Assignment 7

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Python code for solving ODE

2nd order approximation - CDS

1st order approximation - CDS

```
import numpy as np
import matplotlib.pyplot as plt
import time
x0 = 0
xn = 1
dx = 0.1
\#dx = 0.01
n = 10 #Choose when dx = 0.1
#n = 100 #Choose when dx = 0.01
x = np.linspace(x0,xn,n+1)
a = np.zeros(n+1)
b = np.zeros(n+1)
c = np.zeros(n+1)
d = np.zeros(n+1)
phi = np.zeros(n+1)
#boundary conditions
phi[0] = 1.0
phi[n] = 0.0
```

#Calculating coefficients for the first and last node

$$b[1] = -0.2/(dx*dx) - 1$$

$$c[1] = 0.1/(dx*dx) + 1/(2*dx)$$

$$d[1] = phi[0]*(1/(2*dx)-0.1/(dx*dx))$$

$$a[n-1] = 0.1/(dx*dx) - 1/(2*dx)$$

$$b[n-1] = -0.2/(dx*dx) - 1$$

$$d[n-1] = phi[n]*(-1/(2*dx)-0.1/(dx*dx))$$

#Calculating coefficients for the rest

for i in range(2,n-1):

$$a[i] = 0.1/(dx*dx) - 1/(2*dx)$$

$$b[i] = -0.2/(dx*dx) - 1$$

$$c[i] = 0.1/(dx*dx) + 1/(2*dx)$$

#Thomas algorithm

for i in range(2,n):

$$a[i] = a[i]/b[i-1]$$

$$b[i] = b[i] - a[i]*c[i-1]$$

$$d[i] = d[i] - a[i]*d[i-1]$$

$$phi[n-1] = d[n-1] / b[n-1]$$

for i in range(n-2,0,-1):

plt.plot(x, phi)

plt.title("dx = 0.1")

plt.ylabel('phi')

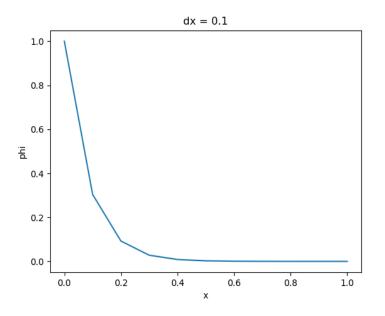
plt.xlabel('x')

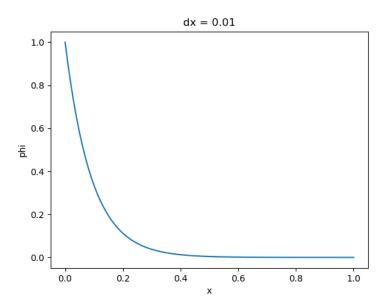
plt.show()

print('CPU time:',time.process_time(),'s')

Output

CPU time: 1.078125 s





2^{nd} order approximation – CDS

1st order approximation - FDS

import numpy as np

import matplotlib.pyplot as plt

import time

x0 = 0

xn = 1

dx = 0.01

n = 100

x = np.linspace(x0,xn,n+1)

a = np.zeros(n+1)

b = np.zeros(n+1)

c = np.zeros(n+1)

d = np.zeros(n+1)

```
phi = np.zeros(n+1)
```

#boundary conditions

phi[0] = 1.0

phi[n] = 0.0

#Calculating coefficients for the first and last node

$$b[1] = -0.2/(dx*dx) - 1 - 1/dx$$

$$c[1] = 0.1/(dx*dx) + 1/(dx)$$

$$d[1] = phi[0]*(-0.1/(dx*dx))$$

$$a[n-1] = 0.1/(dx*dx)$$

$$b[n-1] = -0.2/(dx*dx) - 1 - 1/dx$$

$$d[n-1] = phi[n]*(-1/(dx)-0.1/(dx*dx))$$

#Calculating coefficients for the rest

for i in range(2,n-1):

$$a[i] = 0.1/(dx*dx)$$

$$b[i] = -0.2/(dx*dx) - 1 - 1/dx$$

$$c[i] = 0.1/(dx*dx) + 1/(dx)$$

#Thomas algorithm

for i in range(2,n):

$$a[i] = a[i]/b[i-1]$$

$$b[i] = b[i] - a[i]*c[i-1]$$

$$d[i] = d[i] - a[i]*d[i-1]$$

$$phi[n-1] = d[n-1] / b[n-1]$$

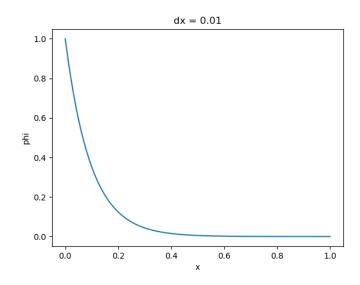
for i in range(n-2,0,-1):

$$phi[i]=(d[i]-c[i]*phi[i+1])/b[i]$$

```
plt.plot(x, phi)
plt.title("dx = 0.1")
plt.ylabel('phi')
plt.xlabel('x')
plt.show()
print('CPU time:',time.process_time(),'s')
```

Output

CPU time: 1.046875 s



2^{nd} order approximation – CDS

$\mathbf{1}^{st}$ order approximation – BDS

import numpy as np import matplotlib.pyplot as plt import time

$$x0 = 0$$

$$xn = 1$$

$$dx = 0.01$$

$$n = 100$$

$$x = np.linspace(x0,xn,n+1)$$

$$a = np.zeros(n+1)$$

$$b = np.zeros(n+1)$$

$$c = np.zeros(n+1)$$

$$d = np.zeros(n+1)$$

#boundary conditions

$$phi[n] = 0.0$$

#Calculating coefficients for the first and last node

$$b[1] = -0.2/(dx*dx) - 1 + 1/dx$$

$$c[1] = 0.1/(dx*dx)$$

$$d[1] = phi[0]*(1/dx -0.1/(dx*dx))$$

$$a[n-1] = 0.1/(dx*dx) - 1/dx$$

$$b[n-1] = -0.2/(dx*dx) - 1 + 1/dx$$

$$d[n-1] = phi[n]*(-0.1/(dx*dx))$$

#Calculating coefficients for the rest

for i in range(2,n-1):

$$a[i] = 0.1/(dx*dx) - 1/dx$$

$$b[i] = -0.2/(dx*dx) - 1 + 1/dx$$

$$c[i] = 0.1/(dx*dx)$$

#Thomas algorithm

for i in range(2,n):

$$a[i] = a[i]/b[i-1]$$

$$b[i] = b[i] - a[i]*c[i-1]$$

$$d[i] = d[i] - a[i]*d[i-1]$$

Output

CPU time: 1.15625 s

