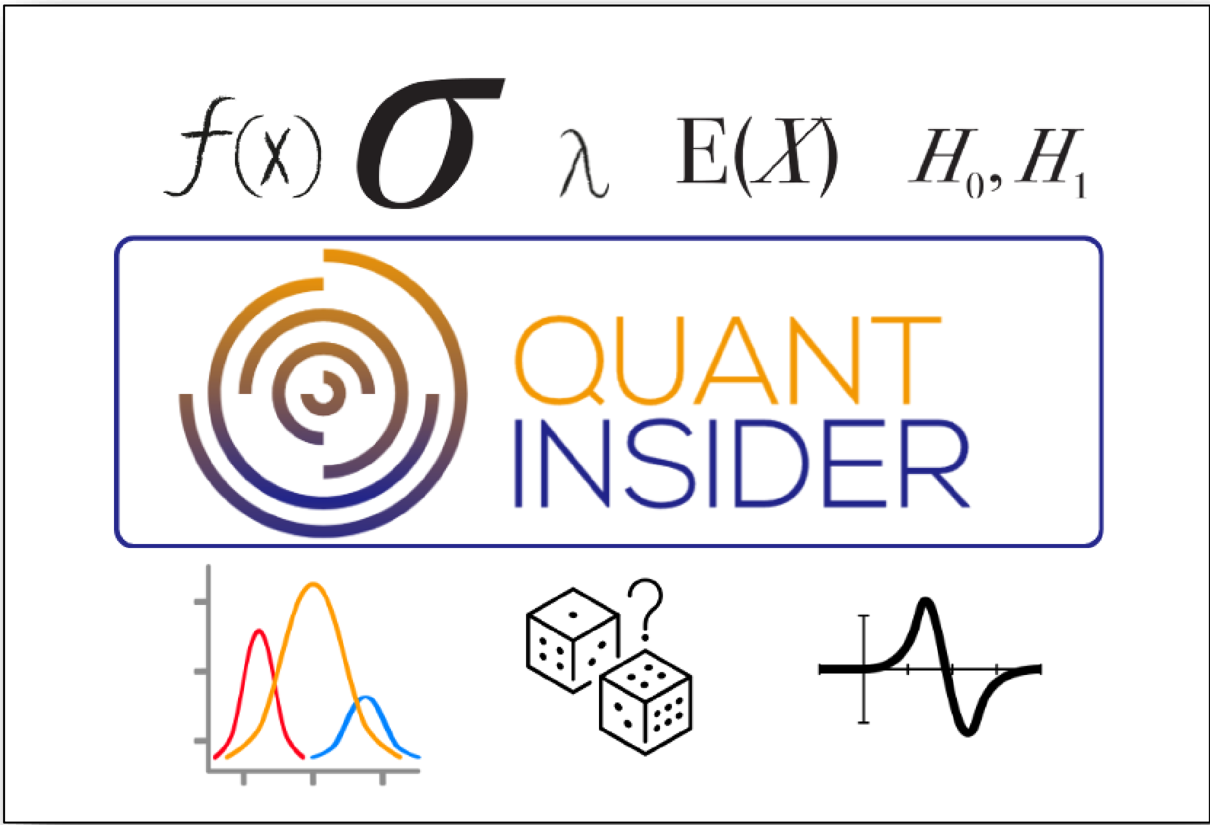

INTEREST RATE DERIVATIVES - 2

The six-month and one-year zero rates are both 10% per annum. For a bond that has a life of 18 months and pays a coupon of 8% per annum (with semiannual payments and one having just been made), the yield is 10.4% per annum. What is the bond's price? What is the 18-month zero rate? All rates are quoted with semiannual compounding.

Suppose the bond has a face value of \$100. Its price is obtained by discounting the cash flows at 10.4%. The price is

$$\frac{4}{1.052} + \frac{4}{1.052^2} + \frac{104}{1.052^3} = 96.74$$

If the 18-month zero rate is R, we must have

$$\frac{4}{1.05} + \frac{4}{1.05^2} + \frac{104}{(1+R/2)^3} = 96.74$$

which gives $R = 10.42\%$.

Assuming that zero rates are as in Problem 4.5, what is the value of an FRA that enables the holder to earn 9.5% for a three-month period starting in one year on a principal of \$1,000,000? The interest rate is expressed with quarterly compounding.

