Forward Contracts

Suppose that John believes the stock price of Vodafone will appreciate consistently over the course of a year. Assume that Vodafone is worth £80 and the 1-year LIBOR rate r is equal to 6%. Also, the dividend yield q is equal to 2% and the borrowing costs are null. John decides to enter into a 1-year forward contract allowing him to buy 1,000 shares of Vodafone in one year at a strike price of £82. After one year, Vodafone's spot price is equal to £86. Did John realize a profit from this transaction?

Discussion

First of all it is interesting to compute the theoretical value of the 1-year forward price F_0 of Vodafone that is given by $F_0 = 80 \times e^{(6\%-2\%)\times 1} = £83.30$. As the theoretical forward price is higher than the strike price K, John has to pay a premium $\text{Forward}_{\text{price}}$ for the forward contract that is equal to the number of shares times the present value of the difference between the forward price and the strike price, as follows:

$$\begin{aligned}\text{Forward}_{\text{price}} &= 1,000 \times (F_0 - K) \times e^{-rT} \\ &= 1,000 \times (83.30 - 82) \times e^{-5\%\times 1} = £1,224\end{aligned}$$

At the end of the year, the forward contract entitles John to receive 1,000 shares of Vodafone at £82 with a market value equal to £86. Therefore, John makes a profit equal to $1,000 \times (86 - 82) = £4,000$ knowing that he paid £1,224 as a forward contract premium.

Interest Rate Swaps

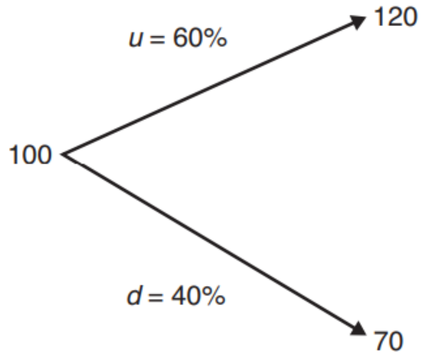
Let E denote the 3-month EURIBOR rate. Consider an interest rate swap contract where Party A pays E to Party B, and Party B pays $24\% - 3 \times E$ to Party A. Let N denote the notional of this swap. Can you express this deal in simpler terms?

Discussion

Party A pays E and receives $24\% - 3 \times E$. This means that Party A receives $24\% - 4 \times E = 4 \times (8\% - E)$. This contract is then equivalent to an interest rate swap arrangement where Party A (the receiver) receives 8% from Party B (the payer), and pays E to Party B. The notional of the equivalent contract is equal to $4 \times N$.

Vanilla Options

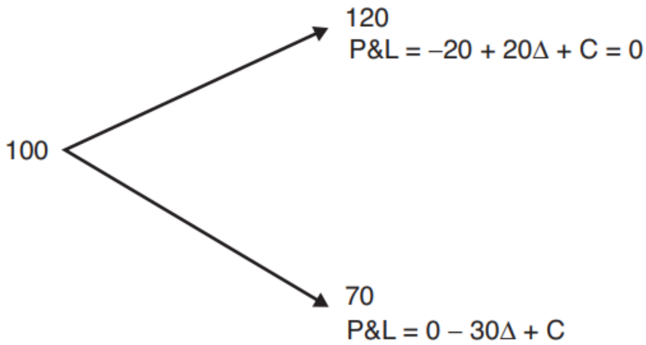
Consider a stock Alpha that can take two specific values 70 and 120 after 5 years. Alpha's initial spot is equal to 100. The equity analysts covering this stock and working for a major financial institution state that there is a 60% probability that the final spot price will be 120 and a 40% probability that the final spot ends at 70. Then, what would be the fair price of a 5-year European at-the-money call option on Alpha? To keep things simple, let's assume that interest rates and dividends are both zero.



Discussion

Since interest rates and dividends are assumed to be null, the price of the call is equal to the expected value of the stock minus the strike (that is equal to 100). If one thinks that the expected value of the stock at maturity is equal to $120 \times 60\% + 70 \times 40\% = 100$, then he is definitely lost. The price of the call would have been equal to $100 - 100 = 0$. One could use probabilities if we are working in a risk-neutral environment, which is not the case of the real world. Therefore, these probabilities are useless in our pricing.

Firstly, the price of an option represents the cost of the hedge. Let's assume that you are short the call, then you have to be long a number delta of stocks in order not to be sensitive to the spot price. In this case, your P&L is always equal to zero no matter what happens to the stock price.



Let C denote the call premium that you received at start date. If the stock price finishes at 120, then the call is exercised and you have to pay 20 to the option's holder. On the other hand, you realized a profit of 20Δ from your stock position. Alternatively, if the final stock price is equal to 70, then the call is not exercised but you lose 30Δ on your stock position.

Since the P&L is null in all cases, we have to solve the following system:

$$\begin{cases} -20 + 20\Delta + C = 0 \\ 0 - 30\Delta + C = 0 \end{cases}$$

This gives us $\Delta = 40\%$ and $C = 12$, which answers our question.

Options Greeks

What impact will an increase in interest rates have on the price of a call option? How about a put option? There is also the question of the effect on dividends.

