Limits

1. For a function $f(x)$ and a point a, describe in words the difference between $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

Look at the definition.

2. For $f(x) = \dfrac{x+3}{2-x}$, compute each of the limits below:

(a) $\lim_{x\to 0} f(x) = \frac{3}{2}$

(b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{2+3}{2-2^-} = +\infty$

why?

(c) $\lim_{x\to 2^+} f(x) = -\infty$

(d) $\lim_{x\to -3} f(x) = 0$

(e) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+\frac{3}{x}}{\frac{2}{x}-1} = -1$

should there be a difference here?

(f) $\lim_{x\to -\infty} f(x)$

3. Compute each of the following limits:

(a) $\lim_{x \to 7} \frac{x^2 - 6x - 7}{x^2 - 14x + 49}$ $= \lim_{x \to 7} \frac{(x+1)(x-7)}{(x-7)^2}$

$= \lim_{x \to 7} \frac{x+1}{x-7} = \lim_{x \to 7} \frac{x-7+8}{x-7} = \lim_{x \to 7} 1 + \frac{8}{x-7} = \text{DNE}$

DNE

why?

(b) $\lim_{x \to 3} \frac{x-3}{\sqrt{x+6} - 3}$ $= \lim_{x \to 3} \frac{(x-3)(\sqrt{x+6}+3)}{x+6-9}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{x-3}}$

$= \lim_{x \to 3} (\sqrt{x+6}+3) = \ldots$

(c) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - 2x + 1$ (NOTE: your final answer should be in terms of x)

$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 1 - (x^2 - 2x + 1)}{h}$

$= 2x - 2$ can you recognize this limit?

4. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all values of x, then what is $\lim_{x \to 1} g(x)$ and how do you know?

lim lim lim

$2 \leq g(x) \leq 1 - 1 + 2 = 2$

$\lim_{x \to 1} g(x) = 2$

5. Use the squeeze theorem and appropriate limit laws to evaluate

$$\lim_{x\to 3}\left((2x^2-12x+18)\cos\left(\frac{1}{x-3}\right)-x+8\right)$$

$$2(x^2-6x+9)\cos\left(\frac{1}{x-3}\right)-x+8$$

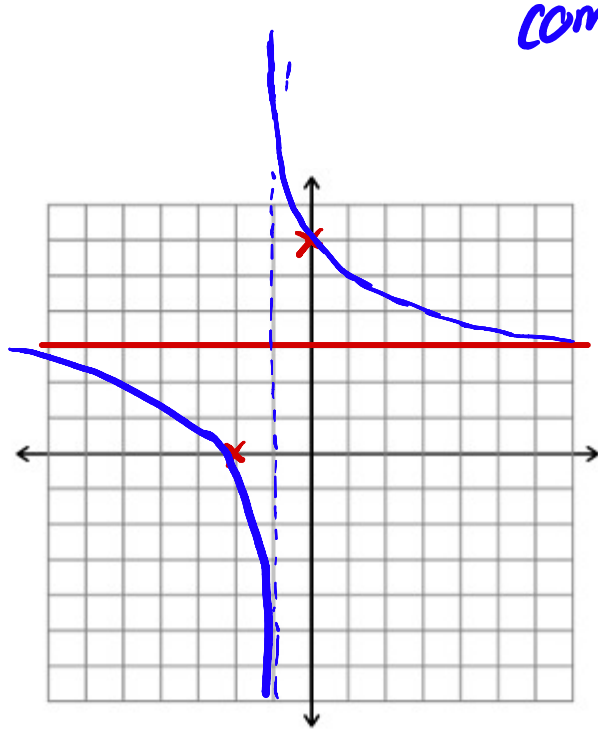
$$-2(x-3)^2-x+8 \le 2(x-3)^2\cos\left(\frac{1}{x-3}\right)-x+8 \le 2(x-3)^2-x+8$$

why? How do I have this ineq?

Then we can squeeze.

6. Sketch the graph of a function $f(x)$ that satisfies the following properties:

- $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$
- $f(0) = 6$
- $f(x)$ has a removable discontinuity at $x = 2$
- $(-2, 0)$ is the only x-intercept of $f(x)$
- $\lim_{x\to -1^-} f(x) = -\infty$
- $\lim_{x\to -1^+} f(x) = \infty$



come talk to me if you need explanation on how I get this graph.

7. Find examples of $f(x), g(x)$, and a so that:

(a) $\lim_{x\to a} f(x) + g(x)$ exists, but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x \leq 0 \end{cases} \quad \Rightarrow f(x) + g(x) = \begin{cases} x & \text{if } x \geq 0 \\ x & \text{if } x \leq 0 \end{cases}$$

why can't limit exists? for f & g

$$g(x) = \begin{cases} x-1 & \text{if } x \geq 0 \\ x+1 & \text{if } x \leq 0 \end{cases}$$

$\Rightarrow f(x)+g(x) = x$

$\lim_{x\to 0}$ exists here

(b) $\lim_{x\to a} f(x) \cdot g(x)$ exists, but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not

$$f(x) = g(x) = \frac{|x|}{x}$$

again come chat with me if you need a reason