Introduction to Financial Engineering and Algorithms Programming Assignment 2 (Spring 2014)

Unless you can provide your own demonstration platform, please specify the programming language you will use and tell the TA. This will be critical for demonstration purpose. The maximum score for this exercise is 160 points.

1. (a) (30 points) Write a computer program that implements the CRR binomial option pricing model to compute European-style call option and put option. A sample test data is $S_0 = 90$, $X = 95$ (strike price), $T = 0.5$ (6 months), $\sigma = 0.30(30\%)$, and $r = 2\%$. The call price is 5.9111 and the put price is 9.9659. You can also verify the price by using put-call parity.

   (b) (20 points) Plot a graph where the x-axis represents the number of partitions n and y-axis represent the option price. What do you see in this plot?

   (c) (10 points) For European-style options, the CRR binomial option pricing model runs in $O(n^2)$ time. However, it is possible to write a $O(n)$-time program. How can this be done?

2. (50 points) Write a computer program the mplements the CRR binomial option pricing model to compute American-style call option and put option. A sample test data is $S_0 = 90$, $X = 95$ (strike price), $T = 0.5$ (6 months), $\sigma = 0.30(30\%)$, and $r = 2\%$. The call price is 5.9111 and the put price is 10.0729.

3. (30 points) The Black-Scholes equation for pricing European options are

$$\begin{aligned} \text{European Call} &= S_0 N(d_1) - Xe^{-rT} N(d_2) \\ \text{European Put} &= Xe^{-rT} N(-d_2) - S_0 N(-d_1), \end{aligned}$$

   where

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S_0}{X}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(\frac{S_0}{X}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}, \end{aligned}$$

   and

$$N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}x^2} dx.$$

   Write a computer program using the Black-Scholes equation to price European options. A sample test data is $S_0 = 90$, $X = 95$ (strike price), $T = 0.5$ (6 months), $\sigma = 0.30(30\%)$, and $r = 2\%$. The call price is 5.9111 and the put price is 10.0729. To compute $N(d)$, please use numerical integration (taught in numerical analysis course). **Do not** call libraries (Excel, Matlab, etc.)!

4. (20 points) In the book "Numerical Recipes in C", there is a way to approximate cumulative normal distribution with a polynomial. Find and use this implementation. (Hint: the **Erf** function can be approximated, and **Erf** is connected to the cumulative normal distribution.)