

# Mathematical Model for FJSSP with Job-Splitting

## Sets and Indices

- $I = \{1, \dots, n\}$ : set of jobs, index  $i, w$ . For each job  $i$ ,  $J_i = \{1, \dots, o_i\}$ : set of operations, index  $j, q$ .
- $K = \{1, \dots, m\}$ : set of machines, index  $k, a$ .
- $L = \{1, \dots, \delta\}$ : sequence positions on each machine (with  $\delta$  an upper bound).

## Parameters

- $g_i$ : lot size of job  $i$ .
- $p_{ijk}$ : unit processing time of operation  $j$  of job  $i$  on machine  $k$ .
- $h_{ijk}$ : setup time of operation  $j$  of job  $i$  on machine  $k$ .
- $u_{ijk} \in \{0, 1\}$ : eligibility, = 1 if machine  $k$  can process operation  $(i, j)$ .
- $s$ : minimum sub-lot size (set  $s = 0$  if no lower bound).
- $M \gg 0$ : a large positive constant (Big- $M$ ).

## Decision Variables

- $x_{ijkl} \in \{0, 1\}$ : = 1 if operation  $(i, j)$  is processed on machine  $k$  in position  $l$ .
- $\alpha_{ijk} \in \mathbb{Z}_+$ : sub-lot size of operation  $(i, j)$  on machine  $k$ .
- $C_{ijk} \geq 0$ : completion time of operation  $(i, j)$  if processed on machine  $k$ .
- $C_{\max} \geq 0$ : makespan.

## Objective Function

$$\min C_{\max}$$

## Constraints

### 1. Completion time (first in sequence).

$$C_{ijk} + M(1 - x_{ijk1}) \geq h_{ijk} + p_{ijk} \alpha_{ijk} \quad \forall i, j \in J_i, k$$

### 2. Completion time (subsequent positions).

$$C_{ijk} + M(2 - x_{ijkl} - x_{wqk(l-1)}) \geq C_{wqk} + h_{ijk} + p_{ijk} \alpha_{ijk}$$

$$\forall i, w, j \in J_i, q \in J_w, k, l > 1$$

**3. Precedence within each job.**

$$C_{ijk} + M(2 - x_{ijkl} - x_{i(j-1)ab}) \geq C_{i(j-1)a} + h_{ijk} + p_{ijk} \alpha_{ijk} \\ \forall i, j = 2, \dots, o_i, k, a, l, b$$

**4. Makespan definition.**

$$C_{\max} \geq C_{ijk} \quad \forall i, j \in J_i, k$$

**5. Lot-size conservation.**

$$\sum_{k \in K} \alpha_{ijk} = g_i \quad \forall i, j \in J_i$$

**6. Sub-lot size consistency.**

$$\alpha_{ijk} \leq g_i \sum_l x_{ijkl}, \quad \alpha_{ijk} \geq s \sum_l x_{ijkl} \quad \forall i, j \in J_i, k$$

**7. Machine capacity per position.**

$$\sum_i \sum_{j \in J_i} x_{ijkl} \leq 1 \quad \forall k, l$$

**8. Operation uniqueness on each machine.**

$$\sum_l x_{ijkl} \leq 1 \quad \forall i, j \in J_i, k$$

**9. Sequencing continuity on each machine.**

$$\sum_i \sum_{j \in J_i} x_{ijkl} - \sum_i \sum_{j \in J_i} x_{ijk(l-1)} \leq 0 \quad \forall k, l = 2, \dots, \delta$$

**10. Machine eligibility.**

$$x_{ijkl} \leq u_{ijk} \quad \forall i, j \in J_i, k, l$$

**Variable Domains**

$$x_{ijkl} \in \{0, 1\}, \quad \alpha_{ijk} \in \mathbb{Z}_+, \quad C_{ijk} \geq 0, \quad C_{\max} \geq 0$$