

# Concepts of Stereo Vision

# Stereo Vision



# Virtual Reality

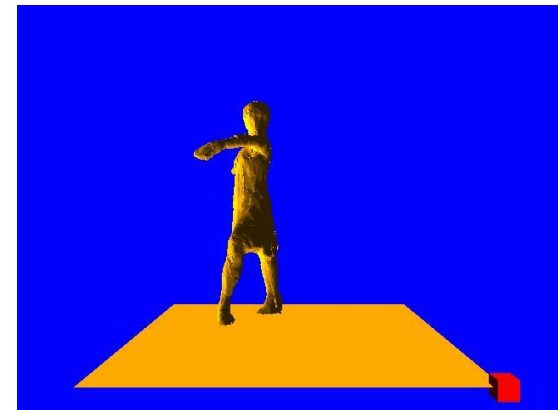
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[Takeo Kanade *et al.*, CMU]

- More than 50 video sequences
- Reconstruct 3D objects from videos

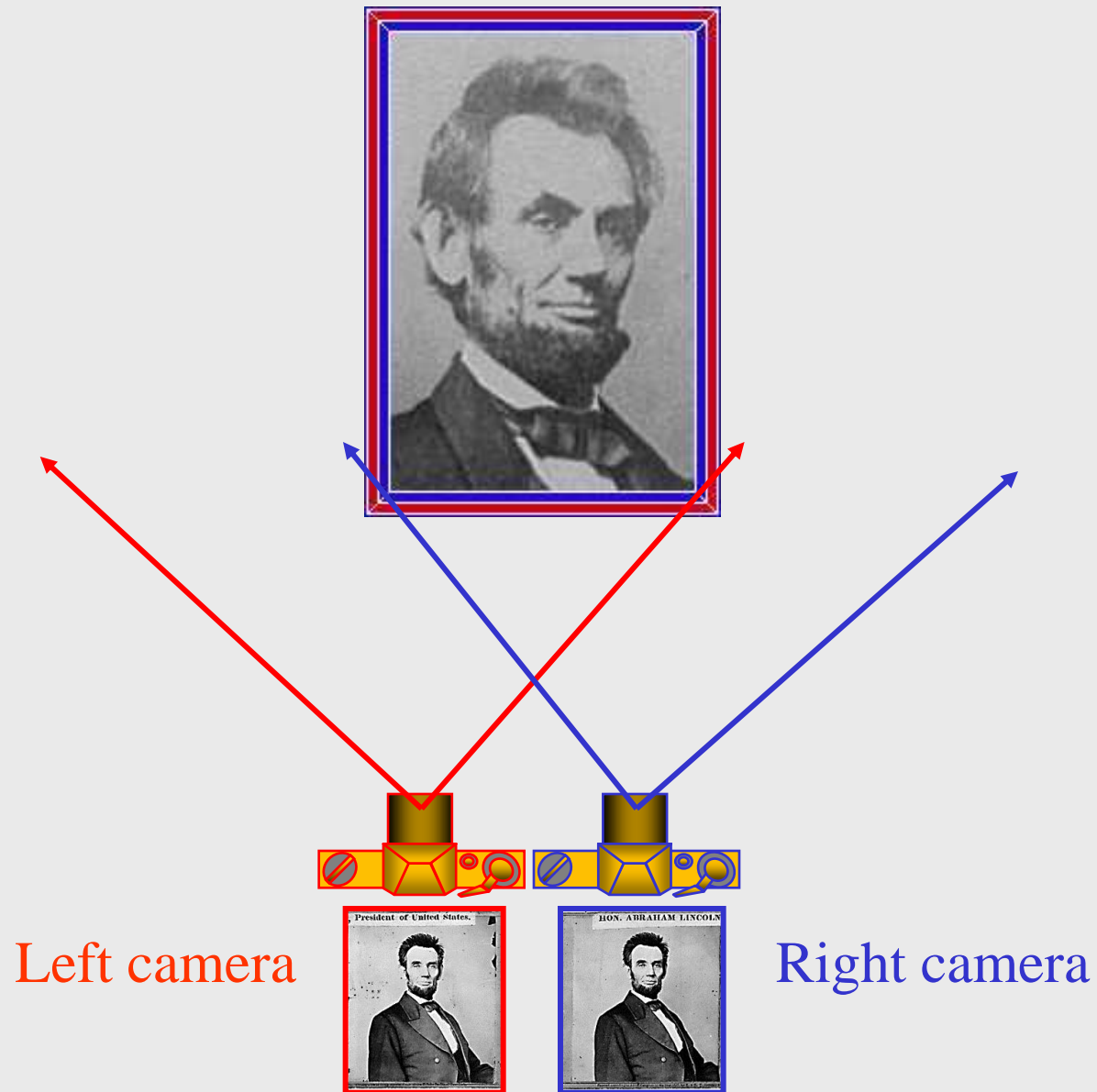


- steerable version used for SuperBowl XXV “eye vision”

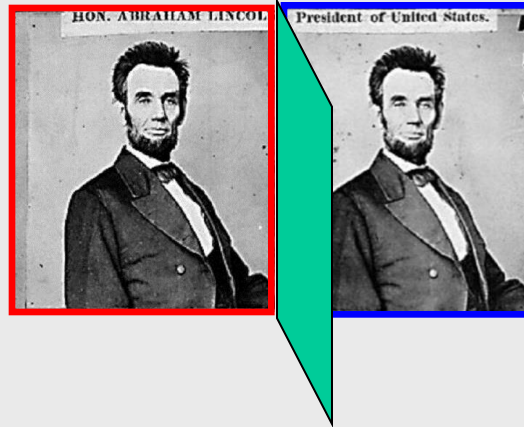


<http://www.cs.cmu.edu/afs/cs/project/VirtualizedR/www/VirtualizedR.html>

# Example of stereo vision



# How to display different images to two eyes ?



Methods:

1) Color Filter Glasses (濾光眼鏡)



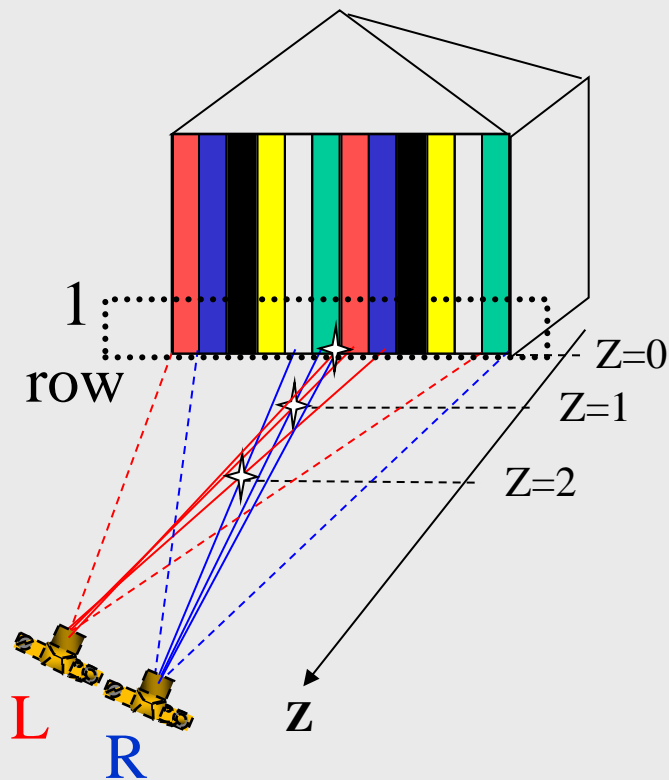
2) Polarized glasses (偏光眼鏡)



3) Synchronization between the screen and glasses

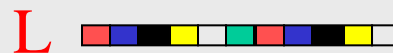


# Parallax and disparity(視差與位差)



Parallax (3D)

Original images



Aligned images



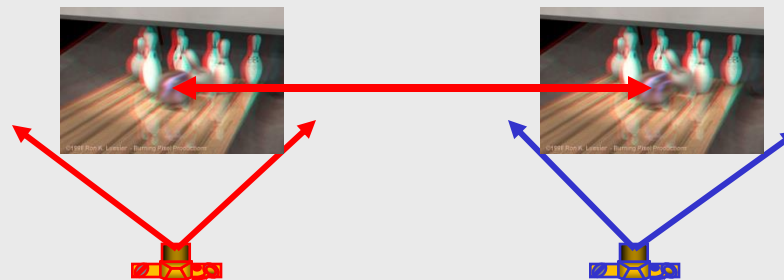
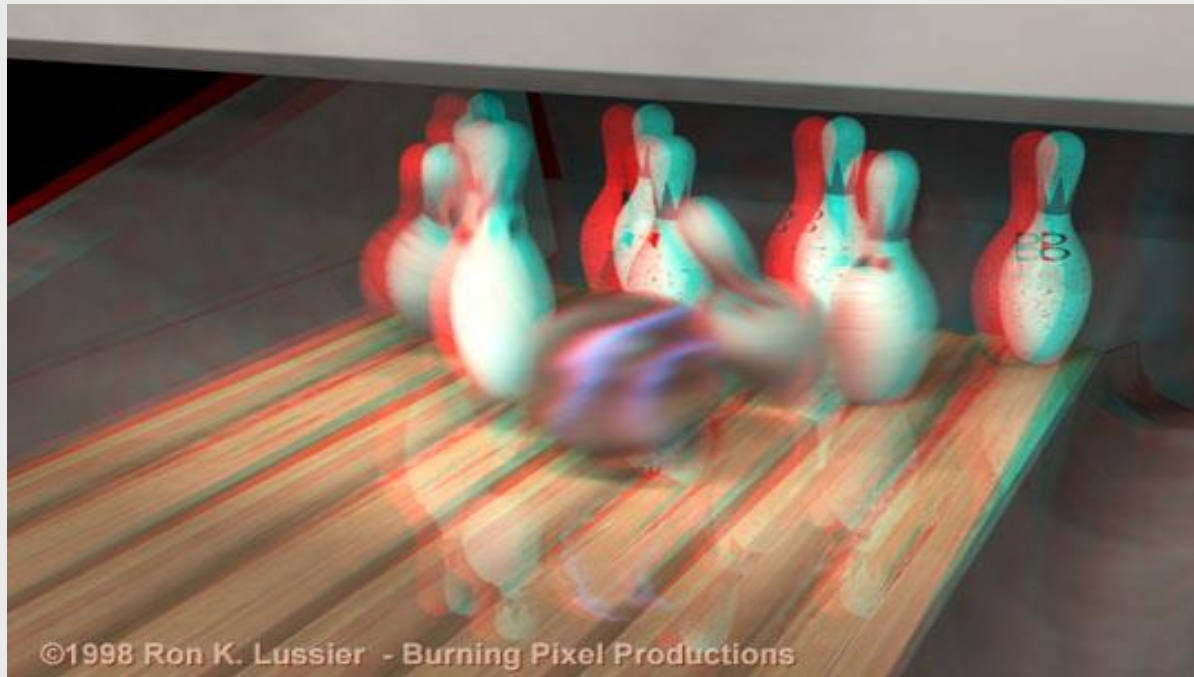
Alignment → Measure depth with respect to this plane

Z=0



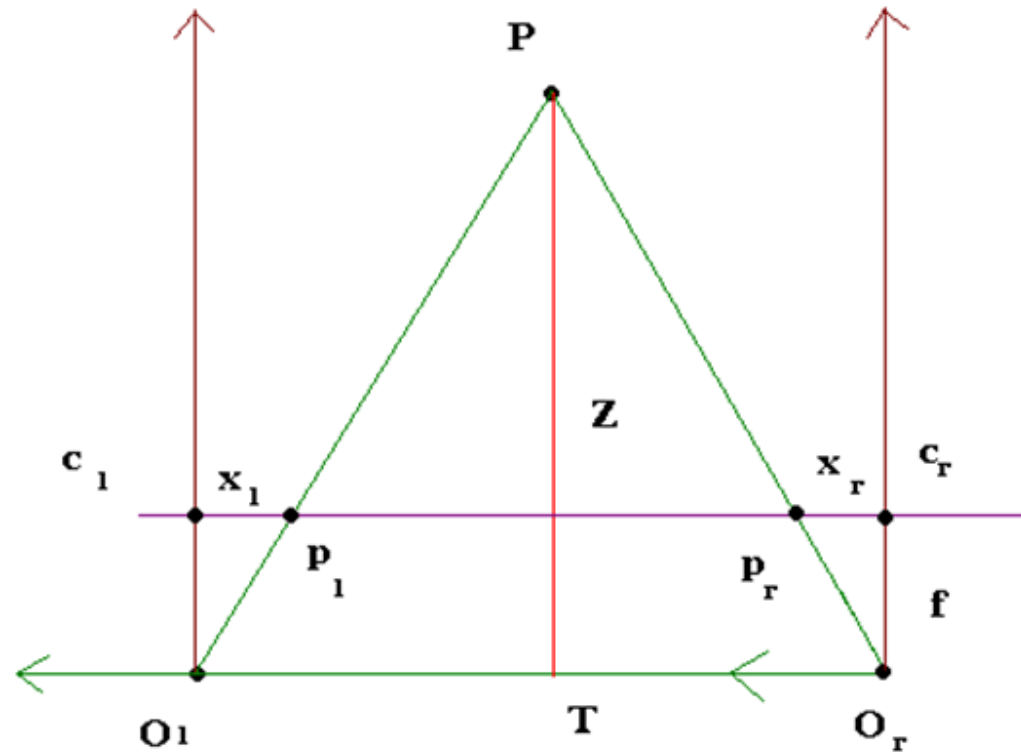
Disparity (1D)

# 位差顯示 (Displaying disparity)



$$disparity \propto \frac{1}{depth}$$

# Recovering depth (reconstruction)



$P$  :its projections  $p_r$ , and  $p_l$

$$x_l = f \frac{X_l}{Z_l}, \text{ or } X_l = \frac{x_l Z_l}{f}$$

$$x_r = f \frac{X_r}{Z_r}, \text{ or } X_r = \frac{x_r Z_r}{f}$$

Cameras are related by the transformation

$$P_r = P_l - T(\text{translation})$$

Using  $Z_r = Z_l = Z$  and  $X_r = X_l - T$ , we have

$$\frac{x_l Z}{f} - T = \frac{x_r Z}{f} \quad \text{or} \quad Z = \frac{Tf}{d}$$

where  $d = X_l - X_r$  is the disparity.

