Applied Computational Methods in Mechanical Sciences

(ME466)

Assignment 8

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Problem Statement:

Obtain the variation of ϕ vs x, given the governing differential equation as:

$$\frac{d\phi}{dx}(\rho u \phi) - \frac{d}{dx}\left[\Gamma \frac{d\phi}{dx}\right] = 0$$

Using central finite difference scheme for the given first order and second order derivatives. Use non- uniform grid using geometric progression:

1. For $r = 0.7$,$n = 10;\ 50;\ 100.$
2. For $r = 0.2$,$n = 10;\ 50;\ 100.$

where n represents the number of grid points and r is the ratio for geometric progression.

The boundary conditions are: $\phi(0) = 0$ and $\phi(1) = 1$.

Value of constants used: $\rho = 1, u = 1, \Gamma = 0.02.$

Python Code:

```
import time

import matplotlib.pyplot as plt




def grid_1d_gp(r,n):

    x = [0 for i in range(n+1)]

    rng=(0,1)

    #gp sum
```

```
    sum=0
    for i in range(n):
        sum = sum + pow(r,i)
    x0 = (rng[1]-rng[0])/sum
    x[0] = rng[0]

    for i in range(n):
        x[i+1] = x[i] + x0*pow(r,i)

    return(x)

def matrix_form(x,bc):

    p=1
    u=1
    t=0.02

    n = len(x)-2
    print(n)
    a = [[0 for i in range(n)] for j in range(n)]
    #rhs vector
    b = [0 for i in range(n)]

    for i in range(1,n+1):
        gamma = ( (-p*u)/(x[i+1]-x[i-1]) ) - ( (2*t)/((x[i]-x[i-1])*(x[i+1]-x[i-1])) )
        alpha = ( (2*t)/((x[i+1]-x[i-1])*(x[i+1]-x[i])) ) + ( (2*t)/((x[i+1]-x[i-1])*(x[i]-x[i-
1])) )
        beta = ( (p*u)/(x[i+1]-x[i-1]) ) - ( (2*t)/((x[i+1]-x[i])*(x[i+1]-x[i-1])) )

        j=i-1
        if(j is 0):
            a[0][0]=alpha
            a[0][1] =beta
```

```
            b[0] = 0 - (gamma*bc[0])
        elif(j is n-1):
            a[n-1][n-1]=alpha
            a[n-1][n-2]=gamma
            b[n-1] =  0 - (beta*bc[1])
        else:
            a[j][j-1] = gamma
            a[j][j] = alpha
            a[j][j+1] = beta

    return(a,b)



def gauss_elm(A,B):
    n= len(B)
    # step 1: Gaussian elimination.
    i=0
    while i < n:
        # pivots
        pivot = A[i][i]
        j=i+1
        while j<n:
            r = A[j][i]/pivot
            # row opreation
            k=i
            while k<n:
                A[j][k] = A[j][k] - A[i][k]*r
                k=k+1

            B[j]=B[j]-B[i]*r
            j=j+1
        i=i+1
```

