Introduction to Financial Engineering and Algorithms

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What is Financial Engineering All About

- We answer two questions:
  - 1. What is the “correct price” of stock options?
    - (We will learn later in the course what stock options are.)
  - 2. What are optimal investment strategies?

What is Financial Engineering All About

- The answer to the first question, the Black-Scholes pricing formula, earned Robert C. Merton and Myron S. Scholes the 1997 Nobel prize in economics.
- William F. Sharpe and Harry M. Markowitz received the 1990 Nobel prize in economics for answering the second question.

What is the “Correct Price”?

- Let’s start by understanding what an *incorrect price* might be with a very simple example.
- Suppose that a barrel of oil trades at £100 (i.e., one can buy it for £100 and one can sell it for £100).
- I want to start trading in oil, too. Would £120 be a correct price?
  - No! Other traders will buy oil in large numbers for £100, sell it to me for £120. Everyone except me will become rich, I end up with a pile of barrels of oil I cant sell.
- Would £90 be a correct price?

What is the "Correct Price"?

- Consider three merchants who are willing to buy and sell bags containing apples and oranges (all of identical size and quality) as follows:

	Bag content	Price
Merchant 1	3 apples, 2 oranges	£5
Merchant 2	2 apples, 3 oranges	£6
Merchant 3	4 apples, 3 oranges	£8

- Are these prices "correct"?

What is the “Correct Price”?

- Lets get rich:
  - Borrow £36 for a short while (with negligible interest);
  - Buy 6 bags from merchant 1 and buy 1 bag from merchant 2 for a total cost of £36;
  - Rearrange the fruit in five bags of 4 apples and 3 oranges each;
  - Sell the five bags to merchant III for £40;
  - Return the £36 loan and pocket a £4 profit.
  - Repeat this process until you are very rich.
  - Lots and lots of people would be buying from merchants 1 and 2 and selling to merchant 3.
  - When this happens the prices of bags 1 and 2 will rise and those of bag 3 will fall, and this process will continue until there are no more easy profits to be made.

What is the “Correct Price”?

- We didn’t assume an intrinsic or objective price of apples and oranges.
- Correct pricing is an equilibrium price.
- We expose incorrect pricing by exhibiting profit-making trading strategies which change price.

Arbitrage

- The making of a certain profit with no investment is called arbitrage.
- Correct pricing = the unique price which does not introduce arbitrage opportunities.
- The correct price of bag 3 would be £36/5 as that is the unique price for which the strategy above (or its opposite) does no produce a profit.
- Arbitrage opportunities = “free lunches”
- There is no such thing as a free lunch!

Portfolio Theory

- Are there investment strategies which are “better” than others?
- But what do we mean by “better”?
- We are greedy, i.e., we want high returns.
- But average high returns are risky!
- Suppose you are offered to make a bet on the outcome of tossing a fair coin:
  - heads– you lose everything you own, including your shirt and are forced to live on the street;
  - tails– you double your wealth.

Portfolio Theory

- Expected gain = 0, but most people won’t bet.
- People are not only greedy– they also have an aversion towards risk.
- Even if a win triples or quadruples their wealth, most people would not place a bet.
- If we used a coin which produces tails with 99% probability some people might choose to place a bet.
- Now the expected gain may tempt some, but not others.

Portfolio Theory

- There is an interplay between expected returns and risk.
- A "good" way to invest is a strategy that gives you an appropriate return and risk depending on your appetite for risk.
- The last part of the course will make this statement precise:
  - We will see that these optimal investments exist and consist of portfolios of bonds and index trackers.

Financial Markets

- Stock markets.
- Bond markets.
- Currency markets or foreign exchange markets.
- Commodity markets.
- Futures and options markets.

Financial Economics: Three Prongs

- Market Efficiency
  - Modeling how prices evolve in (near) efficient markets.
  - e.g., quantifying mismatch risk; probability of market crashes.
- Asset Pricing
  - Factors that drive individual security prices.
  - e.g., comparative assessment of growth versus value indicators; pricing anomalies.
- Corporate Finance
  - Optimum financial management of companies.
  - e.g., capital structure; dividend policy; pension fund investment.

The Math of Finance

- Mathematical finance is the branch of applied mathematics concerned with the financial markets.
- The subject has a close relationship with the discipline of financial economics, which is concerned with much of the underlying theory.
- Generally, mathematical finance will derive, and extend, the mathematical or numerical models suggested by financial economics.

The Math of Finance

- Mathematical finance is not about predicting the price of a stock. What it is about is figuring out the price of options and derivatives.
- In calculus, a derivative gives you a measure of the rate of change of a dependent variable as an independent variable is changed.
- In finance, an option is an example of a derivative, any financial instrument whose value is derived from that of an underlying security.

Mathematical Finance

- Mathematical finance lies at the intersection of
  - Applied Probability
  - Partial Differential Equations
  - Stochastic Differential Equations
  - Economics
  - Statistics
  - Numerical Analysis

Financial Engineering

- Financial engineering contains
  - Mathematical finance
  - Computer algorithms
  - Data structures
  - Numerical analysis

What is a security (證券)?

- A security is a fungible (可互換), negotiable (可磋商修改的) instrument representing financial value.
- Securities are broadly categorized into debt and equity securities such as bonds and common stocks, respectively.

Equity (股票) and Debt (債務)

- Traditionally, securities are divided into debt securities and equity.

Debt

- Debt securities may be called debentures, bonds, notes or commercial paper depending on their maturity and certain other characteristics.
- The holder of a debt security is typically entitled to the payment of principal and interest, together with other contractual rights under the terms of the issue, such as the right to receive certain information.
- Debt securities are generally issued for a fixed term and redeemable by the issuer at the end of that term.