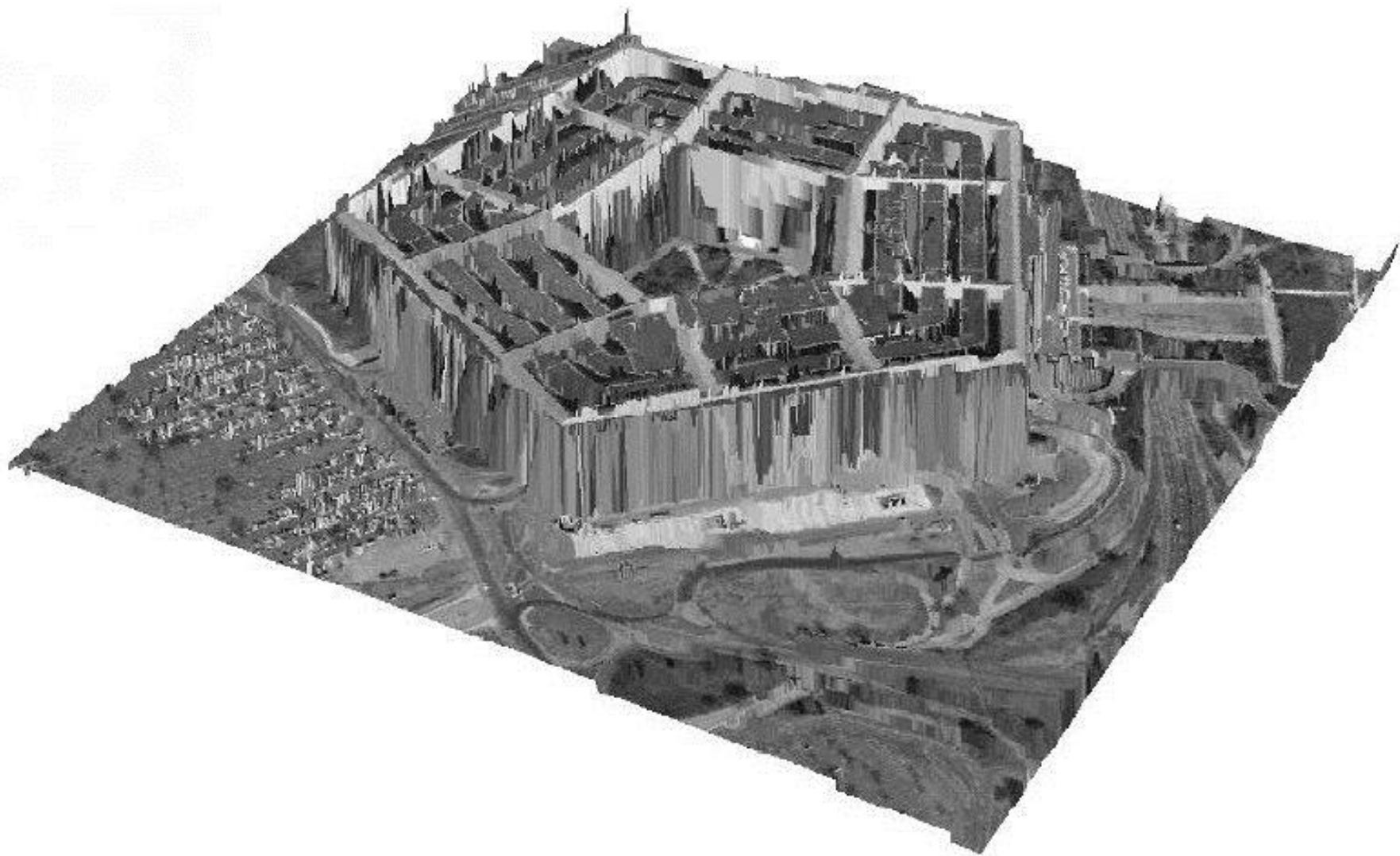
$$\text{disparity} \propto \frac{1}{\text{depth}}$$



# Stereo Vision



$$\textit{disparity} \propto \frac{1}{\textit{depth}}$$



# Feature Matching

What kinds of methods can be used to •  
match

Match “point features”, interpolate –

Match “edge features”, then interpolate –

Match “intensities”, then interpolate –

Optimization Method: –

iterative updating •

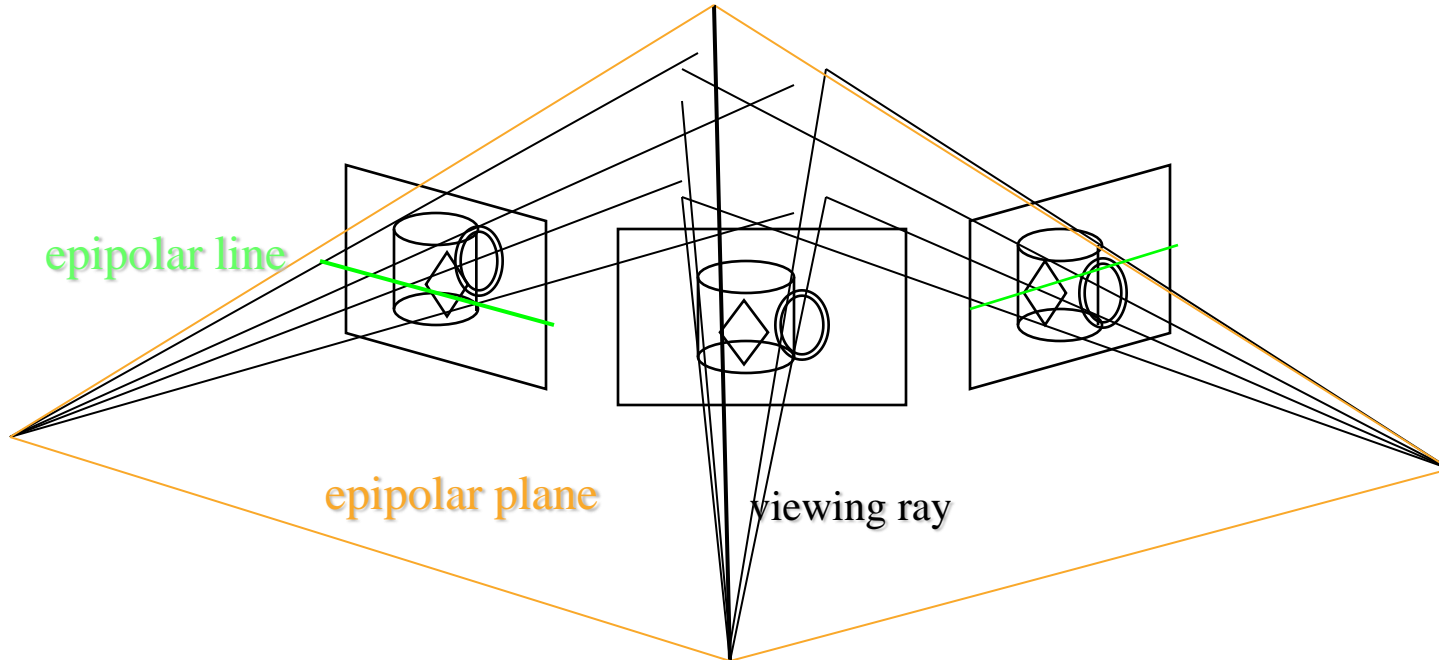
dynamic programming •

energy minimization (regularization, stochastic) •

graph algorithms •

# 極線幾何 (epipolar geometry)

Try to find the best matching pairs along the epipolar line



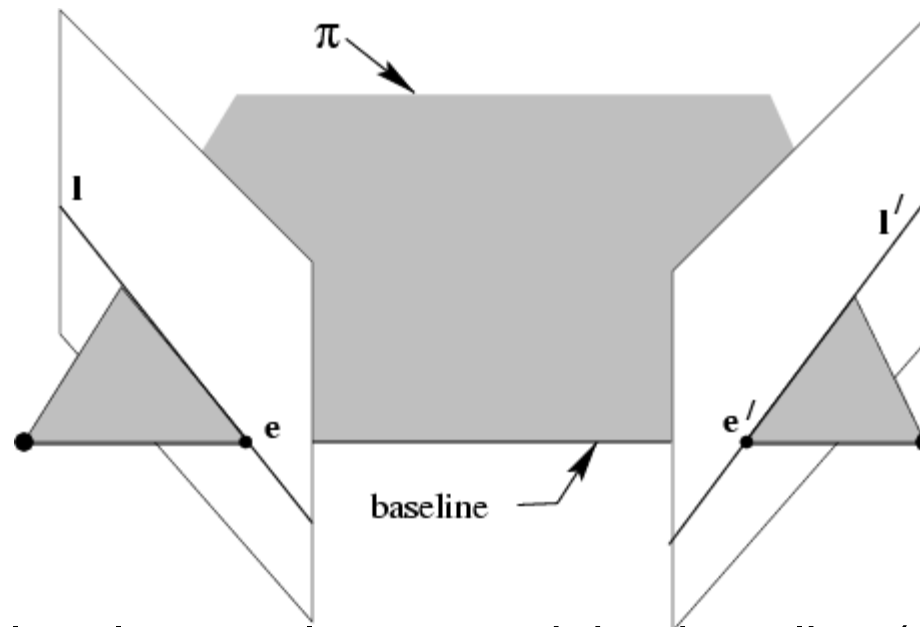
# 極線幾何

epipoles  $e, e'$

= intersection of baseline with image plane

= projection of projection center in other image

= vanishing point of camera motion direction



an epipolar plane = plane containing baseline (1-D family)

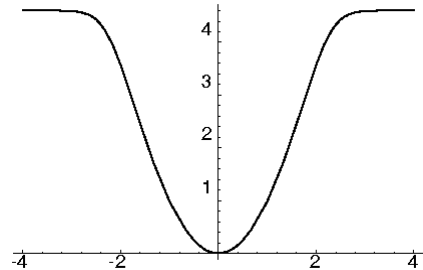
an epipolar line = intersection of epipolar plane with image  
(always come in corresponding pairs)

# Matching technique in Stereo Vision

For every disparity, compute *raw* matching costs .1

$$E_0(x, y; d) = \rho(I_L(x' + d, y') - I_R(x', y'))$$

Why use a robust  
occlusions, oth

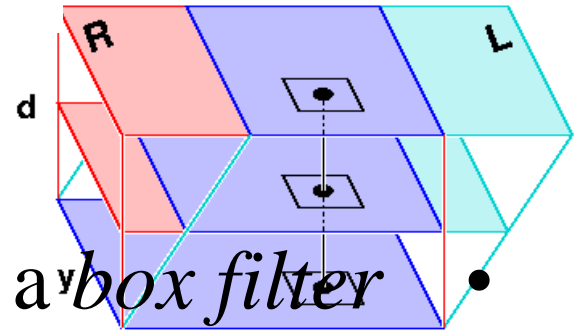


Can also use alternative match criteria •

# Matching technique in Stereo Vision

Aggregate costs spatially .2

$$E(x, y; d) = \sum_{(x', y') \in N(x, y)} E_0(x', y', d)$$



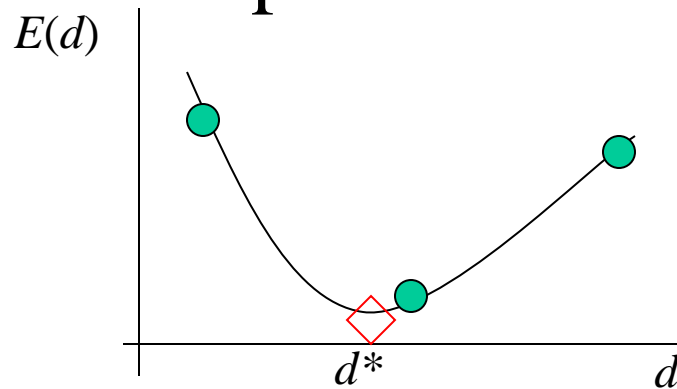
Here, we are using a *box filter*  
(efficient moving average  
implementation)

Can also use weighted average, •  
[non-linear] diffusion...

