Big M for a *max* (*min*) Linear Programming problem:

Step 1. Introduce artificial variables in each row (with no basic variable).

Step 2. Put the artificial variables into the objective function: For max problem $\max z = c^t x - Ma_1 - Ma_2 - \ldots - Ma_m$. (For min problem $\min z = c^t x + Ma_1 + Ma_2 + \ldots + Ma_m$

Step 3. "clean-up" the objective function.

Step 4. Solve the LP by simplex.
If at opt all $a_i = 0$, we got the optimal solution for the original LP.
Otherwise (some $a_i > 0$ at opt) the original LP is infeasible.

$$
\begin{aligned}
\min\ z \ = \quad & 2x_1 + 3x_2 & \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 & \leq 4 \\
& x_1 + 3x_2 & \geq 20 \\
& x_1 + x_2 & = 10 \\
& x_1, x_2 & \geq 0
\end{aligned}
$$

In standard form:

$$
\begin{aligned}
\min \quad & z - 2x_1 - 3x_2 & = 0 \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 + s_1 & = 4 \\
& x_1 + 3x_2 - e_2 & = 20 \\
& x_1 + x_2 & = 10 \\
& x_1, x_2, s_1, e_2 & \geq 0
\end{aligned}
$$

Add artificial variables in constraints 2 and 3:

$$
\begin{aligned}
\min \quad & z - 2x_1 - 3x_2 - Ma_2 - Ma_3 & = 0 \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 + s_1 & = 4 \\
& x_1 + 3x_2 - e_2 + a_2 & = 20 \\
& x_1 + x_2 + a_3 & = 10 \\
& x_1, x_2, s_1, e_2, a_2, a_3 & \geq 0
\end{aligned}
$$

Tableau before "clean-up":

z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	-2	-3	0	0	$-M$	$-M$	0
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

First tableau (after "clean-up"):

z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	$2M-2$	$4M-3$	0	$-M$	0	0	$30M$
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

x_2 enters a_2 leaves the basis. Next tableau:

z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	$(2M-3)/3$	0	0	$(M-3)/3$	$(3-4M)/3$	0	$(60+10M)/3$
0	$5/12$	0	1	$1/12$	$-1/12$	0	$7/3$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3$

x_1 enters a_3 leaves the basis. Next tableau:

z	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	0	0	0	$-1/2$	$(1-2M)/2$	$(3-2M)/2$	25
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

$$
\begin{aligned}
\max\ z = \quad & x_1 + x_2 \\
\text{s.t.} \quad & 2x_1 + x_2 \geq 3 \\
& 3x_1 + x_2 \leq 3.5 \\
& x_1 + x_2 \leq 1 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

In standard form, with an artificial variable in constraint 1:

$$
\begin{aligned}
\max \quad & z - x_1 - x_2 + Ma_1 = 0 \\
\text{s.t.} \quad & 2x_1 + x_2 - e + a_1 = 3 \\
& 3x_1 + x_2 + s_2 = 3.5 \\
& x_1 + x_2 + s_3 = 1 \\
& x_1, x_2, e, a_1, s_2, s_3 \geq 0
\end{aligned}
$$

First tableau (after "clean-up"):

z	x_1	x_2	e	a_1	s_2	s_3	RHS
1	$-2M-1$	$-M-1$	M	0	0	0	$-3M$
0	2	1	-1	1	0	0	3
0	3	1	0	0	1	0	3.5
0	1	1	0	0	0	1	1

x_1 enters a_1 leaves the basis. Next tableau:

z	x_1	x_2	e	a_1	s_2	s_3	RHS
1	0	M	M	0	0	$2M+1$	$-M+1$
0	0	-1	-1	1	0	-2	1
0	0	-2	0	0	1	-3	0.5
0	1	1	0	0	0	1	1

2 phase method for a Linear Programming problem:

Step 1. Introduce artificial variables in each row (with no basic variable).

Step 2. Objective for phase 1: $\min w = a_1 + a_2 + \ldots + a_m$.

Step 3. "clean-up" the objective function.