Equity

- An equity security is a share in the capital stock of a company (typically common stock, although preferred equity is also a form of capital stock).
- The holder of an equity is a shareholder, owning a share, or fractional part of the issuer.
  - Unlike debt securities, which typically require regular payments (interest) to the holder, equity securities are not entitled to any payment.
- •Equity also enjoys the right to profits and capital gain.

Structure of Financial Markets

- Debt Markets
  - Short-Term (maturity < 1 year)
  - Long-Term (maturity > 10 year)
  - Intermediate term (maturity in-between)
  - Represented $41 trillion at the end of 2007.
- Equity Markets
  - Pay dividends, in theory forever.
  - Represents an ownership claim in the firm.
  - Total value of all U.S. equity was $18 trillion at the end of 2005.

Structure of Financial Markets

- Primary Market
  - New security issues sold to initial buyers.
  - Typically involves an investment bank who underwrites the offering.
- Secondary Market
  - Securities previously issued are bought and sold.
  - Examples include the NYSE and NASDAQ.
  - Provide liquidity, making it easy to buy and sell the securities of the companies.
  - Establish a price for the securities.

Structure of Financial Markets

- We can further classify secondary markets as follows:
- Exchanges
  - Trades conducted in central locations (e.g., New York Stock Exchange, CBT)
- Over-the-Counter Markets (OTC)
  - Dealers at different locations buy and sell.
  - Best example is the market for Treasury securities.

Financial Derivatives

- ***Practitioners' Definition****:* Derivative securities are financial contracts that 'derive' their value from cash market instruments such as stocks, bonds, currencies and commodities.
- ***Academic Definition****:* A financial contract is derivative security or contingent claim if its value at expiration date T is determined by the market price of the underlying cash instrument at time T.

Types of Derivative

- Options (選擇權)
- Futures (期貨) and forwards (遠期契約)
- Swaps (交換合約)

- Options, forwards, and futures are basic building blocks.
- Swaps can eventually be decomposed into sets of basic forwards and options.

Course Topics

- Tentative List:
  - Risk-Free Assets
  - Risky Assets.
  - Discrete Time Market Models.
  - Portfolio Management.
  - Forward and Futures Contracts.
  - General Properties of Options.
  - Algorithms for Finance.
  - Variable and Stochastic Interest Rates.

Course Info

- Textbook:
  - Marek Capiński and Tomasz Zastawniak, Mathematics for Finance – An Introduction to Financial Engineering, 2nd Ed., Springer, 2010. (ISBN: 978-0-85729-081-6) (科大)
  - John Hull, Options Futures and Other Derivatives , 8th Ed., Pearson College Div, 2011. (ISBN: 0132164949)
    - 彭崑亮 (02)2701-7353 (有中文版)
- Course Website
  - http://140.121.198.238/wwyhsu/?page_id=359
- Grading (tentative):
  - 70% Homework assignments. (4-6 assignments, including written and programming)
  - 30% Midterm examination.

The Simple Market Model

100 Trillion Dollars!

Simple Market Model

- A UK company is preparing to purchase a piece of equipment in the US for $100,000 in a year's time.
- The price is guaranteed by the producer to remain unchanged.
- Considering the current exchange rate of $1.62 to £1, the manager of the company has reserved £64,000 in the budget to become available at the time of purchase.

- Analyze this decision.

Simple Market Model

- Risk-free and risky security.
  - Risk-free assets: bank deposit, government issued bond, financial institution bond, stable healthy company bond.
  - Risky assets: stocks, foreign currency, gold.
- Time of valuation (t).
  - t = 0 denotes today, current time.
  - t = T denotes some time T in the future.
- The price of one share of a security S is S(t).
  - S(0) = current price, known to all investors.
  - S(T) = some price in the future unknown to all.
    - It can go up or down.
  - The return of the asset is written as: $K_s = \frac{S(T)-S(0)}{S(0)}$

Simple Market Model

- Dynamics of the stock price?
  - How do we model the ups and downs of stock price?
- Our task is to build a mathematical model of a market of financial securities.
  - Compromise between the complexity of the real world and the limitations and simplifications of a mathematical model.
  - To make the problem tractable.
- All stock and bond prices must be strictly positive.
  - $S(t) > 0$ for $t \geq 0$.
- Portfolio is the sum of all assets held.

Example of Simple Market Model Pricing

- If a bond has a value of $S(0) = 100$ and $S(T) = 110$.
  - The return of this investment is 10%.
- If a stock has a value of $S(0) = 50$. At time T, it can take two values:
  - $S(t) = \begin{cases} 52, & \text{with probability } p \\ 48, & \text{with probability } 1-p \end{cases}$
  - For some $0 < p < 1$, the return on the stock will then be
  - $K_s = \begin{cases} 4\% & \text{if the stock goes up} \\ -4\% & \text{if the stock goes d}own \end{cases}$

Terms

- Liquidity: any asset can be bought or sold on demand at the market price in arbitrary quantities.
- Long position: the number of securities of a particular kind held In a portfolio is positive.
- Short position (short selling): investor borrows the stock, sells it, and uses the proceeds to make some other investment.
  - Investors must have sufficient resource to buy the stock back and return it to the owner (closing the short position).

No-Arbitrage Principle

- No arbitrage assumption: the market does not allow risk-free profits with no initial investment.
- Only happens when some market participants make a mistake.

Arbitrage Example

- Suppose dealer A in New York offers to buy British pounds at a rate of $1.62 to £1.
- Suppose dealer B in London sells British pounds at £1 to $1.60.
- Free money!
  - Short position from dealer B and long position to dealer A.
  - Demand for their generous services would quickly compel the dealers to adjust the exchange rate.