# Matching technique in Stereo Vision

Choose winning disparity at each pixel .3  
$$d(x, y) = \arg \min_d E(x, y; d)$$

Interpolate to *sub-pixel* accuracy .4





# Example



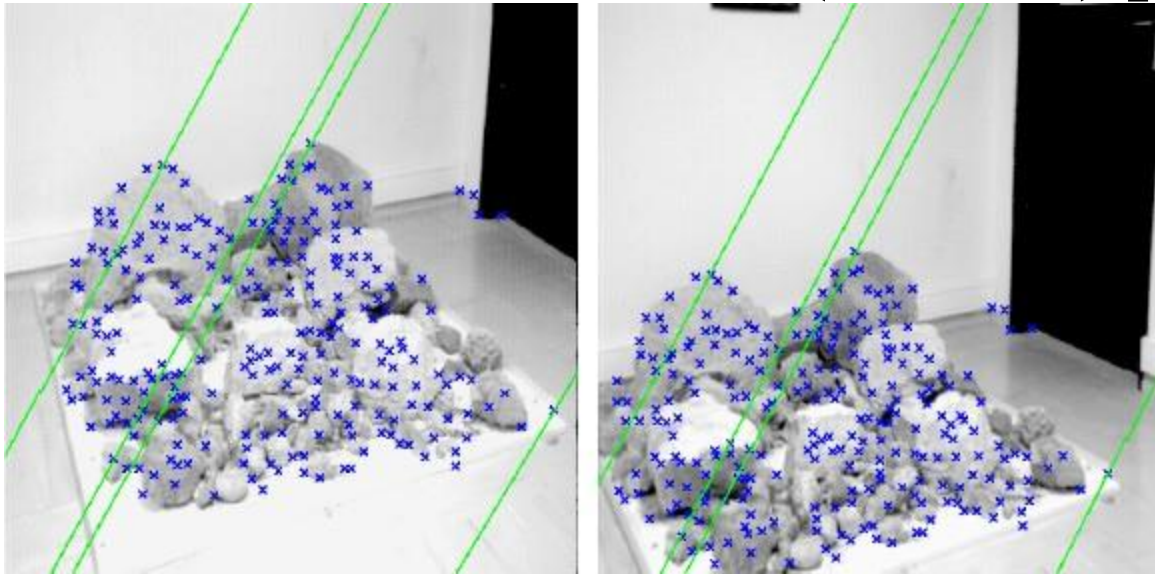
<http://www.rainbowsymphony.com/freestuff.html>

(Wikipedia for images)

**Public Library, Stereoscopic Looking Room, Chicago, by Phillips, 1923**

# Matching technique in Stereo Vision

Match “corner” (interest) points •



Interpolate complete solution •

# Energy minimization

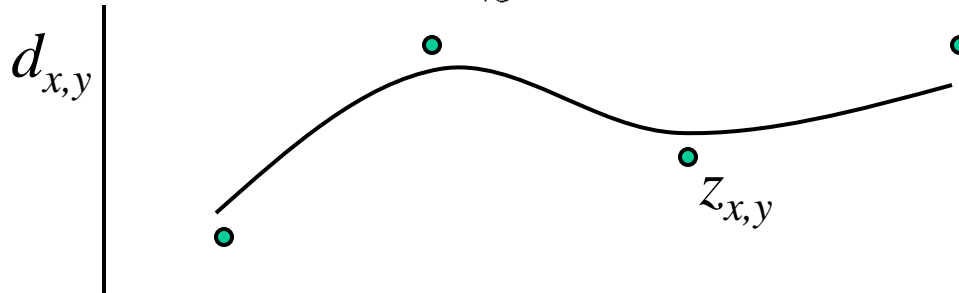
1-D example: approximating splines •

$$E_{\text{total}}(\mathbf{d}) = E_{\text{data}}(\mathbf{d}) + \lambda E_{\text{smoothness}}(\mathbf{d})$$

$$E_{\text{data}}(\mathbf{d}) = \sum_{x,y} (d_{x,y} - z_{x,y})^2$$

$$E_{\text{membrane}}(\mathbf{d}) = \sum_{x,y} (d_{x,y} - d_{x-1,y})^2$$

$$E_{\text{thin plate}}(\mathbf{d}) = \sum_{x,y} (2d_{x,y} - d_{x-1,y} - d_{x+1,y})^2$$



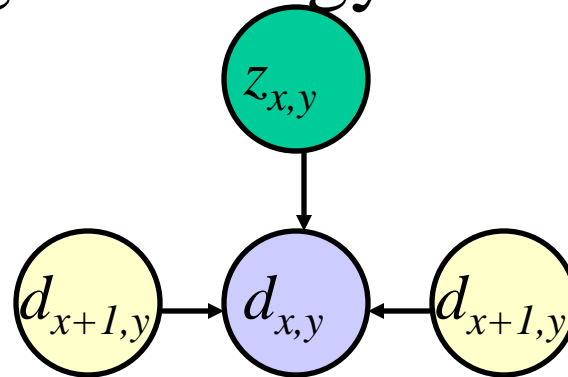
# Relaxation

- How can we get the best solution?
- Differentiate energy function, set to 0

$$\begin{aligned}\frac{\partial E}{\partial d_{x,y}} &= 2(d_{x,y} - z_{x,y}) + \\ &\quad 2\lambda(2d_{x,y} - d_{x-1,y} - d_{x+1,y}) = 0 \\ d_{x,y} &\leftarrow \frac{1}{1 + 2\lambda}(z_{x,y} + d_{x-1,y} + d_{x+1,y})\end{aligned}$$

# Relaxation

Iteratively improve a solution by locally minimizing the energy: *relax* to solution •



Earliest application: WWII numerical simulations •

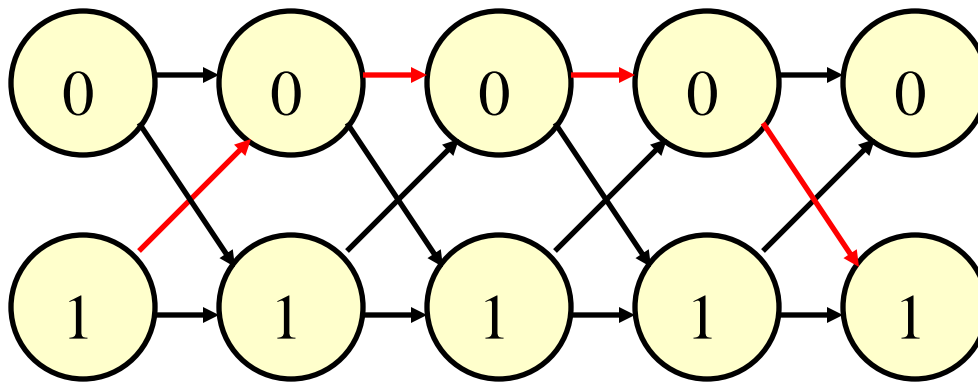
# Dynamic programming

Evaluate best cumulative cost at each pixel •

$$E_{\text{total}}(\mathbf{d}) = E_{\text{data}}(\mathbf{d}) + \lambda E_{\text{smoothness}}(\mathbf{d})$$

$$E_{\text{data}}(\mathbf{d}) = \sum_{x,y} (d_{x,y} - z_{x,y})^2$$

$$E_{\text{smoothness}}(\mathbf{d}) = \sum_{x,y} |d_{x,y} - d_{x-1,y}|$$

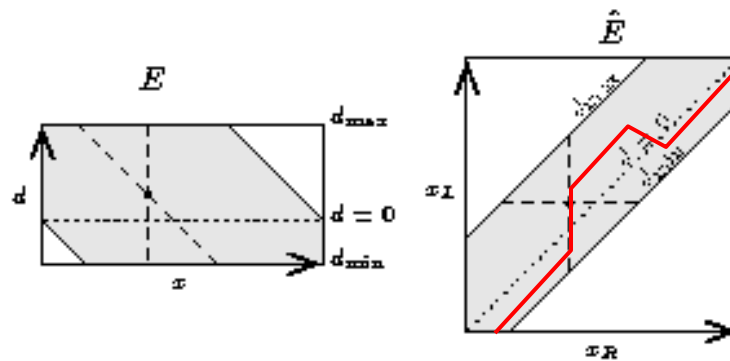


# Dynamic programming

## 1-D cost function •

$$E(\mathbf{d}) = \sum_{x,y} \rho_P(d_{x+1,y} - d_{x,y}) + \sum_{x,y} E_0(x, y; d)$$

$$\tilde{E}(x, y, d) = E_0(x, y; d) + \min_{d'} \left( \tilde{E}(x-1, y, d') + \rho_P(d_{x,y} - d'_{x-1,y}) \right)$$



# Dynamic programming

## Disparity space image and min. cost path •

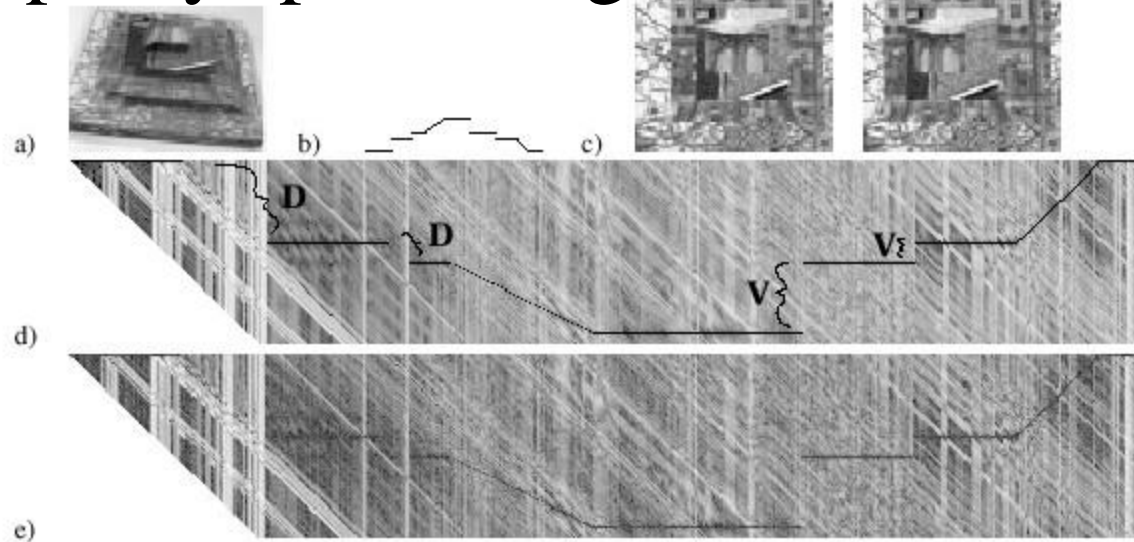


Fig. 4. This figure shows (a) a model of the stereo sloping wedding cake that we will use as a test example, (b) a depth profile through the center of the sloping wedding cake, (c) a simulated, noise-free image pair of the cake, (d) the enhanced, cropped, correlation DSI representation for the image pair in (c), and (e) the enhanced, cropped, correlation DSI for a noisy sloping wedding cake (SNR = 18 dB). In (d), the regions labeled "D" mark diagonal gaps in the matching path caused by regions occluded in the left image. The regions labeled "V" mark vertical jumps in the path caused by regions occluded in the right image.

