

# Introduction to MATLAB Software (1)

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## § 1.0 What is MATLAB? (1)

- Name came from: Matrix laboratory
- Originally created by Prof. **Cleve Moler**, U. of New Mexico, and written in **Fortran** in 1978.
- In 1984, **Jack (or John) Little** rewrote it in **C language** and created *MathWorks Inc.* and published it commercially.
- A high-level programming language with *interactive* environment—responding the results immediately
- A set of tools and facilities: the MATLAB desktop and **Command Window**, a command history, an editor and debugger, a code analyzer and other reports, and browsers for viewing help, the workspace and so forth. These help you use MATLAB functions and files.
- A full-featured scientific calculator—numerical computation

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# What is MATLAB? (2)

- Having programming and graphing capabilities with visualization tool, especially, using the *Handle Graphics* (握把式作圖, from V.4) with GUI (Graphic User Interface)
- A matrix-vector-oriented system and special data structures (from V. 5): *multidimensional arrays* (n-D arrays), *Cell Arrays* (like many drawers in a cabinet), and *structure arrays* (struc(field1, value1, field1, value1,...))
- A **mathematical function library**: a vast collection of *computational algorithms* ranging from *elementary functions* (like sum, sine, cosine) to *more sophisticated functions* (like matrix eigenvalues, Bessel functions, and fast Fourier transforms).



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# What is MATLAB? (3)

- Built-in many *intrinsic functions* and lots of intelligent problem-solving tools for particular applications (called *Toolboxes*) (more than 70 toolboxes so far)
- Having symbolic solutions by using *Symbolic Math Toolbox*), like **Maple** software
- A computational engine with dynamic linking to C, Fortran, and Maple for calling routines.
- MathWorks develops new products: **Simulink** with Blocksets (a real-time dynamic simulation) and **Stateflow** (Finite State Machines or Event-driven Systems)

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# Basic Features

- Basic math working in *command window* (see next page)
- display answers without semicolon
- Nothing display with semicolon (;)
- Remember the variables in *Workspace*
- Variables are not declared by the user but are created on a need-to-use basis by a memory manager
- Save the results as *\*.mat* (or *\*.tex*) files using `>>diary name.mat (or *.tex)`

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# Basic arithmetic operations

addition	+	$5+3$
subtraction	-	$23 - 12$
multiplication	*	$3.14 * 0.85$
division	/ or \	$56/8 = 8 \backslash 56$
power	^	$5^2$

# About Number Display Formats

- MATLAB use double-precision floating-point arithmetic, which is accurate to approximately 15 digits; however, it displays only 5 digits by default. To display more digits, type **format long**.
- The new version of MATLAB (7.0 or upper version) has the *variable precision arithmetic* with **vpa**. You can specify the number of digits as what you desire. (see examples)

# Number Display Formats

Command	average_cost	Comments
format long	35.83333333333334	16 digits format
format short e	3.5833e+01	5 digits plus exponent
format long e	3.583333333333334e+01	16 digits plus exponent
format hex	4041eaaaaaaaaaab	hexadecimal
format bank	35.83	2 decimal digits
format +	+	positive, negative, or zero
format rat	215/6	rational approximation
format short	35.8333	default display

# Variable Naming Rules

Rule	Comments
Variable names are case sensitive (large and small letters are <b>different</b> ).	fruit, Fruit, FrUiT, and FRUIT are all <b>different</b> MATLAB variables.
Variable names can be of <b>any length</b> .	Characters beyond the 63th are ignored.
Variable names must <b>start with a letter</b> , followed by any number of letters, digits, or underscores.	<b>Punctuation characters</b> are <b>not allowed</b> since many have special meaning to MATLAB.