Step 4. Solve the phase 1 LP by simplex.
If at opt all $a_i = 0$, we got a feasible solution for the original LP. Goto step 5.
Otherwise (some $a_i > 0$ at opt) the original LP is infeasible. Stop.

Step 5. Phase 2: Solve the original LP by simplex, with the starting solution found in phase 1.

$$
\begin{aligned}
\min\ z \ = \quad & 2x_1 + 3x_2 & \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 & \le 4 \\
& x_1 + 3x_2 & \ge 20 \\
& x_1 + x_2 & = 10 \\
& x_1, x_2 & \ge 0
\end{aligned}
$$

In standard form:

$$
\begin{aligned}
\min \quad & z - 2x_1 - 3x_2 & = 0 \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 + s_1 & = 4 \\
& x_1 + 3x_2 - e_2 & = 20 \\
& x_1 + x_2 & = 10 \\
& x_1, x_2, s_1, e_2 & \ge 0
\end{aligned}
$$

Add artificial variables in constraints 2 and 3, phase 1 LP:

$$
\begin{aligned}
\min \quad & w - a_2 - a_3 & = 0 \\
\text{s.t.} \quad & (1/2)x_1 + (1/4)x_2 + s_1 & = 4 \\
& x_1 + 3x_2 - e_2 + a_2 & = 20 \\
& x_1 + x_2 + a_3 & = 10 \\
& x_1, x_2, s_1, e_2, a_2, a_3 & \ge 0
\end{aligned}
$$

Tableau before “clean-up”:

w	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	0	0	0	0	-1	-1	0
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

First tableau (after “clean-up”):

w	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	2	4	0	-1	0	0	30
0	1/2	1/4	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

x_2 enters a_2 leaves the basis. Next tableau:

w	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	2/3	0	0	1/3	−4/3	0	10/3
0	5/12	0	1	1/12	−1/12	0	7/3
0	1/3	1	0	−1/3	1/3	0	20/3
0	2/3	0	0	1/3	−1/3	1	10/3

x_1 enters a_3 leaves the basis. Next tableau:

w	x_1	x_2	s_1	e_2	a_2	a_3	RHS
1	0	0	0	0	-1	-1	0
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

End of phase 1. $w = 0$ so?

First tableau of phase 2, before clean-up:

z	x_1	x_2	s_1	e_2	RHS
1	-2	-3	0	0	0
0	0	0	1	$-1/8$	$1/4$
0	0	1	0	$-1/2$	5
0	1	0	0	$1/2$	5

First tableau of phase 2, after clean-up:

z	x_1	x_2	s_1	e_2	RHS
1	0	0	0	$-1/2$	25
0	0	0	1	$-1/8$	$1/4$
0	0	1	0	$-1/2$	5
0	1	0	0	$1/2$	5

End of phase 2.

$$
\begin{aligned}
\max\ z = \quad & x_1 + x_2 & \\
\text{s.t.} \quad & 2x_1 + x_2 & \geq 3 \\
& 3x_1 + x_2 & \leq 3.5 \\
& x_1 + x_2 & \leq 1 \\
& x_1, x_2 & \geq 0
\end{aligned}
$$

In standard form, with an artificial variable in constraint 1:

$$
\begin{aligned}
\min \quad & w - a_1 & = 0 \\
\text{s.t.} \quad & 2x_1 + x_2 - e + a_1 & = 3 \\
& 3x_1 + x_2 + s_2 & = 3.5 \\
& x_1 + x_2 + s_3 & = 1 \\
& x_1, x_2, e, a_1, s_2, s_3 & \geq 0
\end{aligned}
$$

First tableau (after "clean-up"):

w	x_1	x_2	e	a_1	s_2	s_3	RHS
1	2	1	−1	0	0	0	3
0	2	1	−1	1	0	0	3
0	3	1	0	0	1	0	3.5
0	1	1	0	0	0	1	1

x_1 enters a_1 leaves the basis. Next tableau:

w	x_1	x_2	e	a_1	s_2	s_3	RHS
1	0	-1	-1	0	0	-2	1
0	0	-1	-1	1	0	-2	1
0	0	-2	0	0	1	-3	0.5
0	1	1	0	0	0	1	1

End of phase 1.