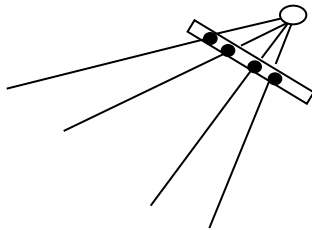


# Concepts of Panoramic Images

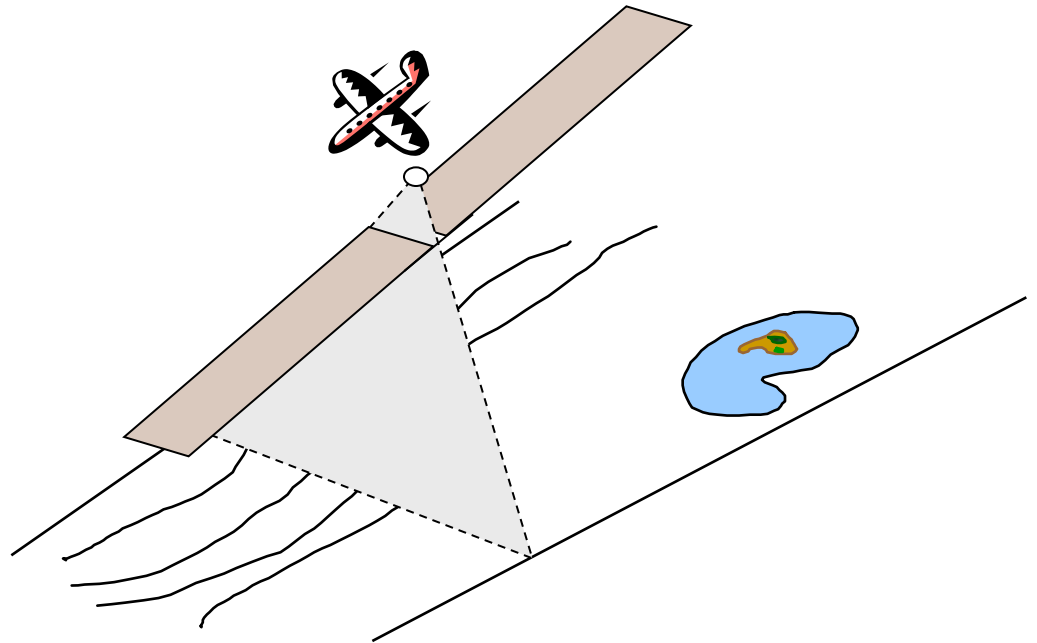
# 使用 Pushbroom camera

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- 1D projective sensor



- ... translating

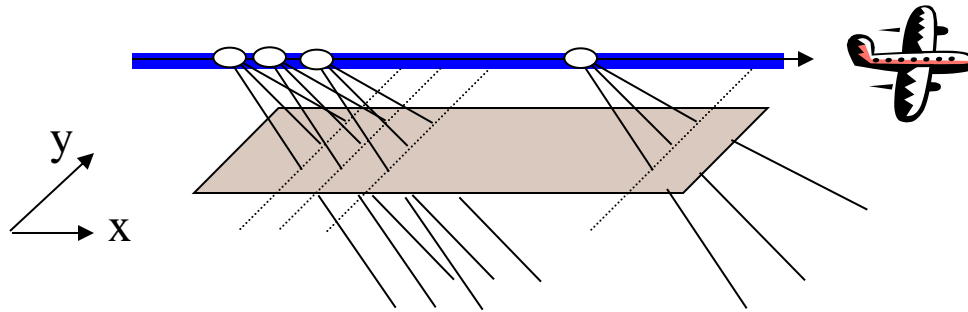


Advantage: large field of view in one dimension

# 使用 Pushbroom camera

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The generalized camera model:



X direction - parallel projection

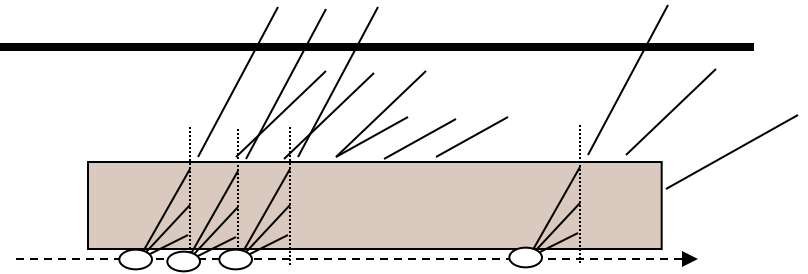
Y direction - perspective projection

Notion *Generator* – the set of all ray origins

*For other cameras, generator can be a 2D surface*

# Using Pushbroom camera

## Imaging process



Images from usual camera  
 $t$



$t+1$



$t+2$



Image from generalized camera



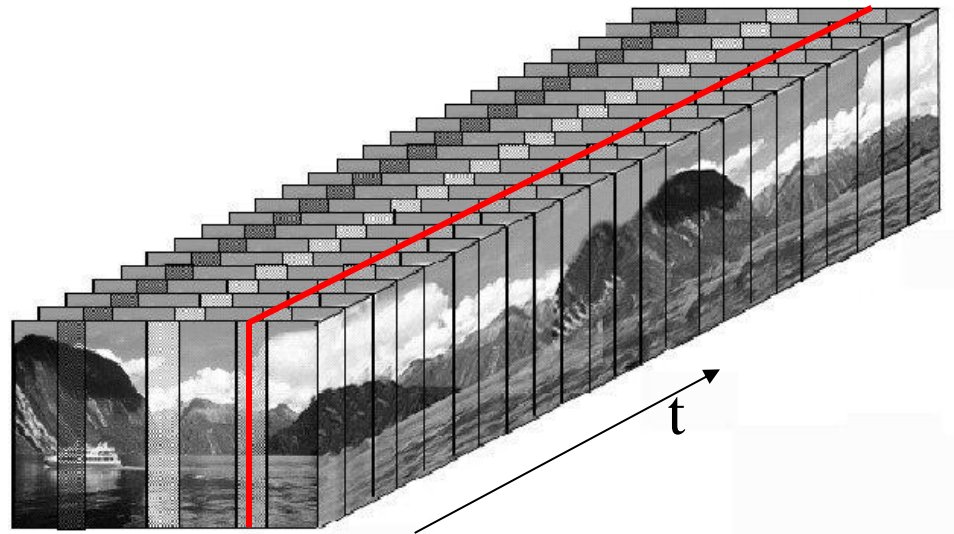
Virtual generalized camera = device (usual camera) + software

# Non-classical cameras

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## Implementation through cuts of 3D video arrays

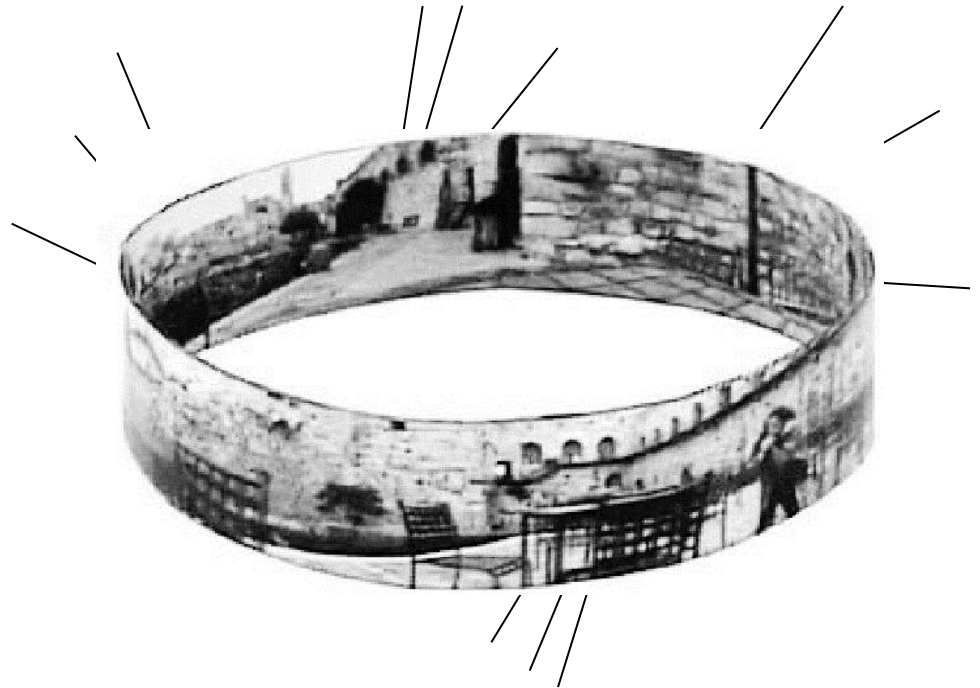
- Take images while moving a usual camera
- Stack them into 3D array
- Take a cut along the “time” dimension



# Using Circular projective camera

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Move “1D sensor” along a circle  
record on a cylinder



Advantage: complete 360° horizontal view

# Using circular projective camera

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- Panoramic Image Stitcher
    - Warping
    - Registration
    - Blending
    - Dewarping
-

# Panoramic Image Stitcher

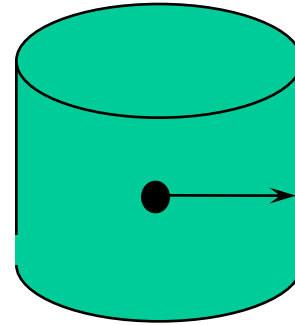
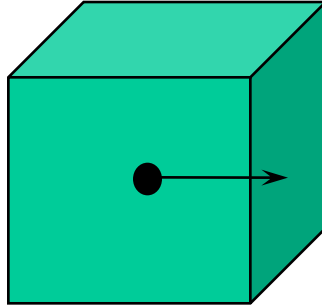
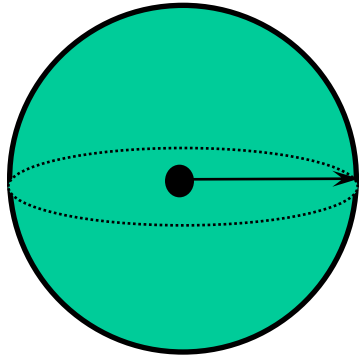
## □ Panoramic Images



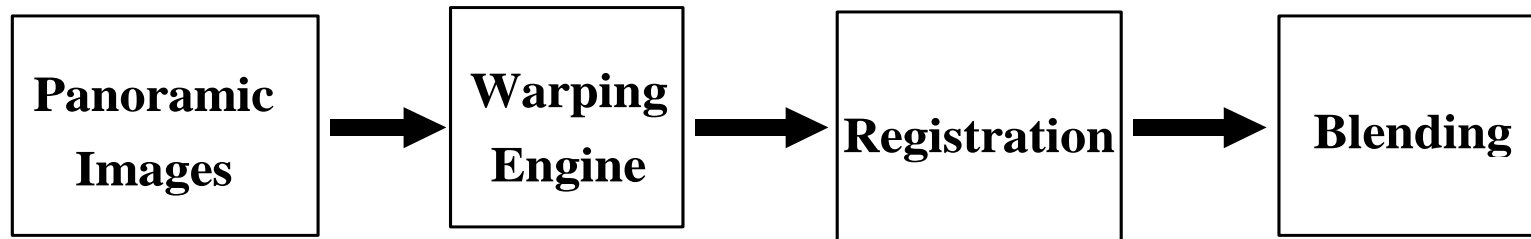


# Panoramic Image Stitcher

## □ Warping Enviroments



## □ Flowchart of the proposed stitcher



A general system for generating a virtual panoramic world.

