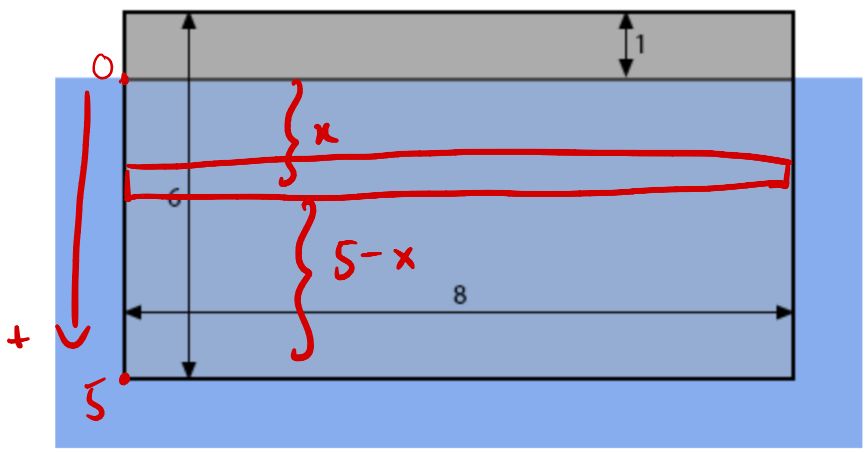
Math 199 CD3 Merit Worksheet 16: Review for Upcoming Midterm

March 25, 2022

1 Hydrostatic Force

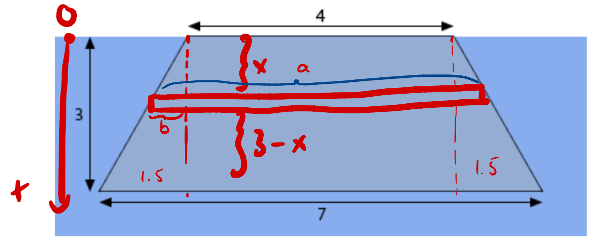
Find the hydrostatic force on the following plates submerged in water as shown in each image. In each case consider the top of the shaded "box" to be the surface of the water in which the plate is submerged. Note as well that the dimensions in many of the images will not be perfectly to scale in order to better fit the plate in the image. The lengths given in each image are in meters.

1. Image:

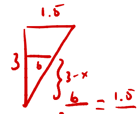


$$\rho \cdot g \int_0^5 8 \cdot x \, dx$$

2. Image:



Consider



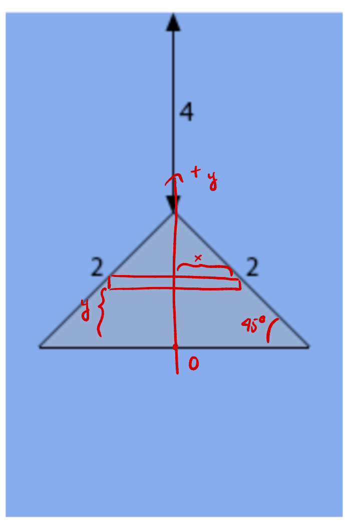
$\frac{b}{3-x} = \frac{1.5}{3}$

$\Rightarrow b = \frac{1}{2}(3-x)$

Call the whole plate width is a, $a = 7 - 2b = 7 - 2\cdot\frac{1}{2}(3-x) = 4+x$

$$\Rightarrow \rho g \int_0^3 x(4+x)\,dx$$

3. Image:



Then use similar Δ, get

$y = \sqrt{2} - x$

But the width of the plate is $2x$

$\Rightarrow 2(\sqrt{2} - y)$ is the width

The depth will be $4 + y = 4 + \sqrt{2} - x$

$$\Rightarrow \rho g \int 2(4+\sqrt{2}-y)(\sqrt{2}-y)\,dy$$

2 Centroid

4. Please state the formula for moments and center of mass before attempt any of the below problems

$$M_x = \rho \int_a^b \frac{1}{2}\left(f^2(x) - g^2(x)\right) dx$$

$$M_y = \rho \int_a^b x\left(f(x) - g(x)\right) dx$$

5. Determine the center of mass of the region bounded by $y = 2\sin(2x)$, $y = 0$ on the interval $[0, \pi/2]$

$$M_x = \int_0^{\pi/2} 2\sin^2(2x)\, dx \quad = \pi/2$$

$$\Rightarrow \text{centroid} = \left(\frac{\quad}{A}, \frac{\quad}{A}\right)$$

area

$$M_y = \int_0^{\pi/2} 2x\sin(2x)\, dx \quad = \pi/2$$

6. Determine the center of mass of the region bounded by $y = x^3$ and $y = \sqrt{x}$

$$M_x = \int_0^1 \frac{1}{2}\left(x - x^6\right) dx \quad = \frac{5}{28}$$

$$M_y = \int_0^1 x\left(\sqrt{x} - x^3\right) dx \quad = \frac{1}{5}$$

$$A = \int_0^1 \sqrt{x} - x^3\, dx = \frac{5}{12}$$

$$\Rightarrow \text{Centroid} = \left(\frac{\quad}{\quad}, \frac{\quad}{\quad}\right)$$

3 Series

Everything up to absolutely convergence and conditionally convergence should be on the exam, please study them.