Discussion

Assume that we sell a call option, then we need to buy Δ of stock. To buy Δ we will need to borrow money, so if the rates go up, it costs us more to borrow money (sell bonds) in order to Delta hedge. Since the option price reflects the cost of hedging, the price of the call must go up if hedging costs go up.

For a put option it is the other way around. If we sell a put then we need to sell Δ of stock. We sell Δ of stock and lend this money (buy bonds), so if rates go up we will make more money from our hedging strategy and thus the price of the put should be lower if rates go up.

We can add the effect of discounting. If rates go up, the discount factor goes down thus lowering the price of options. This is the case for both calls and puts, but in both this effect is generally smaller than the effect of rates on our Delta hedge. In the case of the call, the effect of discounting counters slightly the effect of the rise in the cost of borrowing money. In the case of the put, higher rates mean lower prices and the discount factor lowers them further.

Similar arguments can be made in regards to the price sensitivity to the dividend yield of the underlying asset. If we sell a call option, we need to buy Delta of the asset. If we hold the asset we are long the dividends paid by this asset. If dividends are higher, it means that we make more money on our Delta hedge and thus the cost of hedging is less and the option premium will be less.

Imagine you are in charge of Delta hedging a portfolio of options. And let's assume that you are short skew and volatility goes down. Would you end up buying or selling underlying shares?

Discussion

The skew increases the price of OTM puts and ITM calls; and decreases the price of OTM calls and ITM puts. Being short the skew can mean being short OTM puts, short ITM calls, long OTM calls or long ITM puts. Let's consider the case where you are long OTM call options. If volatility goes down, the Delta of OTM calls goes down. Since you are long the options, the portfolio overall Delta is then negative. Therefore, you have to buy shares to maintain zero sensitivity to the spot price of underlying shares.

Ania Petrova is a Russian vanilla options trader. One of her Delta-neutral portfolios is composed of shares of Gazprom as well as options on this stock. The portfolio daily global $\Theta_{1\mathrm{d}} = -1,000$ RUB. Assuming a realized volatility σ of 16%, what would the daily P&L of Ania be if the stock moves by $\pm 2\%$ during one trading day?

Discussion

Let δ_Π denote the change in the portfolio value. Then,

$$\delta_\Pi = \Theta \times \delta t + \Delta \times \delta S + \frac{1}{2}\Gamma \times \delta S^2$$

Here, Ania is managing a Delta-neutral portfolio, which means that $\Delta = 0$. Therefore

$$\delta_\Pi = \Theta \times \delta t + \frac{1}{2}\Gamma \times \delta S^2$$

This means that for a realized annual volatility of 16%, the loss due to Θ is equal in absolute terms to the gain due to the Γ. In other words, the daily P&L breakeven occurs when the spot moves by $\sigma_{1\mathrm{d}} = \sigma / \sqrt{252} = 1\%$.

$$-1,000 \times 1 + \frac{1}{2} \times \Gamma_{1\mathrm{d}} \times \sigma_{1\mathrm{d}}^2 = 0$$

which implies

$$\frac{1}{2} \times \Gamma_{1\mathrm{d}} \times \sigma_{1\mathrm{d}}^2 = 1,000$$

Since $\sqrt{252}$ is approximately equal to 16, Ania loses money on a daily basis when $|\delta S| < 1\%$ and makes positive P&L when $|\delta S| > 1\%$. Note that the $\Gamma -$ P&L is only dependent on the absolute value of the spot's move and not on its direction.

In this case $|\delta S| = 2\%$, then the daily P&L is as follows:

$$\text{P\&L}_{1\mathrm{d}} = -1,000 \times 1 + \frac{1}{2} \times \Gamma_{1\mathrm{d}} \times \delta S^2$$

Then

$$\text{P\&L}_{1\mathrm{d}} = -1,000 \times 1 + \frac{1}{2} \times \Gamma_{1\mathrm{d}} \times (2 \times \sigma_{1\mathrm{d}})^2$$

$$\text{P\&L}_{1\mathrm{d}} = -1,000 \times 1 + \frac{1}{2} \times \Gamma_{1\mathrm{d}} \times (2 \times \sigma_{1\mathrm{d}})^2$$

Or equivalently

$$\text{P\&L}_{1\mathrm{d}} = -1,000 \times 1 + 4 \times \frac{1}{2} \times \Gamma_{1\mathrm{d}} \times \sigma_{1\mathrm{d}}^2$$

This implies that

$$\text{P\&L}_{1d} = -1,000 \times 1 + 4,000 = 3,000 \text{ RUB}$$

Ania makes an overall profit of 3,000 RUB due to the large move of the underlying spot compared to its realized volatility.

Assuming a volatility of 20%, can you give a quick estimation for the price of a 1-year European at-the-money (ATM) call option? Also, what would you say concerning the cheapest between a basket composed of two 1-year ATM European calls and a single ATM call option expiring in 2 years?