Find the Risk Free Profit

Dealer A	Buy	Sell
€1.0000	$1.0202	$1.0284
£1.0000	$1.5718	$1.5844

Dealer B	Buy	Sell
€1.0000	£0.6324	£0.6401
$1.0000	£0.6299	£0.6375

Another Example

- Dealer A in New York offers to buy British pounds a year from now at a rate of $1.58 for £1.
- Dealer B in London would sell British pounds immediately at a rate of £1 for $1.60.
- Suppose the dollar can be borrowed at an annual rate of 4% and British pound can be invested in a bank account at 6%.
  - Borrow $10,000 and convert to £6,250. (Dealer B)
  - Deposit into bank account for 1 year gaining £375 interest.
  - Convert £6,250+ £375 to $10,467.50. (Dealer A)
  - Repay the loan $10,000+4% of $10,400.
  - $67.50 profit!

Free Bonus are Never Free

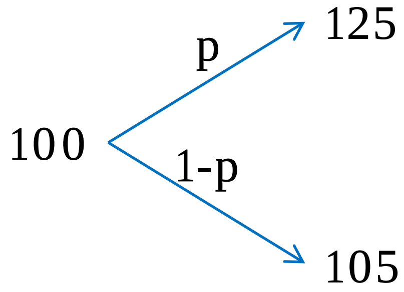
- Premium is already included in the price of the goods.
- Scratch cards as an additional free bonus from purchases.
- Lottery tickets are never free, even the change of winning is very slim.

One-Step Binomial Model

- Suppose that S(0)=100 and S(T) can take two values

$$S(t) = \begin{cases} 125, & \text{with probability } p \\ 105, & \text{with probability } 1 - p \end{cases}$$

- The probability p lies within $0 < p < 1$.
- Bond price A(0)=0 and A(T)=110, giving 10% return.



- We call the price movements going up or going down.
  - Respective to each other (although both prices are higher than the initial price).

No-Arbitrage Principle Binomial Model

- $S(t) = \begin{cases} S^u(T), & \text{with probability } p \\ S^d(T), & \text{with probability } 1-p \end{cases}$
- $S^d(T) < S^u(T)$ and $0 < p < 1$.
- The restriction $\frac{S^d(T)}{S(0)} < \frac{A(T)}{A(0)} < \frac{S^u(T)}{S(0)}$ has to be imposed to prevent arbitrage.

Proof

- Suppose $\frac{A(T)}{A(0)} \leq \frac{S^d(T)}{S(0)}$
  - Borrow the amount S(0) risk free.
  - Buy one share of stock for S(0).
- Holding a portfolio of 1 share of stock and $-\frac{S(0)}{A(0)}$ bonds.
- Thus, V(0)=0 and

$$V(t) = \begin{cases} S^u(T) - \dfrac{S(0)}{A(0)} A(T), & \text{with probability } p \\ S^d(T) - \dfrac{S(0)}{A(0)} A(T), & \text{with probability } 1-p \end{cases}$$

- $V(t) \geq 0$, violating the no arbitrage principle.

Proof

- Suppose $\frac{A(T)}{A(0)} \geq \frac{S^u(T)}{S(0)}$
  - Short sell one share of stock for S(0).
  - Invest the amount S(0) risk free.
- Holding a portfolio of -1 share of stock and $\frac{S(0)}{A(0)}$ bonds.
- Thus, V(0)=0 and

$$V(t) = \begin{cases} -S^u(T) + \frac{S(0)}{A(0)}A(T), & \text{with probability } p \\ -S^d(T) + \frac{S(0)}{A(0)}A(T), & \text{with probability } 1-p \end{cases}$$

- $V(t) \geq 0$, violating the no arbitrage principle.

Risks and Return

- Let A(0)=100, A(T)=110, and S(0)=80 dollars.

$$S(T) = \begin{cases} 100, & \text{with probability } 0.8 \\ 60, & \text{with probability } 0.2 \end{cases}$$

- Suppose we invest $10,000 in 50 shares of S and 60 shares of A.

$$V(T) = \begin{cases} 11600, & \text{if the stock goes up} \\ 9600, & \text{if the stock goes down} \end{cases}$$

$$K_V = \begin{cases} 16\%, & \text{if the stock goes up} \\ -4\%, & \text{if the stock goes down} \end{cases}$$

- Expected return:

$$E[K_V] = 16\% * 0.8 + (-4\%) * 0.2 = 12\%$$

Risks and Return

- The risk of the investment is defined to be the standard deviation of the random variable K_V.
- $\sigma_V = \sqrt{(16\% - 12\%)^2 * 0.8 + (-4\% - 12\%)^2 * 0.2} = 8\%$

- If we invest all in A (which is risk free), the expected return is $E[K_V] = 10\%$, the risk is $\sigma_V = 0\%$.

- If we invest 125 shares of S, the expected return and risk is

$$E[K_V] = 25\% * 0.8 + (-25\%) * 0.2 = 15\%$$

$$\sigma_V = \sqrt{(25\% - 15\%)^2 * 0.8 + (-25\% - 15\%)^2 * 0.2} = 20\%$$

Risks and Return

- Given the choice between portfolios with same expected returns, any investor would definitely prefer the one involving lower risk.
- Give the same level of risks, investors will prefer choosing one with higher returns.
- In general, higher expected return is usually associated with higher risks.

Exercise

- Using the above portfolios A and S, invest 50% in each and compute the expected return and risk.

Forward Contracts

- A forward contract is an agreement to buy or sell a risky asset at a specified future time.
  - The time is called delivery date.
  - The price F fixed at this present moment is called the forward price.
- The buyer takes (enters) a long forward position, the seller takes a short forward position.
- No money is at the time when a forward contract is exchanged.

