INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering
CH 5350 Applied Time-Series Analysis

Assignment 2

Due: Friday, September 05, 2014 11:00 PM

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1. [AR process]

The AR(1) process is given by $x[k] = \phi_1 x[k-1] + e[k]$ where $e[k] \sim \mathcal{N}(0, \sigma_e^2)$ and $|\phi_1| < 1$. For this process,

(a) Verify that the process is not stationary by showing that $\text{corr}(x[k], x[k-l]) = \phi_1^l \left[\dfrac{\text{var}(x[k-l])}{\text{var}(x[k])}\right]^{1/2}$

(b) Show that the solution to $x[k]$ at **large** times is given by

$$x[k] = \sum_{r=0}^{\infty} (\phi_1)^r e[k-r]$$

(`Hint: Write the generic solution as a sum of two terms, particular and complementary solution`).

(c) Consequently, argue that the process is only *asymptotically* (at large times) stationary for arbitrary initial conditions.

(d) How should be the initial condition, *i.e.*, the value of $x[0]$ be chosen so that the process is also stationary and not merely asymptotically stationary?

2. [ACF of MA and AR processes]

(a) The MA(2) process is represented by $H(q^{-1}) = 1 + c_1 q^{-1} + c_2 q^{-2}$. Use the ACVF generating function to arrive at the theoretical expressions for the ACVF of the MA(2) process.

(b) For the process above, find the constraints (regions) on c_1 and c_2 so as to produce admissible ACF.

(c) Write the general expression for the ACF of an AR(2) process $H(q^{-1}) = \dfrac{1}{1 - 1.3q^{-1} + 0.4q^{-2}}$. Compare your answer with results provided from the theoretical as well as estimates from `R` (using `ARMAacf` and `acf` respectively).

3. [Periodicities and ACF]

Periodicities are a common phenomena. A process generates a sinusoidal wave, which is observed with error,

$$y[k] = A\sin(2\pi f_0 k) + e[k]$$

where $e[k]$ is the usual zero-mean unit-variance WN sequence and A, f_0 are suitable constants.

(a) Is the process stationary? Support your answer suitably.

(b) Prove that the time-averaged ACVF[1] of $y[k]$,

$$R_{yy}[l] = \frac{1}{N} \sum_{k=l+1}^{N} (y[k] - \bar{y})(y[k-l] - \bar{y}) \tag{1}$$

where $\bar{y}$ is the sample mean, is **asymptotically** (large samples, $N \to \infty$) also a sinusoidal sequence with frequency f_0.

(c) Is there any advantage of detecting periodicity of the sine wave from its ACF rather than examining $y[k]$ directly?

(d) Verify results in part (b) using R from 1000 samples of a sine wave with frequency $f_0 = 0.2$ cycles/sample with SNR maintained at 10.

4. [PACF]

(a) Compute the theoretical PACF at lags $l = 1, 2$ of two processes, $x_1[k] - 0.7x[k-1] + 0.12x[k-2] = e[k]$ and $x_2[k] = e[k] + 0.4e[k-1]$. Compare your results with those given by `ARMAacf`.

(b) For the data given in datasets `EuStockMarkets (SMI)`, `quakes` (`mag`), `nottem` (`datasets`) and $x[k] = 0.01k + e[k]$ - plot the time-series, comment on the stationarity / periodicity / non-stationarity. Plot the ACF and PACF for each time-series and tabulate your observations with respect to stationarity / periodicity / non-stationarity.

5. [Distribution of ACF estimates]

The ACF of a random process is generally estimated using the expression given in (1). It is clear that at each lag the estimated ACF is also a random variable. Study the distribution of ACF estimates (i.e., fit a suitable distribution) of a white-noise process at three different lags (say $l = 1, 5, 17$) using the `acf` routine in R.

[1] You may assume that the white-noise process is *ergodic*, i.e., the time- and ensemble-averages coincide in the limiting case.