INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH 5350 Applied Time-Series Analysis

Assignment 4

Due: Friday, October 17, 2014 11:00 PM

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1. [Power spectral density estimation]

Generate $N = 1500$ observations of the process:

$$v[k] = H(q^{-1})e[k] \qquad H(q^{-1}) = \frac{1 + 0.4q^{-1}}{1 + 0.25q^{-2}}$$

Estimate the power spectral density using two different approaches: (i) estimating the ACVF and using W-K Theorem, and (ii) fitting a time-series model followed by the expression for the p.s.d. of $v[k]$. With the first method, is the p.s.d. estimate sensitive to the maximum lag included in estimation? Which of these methods do you find easier to use and, in your opinion, yields better results?

2. [Coherency]

Consider a measurement $y[k] = x[k] + v[k]$ of an output $x[k] = H(q^{-1})u[k]$ where $u[k]$ is a stationary signal and $v[k]$ is the net effect of disturbances and measurement error, also assumed to be stationary.

(a) Derive an expression for the cross p.s.d., $\gamma_{yu}(\omega)$, assuming $v[k]$ is uncorrelated with $u[k]$.

(b) From the answer in part (a), arrive at the expression for coherency $\kappa_{yu}(\omega) = \dfrac{\gamma_{yu}(\omega)}{\sqrt{\gamma_{yy}(0)\gamma_{uu}(0)}}$.

(c) Finally, derive an expression for the SNR$(\omega) = \frac{\gamma_{xx}(\omega)}{\gamma_{vv}(\omega)}$ in terms of $|\kappa_{yu}(\omega)|$.

3. [Modelling periodicities]

For the data given In `nottem`, check for periodicities from the PSD. Build a model for the series by first fitting the periodicities and then an ARMA model for the residuals.

4. [Variability of sample mean]

Prove that the sample mean calculated from N samples of a stationary process has the variance

$$\text{var}(\bar{x}) = \frac{1}{N}\left[\sigma_{xx}[0] + 2\sum_{l=1}^{N-1}\left(1 - \frac{|l|}{N}\right)\sigma_{xx}[l]\right]$$

Verify your result by means of Monte-Carlo simulations for an MA(1) process: $x[k] = e[k] + 0.4e[k-1]$

5. [Fisher's information]

(a) Show that there exists no efficient estimator for estimating the parameter λ of a WN process with exponential distribution: $f(\lambda) = \lambda e^{-\lambda y}$

(b) For the same process, show that the inverse of the parameter $1/\lambda$ can be estimated efficiently.