Forward Contracts

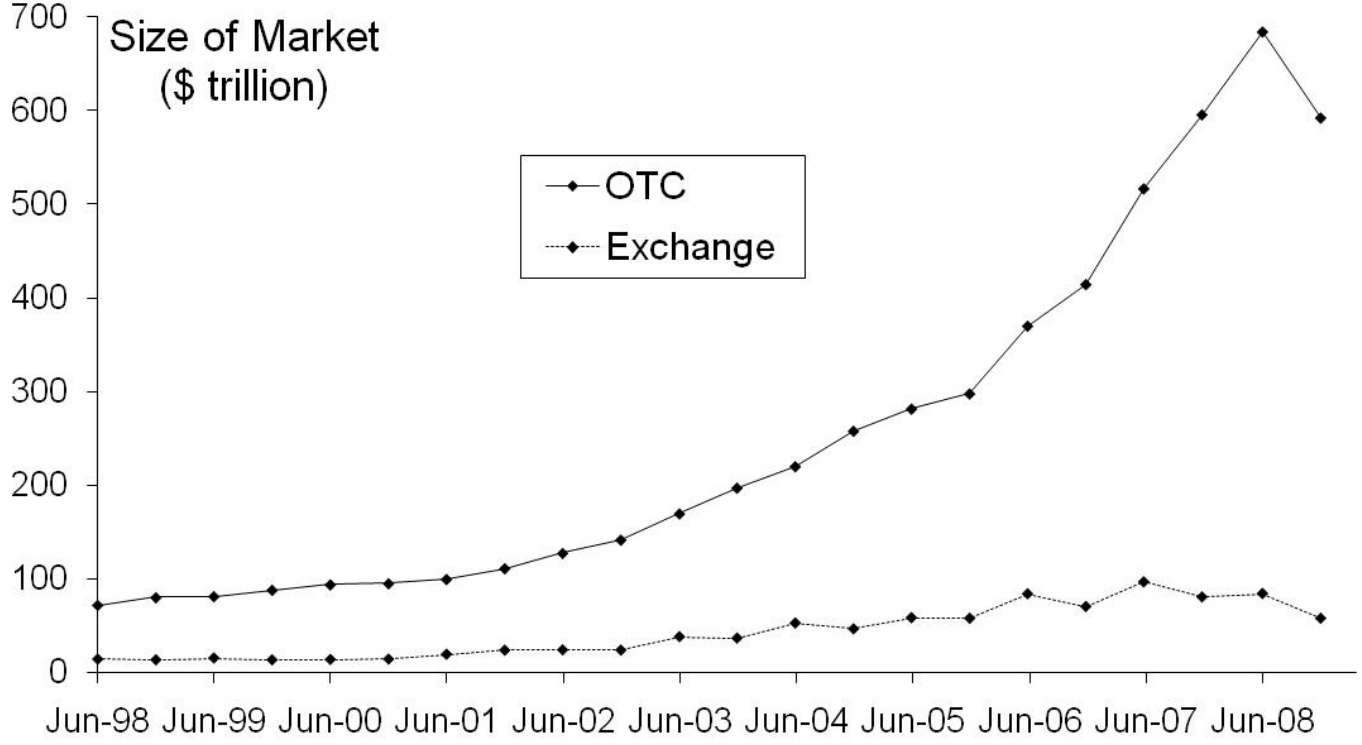
- Forward contracts are similar to futures except that they are traded in the over-the-counter market (see the next two slides)
- Forward contracts are popular on foreign currencies and interest rates
- Foreign currency quotes for GBP (in USD)

July 17, 2009	Bid	Offer (Asked)
Spot (現貨)	1.6382	1.6386
1-month forward	1.6380	1.6385
3-month forward	1.6378	1.6384
6-month forward	1.6376	1.6383

Forward Contracts

- The over-the-counter (OTC) market is an important alternative to exchanges.
  - It is a telephone and computer-linked network of dealers who do not physically meet.
  - The dealers, also known as the market makers, are always prepared to quote both a bid price (at which they are prepared to buy) and an asked price (at which they are prepared to sell).
  - Trades are usually between financial institutions, corporate treasurers, and fund managers.
  - The contracts can be custom-made.
  - Default (違約) risk: the contract could not be honored.

Size of OTC and Exchange-Traded Derivatives Markets (1998-2008)



Example

- Suppose the forward price of iPhone is $22,000NTD.
- If the market price (the spot price) turns out to be $23,500NTD on delivery date, the long forward contract holder will gain $1,500NTD by buying the iPhone for $22,000NTD.
- If the market price turns out to be $20,000NTD, the holder of the long forward contract will lose $2,000NTD for buying the iPhone for $22,000NTD.

Forward Contracts

- In general, the party holding a long forward contract with delivery date T will benefit if the future asset price $S(T)$ rises above the forward price.

Call Options

- Let A(0)=100, A(T)=110, S(0)=100 and

$$S(T)=\begin{cases}120, & \text{with probability } p\\ 80, & \text{with probability } 1-p\end{cases}$$

  - $0 < p < 1$.
- A call option with strike price \$100 and exercise time (maturity date) T is a contract giving the holder the right (but no obligation) to purchase a share of stock for \$100 at time T.
  - If the stock price falls below the strike price, the option would be worthless.
  - If the stock price rises to \$120 at time T, the option will bring a profit of \$20.
  - Exercise the option: But the stock for \$100 and sell it to the market immediately for \$120.

Call Options

- The payoff of a call option at value time T is a random variable

$$C(T) = \begin{cases} 20, & \text{if stock goes up} \\ 0, & \text{if stock goes down} \end{cases}$$

- In general,

$$C(T) = \max(S(T) - X, 0)$$

  - Sometimes denoted as $C(T) = (S(T) - \mathrm{X})^+$
  - X denotes the strike price.
  - $C(0)$ denote the value of the option at time 0 (current time).

Pricing the Call Option

- Construct an investment in x stocks S and y bonds such that the value of the investment at time T is the same as that of the option:

$$xS(T) + yA(T) = C(T)$$

- This step is called replicating the option.