# Panoramic Image Stitcher

## □ Cylindrical Environment Map

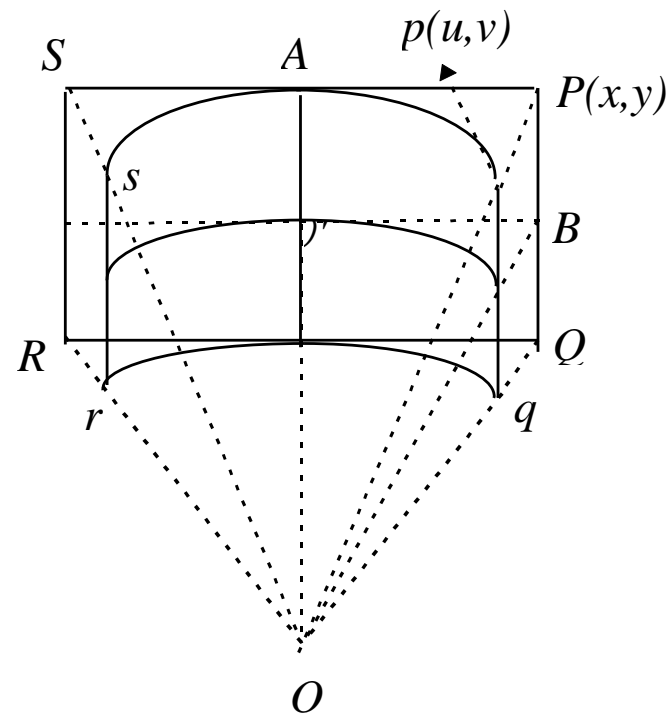
■  $PQRS \rightarrow \blacksquare pqrs$

$P(x,y) \rightarrow p(u,v)$

$$u = f \tan^{-1} \frac{\overline{O'B}}{\overline{OO'}} = f \tan^{-1} \frac{x}{f}$$

$$v = f \frac{\overline{PB}}{\overline{OB}} = f \frac{y}{\sqrt{x^2 + f^2}}$$

$f$ : focal length

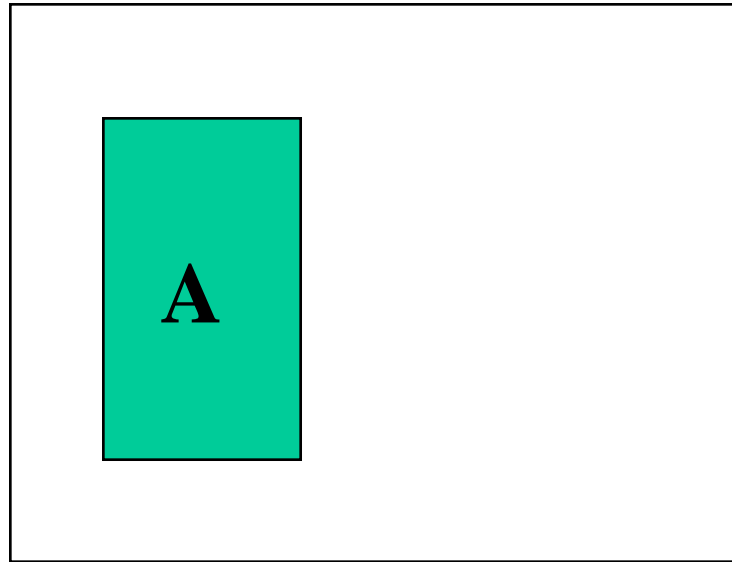
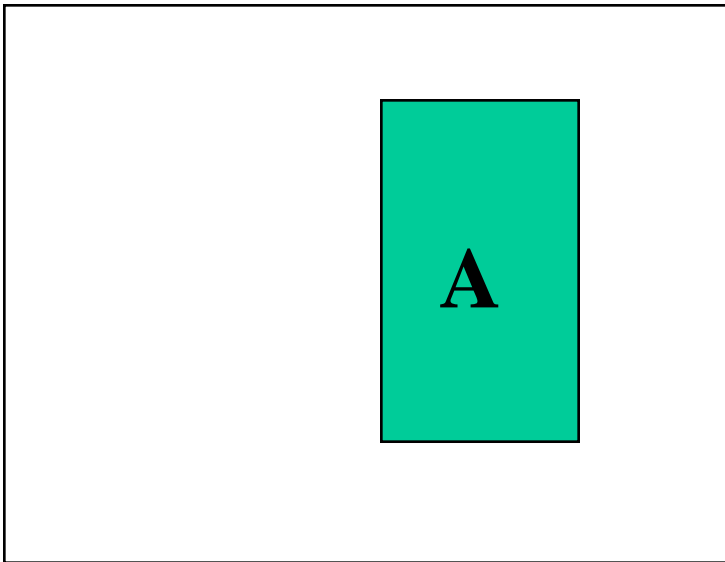


The geometry for cylindrical mapping.

# Panoramic Image Stitcher

## □ Traditional methods:

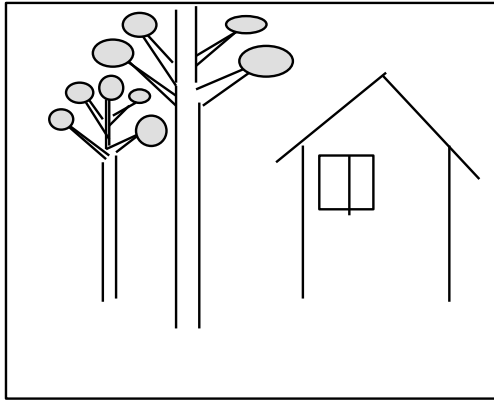
Using a correlation technique and a full search approach to determine the optimal offsets.



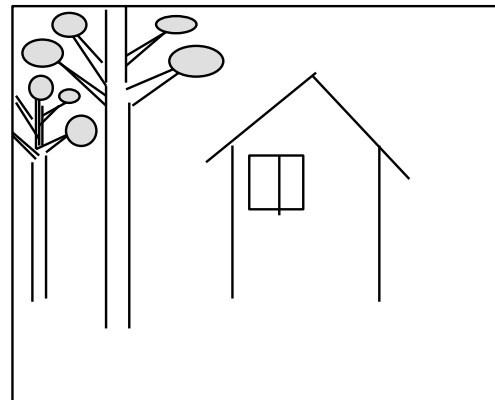
# Panoramic Image Stitcher

□ Our proposed method:

Using edge information to get a set of possible solutions in advance.



(a)



(b)

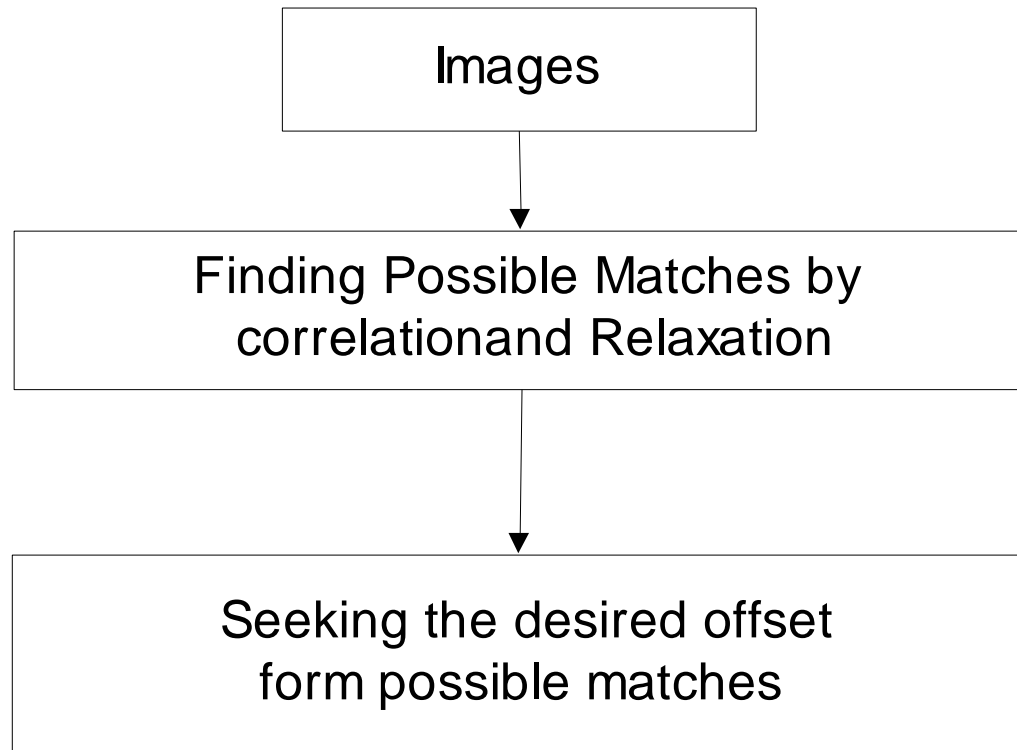
$$P_a = (100, 115, 180, 200, 310, 325, 360, 390, 470)$$

$$P_b = (20, 35, 100, 120, 230, 245, 280, 310, 390)$$

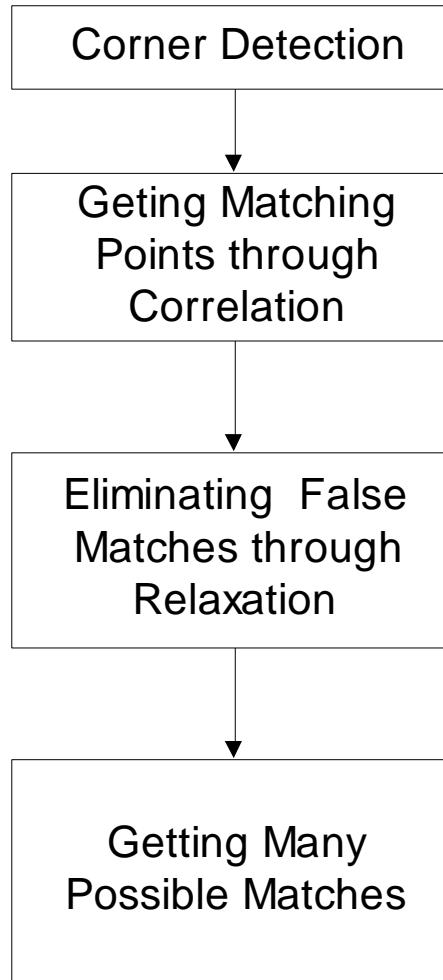
# Panoramic Image Stitcher



# Panoramic Image Stitcher



# Panoramic Image Stitcher



# Panoramic Image Stitcher

## □ Corner Detection

$$R(x, y) = \det[C] - 0.04 \text{trace}^2[C]$$

$$C = \begin{bmatrix} \bar{I}_x^2 & I_x I_y \\ I_x I_y & \bar{I}_y^2 \end{bmatrix}$$

$\bar{I}$ : the smoothing version of image I

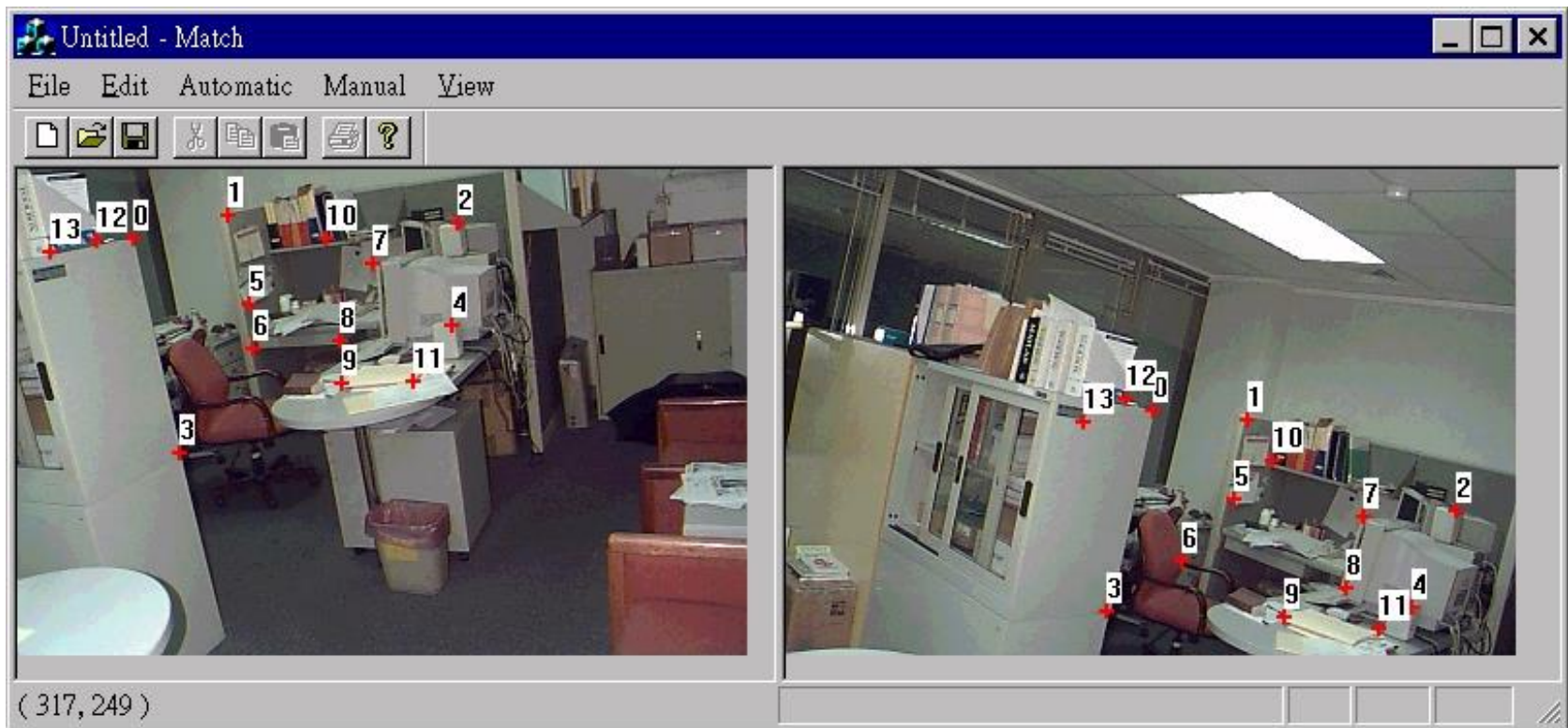
$\bar{I}_x$ : the  $x$  directional derivative of image I

$\bar{I}_y$ : the  $y$  directional derivative of image I



# Panoramic Image Stitcher

## □ Eliminating False Matches through Relaxation



# Panoramic Image Stitcher

## □ Matching through Correlation

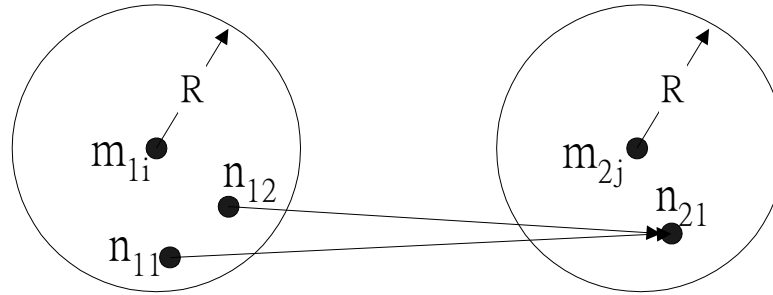
$$\text{Corr}(p; q) = \frac{1}{\sigma_1 \sigma_2 (2M+1)^2} \sum_{x, y=-M}^{x, y=M} [I_1(x + p_x, y + p_y) - u_1][I_2(x + q_x, y + q_y) - u_2]$$

$u_i$  and  $\sigma_i$ : local mean and variance of  $I_i$ , respectively

$(2M+1)^2$ : the area of matching window.

