Financial Engineering Final Exam (Spring 2014)

Please write or print your solutions on A4 papers. Also, remember to write down your department, student ID, and name. This exam contains 5 questions and is an open book exam. The maximum score for this exam is 100 points. You have 100 minutes to complete this exam.

1. (10 points) When trading forwards and futures, the initialization cost is zero. This means that you do not have to pay money to go into a forward or future contract. However, when buying options, you will have to pay a **premium**. Why?

2. (15 points) Consider a European put option with strike price \$80 and premium price of \$5. Draw the profit curve (profit vs. terminal price) of this option.

3. We know that the underlying assumption of stock prices is lognormally distributed. The binomial tree (lattice) model can approximate the Black-Scholes solution when the resolution $n \to \infty$.

   (a) (15 points) When using the CRR binomial tree by Cox, Ross, and Rubinstien, how did they solve for the u and d of the stock price movements by matching the mean and variances of the stock price? (Hint: There are two linear equations, mean and variance)

   (b) (20 points) Given an American put option with the following parameters, spot price $S_0 = 100$, strike price $K = 120$, time to maturity $\tau = 1.0$ years, volatility $\sigma = 25\%$, interest rate $r = 1.85\%$, and number of terms $n = 3$. Tabulate the solution of this option using the CRR binomial tree model (compute the parameters u, d, p, and $e^{-r\Delta t}$ (discount per term).

4. The current spot price of stock A is \$52.25, time to maturity τ is 6 months (0.5 years), and the announced risk free rate r is 1.25%. Assume that we have the following scenario on options on stock A (the prices are exaggerated to many decimal digits):

Strike Price	Call Price	Put Price
50.00	3.74811144	1.18658597
51.00	3.12999734	1.73135625
52.00	2.58056269	2.00658630
53.00	1.92362392	2.51991213
54.00	**1.68689224**	**3.10044473**
55.00	1.33731816	3.74464015

   (a) (10 points) Assume the call and put prices for strike price \$54 is correct. Try to find the implied volatility σ of the stock using this price (up to two decimal digit).

   (b) (10 points) Using the σ in the previous question, your seventh sense tells you that there is some mispricing occurring. Can you detect where is it?

5. Monte Carlo methods is a simulation method to solve or approximate problems with unknown distribution. Explain the following problems.

   (a) (10 points) What is the convergence rate of Monte Carlo methods? Suppose that 1000 paths lead to an accuracy of 0.1, then approximately how many paths will lead to an accuracy of 0.01?

   (b) (10 points) What is quasi Monte Carlo method?