- Compute the time 0 value of the investment. By no arbitrage principle,

$$xS(0) + yA(0) = C(0)$$

- This step is called pricing or valuing the option.

Replicating the Option

- $C(T) = xS(T) + yA(T) = \begin{cases} 120x + 110y \\ 80x + 110y \end{cases}$
- $\begin{cases} 120x + 110y = 20 \\ 80x + 110y = 0 \end{cases}$
- Solving the equation we get $x = \frac{1}{2}$ and $y = -\frac{4}{11}$.
- To replicate the option C, we need to buy $\frac{1}{2}$ shares of stock and take a short position of $-\frac{4}{11}$ in bonds.
  - Borrowing $\frac{4}{11} * 100 = \frac{400}{11}$ dollars.

Pricing the Option

- We can compute the value of the investment in stock and bonds at time 0:

$$C(T) = xS(0) + yA(0) = \frac{1}{2} * 100 - \frac{4}{11} * 100 \cong 13.6364$$

- The option price is 13.6364.

Put Options

- Let A(0)=100, A(T)=110, S(0)=100 and
$$S(T) = \begin{cases} 120, & \text{with probability } p \\ 80, & \text{with probability } 1-p \end{cases}$$
  - $0 < p < 1.$
- A put option with strike price $100 and exercise time T is a contract giving the holder the right (but no obligation) to sell a share of stock for $100 at time T.
  - If the stock price falls below the strike price, the option would bring profit..
  - If the stock price rises above the strike price, the option would become worthless.

Put Options

- The payoff of a call option at value time T is a random variable

$$P(T) = \begin{cases} 0, & \text{if stock goes up} \\ 20, & \text{if stock goes down} \end{cases}$$

- In general,

$$P(T) = \max(X - S(T), 0)$$

  - Sometimes denoted as $P(T) = \left(X - S(T)\right)^{+}$
  - X denotes the strike price.
  - $P(0)$ denote the value of the option at time 0 (current time).

European vs. American Options

- A European option can be exercised only at maturity
- An American option can be exercised at any time during its life.

Google Option Prices (July 17, 2009; Stock Price is 430.25)

	Calls			Puts		
Strike price ($)	Aug 2009	Sept 2009	Dec 2009	Aug 2009	Sept 2009	Dec 2009
380	51.55	54.60	65.00	1.52	4.40	15.00
400	34.10	38.30	51.25	4.05	8.30	21.15
420	19.60	24.80	39.05	9.55	14.70	28.70
440	9.25	14.45	28.75	19.20	24.25	38.35
460	3.55	7.45	20.40	33.50	37.20	49.90
480	1.12	3.40	13.75	51.10	53.10	63.40

- Options prices (or option premiums) are the prices to buy or sell options (not the price to trade the underlying asset).
- On July 17^{th}, an investor pays $28.75 to buy a call option, which gives the investor the right to purchase one share of Google at $440 in Dec. 2009.
- For calls (puts), there are more valuable for lower (higher) strike prices.
- In general, the longer time to maturity implies higher option prices.

Exchanges Trading Options

- Chicago Board Options Exchange (CBOE) (in the U.S.)
  - The first and largest exchange in the world for trading stock options
  - The stock options traded on the CBOE, e.g., the Google example on slide 1.21, are all American-style options
- Pacific Exchange (in the U.S.)
  - Began trading options in San Francisco in 1976 and was merged with NYSE in 2006
- Philadelphia Stock Exchange (in the U.S.)
  - Oldest stock exchange founded in 1790 in the U.S.
  - The pioneer to trade foreign exchange options
  - Merged with NASDAQ in 2007

Exchanges Trading Options

- NYSE Euronext (in the U.S.)
- International Securities Exchange (in the U.S.)
  - Launched as the first fully electronic US options exchange
  - Offers equity, index, and foreign exchange options
- Eurex (in Germany and Swissland)

- Most exchanges offering futures contracts also offer options on these contracts, e.g., the CME group also offers options on corn futures.
- The OTC options market grew rapidly since 1980s and is now bigger than the exchange options market.
- The advantage of the OTC market is the tailored-made option contracts to meet special needs.

Options vs. Futures

- An option gives the holder the right to buy or sell the underlying asset at a certain price.
  - A futures/forward contract gives the holder both the right and the obligation to buy or sell the underlying asset at a certain price.
- For options, in addition to the dimension of different maturities, there are a series of strike prices for each maturity date that traders can choose.
  - Futures/forwards consider only the dimension of different maturities.

Foreign Exchange

- Foreign currency is a type of risky security.
- Investing foreign currency in a bank account can generate extra risk-free income.

Managing Risk with Options

- Suppose S(0)=100, X=100, and

$$S(T) = \begin{cases} 120, & \text{with probability } p \\ 80, & \text{with probability } 1-p \end{cases}$$

- If our initial wealth is \$1000, we can buy 10 shares of S, at time T they will be worth

$$S(T) = \begin{cases} 1200, & \text{if stock goes up} \\ 800, & \text{if stock goes down} \end{cases}$$

- If we buy \$1000/13.6364=73.3333 call options, then

$$73.3333 \times C(T) = \begin{cases} 1466.67, & \text{if stock goes up} \\ 0.00, & \text{if stock goes down} \end{cases}$$

  - Did not consider the cost of premium.

Managing Risk with Options

- If stock goes up, investing call options give 46.67% and investing stock gives 20%.
- If stock goes down, investing call options give -100% and investing stock gives -20%.

Managing Risk with Options

- Options can be employed to reduce risk.
- Consider an investor planning to purchase stock in the future, the current stock price S(0)=100.
  - Funds are available at a future time T.

$$S(T) = \begin{cases} 160, & \text{with probability } p \\ 40, & \text{with probability } 1-p \end{cases}$$

- Assume the bond price A(0)=100 and A(T)=110.