# Several Special Variables

Variable	Value
ans	Default variable name used for <b>results</b>
pi	<b>Ratio of the circumference of a circle</b> to its diameter
eps	<b>Smallest number</b> such that when added to 1 creates a floating-point number greater than 1 on the computer
inf	<b>Infinity</b> , e.g., $1/0$
NaN	Not-a-Number, e.g., $0/0$
$i$ and $j$	$i = j = \sqrt{-1}$
realmin	The <b>smallest</b> usable positive real number
realmax	The <b>largest</b> usable positive real number

# Exercise 1.0

- 1. Calculate the value of the following functions at  $x = 10$  using three different ways and also plot their graphs with "easy way":
  - $\gg f = ' \sin(2 * x + 5) * \cos(3 * x) * (2 * x^2 - 7) '$
  - $\gg f = \text{inline}(' \sin(2 * x + 5) * \cos(3 * x) * (2 * x^2 - 7) ')$

## §1.1 Vectors and Plotting

- **Row vectors:** `>> x=[10.1 20.2 30.3]`
- Column vectors: `>> x=[10.1; 20.2; 30.3]`
- To change the orientation of a vector **from row to column** or **column to row**, use an **apostrophe (')**:  
`>> x=[10.1 20.2 30.3]'`
- Equal spacing vectors:
- `>> x=linspace(<Left Endpoint>, <Right Endpoint>, <Number of Points>)`

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# Vectors

- Row vectors: Use colon notation: `>> x = 20:24`
- Equivalent to `>> x=[20 21 22 23 24]`
- Using the stride: `>> x = 20:2:29`
- Equivalent to `>> x=[20 22 24 26 28]`
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# Examples

- **`>> x=linspace(a, b, n)`**
  - The  $k$ -th point:  $x_k = a + (k - 1) * (b - a) / (n - 1)$
  - `>> x=logspace(a ,b, n)`
  - The  $k$ -th point:  $x_k = 10^{[a+(k-1)*(b-a)/(n-1)]}$

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# Elementary (built-in) functions

- **`>> y=abs(x) % x can be scalars or vectors`**
- `>> y=sin(x); y=cos(x);` and so on.
- `>> y=asin(x); y=acos(x);` % the arcsine and arccosine
- `>> y=log(x)` % natural logarithmic function
- `>> y=log10(x); y=log2(x);` % the common logarithm and base 2 logarithm
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- `>> y=min(x) % find the minimum number in vector x`
- `>> y=max(x); % find the maximum number in vector x`
- `>> y=mean(x); % find the average number in vector x`
- `>> y=sum(x) % find the sum of all numbers in vector x`
- `>> y=sort(x) % sorting the numbers in vector x`



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# Matlab codes

```
n = 21;  
h = 1/(n-1);  
for k=1:n  
    x(k) = (k-1)*h;  
end
```

%%

```
n = 21;  
h = 1/(n-1);  
x = zeros(1, n);  
for k=1:n  
    x(k) = (k-1)*h;  
end
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# Matlab codes

```
%Compute sin(x) for 21 points on [0, 1]
```

```
n = 21;
```

```
x = linspace(0,1,n);
```

```
y = zeros(1,n);
```

```
for k=1:n
```

```
    y(k) = sin(x(k));
```

```
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% Compute sin(x) for 21 points on [0, 2pi]
```

```
n = 21;
```

```
x = linspace(0,1,n);
```

```
y = sin(2*pi*x);
```

# Advantages of Vectorization

- **Speed:** The build-in MATLAB functions provide results of several calls faster if called once with the corresponding vector argument(s).
- **Clarity:** Easier to read a vectorized MATLAB script than its scalar-level counterpart.
- **Education:** requiring to think at the vector level and fostering the style of algorithmic thinking.

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# Exploiting Symmetry

- Example of plotting  $\sin(2\pi x)$  for  $x \in [0, 1]$
- Using the properties:

$$\sin\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} - x\right)$$

- and

$$\sin(\pi + x) = -\sin(x)$$

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# Matlab codes

```
function y = SinValue(n)