The forward rate is 9.0% with continuous compounding or 9.102% with quarterly compounding. From equation (4.9), the value of the FRA is therefore

$$[1,000,000 \times 0.25 \times (0.095 - 0.09102)]e^{-0.086 \times 1.25} = 893.56$$

or \$893.56.

The term structure of interest rates is upward sloping. Put the following in order of magnitude:

a. The five-year zero rate

b. The yield on a five-year coupon-bearing bond

c. The forward rate corresponding to the period between 4.75 and 5 years in the future

What is the answer to this question when the term structure of interest rates is downward sloping?

When the term structure is upward sloping, $c > a > b$. When it is downward sloping, $b > a > c$.

Portfolio A consists of a one-year zero-coupon bond with a face value of \$2,000 and a 10-year zero-coupon bond with a face value of \$6,000. Portfolio B consists of a 5.95-year zero-coupon bond with a face value of \$5,000. The current yield on all bonds is 10% per annum.

a. *Show that both portfolios have the same duration.*
b. *Show that the percentage changes in the values of the two portfolios for a 0.1% per annum increase in yields are the same.*
c. *What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?*

(a) The duration of Portfolio A is

$$\frac{1 \times 2000e^{-0.1\times1} + 10 \times 6000e^{-0.1\times10}}{2000e^{-0.1\times1} + 6000e^{-0.1\times10}} = 5.95$$

Since this is also the duration of Portfolio B, the two portfolios do have the same duration.

(b) The value of Portfolio A is

$$2000e^{-0.1} + 6000e^{-0.1\times10} = 4016.95$$

When yields increase by 10 basis points its value becomes

$$2000e^{-0.101} + 6000e^{-0.101\times10} = 3993.18$$

The percentage decrease in value is

$$\frac{23.77 \times 100}{4016.95} = 0.59\%$$

The value of Portfolio B is

$$5000e^{-0.1\times5.95} = 2757.81$$

When yields increase by 10 basis points its value becomes

$$5000e^{-0.101\times5.95} = 2741.45$$

The percentage decrease in value is

$$\frac{16.36 \times 100}{2757.81} = 0.59\%$$

The percentage changes in the values of the two portfolios for a 10 basis point increase in yields are therefore the same.

(c) When yields increase by 5% the value of Portfolio A becomes

$$2000e^{-0.15} + 6000e^{-0.15\times 10} = 3060.20$$

and the value of Portfolio B becomes

$$5000e^{-0.15\times 5.95} = 2048.15$$

The percentage reduction in the values of the two portfolios are:

$$\text{Portfolio A:} \quad \frac{956.75}{4016.95} \times 100 = 23.82$$

$$\text{Portfolio B:} \quad \frac{709.66}{2757.81} \times 100 = 25.73$$

Since the percentage decline in value of Portfolio A is less than that of Portfolio B, Portfolio A has a greater convexity .

Interest RATE Futures

What is the purpose of the convexity adjustment made to Eurodollar futures rates? Why is the convexity adjustment necessary?

Suppose that a Eurodollar futures quote is 95.00. This gives a futures rate of 5% for the three-month period covered by the contract. The convexity adjustment is the amount by which futures rate has to be reduced to give an estimate of the forward rate for the period The convexity adjustment is necessary because a) the futures contract is settled daily and b) the futures contract expires at the beginning of the three months. Both of these lead to the futures rate being greater than the forward rate.

It is January 30. You are managing a bond portfolio worth \$6 million. The duration of the portfolio in six months will be 8.2 years. The September Treasury bond futures price is currently 108-15, and the cheapest-to-deliver bond will have a duration of 7.6 years in September. How should you hedge against changes in interest rates over the next six months?

The value of a contract is $108\frac{15}{32} \times 1,000 = \$108,468.75$. The number of contracts that should be shorted is

$$\frac{6,000,000}{108,468.75} \times \frac{8.2}{7.6} = 59.7$$

Rounding to the nearest whole number, 60 contracts should be shorted. The position should be closed out at the end of July.

Suppose that the Treasury bond futures price is 101-12. Which of the following four bonds is cheapest to deliver?