Strategies

- Wait until time T and the funds become available, purchase the stock for S(T).

- At time 0, borrow money to buy a call option with strike price $100.
- At time T, repay the loan with interest and purchase the stock, in addition, exercise the option if the stock price goes up.

Strategy

- The first strategy puts the investor in considerable risk.

- The second strategy, investor will borrow $C(0) \cong 31.8182$ to pay for the option (Solved by replication portfolio).
  - At time T, pay $35 to clear the loan.
  - Use the option to buy the stock.
- The cost of purchasing one share will be

$$S(T) - C(T) + 35 = \begin{cases} 135, & \text{if stock goes up} \\ 75, & \text{if stock goes down} \end{cases}$$

- Risk is reduced as the spread is narrowed.

Types of Traders

- Hedgers (避險者): use derivatives to reduce the risk that they face from potential future movements in a market variable
- Speculators (投機者): use derivatives to bet on the future direction of a market variable (Note that the leverage effect of derivatives can amplify the gains or losses in returns)
- Arbitrageurs (套利者): take offsetting positions in two or more instruments to lock in profit

- The participation of these three types of traders provide the great liquidity for derivatives markets

Types of Traders

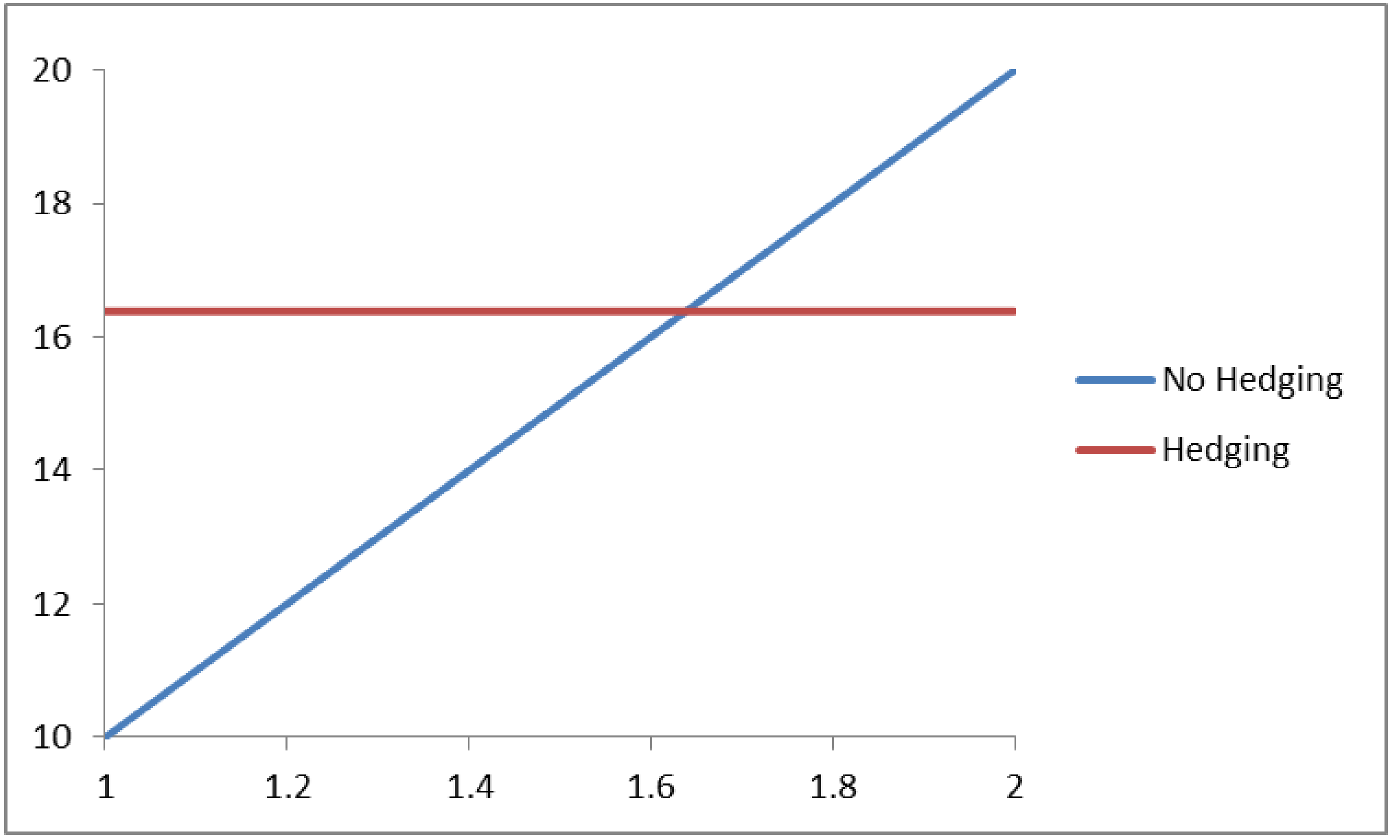
- For most financial institutions:
  - The need to hedge the interest rate and foreign exchange risk → hedgers.
  - To earn trading profit with speculation → speculators.
  - To earn trading profit with arbitrage → arbitrageurs.

- Some of the largest trading losses in derivatives have occurred because a trader who had a mandate to be hedgers or arbitrageurs switched to being speculators

Hedging Examples

- Using forward:
  - A US company will pay £10 million for imports from Britain in 3 months and decides to hedge the foreign exchange risk using a long position in a forward contract (with the delivery price to be $1.6384/GBP).