% Compute sin(x) for 21 points on [0, 2pi] with symmetry
% n must be a positive integer of multiple of 4

m = (n+1)/4; % m = 5; n = 4*m+1;
x = linspace(0,1,n);
a = x(1:m+1);
y = zeros(1, n);
y(1:m+1) = sin(2*pi*a);
y(2*m+1:-1:m+2) = y(1:m);
y(2*m+2:n) = -y(2:2*m+1);
```

# Displaying Tables

## ● Creating a script file

- Use '%' notation to give *comments* for the command codes.
- `disp(' *** ')`: to display strings enclosed by single quotes.
- `sprintf`: to produce a string that includes the values of named variables.

`sprintf(< String with Format Specification >, < List of Variables >`

- `disp(sprintf(' *** ', names of variables))`

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- Use '%' notation to give *comments* for the command codes.
- **disp(' \*\*\* '): to display strings enclosed by single quotes.**
- **sprintf:** to produce a string that includes the values of named variables.

`sprintf(< String with Format Specification >, < List of Variables >`

- `disp(sprintf(' *** ', names of variables))`



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# Matlab codes

```
% Script File: SineTable
% Prints a short table of sine evaluations.
clc      % clear the command window and home the cursor.
n = 21;   x = linspace(0,1,n);
y = sin(2*pi*x);
disp('')
disp('k      x(k)      sin(x(k))')
disp('-----')
for k=1:21
    degrees = (k-1)*360/(n-1);
    disp(sprintf('%2.0f %3.0f %6.3f', k, degrees, y(k)));
end
disp('');
disp('x(k) is given in degrees.')
disp(sprintf('One Degree = %5.3e Radians', pi/180))
```

# plotting $y = \sin(x)$

- Use **'plot'** command to create a *figure*.
- Use **'title'**, **'xlabel'**, and **'ylabel'** to *comment* the plot.
- Use **'pause(1)'** command permits a 1-second viewing of each plot.

# Matlab codes

%1.1.5 A simple plot of  $y = \sin(x)$

```
n = 21;  
x = linspace(0, 1, n);  
y = sin(2*pi*x);  
plot(x, y)  
title('The Function  $y = \sin(2\pi x)$ ')  
xlabel('x (in radians)')  
ylabel('y')
```

# Matlab codes

```
n = 200;  
x = linspace(0, 1, n);  
y = sin(2*pi*x);  
plot(x, y)  
title('The function  $y = \sin(2\pi x)$ ')  
xlabel('x (in radians)')  
ylabel('y')
```

# Matlab codes of SinePlot

```
% Script File: SinePlot
% Displays increasingly smooth plots of sin(2*pi*x).

close all          % Close all windows.
for n = [4 8 12 16 20 50 100 200 400]
    x = linspace(0, 1, n);
    y = sin(2*pi*x);
    plot(x,y)
    title(sprintf('Plot of sin(2*pi*x) based upon...
                  n = \%.30f points.', n))
    pause(1)
end
```

# Exercise 1.1

- 1. Calculate the length of the power cable in Lecture 1 by locating root function **fzero** and *arc length formula*.
- 2. plot  $y = \cos(x)$  for  $x \in [0, 2\pi]$  by applying vectorization and symmetry.
- 3. Display the results of problem 2 in a table.
- 4. Plot the function  $y = \sin(x)$  for  $x \in [-\pi, \pi]$  and their Taylor polynomials

$$S_1(x) = x, \quad S_2(x) = x - \frac{x^3}{3!}, \quad S_3(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!},$$



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## § 1.2 More Vectors, More Plotting, and Matrices

- **1.2.1 Vectorizing Function Evaluations**
- 1.2.2 Scaling and Superpositioning
- 1.2.3 Plotting Polygons
- 1.2.4 Some Matrix Computations

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# Plotting the Rational Function

- Plotting the Rational Function

$$f(x) = \left( \frac{1 + \frac{x}{24}}{1 - \frac{x}{12} + \frac{x^2}{384}} \right)^8, \quad x \in [0, 1]$$

- An approximation to the function  $e^x$ .



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- An approximation to the function  $e^x$ .

# Matlab codes of ExpPlot