Bond	*Price*	*Conversion Factor*
1	*125-05*	*1.2131*
2	*142-15*	*1.3792*
3	*115-31*	*1.1149*
4	*144-02*	*1.4026*

The cheapest-to-deliver bond is the one for which

$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

is least. Calculating this factor for each of the 4 bonds we get

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$
$$\text{Bond 2: } 142.46875 - 101.375 \times 1.3792 = 2.652$$
$$\text{Bond 3: } 115.96875 - 101.375 \times 1.1149 = 2.946$$
$$\text{Bond 4: } 144.06250 - 101.375 \times 1.4026 = 1.874$$

Bond 4 is therefore the cheapest to deliver.

An investor is looking for arbitrage opportunities in the Treasury bond futures market. What complications are created by the fact that the party with a short position can choose to deliver any bond with a maturity of over 15 years?

If the bond to be delivered and the time of delivery were known, arbitrage would be straightforward. When the futures price is too high, the arbitrageur buys bonds and shorts an equivalent number of bond futures contracts. When the futures price is too low, the arbitrageur sells bonds and goes long an equivalent number of bond futures contracts.

Uncertainty as to which bond will be delivered introduces complications. The bond that appears cheapest-to-deliver now may not in fact be cheapest-to-deliver at maturity. In the case where the futures price is too high, this is not a major problem since the party with the short position (i.e., the arbitrageur) determines which bond is to be delivered. In the case where the futures price is too low, the arbitrageur's position is far more difficult since he or she does not know which bond to buy; it is unlikely that a profit can be locked in for all possible outcomes.

Suppose that a bond portfolio with a duration of 12 years is hedged using a futures contract in which the underlying asset has a duration of four years. What is likely to be the impact on the hedge of the fact that the 12-year rate is less volatile than the four-year rate?

Duration-based hedging schemes assume parallel shifts in the yield curve. Since the 12-year rate tends to move by less than the 4-year rate, the portfolio manager may find that he or she is over-hedged.

Assume that a bank can borrow or lend money at the same interest rate in the LIBOR market. The 90-day rate is 10% per annum, and the 180-day rate is 10.2% per annum, both expressed with continuous compounding and actual/actual day count. The Eurodollar futures price for a contract maturing in 91 days is quoted as 89.5. What arbitrage opportunities are open to the bank?

The Eurodollar futures contract price of 89.5 means that the Eurodollar futures rate is 10.5% per annum with quarterly compounding and an actual/360 day count. This becomes $10.5 \times 365/360 = 10.646\%$ with an actual/actual day count. This is

$$4 \ln(1 + 0.25 \times 0.10646) = 0.1051$$

or 10.51% with continuous compounding. The forward rate given by the 91-day rate and the 182-day rate is 10.4% with continuous compounding. This suggests the following arbitrage opportunity:

1. Buy Eurodollar futures.
2. Borrow 182-day money.
3. Invest the borrowed money for 91 days.

A portfolio manager plans to use a Treasury bond futures contract to hedge a bond portfolio over the next three months. The portfolio is worth \$100 million and will have a duration of 4.0 years in three months. The futures price is 122, and each futures contract is on \$100,000 of bonds. The bond that is expected to be cheapest to deliver will have a duration of 9.0 years at the maturity of the futures contract. What position in futures contracts is required?

a. *What adjustments to the hedge are necessary if after one month the bond that is expected to be cheapest to deliver changes to one with a duration of seven years?*

b. *Suppose that all rates increase over the next three months, but long-term rates increase less than short-term and medium-term rates. What is the effect of this on the performance of the hedge?*

The number of short futures contracts required is

$$\frac{100,000,000 \times 4.0}{122,000 \times 9.0} = 364.3$$

Rounding to the nearest whole number 364 contracts should be shorted.

(a) This increases the number of contracts that should be shorted to

$$\frac{100,000,000 \times 4.0}{122,000 \times 7.0} = 468.4$$

or 468 when we round to the nearest whole number.

(b) In this case the gain on the short futures position is likely to be less than the loss on the bond portfolio. This is because the gain on the short futures position depends on the size of the movement in long-term rates and the loss on the bond portfolio depends on the size of the movement in medium-term rates. Duration-based hedging assumes that the movements in the two rates are the same.

SWAPS

A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on £20 million for interest at 6% on \$30 million once a year. The term structure of interest rates in both the United Kingdom and the United States is currently flat, and if the swap were negotiated today the interest rates exchanged would be 4% in dollars and 7% in sterling. All interest rates are quoted with annual compounding. The current exchange rate (dollars per pound sterling) is 1.8500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?