Discussion

Concerning the pricing of European at-the-money calls, you should be able to give an accurate straightforward estimation using formula (5.6). Then, the price of a 1-year ATM European call is as follows:

$$C_{1y} = 0.4 \times \sigma \times \sqrt{1} = 0.4 \times 20\% \times 1 = 8\%$$

Now, still using the same approximation, we get the price C_{2y} of an ATM European call option with a maturity of 2 years:

$$C_{2y} = 0.4 \times \sigma \times \sqrt{2}$$

On the other hand, the price B_{1y} of a basket composed of two ATM European calls having a maturity of 1 year is equal to

$$B_{1y} = 2 \times C_{1y} = 2 \times 0.4 \times \sigma \times \sqrt{1} = \sqrt{2} \times C_{2y}$$

This implies that $B_{1y} > C_{2y}$. We can conclude that a 2-year ATM European call option is cheaper than two 1-year ATM European calls.

RATIO SPREAD

Discuss scenarios where the upside risk in a call ratio spread might be mitigated or managed.

Scenarios and Mitigation Strategies:

Neutral to Slightly Bullish Market:

Illustration: Consider a call ratio spread with K1 at $100 and K2 at $110. The investor buys 1 call at $100 and sells 2 calls at $110.

Mitigation: In a market scenario where the asset's price stays below $110 (the higher strike), the risk is mitigated. The strategy profits as the sold calls expire worthless.

Upside Risk Mitigation:

Illustration: If the market rises significantly above the higher strike, say to $120.

Mitigation: The upside risk is mitigated by capping the profit potential. The long call at the lower strike (K1) provides some profit, but the gains are limited due to the short calls at the higher strike (K2).

Adjustments for Upside Breakouts:

Illustration: If the market shows a strong bullish trend, breaching the higher strike.

Mitigation: Investors might choose to manage the upside risk by adjusting the position. This could involve closing the short calls, rolling them to higher strikes, or adding further long calls to create a more balanced spread.

Monitoring and Exit Strategies:

Illustration: Suppose the market is experiencing a steady increase, but the investor is concerned about unlimited upside risk.

Mitigation: Continuous monitoring is crucial. Exiting the position before significant losses occur or implementing dynamic adjustments based on market conditions can help manage risks effectively.

Impact of Changes in Volatility on Call Ratio Spread Profitability:

Question:

Illustration: Consider a call ratio spread with a long call at-the-money (K1) and two short out-of-the-money calls (K2) struck at 120% of the underlying asset's initial value. The current volatility is 20%, and the market is experiencing a steady increase in volatility.

How would an increase in volatility impact the profitability of the call ratio spread?

Answer:

As volatility rises, the value of options generally increases. In the case of a call ratio spread, the impact can be illustrated as follows:

Initial Scenario (Volatility = 20%):

The premium for the at-the-money call (K1) and the out-of-the-money calls (K2) is relatively lower due to lower implied volatility.

Profitability is influenced by the difference between the premium received from selling the two out-of-the-money calls and the premium paid for the at-the-money call.

Increased Volatility Scenario (Volatility = 30%):

Option premiums, especially for out-of-the-money options, increase with higher volatility.

The premium received from selling the out-of-the-money calls is now higher.

The overall profitability of the call ratio spread improves due to the increased value of the short options.

Effect of Increased Volatility on Risk Profile and Hedging Strategies:

Question:

Illustration: Assume the same call ratio spread as before, with a focus on the risk profile. The investor is concerned about the potential losses resulting from an unexpected spike in volatility.

How might an increase in volatility affect the risk profile of the call ratio spread?

What strategies could an investor employ to hedge against increased volatility?

Answer:

Impact on Risk Profile:

With an increase in volatility, the potential losses on the short out-of-the-money calls (K2) may escalate.

The unlimited upside risk becomes more pronounced as the higher volatility contributes to larger price swings in the underlying asset.

Hedging Strategies:

Long Volatility Instruments: Investors might consider buying additional long volatility instruments, such as long calls or long straddles, to offset potential losses from the short options in the ratio spread.

Dynamic Position Adjustments: Periodic adjustments to the position, such as rolling the short calls to higher strikes or closing out portions of the position, can help manage risk in response to changing volatility.

Impact of Dividends on Ratio Spread Performance:

Question:

Illustration: Suppose an investor establishes a call ratio spread on a stock currently priced at $100. The investor goes long one at-the-money call (K1 = $100) and short two out-of-the-money calls (K2 = $120). The stock pays a quarterly dividend of $1.50 per share.

How would dividends impact the performance of the call ratio spread, especially considering the strike prices and the quantity of options involved?

Answer:

Initial Scenario (No Dividend):

The investor is receiving a premium for selling two out-of-the-money calls and paying a premium for the at-the-money call.

The impact of dividends is negligible in this scenario.

Dividend Payment Scenario:

When the stock pays a dividend, the stock price tends to decrease by the dividend amount ($1.50 in this case).

The long call at-the-money (K1) is impacted negatively as the stock price drops.

The short out-of-the-money calls (K2) benefit from the decrease in stock price.

Net Impact on the Call Ratio Spread:

The net impact depends on the magnitude of the dividend relative to the strikes and premiums involved.