```
% Script File: ExpPlot1
% Approximate exp(x) by the function:
%  $f(x) = ((1+x/24)/(1-x/12+(x^2/384)))^8$  across [0, 1].
% Scalar operations--using for-loop.

close all          % Close all windows.
n = 200;
x = linspace(0, 1, n);
y = zeros(1,n);
for k = 1:n
    y(k) = ((1+x(k)/24)/(1-x(k)/12+(x(k)/384*x(k)))^8;
end
plot(x, y)
```

## 1.2.1 Vector Operations

- Operations of vector scale, vector add, vector subtract
- Operation of pointwise vector multiply `'.*'`
- Operation of pointwise vector divide  `'./ '`
- Operation of pointwise vector exponentiation  `'.^ '`
- Write your programs with MATLAB using *vector* or *matrix* operations

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- Write your programs with MATLAB using *vector* or *matrix* operations

# Matlab codes of ExpPlot

```
% Script File: ExpPlot
% Approximate exp(x) by the function:
% f(x)=((1+x/24)/(1-x/12+(x^2/384)))^8 across [0, 1].
% Vector operations--Using pointwise vector operations.

close all           % Close all windows.
x = linspace(0, 1, 200);
num = 1 + x/24;
denom = 1 - x/12 + (x/384).*x;
quot = num ./ denom;
y = (quot).^8;
plot(x, y, x, exp(x))
```



# Plotting the $\tan(x)$ Function

- Plot the graph of a function using **plot** with autoscaling feature.
- Example: Plot the  $\tan(x)$  function

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \quad x \in [-\pi/2, 11\pi/2]$$

- Using the **axis** function to scale the axes manually.
- Syntax: `>> axis([xmin xmax ymin ymax])`

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- Syntax:** `>> axis([xmin xmax ymin ymax])`

# Matlab codes for TangentPlot

```
% Script File: TangentPlot1
% Plots the function tan(x), -pi/2 <= x <= 11pi/2

close all
x = linspace(-pi/2,11*pi/2,200);
y = tan(x);
plot(x, y)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

x = linspace(-pi/2,11*pi/2,200);
y = tan(x);
plot(x, y)
axis([-pi/2 9*pi/2 -10 10])
```

## Matlab codes for TagentPlot

```
% Script File: TagentPlot1
% Plots the function tan(x), -pi/2 <= x <= 11pi/2
close all
x = linspace(-pi/2, pi/2, 40);
y = tan(x); plot(x, y)
ymax = 10;
axis([-pi/2 9*pi/2 -ymax ymax])
title('The Tangent Function'),
xlabel('x'), ylabel('tan(x)')
hold on
for k=1:4
    xnew = x + k*pi;
    plot(xnew, y);
end
hold off
```

## 1.2.2 Superpositioning (Multi-Graph)

- **hold on:** to superimpose (疊置在上面 (使重疊); 加上 = superpose) all subsequent plots on the current figure window.
- **hold off:** to shut down the superpositioning feature and set the stage for normal plotting thereafter.
- Another way to superimpose in the same plot is by calling **plot** with an extended parameter list.
- The syntax of multi-graph's **plot**:  
`plot(<First graph specification>, ..., <Last graph specification>)`
- The form of each graph specification:  
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# Matlab codes for Plotting with superposition

```
% Script File: SinAndCosPlot
% Plots the functions sin(2*pi*x) and cos(2*pi*x)
% across [0,1] and marks their intersection.