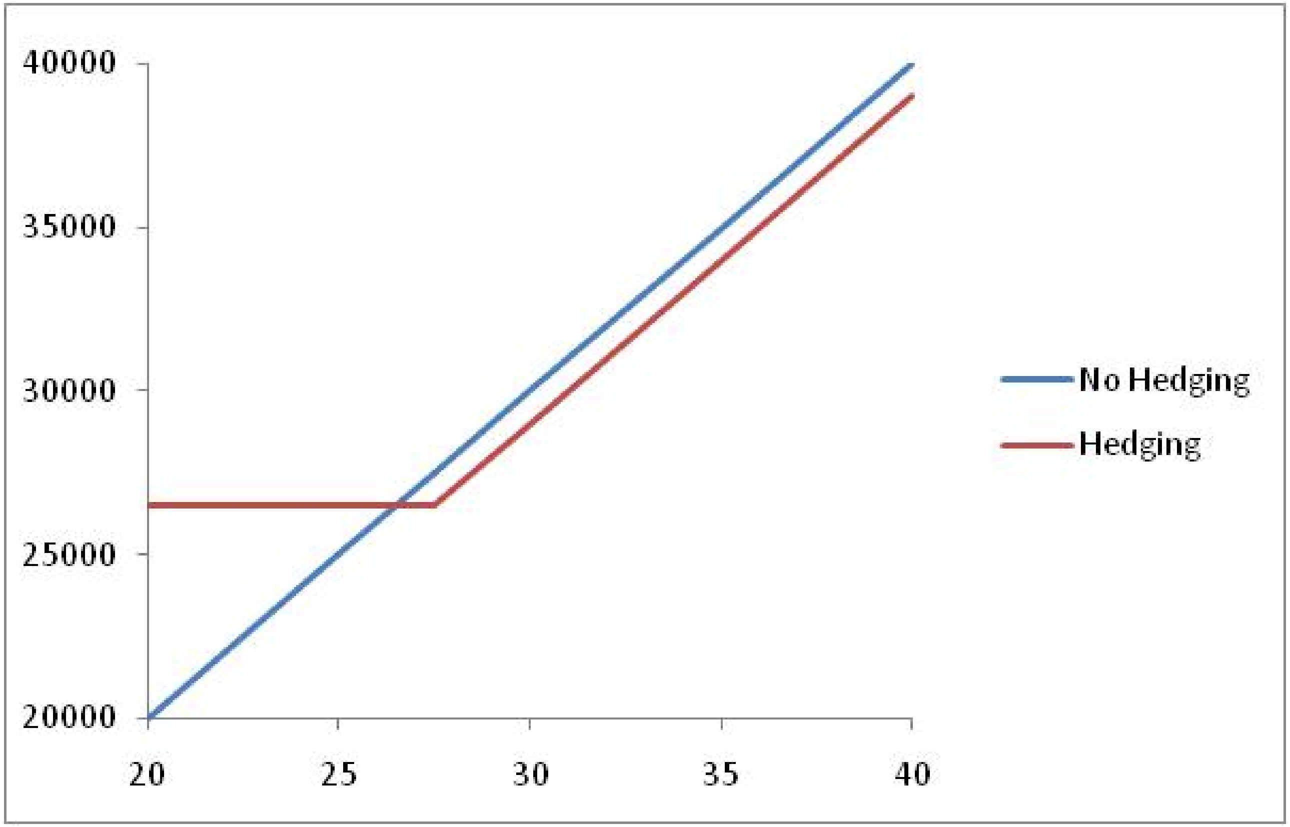
Hedging Examples



Hedging Examples

- Using options:
  - An investor owns 1,000 Microsoft shares currently worth $28 per share.
  - A two-month put with a strike price of $27.50 costs $1.
  - The investor decides to hedge by buying 10 contracts, each of which can sell 100 shares.

Hedging Examples



Speculation Example

- Using futures:
  - In February, an investor thinks that the GBP will strengthen relative to the USD over the next two months.
  - He considers to purchase £250,000 in the spot market or to take a long position in futures contracts on GBP, each of which is for the purchase of £62,500 with the delivery price 1.6410 in April and with the margin requirement to be $5,000.

	Possible Trade	
	Buy £250,000 *Spot price = 1.6470*	*Buy 4 futures contracts* *Futures price = 1.6410*
Investment	$411,750	$20,000
Profit if April spot = 1.9000	$13,250	$14,750
Profit if April spot = 1.8000	–$11,750	–$10,250

Speculation Example

- Using options:
  - An investor with $2,000 to invest feels that a stock price will increase over the next 2 months.
  - The current stock price is $20 and the price of a 2-month call option with a strike of $22.50 is $1.

	December stock price	
Investor's strategy	*$15*	*$27*
Buy 100 shares	–$500	$700
Buy 2,000 call options	–$2,000	$7,000

Speculation Example

- Note that the cost to purchase options is a sunk cost, which cannot be recovered even if the investor does not exercise options.
- With these call options, the downside risk is the losses of $2,000, but the upside gains could be infinitely large.
- The leverage effect amplifies the returns in gains and losses generally.

Arbitrage Example for Spots

- A stock price is quoted as £100 in London and $162 in New York. In addition, the current exchange rate is known to be 1.6500.
- The strategy to arbitrage from the distortion:
  - Buy stock shares at $162/share in New York and sell stock shares in London at £100/share, which is equivalent to $165/share → earn $3/share.
- The arbitrage opportunity disappears quickly
  - The buying behavior bids the share price in NY up and the selling behavior drive the share price in London down until the price parity holds again.

Arbitrage Example for Spot and Futures without Storage Costs

- Suppose that:
  - The spot price of gold is US$1,000.
  - The quoted 1-year futures price of gold is US$1,100.
  - The 1-year US$ interest rate is 5% per annum.
  - No income or storage costs for gold.
- The arbitrage strategy
  - Buy the gold spot with the borrowed fund and enter a short position of selling gold after 1 year.
  - The cost to acquire the gold spot is $\$1,000 \times (1+5\%) = \$1,050$, and the payoff to sell gold via futures is $1,100 → earn $50.