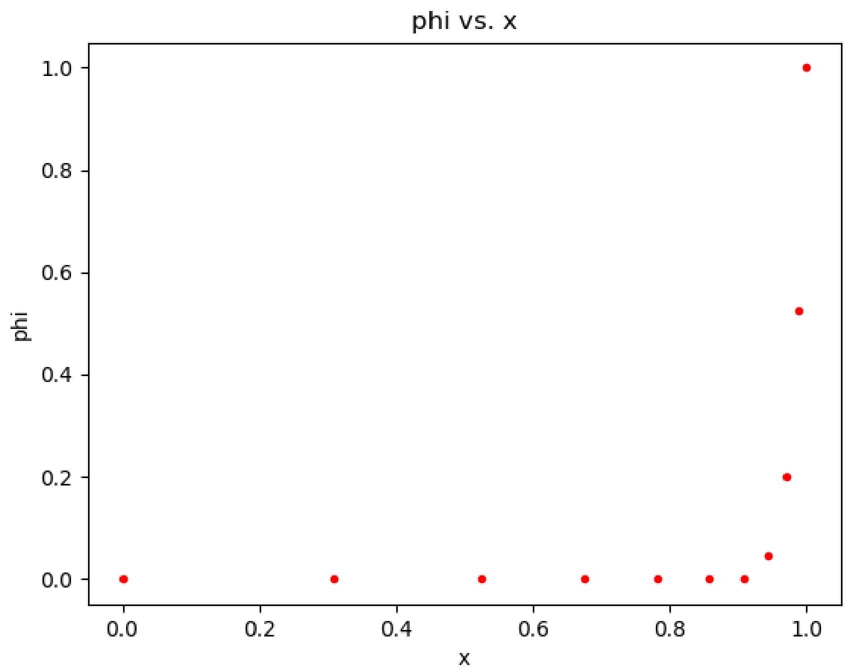
```
    #Back Substitution from nth row
    x= [0 for i in range(n)]

    i = n-1
    x[i] = B[i]/A[i][i]
    i=i-1
    while i>=0:
        sum = 0
        k=i+1
        while k<n:
            sum = sum + A[i][k]*x[k]
            k=k+1
        x[i]=(B[i]-sum)/A[i][i]
        i=i-1
    return(x)


def solver(r,n):
    bc= (0,1)
    x = grid_1d_gp(r,n)
    a,b = matrix_form(x,bc)
    phi = gauss_elm(a,b)

    mod_phi = [0 for i in range(n+1)]

    for i in range(len(phi)):
        mod_phi[i+1] = phi[i]
    mod_phi[0] = bc[0]
    mod_phi[n]= bc[1]

    print ("\n CPU time: ", time.process_time(),'s')
```

```
    return(mod_phi,x)


def plotter(phi,x):
    #plotting
    plt.plot(x,phi,'r.')
    plt.xlabel('x')
    plt.ylabel('phi')
    plt.title('phi vs. x')
    plt.show()


#main


p,x = solver(0.7,10)
plotter(p,x)
```

Results:

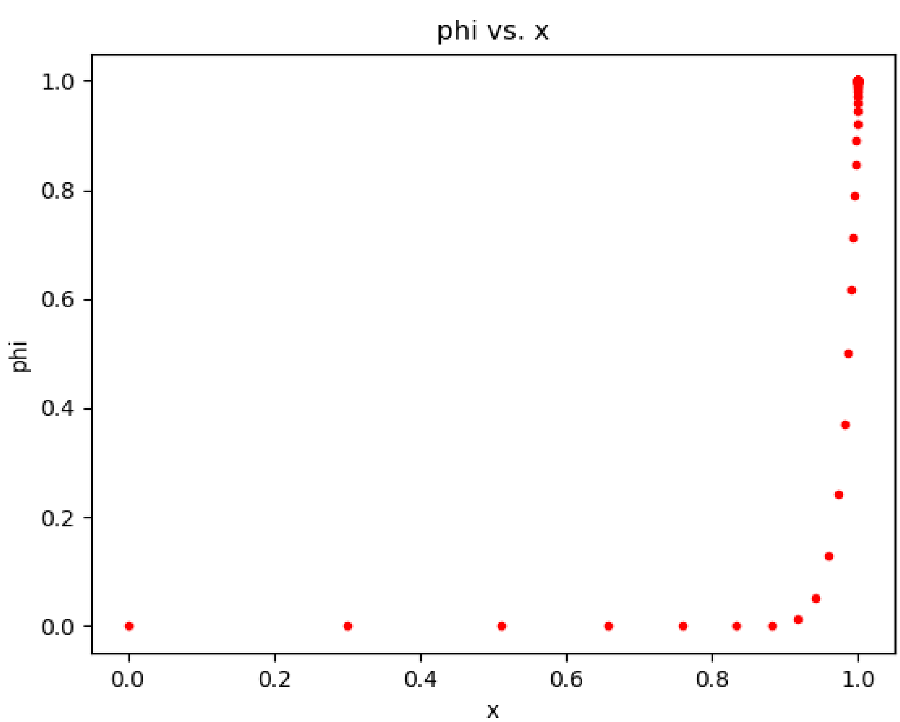
1. **With r=0.7 and n = 10:**

```
CPU time:  0.6875 s
```