close all
x = linspace(0,1,200);
y1 = sin(2*pi*x);
y2 = cos(2*pi*x);
plot(x,y1,x,y2,'--',[1/8,5/8],[1/sqrt(2),-1/sqrt(2)],'*')
```

## Polygons with $n$ Vertices

- If  $x$  and  $y$  are column vectors that contain the *coordinate values*, then **plot**( $x, y$ ) does not display the polygon because  $(x_n, y_n)$  is not connected to  $(x_1, y_1)$ . Need to make a **concatenation** of vectors.
- If  $r_1, r_2, \dots, r_m$  are row vectors, then

$$v = [r_1 r_2 \cdots r_m]$$

is also a row vector.

- If  $c_1, c_2, \dots, c_m$  are column vectors, then

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## Some Commands for Plotting

- The command **axis equal** ensures that the x-distance per pixel is the same as the y-distance per pixel.
- The command **axis off** does **not** display the coordinate axes.
- The command **subplot( $m, n, k$ )** breaks up the current figure into a  $m$ -by- $n$  array of sub-windows, and place the next plot in the  $k$ th one of these. They are indexed **left to right** and **top to bottom**. see **polygons.m**



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## 1.2.4 Some Matrix Computations

- Consider the problem of plotting the function

$$f(x) = 2 \sin(x) + 3 \sin(2x) + 7 \sin(3x) + 5 \sin(4x)$$

across the interval  $[-10, 10]$ .

- 1. Scalar-level script (see p.23)
- 2. Vector-level script
- 3. Matrix-level script
- Ideas: A linear combination of vectors is equivalent to matrix-vector multiplication.

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# Matrix-Vector Product

$$2 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 7 \\ 2 \\ 8 \end{bmatrix} + 3 \begin{bmatrix} 5 \\ 0 \\ 3 \\ 8 \\ 4 \\ 2 \end{bmatrix} + 7 \begin{bmatrix} 8 \\ 3 \\ 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} + 5 \begin{bmatrix} 1 \\ 6 \\ 8 \\ 7 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 8 & 1 \\ 1 & 0 & 3 & 6 \\ 4 & 3 & 3 & 8 \\ 7 & 8 & 1 & 7 \\ 2 & 4 & 1 & 0 \\ 8 & 2 & 1 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 7 \\ 5 \end{bmatrix}$$



# Creating A Matrix

- 1. Typing all entries row by row.
- 2. Using two for-loops to initialize a matrix by creating a zeros matrix first.
- 3. Aggregating its columns to form a matrix:

$$[< Column1 > < Column2 > \dots < Columnm >]$$

Note: all column vectors must have the same length.

- 4. Using a single loop whereby each pass sets up a single column (test sumOfSines).
- Note: Creating a  $m \times n$  zero matrix:  $\gg A = \text{zeros}(m, n).$

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## Another Example of Matrix Computations

- Consider the problem of plotting the two functions

$$f(x) = 2 \sin(x) + 3 \sin(2x) + 7 \sin(3x) + 5 \sin(4x)$$

$$g(x) = 8 \sin(x) + 2 \sin(2x) + 6 \sin(3x) + 9 \sin(4x)$$

over the interval  $[-10, 10]$ . (See SumOfSines2)

- In general, if the function

$$f(x) = \alpha_1 \sin(x) + \alpha_2 \sin(2x) + \alpha_3 \sin(3x) + \alpha_4 \sin(4x)$$

We want to find  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  so that  $f(1) = -2, f(2) = 0, f(3) = 1$ , and  $f(4) = 5$ .

- This arises the problem of solving a linear system (see p.27).

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## Exercise 1.2

- 1. Calculate the approximations of  $e^{1/2}$  and  $e^8$  by its Taylor polynomial with 8 terms and compare their absolute errors.
- 2. Calculate the approximations of  $\ln 2$  by the two Taylor polynomials (shown in the Lecture Notes of Chapter 1) with 10 terms and compare their absolute errors.

## § 1.3 Building Exploratory Environments

- **1.3.1 The Up/Down Sequence**
- 1.3.2 Random Processes
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## § 1.3 Building Exploratory Environments

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## An Example of Up/Down Sequence

- Suppose  $x_1$  is given positive integer and that we define the sequence as

$$x_k = \begin{cases} x_k/2, & \text{if } x_k \text{ is even,} \\ 3x_k + 1, & \text{if } x_k \text{ is odd.} \end{cases}$$

- The sequence is: 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ..., which is called *up/down* sequence.
- The **input** command:

`x = input('< string message >')`

- The **while-loop** form (see p.30)
- The **if-then-else** structures (see p.30)

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## *while* loops and *if-then-else* controls

```
k = 0;
while k <= 100
{command statements}
k = k +1;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if A > B,
    'greater'
elseif A < B,
    'less'
elseif A == B,
    'equal'
else
    error('A and B are different data')
end
```

## *switch* control

```
switch sign(A-B)
    case 0
        'A = B'
    case 1
        'A > B'
    case -1
        'A < B'
    otherwise
        error('A and B are different data type')
end
```

## Relation Operators in MATLAB

- ' < ' less than and ' > ' greater than
- ' <= ' less than or equal, ' >= ' greater than or equal
- ' == ' equal, ' ~= ' not equal
- ' & ' and, ' | ' or, ' ~ ' not

## Some Build-in Functions

- The **rem** function: **rem(a, b)** returns the remainder when  $b$  is divided into  $a$ .
- In the command **disp(sprintf('%-5.0f', x))**, the **minus sign left justifies** the display of the value.
- The **max** function: **max(v)** returns the maximum value and the index where it occurs.
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- The function **rand(n, 1)**: creates a length- $n$  column vector of real numbers chosen randomly from the interval (0, 1).
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- The histogram function: **hist** can be used in several ways and the script shows two possibilities:
- **hist(x, n)**: reports the distribution of the  $x$ -values according to where they "belong" with respect to  $n$  equally spaced bins spread across the interval **[min(x), max(x)]**.
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# Generating Random Integers

- Using **rand** or **randn** functions plus the **floor** or **ceil** functions to generate random integers.
- An example—the **dice** problem: simulating 1000 rolls of a pair dice and displaying the outcome in histogram form.
- The command **z = floor(6\*rand(n, 1) + 1)** computes a random vector of integers selected from  $\{1, 2, 3, 4, 5, 6\}$  and assigns them to **z**.
- **Note:**

$$\text{floor}(x) + 1 = \text{ceil}(x) \quad \forall x \in (-\infty, \infty)$$

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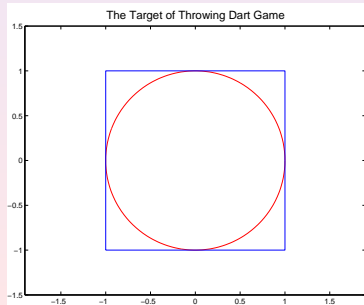
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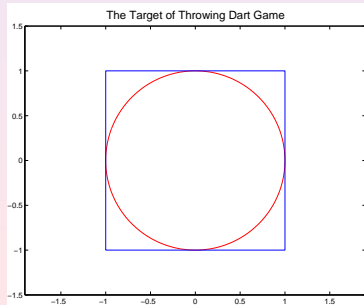
## Solving Non-random Problems

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- Assume that the darts land anywhere on the square with *equal probability*.
- After a large number of throws, the fraction (probability) of the darts that land *inside the circle* should be approximately equal to  $\pi/4$ , the ratio of the circle area to the the square's area.
- By simulating the throwing of a large number of darts, we can produce an estimate of  $\pi$ :

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- The command `rand('seed', .123456)`: starts the random number sequence with a *prescribed seed*, which enables to repeated the random simulation with exactly the same sequence of underling random numbers.
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# Polygon Smoothing

- Let  $x$  and  $y$  are  $(n + 1)$ -vectors and  $x_1 = x_{n+1}$  and  $y_1 = y_{n+1}$  then

`plot(x, y, x, y, 's')`

display a polygon.

- If we compute

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xnew = [(x(1 : n) + x(2 : n + 1))/2; (x(1) + x(2))/2];  
ynew = [(y(1 : n) + y(2 : n + 1))/2; (y(1) + y(2))/2];  
plot(xnew, ynew)
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then a new polygon is displayed that is obtained by *connecting the side midpoints* of the original polygon.

- We wish to explore what happens when *this process is repeated*.

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