The strategy may experience a temporary setback due to the negative impact on the long call, but the short calls may partially offset this.

Adjusting the Strategy for Dividend Changes:

Question:

Illustration: Continuing with the scenario, assume the company announces an increase in its quarterly dividend to $2.00 per share.

How can the investor adjust their call ratio spread strategy to account for this change in dividends?

Answer:

Impact of Increased Dividend:

The increased dividend further amplifies the negative impact on the long call at-the-money (K1) and benefits the short out-of-the-money calls (K2).

Adjustment Strategies:

Dynamic Strike Adjustments: The investor might consider adjusting the strike prices of the options to reflect the increased dividend impact, potentially by rolling the short calls to even higher strikes.

Position Size Modification: If the increased dividend significantly affects the risk profile, the investor might adjust the quantity of options involved in the ratio spread.

STRANGLES AND STRADDLES

Explain the role of delta, gamma, theta, and vega in a short strangle strategy. How do these Greeks influence the position at different market conditions?

Answer

Role of Greeks in a Short Strangle Strategy:

Delta:

Delta measures the sensitivity of an option's price to changes in the underlying asset's price. In a short strangle strategy, which involves selling both an out-of-the-money (OTM) call and an OTM put, the overall delta is typically near zero initially. As the underlying asset's price moves, delta becomes more negative as the put option's delta increases and the call option's delta decreases. This means the position becomes more short as the underlying price falls and more neutral as it rises.

Gamma:

Gamma represents the rate of change of delta. In a short strangle, gamma is at its highest when the underlying price is near the strike prices of the sold options. As the underlying moves, gamma adjusts the delta, causing the position to become more short or more neutral. Traders need to monitor and manage gamma exposure to avoid unexpected changes in delta and to adjust the hedge dynamically.

Theta:

Theta, or time decay, is crucial in non-directional strategies like short strangles. Since the strategy profits from time decay, theta is generally positive. However, it's important to note that as expiration approaches, theta accelerates. Traders should be mindful of the diminishing time decay benefit and consider adjustments or closing the position before theta erodes too much premium.

Vega:

Vega measures the sensitivity of an option's price to changes in implied volatility. In a short strangle, vega is typically negative. An increase in implied volatility raises the value of both the call and put options, negatively impacting the position. Conversely, a decrease in implied volatility benefits the position. Traders need to consider the impact of changes in market volatility and adjust their risk exposure accordingly.

Illustrate how changes in implied volatility can impact the risk and reward profile of a short strangle. Provide specific numerical examples to support your explanation.

Implied volatility (IV) plays a crucial role in the risk and reward profile of a short strangle. The following numerical example illustrates this impact:

Let's consider a trader who sells a strangle on stock XYZ with the following details:

Stock price: $100

Call option (strike $110) premium: $3

Put option (strike $90) premium: $2

Implied volatility: 20%

Scenario 1: Increase in Implied Volatility

If implied volatility increases to 25%, the new option premiums might be:

Call option premium: $4.50

Put option premium: $3.50

The total premium received initially was $5 ($3 + $2). After the IV increase, the new premium is $8 ($4.50 + $3.50). The position's value has declined, resulting in a loss for the trader.

Scenario 2: Decrease in Implied Volatility

If implied volatility decreases to 15%, the new option premiums might be:

Call option premium: $2.50

Put option premium: $1.50

Now, the total premium is $4 ($2.50 + $1.50). The position has gained value, resulting in a profit for the trader.

In summary, an increase in implied volatility erodes the premium received, leading to potential losses, while a decrease enhances the position's value, resulting in potential profits. Traders need to monitor and manage the impact of changes in implied volatility to effectively navigate short strangle positions.

Discuss the concept of dynamic hedging in the context of a non-directional option strategy. How would you adjust the hedge dynamically for a short straddle as market conditions evolve?

Initial Setup:

Imagine a trader initiates a short straddle on a stock trading at $100. The trader simultaneously sells an at-the-money (ATM) call with a strike of $100 and an ATM put with the same strike of $100. The total premium received is $8.

Call premium: $4

Put premium: $4

Market Movement:

Now, let's consider two scenarios:

Scenario A: Upward Price Movement

The stock's price rises to $110.

The call option is in-the-money (ITM), and its premium increases to $8.

The put option is out-of-the-money (OTM), and its premium decreases to $1.

Adjustment:

To dynamically hedge, the trader would buy shares of the underlying stock to neutralize the delta from the ITM call. This mitigates directional risk.

The trader might also sell additional OTM puts to collect more premium and balance the position's overall delta.

Scenario B: Downward Price Movement

The stock's price falls to $90.

The put option is now ITM with a premium of $8.

The call option is OTM with a premium of $1.

Adjustment:

In this case, the trader would sell shares of the underlying to offset the delta from the ITM put, neutralizing directional exposure.

Additionally, the trader might sell more OTM calls to collect premium, balancing the position's overall delta.

Time Decay and Theta:

As time progresses, theta, or time decay, comes into play. Suppose the stock remains around $100.

Both the call and put premiums decay due to time passing.

The trader can capture profits by buying back the options at a lower premium than initially sold, locking in gains.

Volatility Changes:

If there's a sudden increase in volatility, impacting the premiums:

A volatility spike could increase both call and put premiums.

To manage risk, the trader might adjust the hedge by buying or selling additional options or adjusting the position size.

Describe the concept of volatility skew and its impact on option prices. How would you incorporate volatility skew into the selection of strikes for a non-directional strategy?

Consider stock XYZ currently trading at $100. The implied volatility for options with a $110 strike (OTM call) is 20%, while the implied volatility for options with a $90 strike (OTM put) is 25%. This difference in implied volatility levels is referred to as volatility skew.

Answer

Concept of Volatility Skew:

Volatility skew is the uneven distribution of implied volatility across different strike prices or expiration dates. In this case, the higher implied volatility for OTM puts compared to OTM calls indicates a negative volatility skew.

Impact on Option Prices:

OTM Call Option (Strike $110):

Implied Volatility: 20%

Option Premium: $3

OTM Put Option (Strike $90):

Implied Volatility: 25%

Option Premium: $4

<u>Incorporating Volatility Skew into Non-Directional Strategy:</u>

For a non-directional strategy, such as a short straddle or strangle, where one sells both OTM call and put options simultaneously, the trader needs to consider volatility skew for optimal strike selection.

<u>Strike Selection:</u>

In a scenario with negative volatility skew, where puts have higher implied volatility than calls, the trader may choose to sell the OTM put with the higher premium (in this case, the $90 put with a premium of $4).

The trader may then sell the OTM call with a lower premium (in this case, the $110 call with a premium of $3).

<u>Reasoning:</u>

Selling the higher premium put takes advantage of the elevated implied volatility, potentially providing a higher credit for the position.

Simultaneously, selling the lower premium call helps balance the position and manage the risk, considering the lower implied volatility for calls.

Adjustment for Skew Changes:

If the volatility skew were to shift, the trader would need to reassess and potentially adjust the strike selection.

For instance, if the skew becomes more pronounced, favoring even higher implied volatility for puts, the trader might consider adjusting the put strike even further OTM to capture more premium.

You are managing a non-directional options strategy involving a short straddle on two correlated stocks, A and B. The current stock prices are $120 for A and $90 for B. You have sold both an at-the-money (ATM) call and an ATM put for each stock, collecting a total premium of $10 for stock A and $8 for stock B. Additionally, the correlation coefficient between stocks A and B is 0.8.

Question:

Explain how you would use correlation analysis to enhance the risk-return profile of the non-directional strategy involving the short straddle on stocks A and B. Provide specific quantitative adjustments based on the given scenario.

Solution:

Role of Correlation in Non-Directional Strategies:

Correlation measures the degree to which two assets move in relation to each other. In non-directional strategies involving multiple assets, such as a short straddle on correlated stocks A and B, correlation analysis helps manage risk and optimize the overall position.

Step 1: Initial Position

Stock A:

Premium: $5 (Call) + $5 (Put) = $10

Stock B:

Premium: $4 (Call) + $4 (Put) = $8

Step 2: Correlation Adjustment

Calculate the correlation-adjusted position:

Adjusted Premium for Stock B = Original Premium * Correlation (A, B)

Adjusted Premium for Stock B = $8 * 0.8 = $6.40

Step 3: Reassess Position

Compare the adjusted premiums:

Adjusted Premium for Stock A = $10 (unchanged)

Adjusted Premium for Stock B = $6.40

Enhancing Risk-Return Profile:

Correlation Benefit:

A higher correlation suggests that the price movements of stocks A and B are more synchronized. This can be advantageous in a non-directional strategy.

The correlation-adjusted premium for stock B is lower, allowing for a potentially more favorable risk-return profile.

Risk Management:

The adjusted premium reflects the impact of correlation on the position. The trader might consider adjusting the position size or strike selection to balance the overall risk.

Example Adjustment:

If the original position size was 100 contracts for each stock, the trader might adjust the position size for stock B to 80 contracts (correlation-adjusted premium divided by original premium for stock B).

Long Straddle/Strangle Around Earnings

Scenario:

You are considering a long straddle or strangle strategy for Company X, which historically experiences significant price movements post-earnings announcements. The current stock price is $150, and the upcoming earnings release is expected to be highly impactful. The historical average price movement post-earnings is 10%, and the implied volatility is currently at 30%.

Question:

Explain how you would employ a long straddle or strangle strategy around the earnings announcement for Company X. Provide a quantitative, step-by-step breakdown of the strategy, including entry and exit points. Additionally, considering the historical volatility, suggest how you would adjust the strike selection and expirations for the straddle or strangle.

Solution:

1. Strategy Selection:

Given the anticipated large price movements post-earnings, a long straddle or strangle strategy is appropriate. This involves buying both a call and a put option to capitalize on potential volatility.

2. Quantitative Analysis:

Step 1: Determine Potential Price Movement

Historical average price movement: 10%

Expected price movement post-earnings = Current stock price * Historical average = $150 * 0.10 = $15

Step 2: Strike SelectionTo account for the expected $15 price movement, select strikes that are $15 away from the current stock price.

