1 Gaussian Distribution and its properties

Ex1 Σ : symmetric, then Σ^{-1} is symmetric
We have $\Sigma\Sigma^{-1} = I$

$$I = I^T$$
$$\Sigma\Sigma^{-1} = (\Sigma\Sigma^{-1})^T$$
$$\Sigma\Sigma^{-1} = (\Sigma^{-1})^T\Sigma^T$$
$$\Sigma^{-1}\sigma(\Sigma^{-1}) = (\Sigma^{-1}\Sigma(\Sigma^{-1}))$$
$$\Sigma^{-1} = (\Sigma^{-1})^T$$

Then Σ^{-1} is symmetric

Ex2 A is symmetric then eigenvectors for A corresponding to different eigenvalues must be orthogonal.
2 vectors u and v are orthogonal if their dot product $(\ u \cdot v \ ) = u^T v = 0$
We have $Au = \lambda_1 u \quad Av = \lambda_2 v$

$$\lambda_1(u{\cdot}v) = (\lambda_1 u)v = (Au){\cdot}v = (Au)^T v = u^T A^T v = u^T Av = u^T(\lambda_2 v) = \lambda_2 u^T v = \lambda_2(u{\cdot}v)$$
$$\lambda_1(u \cdot v) = \lambda_2(u \cdot v)$$
$$(\lambda_1 - \lambda_2)(u \cdot v) = 0$$

$\lambda_1 \ != \lambda_2, \ then \ (u \cdot v) = 0$

Ex3 $\Sigma = \sum_{i=i}^{D} \lambda_i u_i u_i^T$ then $\Sigma^{-1} = \sum_{i=1}^{D} \frac{1}{\lambda_i} u_i u_i^T$
Σ is real, symmetric matrix its eigenvalues
We have $\Sigma = \sum_{i=1}^{D} \lambda_i u_i u_i^T = USU^T$ where U is $D \times D$ matrix with eigenvector as its columns and S is a diagonal matrix with the eigenvalue λ along its diagonal
Because U is a orthogonal matrix $U^{-1} = U^T$

$$\Sigma^{-1} = (USU^{T-1}) = (U^T)^{-1}S^{-1}U^{-1} = US^{-1}U^T = \sum_{i=1}^{D} \frac{1}{\lambda_i} u_i u_i^T$$