# Panoramic Image Stitcher

## □ Eliminating False Matches through Relaxation



Measure of support for a match:

$$1. \text{Strength of match} \quad S_M(m_{1i}, m_{2j}) = c_{ij} \sum_{n_{1k} \in N(m_{1i})} \left[ \max_{n_{2l} \in N(m_{2j})} \frac{c_{kl} \delta(m_{1i}, m_{2j}; n_{1k}, n_{2l})}{1 + \text{dist}(m_{1i}, m_{2j}; n_{1k}, n_{2l})} \right]$$

$$\text{dist}(m_{1i}, m_{2j}; n_{1k}, n_{2l}) = [d(m_{1i}, n_{1k}) + d(m_{2j}, n_{2l})] / 2$$

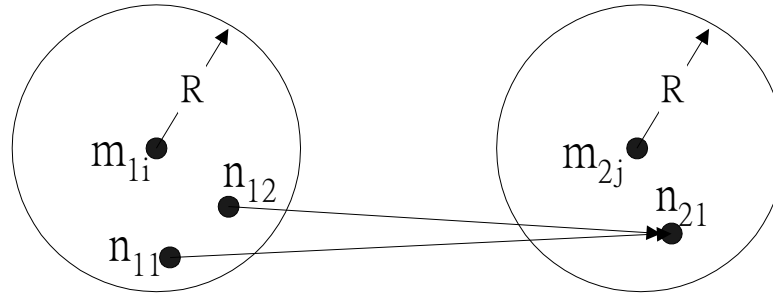
$$\delta(m_{1i}, m_{2j}; n_{1k}, n_{2l}) = \text{a function of } d(m_{1i}, n_{1k}) \text{ and } d(m_{2j}, n_{2l})$$

2. Angle consistence:

The angle between  $m_{1i}n_{1k}$  and  $m_{2j}n_{2k}$  should be less than a threshold  $\theta$ .

# Panoramic Image Stitcher

## □ Eliminating False Matches through Relaxation



$\delta(m_{1i}, m_{2j}; n_{1k}, n_{2l})$  = a function of  $d(m_{1i}, n_{1k})$  and  $d(m_{2j}, n_{2l})$

$$\delta(m_{1i}, m_{2j}; n_{1k}, n_{2l}) = \begin{cases} e^{-r/\varepsilon} & \text{if } r < \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

$$r = \frac{|d(m_{1i}, n_{1k}) - d(m_{2j}, n_{2l})|}{\text{dist}(m_{1i}, m_{2j}; n_{1k}, n_{2l})}$$

$$\text{dist}(m_{1i}, m_{2j}; n_{1k}, n_{2l}) = [d(m_{1i}, n_{1k}) + d(m_{2j}, n_{2l})] / 2$$

# Panoramic Image Stitcher

## □ Eliminating False Matches through Relaxation

Relaxation Process:

Iterate {

    Compute SM for each match

    Delete matches whose SM value is less than the  
    k%-smallest matching strength

    Check angle consistence

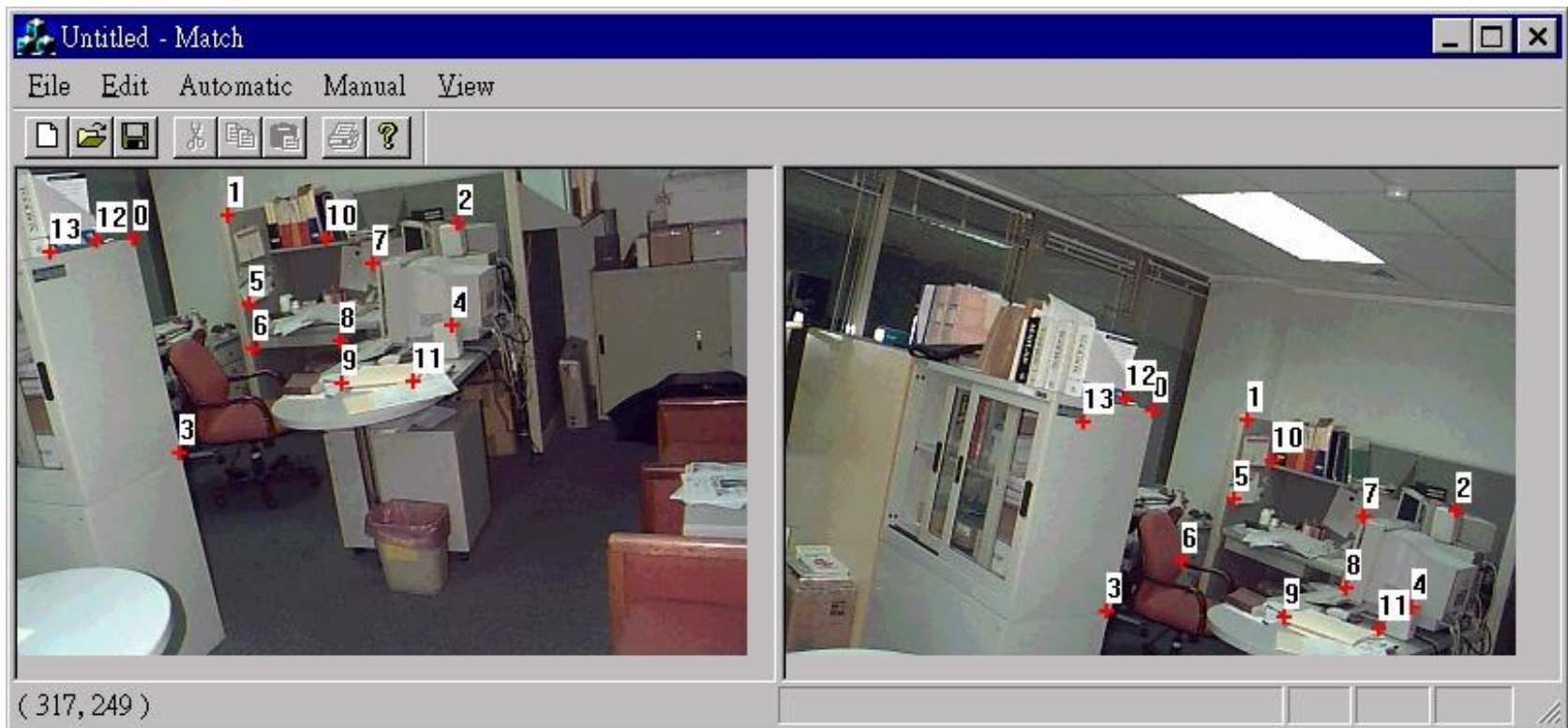
    Update SM with the remained matches

}

until iteration times  $<$  a threshold

# Panoramic Image Stitcher

## □ Eliminating False Matches through Relaxation



# Panoramic Image Stitcher

## □ Getting Offsets from Matching Candidates

Using “votes” to get the best offset

1. With matching strength(SM) as a measure, choose top 10 matches as candidates to get a set of possible offsets.

$\text{Offset}[I].x = \text{Matches}[I].p.x - \text{Matches}[I].q.x;$

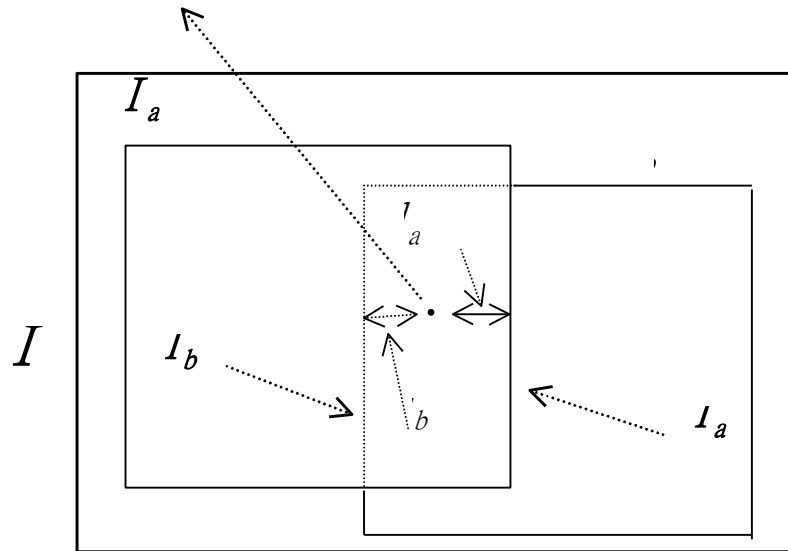
$\text{Offset}[I].y = \text{Matches}[I].p.y - \text{Matches}[I].q.y;$

2. With the set of possible offsets, get the best offset with voting from all the matches.
3. Fine-tuning with correlation.

# Panoramic Image Stitcher

## □ Blending Method

$p_i, q_i, r_i$  (located at the same location)

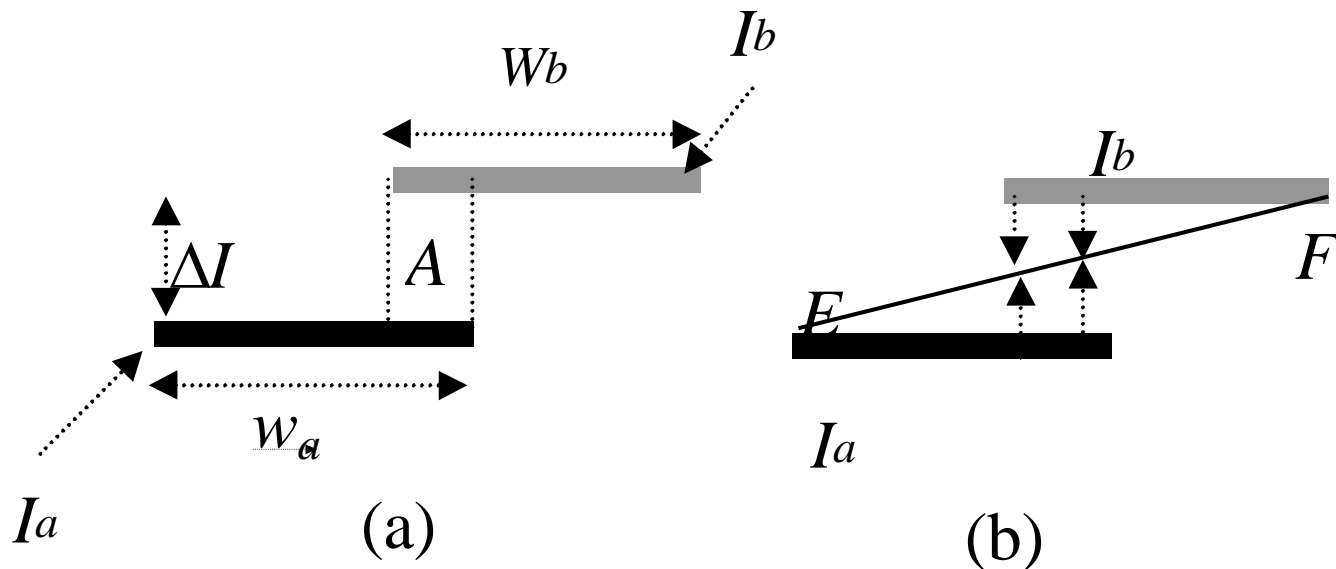


$$I(r_i) = \frac{d_b^t I_a(p_i) + d_a^t I_b(q_i)}{d_a^t + d_b^t}$$



# Panoramic Image Stitcher

## □ Blending Method



$$\Delta I = \frac{1}{|A|} \sum_{i \in A} (I_b(q(i)) - I_a(p(i))),$$

$$I_a(p(x, y)) = I_a(p(x, y)) + x \frac{\Delta I}{2w_1}$$

$$I_b(q(x, y)) = I_b(q(x, y)) + (x - w_2) \frac{\Delta I}{2w_2}$$

# Panoramic Image Stitcher



A series of panoramic images for testing.