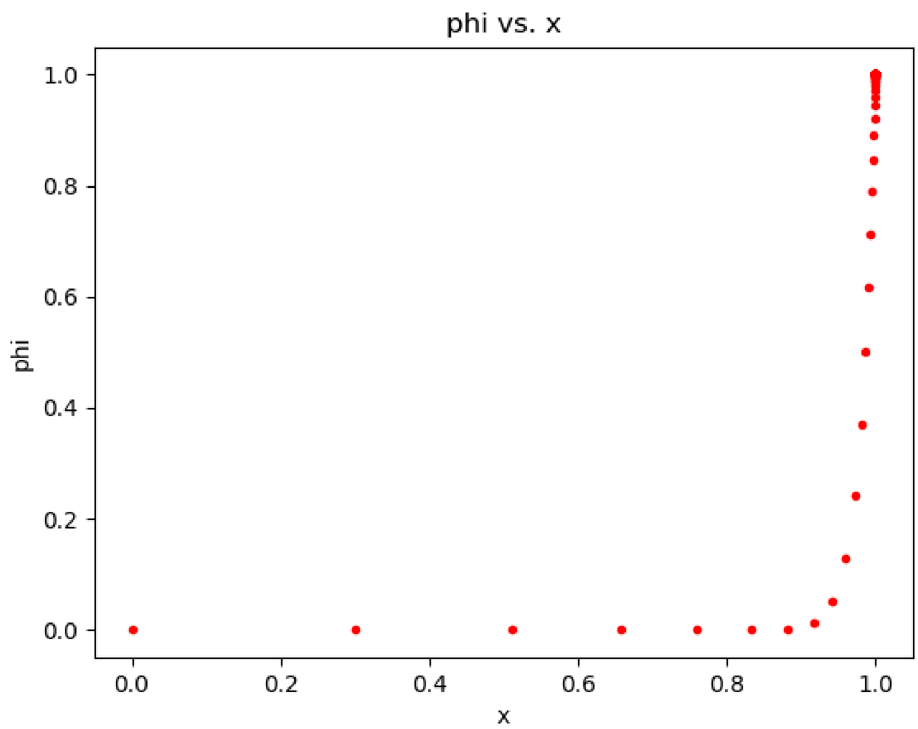
2. **With r=0.7 and n = 50:**

```
CPU time:  0.65625 s
```



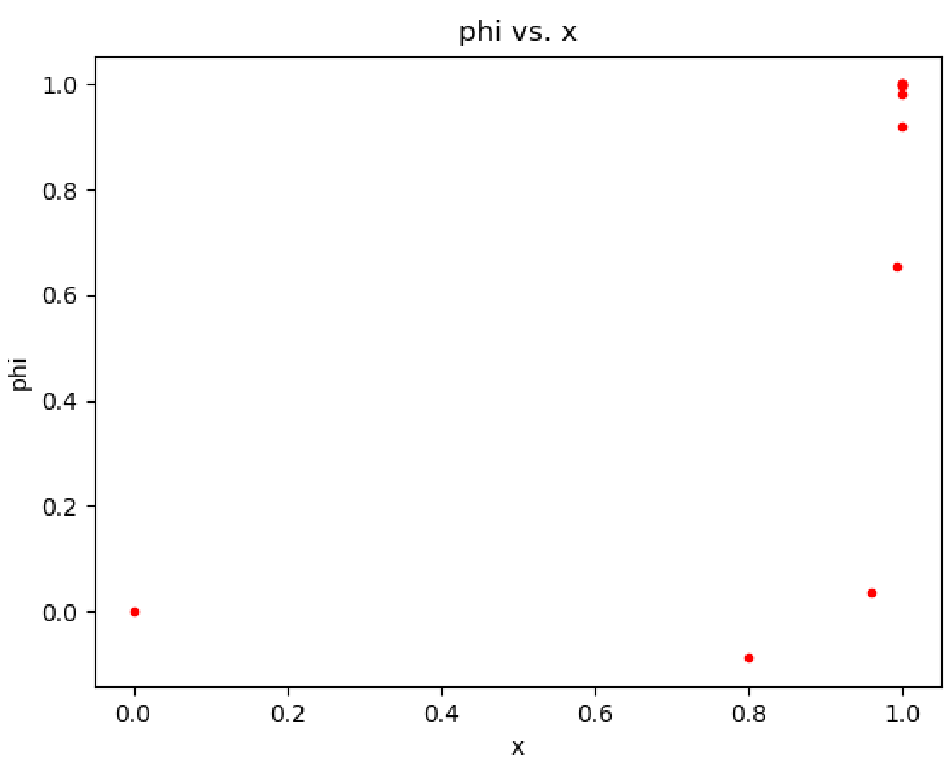
3. With r=0.7 and n = 100:

```
CPU time:  0.6875 s
```



4. With r=0.2 and n = 10:

```
CPU time:  0.375 s
```



5. **With r=0.2 and n = 50:**
   Truncation error, too fine grid at x~1.0
6. **With r=0.2 and n = 100:**
   Truncation error, too fine grid at x~1.0

Additional Result:

With r=0.95 and n = 100: