Chapter 4. Part 4

February 14, 2022

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Theorem

If $X = B(n, p)$ is a binomial random variable, then

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximated standard normal.

We can approximate

$$P(X \le a) = P(X \le a + 0.5) \approx P\left(Z \le \frac{a + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$P(X \ge a) = P(X \ge a - 0.5) \approx P\left(Z \ge \frac{a - 0.5 - np}{\sqrt{np(1-p)}}\right)$$

The approximation is good for $np > 5$ and $n(1-p) > 5$.

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Answer: Let X be the number of defective chips from the lot. Then $X = B(800, 0.03)$. We want to approximate $P(X \geq 31)$ by the normal distribution.

$$P(X \geq 31) = P(X \geq 30.5) \approx P(Z \geq \frac{30.5 - 800 * 0.03}{\sqrt{800 * 0.03 * 0.97}})$$
$$= P(Z > 1.347) = 1 - \Phi(1.347) = 1 - 0.911 = 0.089.$$

In fact,

$$P(X \geq 31) = \sum_{x=31}^{800} \binom{800}{x} 0.03^x 0.97^{800-x} = 0.09254$$

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$$= P(-0.725 \leq Z \leq 1.14) = \Phi(1.14) - \Phi(-0.725) = 0.873 - 0.234 = 0.639$$

In fact,

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Theorem

If X is a Poisson random variable with parameter λ, then

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

is approximately a standard normal random variable.

We can approximate:

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$$P(X \geq 11) = P(X \geq 10.5) \approx P\left(Z \geq \frac{10.5 - 9.6}{\sqrt{9.6}}\right)$$
$$= P(Z \geq 0.29) = 1 - \Phi(0.29) = 1 - 0.614 = 0.386$$

In fact,

$$P(X \geq 11) = \sum_{x=11}^{\infty} \frac{e^{-9.6} 9.6^x}{x!} = 0.367$$

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