# Panoramic Image Stitcher

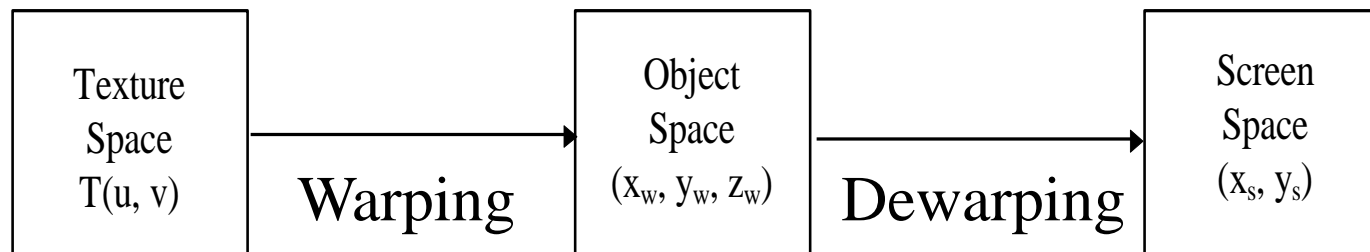


The stitching result obtained by our intelligent stitcher.

# Panoramic Image Stitcher

## □ Dewarping Method

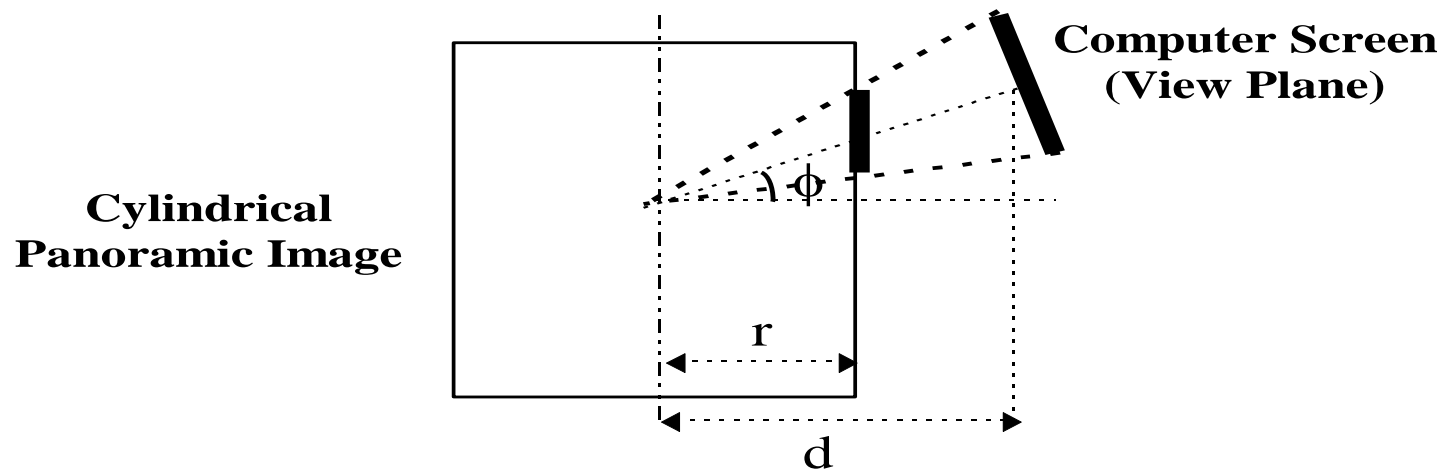
- 相關之技術領域
  - Texture Mapping



# Panoramic Image Stitcher

## □ Dewarping Method

- 環場影像虛擬實境



固定觀測點於幾何中心 + 可動式投影平面

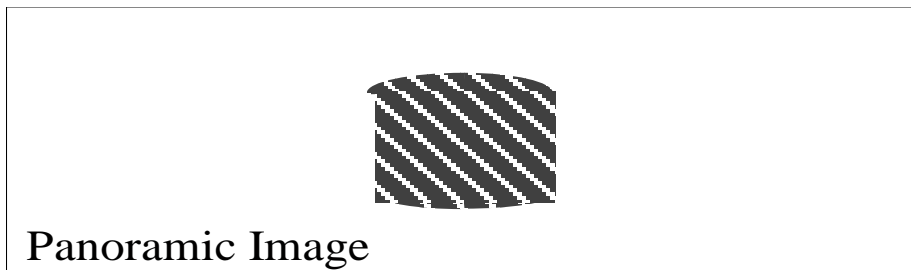
# Panoramic Image Stitcher

## 柱狀環場影像範例

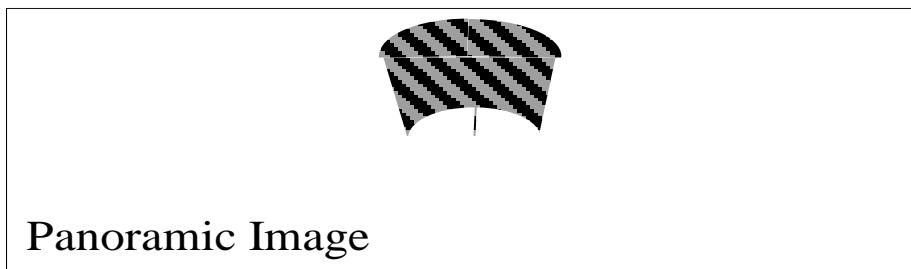


# Panoramic Image Stitcher

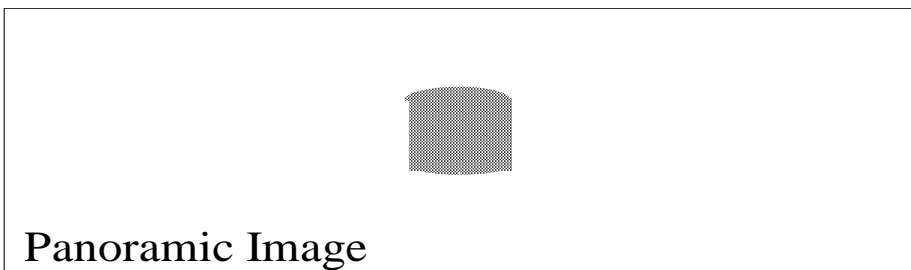
## 影像反扭曲 (Image Dewarping)



平視



抬頭



鏡頭拉近

# Panoramic Image Stitcher

## 查表法之使用

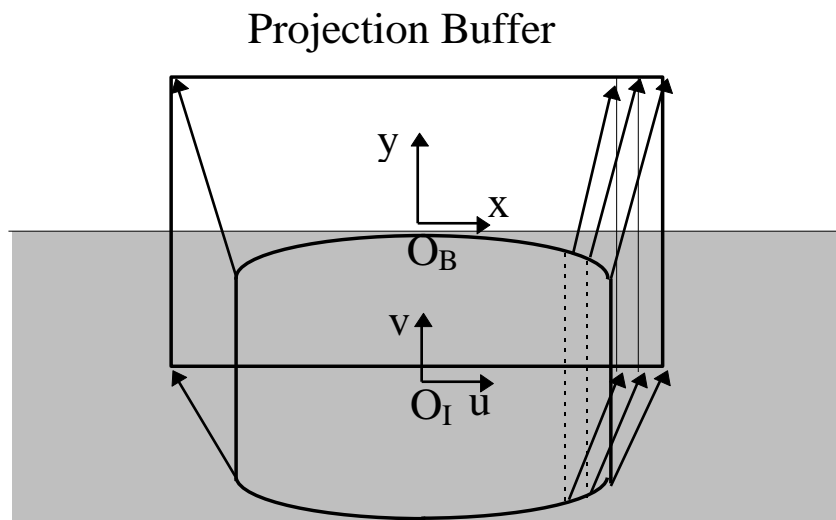
- 記錄反扭曲區域內之各**垂直掃描線**的起點位置及終點位置
- 表中資料在抬頭/ 低頭，以及拉近/拉遠鏡頭時必須重算