Call strike: $150 + $15 = $165

Put strike: $150 - $15 = $135

Step 3: Expirations

Choose expirations that include the earnings announcement date. Typically, options expiring shortly after the earnings release are preferred to capture heightened volatility.

Expiration for both call and put: Closest monthly expiration following the earnings announcement.

Step 4: Entry Points

Execute the strategy just before the earnings release to benefit from the pre-earnings volatility buildup.

Monitor the implied volatility to ensure it is not excessively inflated, as this can impact the options' prices.

Step 5: Exit Points

Plan exit points based on the expected price movement. Consider setting profit targets and stop-loss orders.

Exit just after the earnings release or when the desired price movement is achieved.

3. Adjusting for Historical Volatility:

Given the historical volatility of 10%, which is lower than the current implied volatility of 30%, consider adjusting the strategy to mitigate potential overpricing of options.

Adjust strikes slightly closer to the current stock price or explore different expirations to optimize the risk-return profile.

Enhancing Risk-Return Profile with Correlation in a Non-Directional Option Strategy

Scenario:

You are considering a non-directional option strategy involving a straddle or strangle for two correlated assets, Stock A and Stock B. The current prices are $120 for Stock A and $90 for Stock B. The historical correlation coefficient between the two stocks is 0.7. You want to construct a strategy to take advantage of this correlation in order to enhance the risk-return profile.

Question:

Explain how you would use correlation analysis to construct a non-directional option strategy, specifically a straddle or strangle, for correlated assets Stock A and Stock B. Provide a quantitative, step-by-step breakdown of the strategy, including strike selection and position sizing.

Solution:

1. Strategy Selection:

Given the correlation between Stock A and Stock B, a non-directional strategy like a straddle or strangle can be constructed to take advantage of potential relative price movements.

2. Quantitative Analysis:

Step 1: Determine Correlation-Adjusted Strikes

Calculate the correlation-adjusted expected price movement for Stock B based on the historical correlation coefficient.

$$Correlation - AdjustedExpectedMovementforStockB = \text{Correlation Coefficient} \times \text{Expected Movement for Stock A} = 0.7 \times (\text{Current Price of Stock A} \times \text{Historical Expected Movement Percentage})$$

Step 2: Strike Selection

Choose strikes for Stock A and Stock B options based on the correlation-adjusted expected movement.

For a straddle: Select strikes equidistant from the current stock prices.

For a strangle: Choose OTM strikes based on the adjusted expected movements.

Step 3: Position Sizing

Consider the relative volatility and position size accordingly.

Calculate the notional value of the position for each stock and adjust based on the correlation.

$$\text{Correlation-Adjusted Position Size for Stock B} = \frac{\text{Correlation Coefficient} \times \text{Position Size for Stock A}}{\text{Volatility Ratio (Stock B/Stock A)}}$$

3. Adjusting for Relative Volatility:

Given the historical volatility ratio between Stock B and Stock A, adjust the position size to account for differences in volatility.

The volatility ratio is calculated as

$$\text{Volatility Ratio (Stock B/Stock A)}.$$

Correlation analysis is integral to constructing a non-directional option strategy for correlated assets. In this scenario, the strategy involves adjusting strikes based on correlation-adjusted expected movements, selecting position sizes according to the correlation, and factoring in the relative volatility between the assets. This quantitative approach ensures the strategy is tailored to the specific correlation dynamics between Stock A and Stock B, aiming to enhance the risk-return profile of the overall position.

Risk Management in Non-Directional Option Strategy

Scenario:

You have implemented a short strangle strategy on stock XYZ, with the stock trading at $100. The short strangle consists of selling an OTM call with a strike of $110 and an OTM put with a strike of $90. The premium received for each option is $3. The current implied volatility is 25%. Unexpectedly, the stock price moves significantly, reaching $120.

Question:

Discuss the risk management considerations when implementing a non-directional option strategy, particularly a short strangle. How would you set stop-loss levels and manage position size in such a scenario? Provide a quantitative analysis of the risk exposure and propose adjustments to the risk management approach.

Solution:

1. *<u>Risk Management Considerations:</u>*

Implied Volatility and Premium Decay

Monitor implied volatility levels. If implied volatility increases significantly, it can impact option prices.

Factor in premium decay. Options lose value over time, so time decay should be considered in setting stop-loss levels.

Price Movement Limits:

Define acceptable price movement limits. If the underlying stock moves beyond a certain threshold, it may trigger risk management actions.

Dynamic Hedging:

Implement dynamic hedging to adjust deltas and manage directional risk as the stock price moves.

<u>2. Quantitative Risk Analysis:</u>

Initial Position:

Premium received for the call: $3

Premium received for the put: $3

Total premium received: $6

Scenario: Stock Price Moves to $120:

Call option loss: $10 (current stock price - call strike)

Put option loss: $30 (put strike - current stock price)

Total loss: $40

Implied Volatility Impact:

If implied volatility increases, it may exacerbate losses. For example, a 10% increase in implied volatility could lead to an additional $6 loss.

