Math 221, Merit Worksheet 8

September 11, 2021

1. **Derivatives and Inverse Functions.** Let $f(x)$ be a continuous, increasing function, and let $g(x)$ be the inverse of $f(x)$. Suppose $f(a) = b$.

   (a) Sketch a figure of f and g on the same plot, and draw in a, $f(a)$, b and $g(b)$ as well.

   (b) Using your sketch, what should be the relationship between $f'(a)$ and $g'(b)$? Remember that slope is $\frac{\Delta y}{\Delta x}$, and the difference between f and g is that the roles of x and y have been reversed.

   (c) Recall that $g(f(x)) = x$. Prove the relationship found in the previous part by differentiating this equation at $x = a$.

2. **Some Nice Calculations.** Find the derivatives of the following functions.

   (a) $f(x) = a^x$ (Hint: express a as e to some power)

(b) $f(x) = x^x$

(c) $f(x) = \arctan(x)$ (Hint: use problem 1)

3. **Damped Harmonic Oscillator.** In physics, the equation of a damped harmonic oscillator is $y(t) = Ae^{-kt}\cos(\omega t + \phi)$, for constants A, k, ω, ϕ. It's a good model for things that move back and forth but lose energy over time, like a bad car suspension hitting a bump, or a guitar string.

   (a) Find $y'(t)$.

   (b) Find $y''(t)$.

   (c) The motion is called 'undamped' if $k = 0$. In this case, there is a simple relationship between y and y''. What is that relationship?

   (d) The motion is called 'critically damped' if $\omega = \phi = 0$. In this case, there is a simple relationship between y and y'. What is that relationship?