INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering
CH 5350 Applied Time-Series Analysis

Assignment 6

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1. [Hannan-Rissanen algorithm]

   (a) Write an R function file to implement the H-R algorithm for the estimation of the ARMA model parameters. The user is expected to supply the orders of the respective components.

   (b) Test your code on two different processes: (i) MA(2) process with $c_1 = 1,\ c_2 = 0.21$ and (ii) ARMA(1,2) process with $d_1 = 0.4,\ c_1 = 0.7,\ c_2 = 0.12$.

   (c) Compare the results of your routine with the `arma` routine of the `tseries` package in R.

2. For the Burg's method, [Burg's method of estimation]

   (a) First, starting from D-L equations establish the following recursive relation

$$\boldsymbol{\theta}^{(p)} = \begin{bmatrix} \boldsymbol{\theta}^{(p-1)} + \kappa_p \bar{\boldsymbol{\theta}}^{(p-1)} \\ \kappa_p \end{bmatrix}$$

   where $\kappa_p \triangleq -\phi_{pp}$ is the *reflection coefficient*.

   (b) Next, establish the recursive relation for the forward and backward prediction errors

$$\epsilon_F^{(p)}[k] = \epsilon_F^{(p-1)}[k] + \kappa_p \epsilon_B^{(p-1)}[k-p], \qquad \epsilon_B^{(p)}[k-p] = \epsilon_B^{(p-1)}[k-p] + \kappa_p \epsilon_F^{(p-1)}[k]$$

   (c) Finally, determine the optimal reflection coefficients by solving

$$\min_{\kappa_p} \sum_{k=p}^{N-1} (\epsilon_F^2[k] + \epsilon_B^2[k-p])$$

3. [Predictions]

   For the MA(1) process $x[k] = e[k] - e[k-1]$ where $e[k]$ is the usual zero-mean WN. Suppose we consider the problem of predicting $x[k+1]$ using only $\{x[1],\ x[2],\ \cdots,\ x[k]\}$.

   (a) Arrive at the best BLP of $x[k+1]$ using the Projection Theorem.

   (b) Show that the mean square error in the prediction of $x[k+1]$ is

$$E((x[k+1] - \hat{x}[k+1|k])^2) = \frac{k+2}{k+1}\sigma_e^2$$

4. [Spectral Densities]

   (a) Determine the theoretical power spectral density of the series formed by

$$x[k] = e[k-2] + 2e[k-1] + 4e[k] \quad e[k] \sim \mathcal{N}(0,1)$$

   (b) Generate 2000 samples of $x[k]$. Estimate the power spectrum using different estimators, namely, (i) periodogram (ii) smoothed periodogram using Daniell's smoother (iii) Welch's averaged periodogram method and (iv) parametric method. Compare the estimates with the theoretical one obtained in part (4a) above.