3. Setting Stop-Loss Levels:

Percentage of Initial Premium:

Set a stop-loss level as a percentage of the initial premium received. For example, a 50% stop-loss would trigger an exit if the total loss reaches $3.

4. Managing Position Size:

Notional Value Limits:

Establish notional value limits based on the trader's risk tolerance. For instance, limit the position size to 2% of the total portfolio value.

5. Scenario Adjustment:

Adjusting Strikes:

If the stock price moves significantly, consider adjusting the strikes to bring the position closer to delta neutrality.

Rolling or Closing Positions:

Evaluate rolling or closing the position if the risk exposure becomes excessive. This involves buying back the existing options and selling new ones with different strikes or expirations.

Correlation Skew

Can you explain the concept of an implied volatility skew and an implied correlation skew in the context of options trading?

Implied Volatility Skew:

Explanation:

Implied volatility skew is a phenomenon observed in the options market where implied volatility levels vary across different strike prices or expiration dates for a particular underlying asset. Instead of a flat volatility surface, the curve exhibits a slope, reflecting market expectations. Typically, the implied volatility of out-of-the-money (OTM) options differs from that of at-the-money (ATM) options. Traders often witness a higher implied volatility for OTM options, indicating an anticipation of larger price movements.

Relevance:

The implied volatility skew is crucial in options pricing and trading strategies. It provides insights into market sentiment and expectations regarding potential future price movements. Traders use the skew to adjust their strategies, especially when anticipating specific market events or changes in volatility.

Implied Correlation Skew:

Explanation:

Implied correlation skew extends the concept of implied volatility skew to options on baskets of assets, such as indices or multiple correlated stocks. This occurs when quotes for basket options with different strikes are available, and the implied volatility skews for each constituent are considered. Using a formula to imply correlation for each strike, a correlation skew curve is generated. This curve illustrates how implied correlation varies across different strikes.

Distinguishing from Standard Implied Correlation:

Unlike a standard implied correlation parameter, which assumes a constant correlation across all strikes, implied correlation skew acknowledges that correlation can vary based on the specific strikes chosen. It captures the nuanced relationship between the underlying assets in a basket.

Relevance:

Implied correlation skew is particularly relevant in options pricing for exotic products and complex strategies involving multiple assets. It offers a more granular and accurate representation of risk, especially in scenarios where underlying assets may not move uniformly. Traders and risk managers can benefit from this concept to refine their understanding of how correlation dynamics impact option prices.

Conclusion- While implied volatility skew addresses the unevenness in expected price movements across different options for a single asset, implied correlation skew extends this idea to capture the dynamic nature of correlation in complex, multi-asset options trading scenarios. Both concepts are instrumental in shaping options pricing strategies and risk management approaches.

How does the implied correlation skew differ from a standard implied correlation parameter, and why might it be relevant in options pricing?

From Standard Implied Correlation Parameter:

The standard implied correlation parameter typically assumes a constant correlation value across all strikes and maturities. It simplifies the complex relationship between multiple underlying assets by providing a single, averaged correlation level for the entire basket.

Implied Correlation Skew's Approach:

In contrast, the implied correlation skew recognizes that correlation is not a static, uniform parameter. It acknowledges that correlation can vary at different strike prices for options on a basket of assets. This skew captures the nuanced dynamics between the underlying assets and adjusts the correlation parameter based on the specific strikes chosen.

Relevance in Options Pricing:

Capturing Dynamic Correlation:

The relevance of the implied correlation skew lies in its ability to capture the dynamic nature of correlation within a basket of assets. Standard implied correlation parameters may oversimplify the true correlation dynamics, especially in scenarios where the relationship between the underlying assets is not constant.

Granular Risk Assessment:

Implied correlation skew provides a more granular and accurate representation of risk in options pricing. It recognizes that different strike prices may exhibit varying degrees of correlation, offering traders and risk managers a more sophisticated tool to assess and manage risk in complex, multi-asset portfolios.

Options on Baskets and Exotic Products:

In options pricing for baskets of assets or exotic products, where the relationship between the underlying assets can be intricate, the implied correlation skew becomes particularly relevant. It allows for a more nuanced understanding of how changes in correlation at different strikes impact option prices.

Reflecting Market Sentiment:

The implied correlation skew also reflects market sentiment and expectations regarding the correlation structure of a basket of assets. It provides traders with additional information to refine their strategies based on anticipated changes in the correlation dynamics.

Conclusion - While a standard implied correlation parameter offers a simplified view of correlation, the implied correlation skew embraces the complexity of correlation dynamics. Its relevance in options pricing stems from the need for a more accurate and nuanced representation of risk, especially in scenarios involving multiple assets where correlation is not constant. Traders and risk managers can benefit from the implied correlation skew to enhance their understanding and decision-making in complex options trading scenarios.

BASKET OPTIONS

You are tasked with pricing basket options for a portfolio comprising three correlated assets: Stock A, Stock B, and Stock C. The current prices are $120, $90, and $150, respectively. The correlation coefficients between A and B, B and C, and A and C are 0.7, 0.5, and 0.6, respectively. You are using a mathematical model that incorporates stochastic volatility and jump diffusion processes.