# Panoramic Image Stitcher

## 投影關係 (一)

環場影像與投影緩衝區：



Panoramic Image  $\leftrightarrow$  Projection Buffer

$$u = r \tan^{-1}(x / r),$$

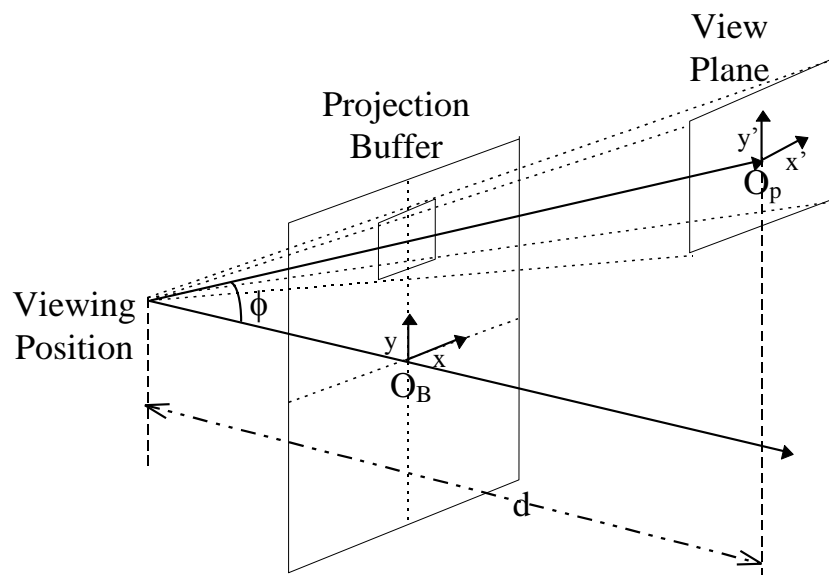
$$v = yr / \sqrt{x^2 + r^2}.$$

Panoramic  
Image

# Panoramic Image Stitcher

## 投影關係 (二)

投影緩衝區與成像平面：



Projection Buffer  $(x', y')$   $\leftrightarrow$  View Plane  $(x', y')$

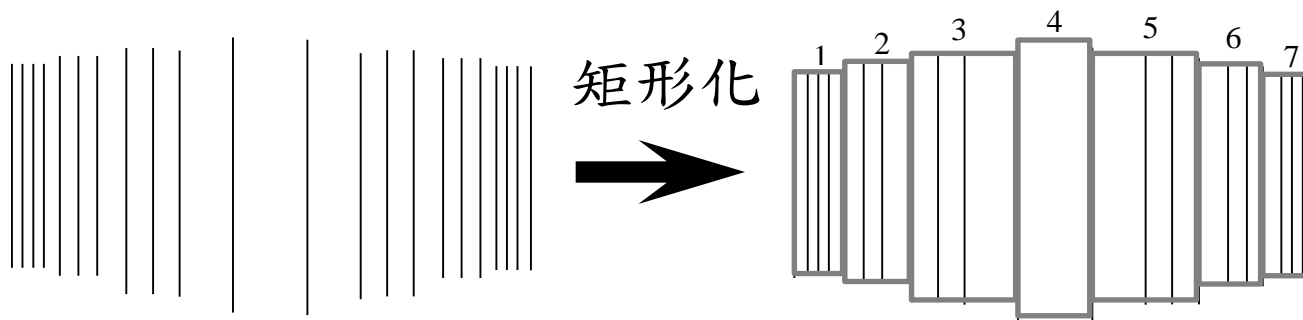
$$x = \frac{r}{d} x',$$

$$y = \frac{r}{d} (y' + d \tan \phi)$$

# Panoramic Image Stitcher

## 查表法改進

- 使用矩形化反扭曲技術



優點：

- ( 1 ) 可充分利用圖形顯示卡之硬體加速器 (如Blitter)
- ( 2 ) 可大量降低資料表之大小

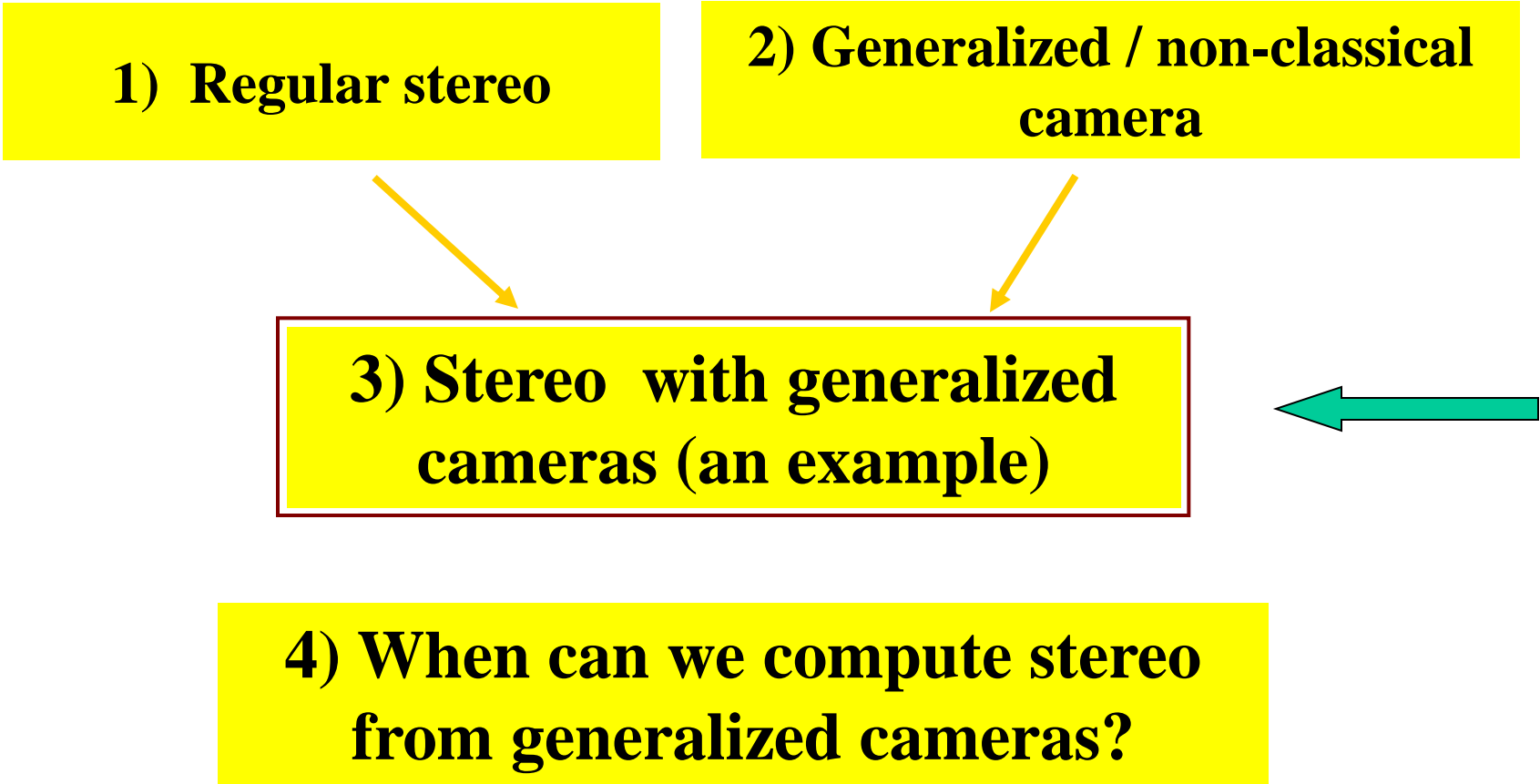
# Panoramic Image Stitcher



# 三維環場影像

**1) Regular stereo**

**2) Generalized / non-classical  
camera**

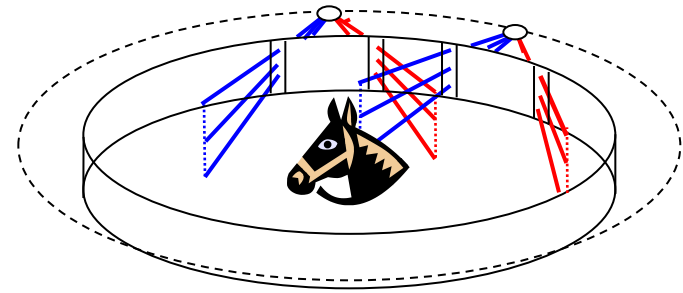
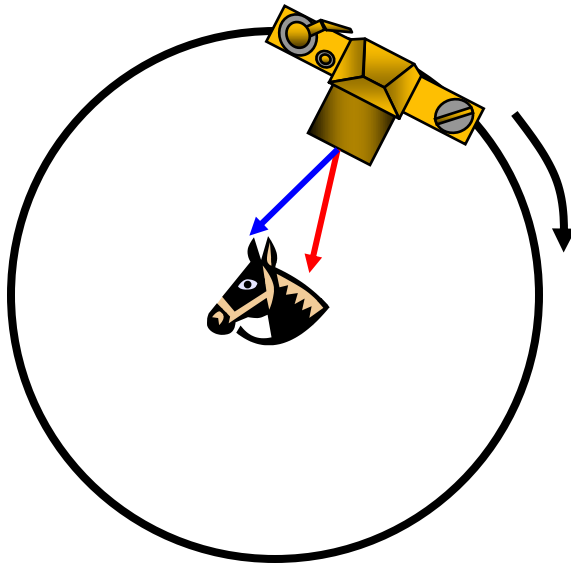


**3) Stereo with generalized  
cameras (an example)**

**4) When can we compute stereo  
from generalized cameras?**

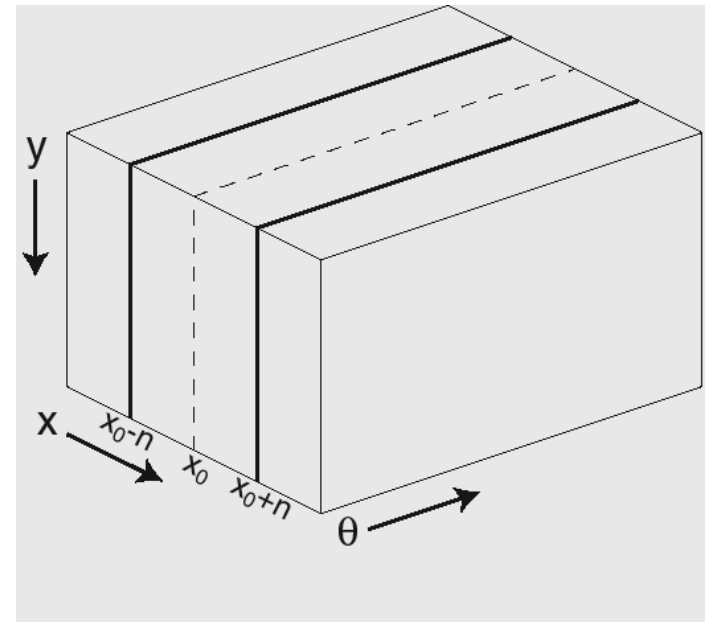
# 沿著一個圓相機往內拍(Object Video)

Inward-facing camera, moving around an object



# Images for both eyes

- Input: video sequence
- Output: 2 symmetric cuts of 3D video array



# Results: red-blue stereo image

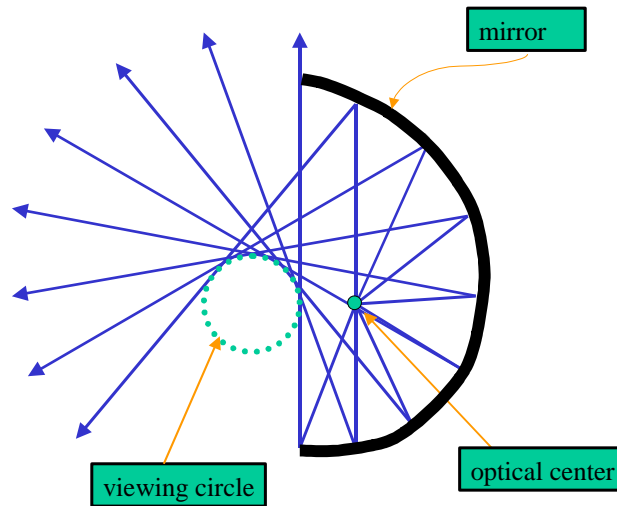


*thanks to Steve Seitz, University of Washington*

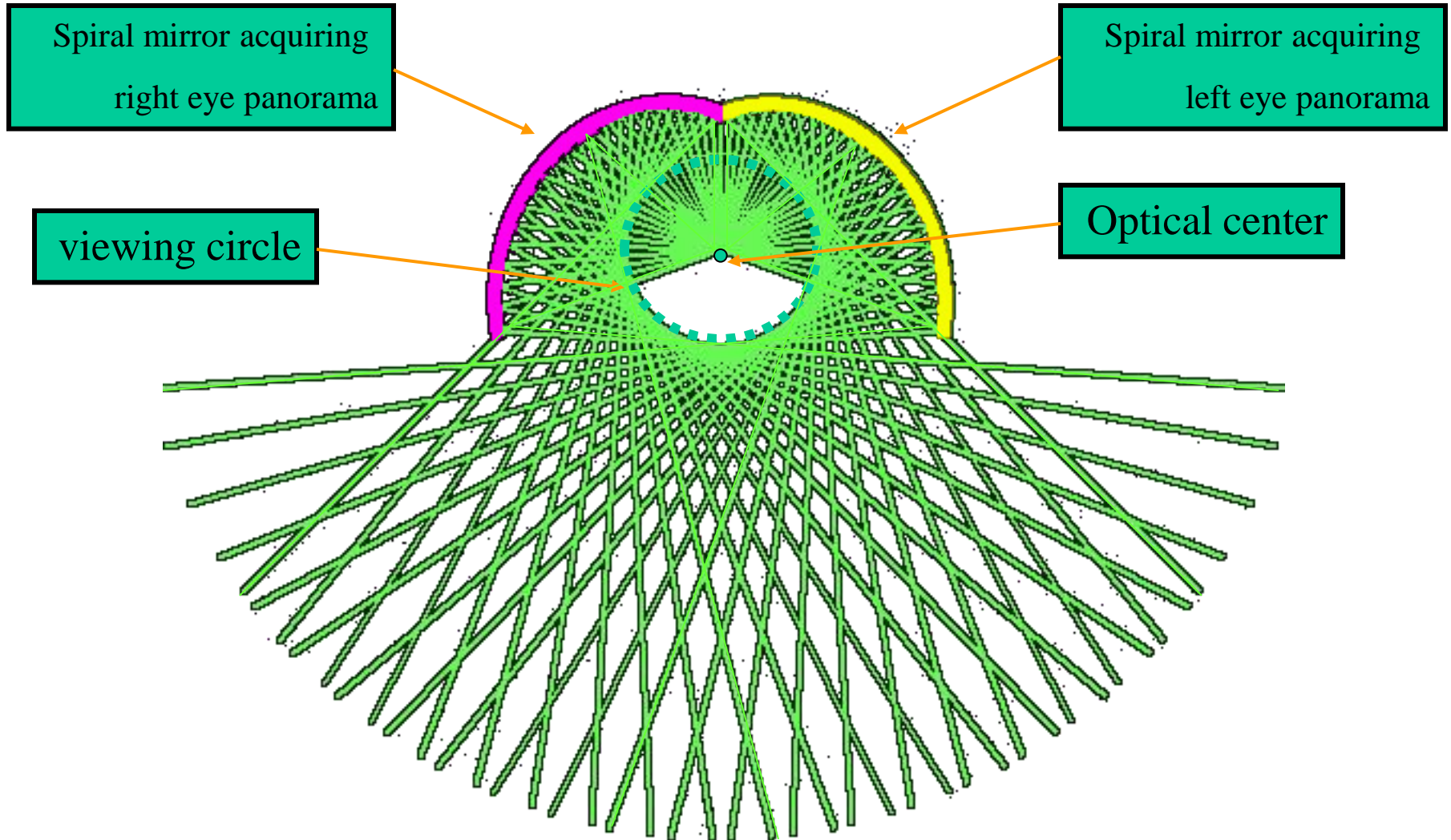


# OmniStereo with Mirrors

## *Dynamic Scenes*



# Stereo Image



# Stereo Image

