Limits

1. For a function $f(x)$ and a point a, describe in words the difference between $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.

Look at the definition.

2. For $f(x) = \dfrac{x+3}{2-x}$, compute each of the limits below:

(a) $\lim_{x\to 0} f(x) = \dfrac{3}{2}$

(d) $\lim_{x\to -3} f(x) = 0$

(b) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \dfrac{2+3}{2-2^-} = 5$

why?

(e) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{1+\frac{3}{x}}{\frac{2}{x}-1} = 1$

should there be a difference here?

(c) $\lim_{x\to 2^+} f(x) = -5$

(f) $\lim_{x\to -\infty} f(x)$

3. Compute each of the following limits:

(a) $\lim_{x\to 7} \frac{x^2-6x-7}{x^2-14x+49}$

$= \lim_{x\to 7} \frac{(x+1)(x-7)}{(x-7)^2}$

$= \lim_{x\to 7} \frac{x+1}{x-7} = \lim_{x\to 7} \frac{x-7+8}{x-7} = \lim_{x\to 7} 1 + \frac{8}{x-7} = \text{DNE}$

DNE

why?

(b) $\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-3}$

$= \lim_{x\to 3} \frac{(x-3)(\sqrt{x+6}+3)}{x+6-9}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{x-3}}$

$= \lim_{x\to 3} (\sqrt{x+6}+3) = \ldots$

(c) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = x^2-2x+1$ (NOTE: your final answer should be in terms of x)

$\lim_{h\to 0} \frac{(x+h)^2-2(x+h)+1-(x^2-2x+1)}{h}$

$= 2x-2$ can you recognize this limit?

4. If $2x \le g(x) \le x^4 - x^2 + 2$ for all values of x, then what is $\lim_{x\to 1} g(x)$ and how do you know?

lim lim lim

$2 \le g(x) \le 1-1+2 = 2$

$\lim_{x\to 1} g(x) = 2$

5. Use the squeeze theorem and appropriate limit laws to evaluate

$$\lim_{x\to 3}\left(\left(2x^2-12x+18\right)\cos\left(\frac{1}{x-3}\right)-x+8\right)$$

$$2(x^2-6x+9)\cos\left(\frac{1}{x-3}\right)-x+8$$

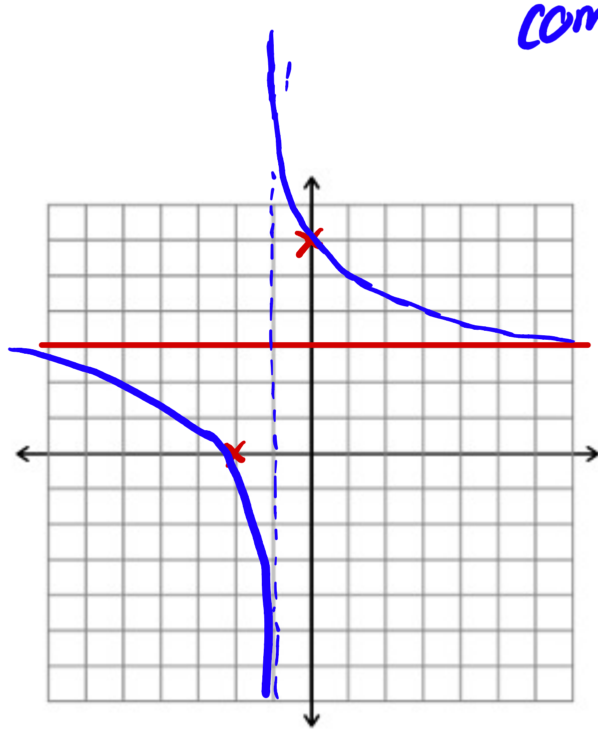
$$-2(x-3)^2-x+8 \le 2(x-3)^2\cos\left(\frac{1}{x-3}\right)-x+8 \le 2(x-3)^2-x+8$$

why? How do I have this ineq?

Then we can squeeze.

6. Sketch the graph of a function $f(x)$ that satisfies the following properties:

- $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 3$
- $f(0) = 6$
- $f(x)$ has a removable discontinuity at $x = 2$
- $(-2, 0)$ is the only x-intercept of $f(x)$
- $\lim_{x\to-1^-} f(x) = -\infty$
- $\lim_{x\to-1^+} f(x) = \infty$



come talk to me if you need explanation on how I get this graph.

7. Find examples of $f(x), g(x)$, and a so that:

(a) $\lim_{x\to a} f(x)+g(x)$ exists, but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not

$$f(x)=\begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x \leq 0 \end{cases}$$

why can't limit exists ? for f & g

$$g(x)=\begin{cases} x-1 & \text{if } x \geq 0 \\ x+1 & \text{if } x \leq 0 \end{cases}$$

$$\Rightarrow f(x)+g(x)=\begin{cases} x & \text{if } x \geq 0 \\ x & \text{if } x \leq 0 \end{cases}$$

$\Rightarrow$ $f(x)+g(x)=x$

$\lim_{x\to 0}$ exists here

(b) $\lim_{x\to a} f(x)\cdot g(x)$ exists, but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ do not

$$f(x)=g(x)=\frac{|x|}{x}$$

again come chat with me if you need a reason