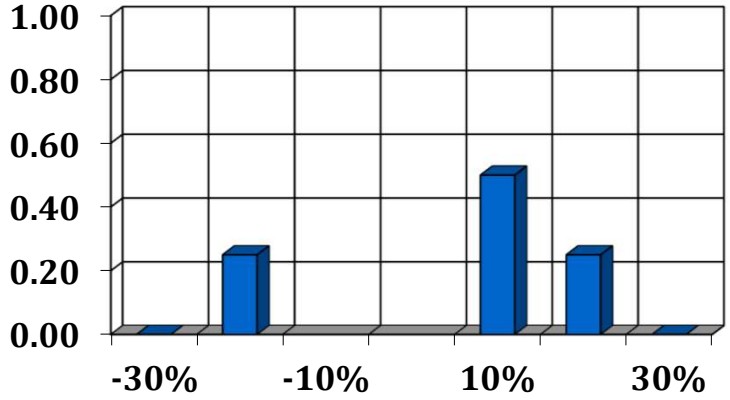
$$= (1)x(0.05) = 0.05 = 5\%$$

Expected Rates of Return

Example 2: Risky Investment 1

Computing Expected Rate of Return

Risky Investment with 3 Possible Returns



$$E(R_i) = \sum_{i=1}^{n} (\text{Probability of Return}) \times (\text{Possible Return})$$

$$= (0.25)(0.20)+(0.25)(-0.20)+(0.50)(0.10)$$

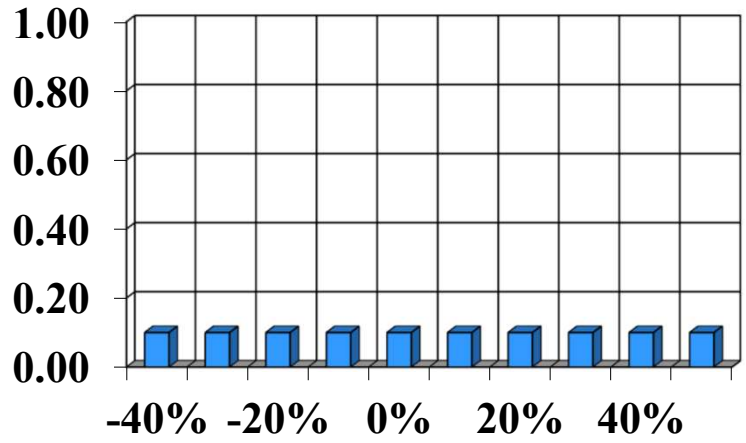
$$= 5\%$$

Expected Rates of Return

Example 3: Risky Investment 2

Computing Expected Rate of Return

Risky Investment with 10 Possible Returns



$$E(R_i) = \sum_{i=1}^{n} (\text{Probability of Return}) \times (\text{Possible Return})$$

$$= (0.1)\text{x}[(-0.4)+(-0.3)+(-0.2)+(-0.1)+(0)+(0.1)+(0.2)+(0.3)+(0.4)+(0.5)]$$

$$= 5\%$$

Risk of Expected Return

Risk refers to the **uncertainty of an investment**;

therefore, the measure of risk should reflect the degree of the uncertainty.

> The risk of expected return reflects the degree of uncertainty that the **actual return** will be different from the **expect return**.
>
> The common measures of risk are based on the **variance** (or standard deviation) of rates of return distribution of an investment

Value at risk (VaR) is the worst possible loss over a specific time horizon, at a given probability

Measuring the Risk of Expected Return

- **The Variance Measure**

$$Variance\ \sigma^2 = \sum_{i=1}^{n} (Probability) \times (Possible\ Return - Expected\ Return)^2$$

- **Standard Deviation (σ):** It is the square root of the variance and measures the total risk

$$\sigma = \sqrt{\sum_{i=1}^{n} P_i [R_i - E(R_i)]^2}$$

- **Coefficient of Variation (CV):** It measures the risk per unit of expected return and is a relative measure of risk.