3.1 These problems should just be routine, please try to do all of them

Determine whether the following series converge or diverge. NOte that it is not always possible to use alternating series test. After you decided the series diverge or converge, pick your favorite numer n and calculate the error up to its first n terms if possible (some expression might be too hard to integrate then you can skip). Please state any inequality you are going to use for estimating, but more importantly, tell me what it means

7.
$$\sum_{0}^{\infty}(-1)^n \frac{n+1}{2n+1}$$

Alternating series test would fail ? Why?

However, every n^{th} term will not $\to 0$

$\Rightarrow$ Cannot converge

8. Estimate the error by its first 10 terms,
$$\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n^2}$$

Using

The error of alternating series is less than the magnitude of the next terms

10 terms

$\hookrightarrow n = 11 \Rightarrow \frac{1}{11^2} = \frac{1}{121}$ is the error

9.
$$\sum_{0}^{\infty}\frac{n!}{(2n)!}$$

$$\lim \left| \frac{(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{n!} \right| = \lim \left| \frac{n+1}{(2n+1)(2n+2)} \right| = 0 < 1$$

$\Rightarrow$ Converge

10.
$$\sum_{1}^{\infty} n(3/4)^n$$

$$\lim \frac{(n+1)}{(3/4)^{n+1}} \cdot \frac{(3/4)^n}{n} = \frac{4}{3} \lim \left|\frac{n+1}{n}\right|$$

$$= \frac{4}{3} > 1$$

$\Rightarrow$ Diverge.

11. Study this series

$$1 - \frac{2^2+1}{2^3+1} + \frac{3^2+1}{3^3+1} \overset{-}{\cancel{+}} \cdots$$

sorry typo

See if $\sum (-1)^{n+1} \frac{n^2+1}{n^3+1}$ converge

By alternating series test.

$\boxed{\text{Conditionally convergence}}$

12.

$$\sum_1^\infty \frac{n^3}{(\ln 2)^n}$$

Ratio test

diverges

13.

$$\sum_1^\infty \frac{n^3}{(\ln 3)^n}$$

Ratio test

Converges

14. $\sum \frac{x^n}{n!}$

Ratio test
But conditionally converges.
when $|x| < 1$

15. $\sum n! x^n$

Ratio test

But also conditionally
converges when $x = 0$

16. $\sum \frac{2}{3+5n}$

integral test with
$f(x) = \frac{2}{3+5x}$

17. $\sum \frac{n^2}{n^3+1}$

$f(x) = \frac{x^2}{x^3+1}$

3.2 Conditionally convergence

Determine if the series converge conditionally or absolutely, using both alternating series test or LCT

18.

$$\sum_{1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}+\sqrt{n}}$$

Alternating series test

- $\lim \to 0$ ✓
- $a_n = \frac{1}{\sqrt{}+\sqrt{}} > 0$ ✓
- Decreasing ?

$\frac{1}{\sqrt{n+1}+\sqrt{n}}$ always $\downarrow$ $\Rightarrow$ Abs converges.

19.

$$\sum_{1}^{\infty} (-1)^{n+1} \frac{n^2}{e^n}$$

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again, alternating series test.

$\left(\frac{n^2}{e^n}\right)' = \frac{2ne^n - n^2e^n}{e^{2n}} < 0$ if $2ne^n - n^2e^n < 0$

$(2n - n^2)e^n < 0$

e^n always > 0

$\Rightarrow 2n - n^2 < 0$

$\Leftrightarrow n(2-n) < 0$

$\Rightarrow \begin{cases} n < 0 \\ 2-n > 0 \end{cases}$ or $\begin{cases} n > 0 \\ 2-n < 0 \end{cases}$

$\Rightarrow \begin{cases} n < 0 \\ n < 2 \end{cases}$ or $\begin{cases} n > 0 \\ n > 2 \end{cases}$

$\Rightarrow n < 0$ or $n > 2$

for the series to converge. ✓

But since we sum from 1

$\Rightarrow n > 2$.