The swap involves exchanging the sterling interest of $20 \times 0.10 = 2.0$ million for the dollar interest of $30 \times 0.06 = \$1.8$ million. The principal amounts are also exchanged at the end of the life of the swap. The value of the sterling bond underlying the swap is

$$\frac{2}{(1.07)^{1/4}} + \frac{22}{(1.07)^{5/4}} = 22.182 \text{ million pounds}$$

The value of the dollar bond underlying the swap is

$$\frac{1.8}{(1.04)^{1/4}} + \frac{31.8}{(1.04)^{5/4}} = \$32.061 \text{ million}$$

The value of the swap to the party paying sterling is therefore

$$32.061 - (22.182 \times 1.85) = -\$8.976 \text{ million}$$

The value of the swap to the party paying dollars is $+\$8.976$ million. The results can also be obtained by viewing the swap as a portfolio of forward contracts. The continuously compounded interest rates in sterling and dollars are 6.766% per annum and 3.922% per annum. The 3-month and 15-month forward exchange rates are $1.85e^{(0.03922-0.06766)\times 0.25} = 1.8369$ and $1.85e^{(0.03922-0.06766)\times 1.25} = 1.7854$. The values of the two forward contracts corresponding to the exchange of interest for the party paying sterling are therefore

$$(1.8 - 2 \times 1.8369)e^{-0.03922\times 0.25} = -\$1.855 \text{ million}$$

$$(1.8 - 2 \times 1.7854)e^{-0.03922\times 1.25} = -\$1.686 \text{ million}$$

The value of the forward contract corresponding to the exchange of principals is

$$(30 - 20 \times 1.7854)e^{-0.03922\times 1.25} = -\$5.435 \text{ million}$$

The total value of the swap is $-\$1.855 - \$1.686 - \$5.435 = -\$8.976$ million.

Suppose that the term structure of interest rates is flat in the United States and Australia. The USD interest rate is 7% per annum and the AUD rate is 9% per annum. The current value of the AUD is 0.62 USD. Under the terms of a swap agreement, a financial institution pays 8% per annum in AUD and receives 4% per annum in USD. The principals in the two currencies are \$12 million USD and 20 million AUD. Payments are exchanged every year, with one exchange having just taken place. The swap will last two more years. What is the value of the swap to the financial institution? Assume all interest rates are continuously compounded.

The financial institution is long a dollar bond and short a USD bond. The value of the dollar bond (in millions of dollars) is

$$0.48e^{-0.07\times1} + 12.48e^{-0.07\times2} = 11.297$$

The value of the AUD bond (in millions of AUD) is

$$1.6e^{-0.09\times1} + 21.6e^{-0.09\times2} = 19.504$$

The value of the swap (in millions of dollars) is therefore

$$11.297 - 19.504 \times 0.62 = -0.795$$

or −\$795,000.

As an alternative we can value the swap as a series of forward foreign exchange contracts. The one-year forward exchange rate is $0.62e^{-0.02} = 0.6077$. The two-year forward exchange rate is $0.62e^{-0.02\times2} = 0.5957$. The value of the swap in millions of dollars is therefore

$$(0.48 - 1.6 \times 0.6077)e^{-0.07\times1} + (12.48 - 21.6 \times 0.5957)e^{-0.07\times2} = -0.795$$

which is in agreement with the first calculation.

A corporate treasurer tells you that he has just negotiated a five-year loan at a competitive fixed rate of interest of 5.2%. The treasurer explains that he achieved the 5.2% rate by borrowing at six-month LIBOR plus 150 basis points and swapping LIBOR for 3.7%. He goes on to say that this was possible because his company has a comparative advantage in the floating-rate market. What has the treasurer overlooked?

The rate is not truly fixed because, if the company's credit rating declines, it will not be able to roll over its floating rate borrowings at LIBOR plus 150 basis points. The effective fixed borrowing rate then increases. Suppose for example that the treasurer's spread over LIBOR increases from 150 basis points to 200 basis points. The borrowing rate increases from 5.2% to 5.7%.

A bank finds that its assets are not matched with its liabilities. It is taking floating-rate deposits and making fixed-rate loans. How can swaps be used to offset the risk?

The bank is paying a floating-rate on the deposits and receiving a fixed-rate on the loans. It can offset its risk by entering into interest rate swaps (with other financial institutions or corporations) in which it contracts to pay fixed and receive floating.