Arbitrage Example for Spot and Futures without Storage Costs

- Suppose that:
  - The spot price of gold is US$1,000.
  - The quoted 1-year futures price of gold is US$990.
  - The 1-year US$ interest rate is 5% per annum.
  - No income or storage costs for gold.
- The arbitrage strategy
  - Short sell the gold spot, deposit the proceeds, and enter a long position of buying gold after 1 year.
  - The FV of the proceeds of selling the gold spot is $\$1,000\times(1+5\%)=\$1,050$, and the cost to buy gold back via futures is \$990 $\rightarrow$ earn \$60 (note that the purchased gold should be returned to the lender).

Theoretical Futures Price without Storage Costs

- If the spot price of gold is S and the futures price for a contract deliverable in T years is F, then the following relationship should hold

$$F = S(1 + r)^T$$

where r is the 1-year (domestic currency) risk-free rate of interest

- In the previous examples, S =1000, T=1, and r=0.05, so that the theoretical futures price is

$$F = 1000(1 + 0.05)^1 = 1050$$

Arbitrage Example for Spot and Futures with Storage Costs

- Suppose that:
  - The spot price of oil is US$70.
  - The quoted 1-year futures price of oil is US$80.
  - The 1-year US$ interest rate is 5% per annum.
  - The storage costs of oil are 2% per annum.
- The arbitrage strategy
  - Buy the oil spot with the borrowed fund and enter a short position of selling oil after 1 year.
  - The cost to acquire and store the oil is $\$70 \times (1+5\%+2\%) = \$74.9$, and the payoff to sell oil via futures is $80 → earn $5.1.

Arbitrage Example for Spot and Futures with Storage Costs

- Suppose that:
  - The spot price of oil is US$70.
  - The quoted 1-year futures price of oil is US$65.
  - The 1-year US$ interest rate is 5% per annum.
  - The storage costs of oil are 2% per annum.
- The arbitrage strategy
  - (Short) sell the oil spot, deposit the proceeds, and enter a long position of buying oil after 1 year.
  - The sum of the FV of the proceeds of selling the oil spot and the save of the storage cost is $\$70\times(1+5\%+2\%)=\$74.9$, and the cost to buy oil back via futures is $65 → earn $9.9.

Theoretical Futures Price with Storage Costs

- If the spot price of gold is S and the futures price for a contract deliverable in T years is F, then the following relationship should hold
$$F = S(1 + r + \text{storage cost})^T$$
where r is the 1-year (domestic currency) risk-free rate of interest.
- In the previous examples, S =70, T=1, r=0.05, and the storage cost is 2%, so that the theoretical futures price is
$$\$70\times(1+5\%+2\%)=\$74.9$$

Ways Derivatives are Used

- To hedge risks.
- To speculate (take a view on the future direction of the market).
- To lock in an arbitrage profit.
- To change the nature of a liability or an investment with swaps
  - Interest rate swaps: from floating to fixed interest rates debt.
  - Equity swaps: exchange equity returns with LIBOR plus a spread.

Case 1: Discussion

- If the exchange rate of USD to GWP falls below 1.5625, the funds will be insufficient.
- The company must hedge the interest rate risk.

First Approach – Using Forwards

- Forward contracts are used to convert £ into $ at the end of the year.
- Suppose the risk-free returns of bonds in £ and $ turns out to be $K_{\$} = 5\%$ and $K_{£} = 3\%$.
- Forward price F of £ is

$$F = S(0)\frac{1+K_{\$}}{1+K_{£}},\ 1 + K_x = \frac{A_h(T)}{A_h(0)}$$

- Suppose that F=1.6515, £64,000 can be converted to $105,693.20.
  - Equipment can be purchased with an $5,693.20 surplus.
  - Exchange rate will not affect this result.

Second Approach – Put Options

- Suppose the spot rate S(0)=1.62 dollars to a pound.
- Suppose the risk-free returns of bonds is $K_{\$} = 5\%$ and $K_{£} = 3\%$.
- Two future rates $S^u(1)$ and $S^d(1)$ needs to be calibrated to match market data.
- We must used traded option on the market to do so.
- Suppose that the calls with strike prices of 1.6 and 1.7 dollars to a pound is sold at 1.1178 and 0.0624 dollars pound are most liquid.
  - We can solve $S^u(1) \cong 1.8126$ and $S^d(1) \cong 1.4273$ dollars to a pound. (Refer to textbook)

Second Approach – Put Options

- We need to determine the strike price X.
  - This will affect the option price $P(X)$.
  - We need to purchase 64,000 puts to convert £64,000 to $.
  - This will cost $64{,}000 \times P(X)$ dollars. (Borrow!)
  - Suppose the interest rate of loans is 10% for US dollar.
- At the end of the year, we need to repay $P(X) \times 64{,}000 \times 1.1$ dollars in addition the price of the equipment.
- The equation for the strike price is

$$100{,}000 + P(X) \times 64{,}000 \times 1.1 = 64{,}000 \times X$$

- Suppose the ideal solution is $X = 1.6680$ pounds to a dollar, and this put option is sold at $P(X) = 0.0959$ dollars.

Second Approach – Put Options

- We borrow $P(X) \times 64{,}000 \cong 6135.32$ dollars to purchase the put option.
- We need to repay $P(X) \times 64{,}000 \times 1.1 = 6748.85$. (Including interest)
- At the end of the year,
  - If the exchange rate goes up, we sell £ at the market price.
  - Additional profit $64{,}000 \times 1.8126 - 106{,}748.85 \cong 9259.81$
  - If the exchange rate goes down, we exercise the option and break even.