$$CV = \frac{Standard\ \ Deviation\ \ of\ \ Return}{Expected\ \ Rate\ of\ \ Return}$$

$$= \sigma / E(R)$$

Risk of Expected Return

Example 1: Risk-free Investment

Computing risk of expected return

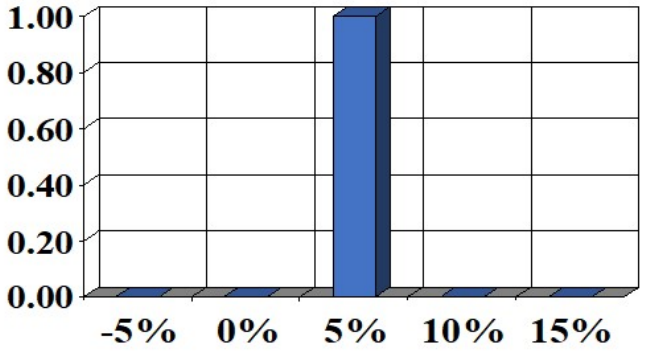
$$Variance = \sum_{i=1}^{n} P_i[R_i - E(R_i)]^2$$

$$= \sum_{i=1}^{n} (1.0) \times (0.05 - 0.05)^2 = 0$$

Standard Deviation:

CV:

Risk-free Investment



Risk of Expected Return

Example 2: Risky Investment 1

Computing risk of expected return

Variance

$$(0.25)(-0.2-0.05)^2+(0.5)(0.1-0.05)^2+(0.25)\,(0.2\ -0.05)^2$$

$$= 0.0225$$

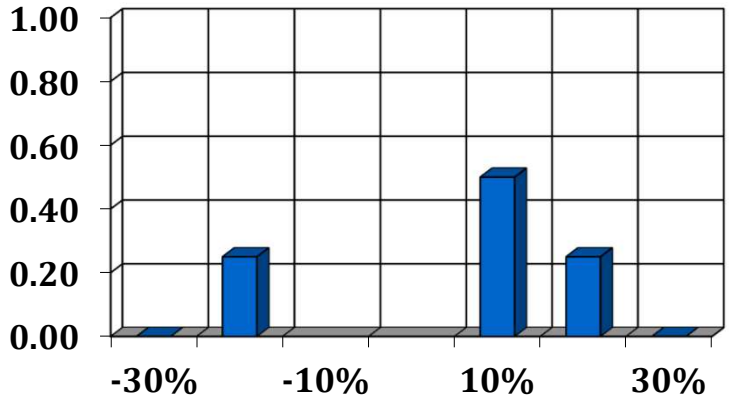
Standard Deviation:

$$\sqrt{0.0225} = 15\%$$

CV:

$$\frac{\sigma_i}{E(R)} = 0.15/0.05 = 3$$

Risky Investment with 3 Possible Returns



Risk of Expected Return

Example 3: Risky Investment 2

Computing risk of expected return

Variance

$$(0.1)(0.4 - 0.05)^2 + (0.1)(-0.3 - 0.05)^2 + \cdots + (0.1)(0.50 - 0.05)^2$$

$$= 0.0825$$

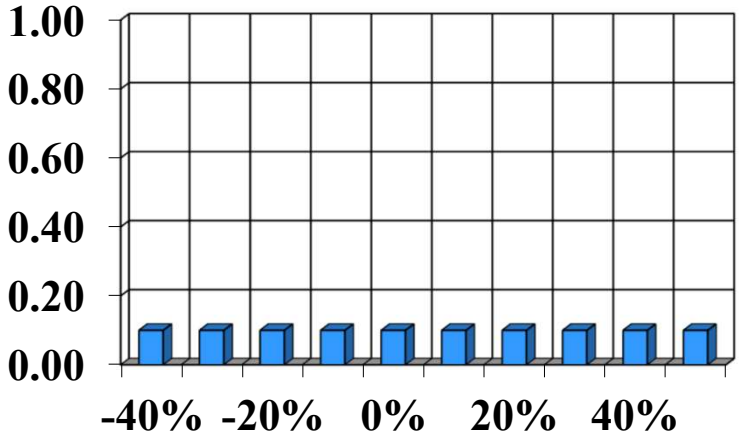
Standard Deviation:

$$\sqrt{0.0825} = 28.72\%$$

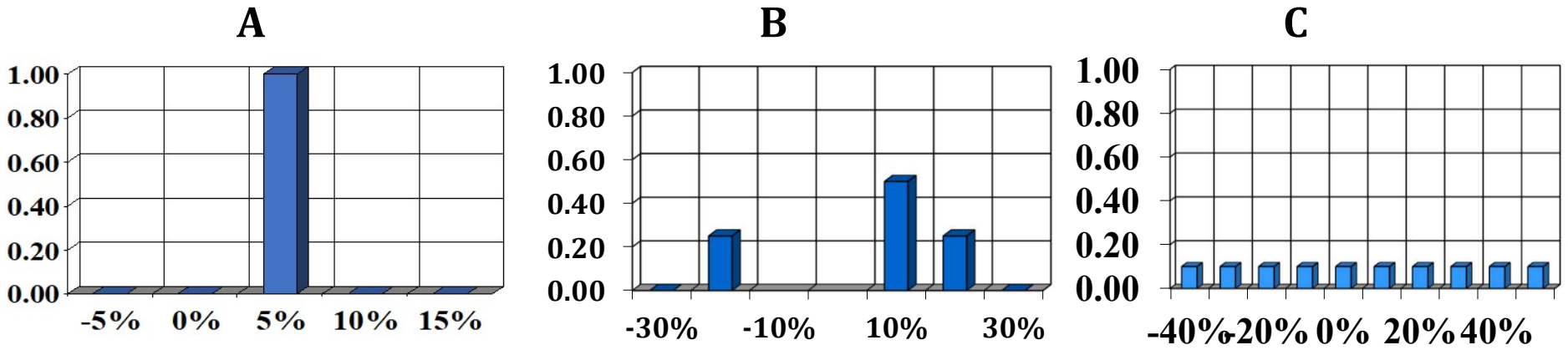
CV:

$$\frac{\sigma_i}{E(R)} = 0.2872/0.05 = 5.745$$

Risky Investment with 10 Possible Returns



Risk of expected return



	Investment A	Investment B	Investment C
Expected Return	0.05	0.05	0.05
Standard Deviation	0	0.1187	0.2872
CoV	0	1.6957	5.7446

⇒ *The more spread out the distribution of possible payoffs from an investment, the higher the standard deviation and the bigger the risk*

Different attitudes towards risk

Risk averse:

> a risk-averse investor will always prefer an investment with a certain return to one with the same expected return but any amount of uncertainty

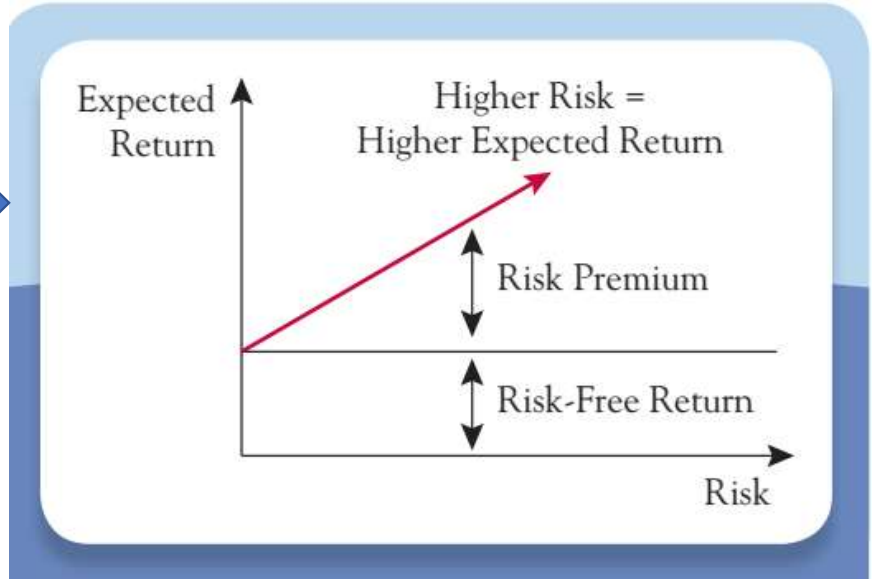
Risk neutral:

> a risk-neutral investors wouldn't care as long as the expected return is the same

Risk seeking:

> a risk-seeking investors

Figure 5.3 The Tradeoff between Risk and Expected Return



Investments with Different Returns and Risks

	Investment A	Investment B	Investment C
Expected Return	0.05	0.05	0.05
Standard Deviation	0	0.1187	0.2872
CoV	0	1.6957	5.7446

Sources of risk

Idiosyncratic risk

Idiosyncratic risks, or unique risks, are those affecting a small number of people/industry but no one else

Systematic risk

Idiosyncratic risks, or economywide risks, are those affecting everyone

Reducing risk through diversification

Not to venture all eggs in one basket

Hedging risk

The strategy of reducing idiosyncratic risk by making two investments with opposing risk

Spreading risk

Find investments whose payoffs are unrelated