Question:

Explain how you would approach pricing basket options for this portfolio. Consider the provided correlation coefficients and the characteristics of each asset. Discuss the factors influencing the pricing, and elaborate on how the chosen mathematical model accounts for correlation dynamics. Provide a quantitative example by calculating the price of a European basket call option with a strike of $200, assuming a maturity of 3 months and an implied volatility of 20%.

Solution:

Approach to Pricing Basket Options:

Correlation Dynamics:

Utilize the provided correlation coefficients (0.7, 0.5, 0.6) to capture the correlation dynamics among the assets. Recognize that these correlations influence the joint movement of the assets, impacting the pricing of basket options.

Stochastic Volatility and Jump Diffusion:

Employ a mathematical model that incorporates stochastic volatility and jump diffusion processes. This advanced model captures the dynamics of volatility changes over time and considers the possibility of abrupt jumps in asset prices, providing a more realistic representation of market behavior.

Factors Influencing Pricing:

Underlying Asset Characteristics:

Consider the individual characteristics of each asset, such as current prices, volatility, and potential jump magnitudes. These factors contribute to the overall risk and return profile of the portfolio, influencing the pricing of basket options.

Correlation Structure:

The correlation coefficients between assets significantly impact the pricing. Higher correlations may lead to increased joint movements, affecting option prices. The chosen model should account for these correlations dynamically over the option's life.

Implied Volatility:

Implied volatility, set at 20%, plays a crucial role in pricing. Ensure the model incorporates this volatility level, adjusting dynamically based on market conditions.

Quantitative Example:

Assuming a European basket call option with a strike of $200, a maturity of 3 months, and an implied volatility of 20%, the quantitative steps involve:

Calculate the Joint Distribution:

Utilize the correlation coefficients to establish the joint distribution of asset prices over the option's maturity.

Implement Stochastic Volatility and Jump Diffusion:

Integrate the chosen mathematical model, considering stochastic volatility and jump diffusion processes. Simulate possible price paths for each asset, accounting for potential jumps.

Pricing the Basket Option:

Utilize the simulated price paths to calculate the payoff of the basket call option at maturity. Discount the expected payoff back to present value to obtain the option price.

Sensitivity Analysis:

Conduct sensitivity analyses to understand how changes in individual asset prices, volatilities, and correlations impact the option price.

Dynamic Hedging for Basket Options

Scenario:

You are tasked with implementing a dynamic hedging strategy for a basket option on a portfolio comprising Stock A, Stock B, and Stock C. The current prices are $120, $90, and $150, respectively. The correlation coefficients between A and B, B and C, and A and C are 0.7, 0.5, and 0.6, respectively. You have a European basket call option with a strike of $200, a maturity of 3 months, and an implied volatility of 20%. Implement a dynamic hedging strategy to manage the delta and gamma exposures of the basket option, considering potential changes in correlation over time.

Question:

Explain a dynamic hedging strategy to manage the delta and gamma exposures of the given basket option. Discuss the factors influencing the dynamic hedge and how you would adjust the hedge as correlation evolves. Provide a quantitative example by calculating the adjustments t o the hedge when the correlation between A and B increases from 0.7 to 0.8 after 1 month.

Solution:

Dynamic Hedging Strategy:

Initial Hedge Setup:

Begin by establishing an initial delta and gamma hedge based on the current asset prices, strike, maturity, and implied volatility. This involves buying or selling a combination of underlying assets and/or options to offset the option's risk.

Continuous Monitoring:

Continuously monitor the option's delta and gamma exposure. Given that the correlation between underlying assets can change, it is crucial to stay vigilant about the evolving risk profile of the basket option.

Delta Hedging:

Adjust the delta hedge by buying or selling underlying assets or options to maintain a delta-neutral position. This involves calculating the current delta of the option and offsetting it with changes in the underlying asset prices.

Gamma Hedging:

Address gamma exposure by periodically rebalancing the hedge to adapt to changes in volatility. This may involve adjusting the position size or repositioning options to offset changes in the option's gamma.

Adjusting the Hedge with Changing Correlation:

Quantitative Adjustment Example:

Assume the correlation between A and B increases from 0.7 to 0.8 after 1 month. Calculate the new correlation-adjusted delta and gamma for the basket option using the Black-Scholes model or a more sophisticated multi-asset pricing model.

Delta Hedge Adjustment:

Adjust the delta hedge to account for the new correlation. If the correlation increase positively impacts the option's delta, the hedge might need to be adjusted by reducing the position size to maintain delta neutrality.

Gamma Hedge Adjustment:

Evaluate the impact of the correlation change on gamma. Depending on the correlation's effect on gamma, adjust the hedge by rebalancing options or underlying assets to manage the updated gamma exposure.

Continuous Adaptation:

As correlation continues to evolve, repeat the process of adjusting the delta and gamma hedge. Continuously monitor the option's risk factors and make necessary modifications to ensure the hedge remains effective in offsetting changes in the option's exposure.