Implicit and Logarithmic Differentiation

1. Fill in the blanks: To use implicit differentiation, first, take the derivative of every term with respect to __________. Whenever the term __________ appears, use ____________________. Finally, solve for __________.

2. Consider the curve given by $y^3 = \sin x$.

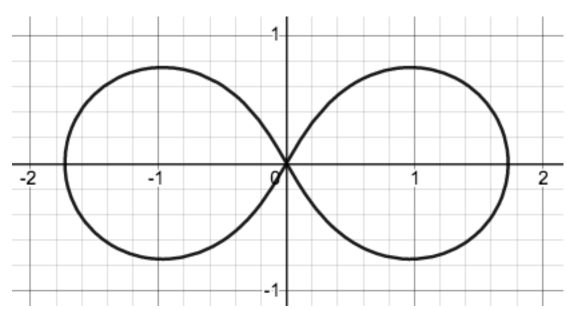
   (a) Solve for y first and then compute $\dfrac{dy}{dx}$ using the chain rule.

   (b) Compute $\dfrac{dy}{dx}$ using implicit differentiation without first solving for y.

   (c) Verify that your answers from parts a and b are the same (you may need to use your expression for y in terms of x from part a in order to do so).

3. The figure below at right is the curve given by $x^4 - 3x^2 + y^4 + y^2 + 2x^2y^2 = 0$.

   (a) Use implicit differentiation to find $\dfrac{dy}{dx}$ (do not try to simplify after solving for it!).

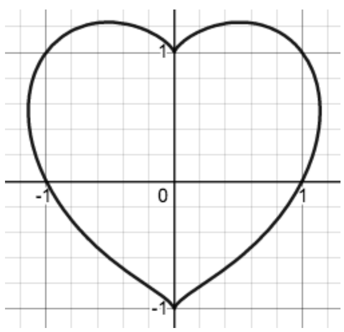


(b) What condition needs to be satisfied for the tangent line to the curve to be vertical? Sketch on the graph where this will occur.

(c) What condition needs to be satisfied for the tangent line to the curve to be horizontal? Sketch on the graph where this will occur.

4. The figure below at left is curve given by $(x^2 + y^2 - 1)^3 - x^2y^3 = 0$.

(a) Differentiate each term with respect to x to obtain an equation in terms of x, y, and $\frac{dy}{dx}$, but do not try to simplify or solve for $\frac{dy}{dx}$.



(b) Use your equation from part a to find the value of $\frac{dy}{dx}$ at the point $(1, 1)$ by plugging in $x = 1, y = 1$ and solving.

(c) Use your result from part b to find the equation of the tangent line to the curve at the point $(1, 1)$. Sketch the tangent line on the graph.

5. Recall that f^{-1} represents the inverse function of f.

   (a) Discuss with your group what it means for two functions to be inverses of each other.

   (b) If $y = f(x)$, what is the relationship between x, y, and f^{-1}?

   (c) If $y = f^{-1}(x)$, what is $\dfrac{dy}{dx} = (f^{-1})'(x)$? You may use your notes to find the equation, or derive it through implicit differentiation.

   (d) If $f(4) = 5$ and $f'(4) = \dfrac{2}{3}$, find $(f^{-1})'(5)$.

   (e) If $f(x) = x + e^x$, find $(f^{-1})'(1)$.

6. Fill in the blanks: We use logarithmic differentiation when we want to find the derivative of a function of the form __________. First, we set __________ = __________. Next, we take the natural logarithm of both sides and move __________ out of the logarithm. We can now apply ______________________________ to solve for __________. Finally, we replace __________ with __________.

7. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for each of the following equations:

(a) $y = x^x$

(b) $y = (\sin x)^{\ln x}$

(c) $x^y = y^x$

8. Find the equation of the tangent line to the curve $y = \ln(x^2 + y^2)$ at the point $(1, 0)$.