INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering
CH 5350 Applied Time-Series Analysis

Assignment 5

Due: Friday, November 07, 2014 11:00 PM

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1. [Least Squares Estimation]

   (a) Prove that in the LS estimation of θ with the model $y[k] = x^T[k]\theta + e[k]$, $E(\hat{\sigma}_\epsilon^2) = \sigma_e^2$, where $\hat{\sigma}_\epsilon^2 = \dfrac{SSE}{N-p}$ .Satte that any assumptions that you have to make.

   (b) The LS estimator and the Y-W estimator produce almost identical estimates of an AR(P) model, with a subtle difference. The subtle difference lies in the way each of these methods handle non-zero mean of a series. Show that this difference vanishes for large N.

   (c) Verify parts (a) and (b) using simulated series in R for an AR(2) model with $d_1 = -1.1,\ d_2 = 0.28$.

2. [MLE and Least Squares]

   (a) Given two samples $x[1]$ and $x[2]$ (with different magnitudes) of a series, fit an AR(1) model using the MLE method (ITSM, Brockwell and Davis).

   (b) For the process above, arrive at the Least Squares solution and compare it with the MLE.

3. [Problem from Schumway and Stoffer]

   Let M_t represent the cardiovascular mortality series discussed in Chapter 2, Example 2.2. Fit an AR(2) model to the data using linear regression and using Yule–Walker.

   (a) Compare the parameter estimates obtained by the two methods.

   (b) Compare the estimated standard errors of the coefficients obtained by linear regression with their corresponding asymptotic approximations.

4. [Fitting a seasonal model]

   (a) For the data given in `sarima_data.Rdata`, fit a model by the classical approach of decomposing the series into seasonal, trend and stationary components (use the stl routine in R to achieve this decomposition). Fit an ARMA model to the stationary series.

   (b) For the same data set, build a traditional SARIMA model in a systematic manner.

5. [Conditional Expectation]

   (a) Prove that the solution to $\min_{g(x)} E((Y - g(x))^2)$ is the conditional expectation $E(Y|X)$.

   (b) Show that if X and Y are jointly Gaussian with unconditional expectations μ_X and μ_Y respectively, the conditional expectation $E(Y|X)$ is a linear function of x.

   (c) Consider the random variable $y = x^2 + z$ where x and z are independent zero-mean processes with unit variance. Find the MSE approximation of y with respect to x. What is the value of MSE?

   (d) Suppose a linear approximation of y in part (c) is sought. What is the MSE obtained with the best linear model?